African Journal of Health, Nursing and Midwifery
ISSN: 2689-9418
Volume 7, Issue 4, 2024 (pp. 11-30)
www.abjournals.org

MATHEMATICAL MODELLING OF THE IMPACTS OF MORNING FATIGUE ON ACADEMIC PERFORMANCE: A CASE STUDY OF FEDERAL UNIVERSITY WUKARI, TARABA STATE.

Okorie Charity Ebelechukwu, Jinan Gommam Bongcir,

and Ochigbo Josephine Elanma

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.

Cite this article:

Okorie, C. E., Jinan, G. B., Ochigbo, J. E. (2024), Mathematical Modelling of the Impacts of Morning Fatigue on Academic Performance: A Case Study of Federal University Wukari, Taraba State. African Journal of Health, Nursing and Midwifery 7(4), 11-30. DOI: 10.52589/AJHNM-BUFOHVGK

Manuscript History

Received: 12 Jul 2024

Accepted: 13 Sep 2024

Published: 24 Sep 2024

ABSTRACT: *Several authors have researched the phenomenon of sleep deprivation among students. In their studies, we discovered that they did not consider morning fatigue. So, we decided to fill the gap in the literature by using mathematical models to study the impact of morning fatigue. The aim is to develop mathematical modelling for the impact of morning fatigues on the academic performances of the students, a case study of the Federal University Wukari in Taraba State. In this project work, we formulated a mathematical model based on a system of ordinary differential equations to study the impact of morning fatigue on academic performance. We tested the model for existence and uniqueness and discovered that the model exists and that it is unique. The basic reproduction number was computed using the next-generation matrix approach. Questionnaires were distributed to 400 students and the data for this research were collected from the responses of the students. The result of the basic reproduction number shows that morning fatigue can be controlled. Using the Routh-Hurwitz criterion for local stability, the fatigue-free equilibrium (FFE) states of the model were established and proved to be locally asymptotically stable. Sensitivity analysis was then carried out to determine which parameters should be targeted by control intervention strategies of which the result shows that an increase in the acceptance of the control measure rate leads to a reduction in the prevalence of fatigue. Finally, a numerical simulation of the model was carried out and the result shows that Morning fatigue has a great impact on the academic performance of students. This means that there is a need for students to avoid reading till daybreak which in turn brings morning fatigue.*

KEYWORDS: Morning fatigue, Simulation, Basic reproduction number, Locally asymptotically stable, parameters.

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

INTRODUCTION

Fatigue is a feeling of tiredness that may also occur with other symptoms, such as lack of motivation and energy. Tired all the time is a popular complaint, tiredness and fatigue are common problems often, it is not a medical issue but one that can be reversed by a change of lifestyle and Tiredness is also known as fatigue. Fatigue and workplace sleepiness are consequences of modern industrial society. Fatigue is a complex biological phenomenon that occurs as a function of time awake, time of day, workload, health, and off-duty lifestyle. Fatigue is a function of two major biological factors – the homeostatic drive for sleep and the circadian rhythm of sleepiness (Caldwell et al., 2019).

Fatigue is the state of feeling very tired, weary or sleepy resulting from insufficient sleep, prolonged mental or physical work, or extended periods of stress or anxiety. Boring or repetitive tasks can intensify feelings of fatigue. Fatigue can be described as either acute or chronic." (Canadian Centre for Occupational Health and Safety, 2017). Fatigue from lack of sleep will be the focus of this review. Fatigue can also result from intense or monotonous cognitive activities or physical demands; however, these issues will not be addressed here. Krausman et al. (2002) provide a detailed account of the effects of physical fatigue on cognition. Mental and physical fatigue result from different conditions and have different symptoms, and it is important to distinguish between the two (Lieberman, 2011).

Okano et al. (2019) observed that better quality, longer duration, and greater consistency of sleep are strongly associated with better academic performance in college.

Morning fatigue has been an important topic of research for decades within psychology. Behavioural studies of the impact of fatigue have a long history and have documented the negative effects associated with lack of sleep across a broad range of tasks and domains (Lim and Dinges, 2010). As technological advancement enabled more sophisticated neuroimaging research, investigations of fatigues focused more extensively on neurophysiologic and neuropsychological data, uncovering a variety of changes in brain functioning associated with fatigue circadian rhythms (Drummond, 2004). Most recently some efforts have been invested in using the extension of empirical literature to develop a computational account of fatigue and circadian rhythms and the impact of those factors on cognitive functioning (Wesenten 2012). Inadequate sleep often results in impaired cognition which negatively affects health and well-being (Rupp, 2010). Satisfactory sleep is essential to fulfilling social and job-related duties. Sleep deprivation may result in reduced performance and an increased level of anxiety (Gilbert 2010).

Heuer (2003) emphasized that one night of total sleep deprivation decreases the chance of obtaining and storing account information.

Several studies have pointed out the possible challenges to optimal functioning when people experience sleep deprivation for example, Jovanovski (2007) conducted a study that investigated the relationship between the function of sleep as they related to morning fatigue in the Academic performance of students. In this study, the head dressed the gap in the literature for several reasons. First, no researchers had identified the impact of morning fatigue on the academic performance of students, primarily because of the relative novelty of online education (Becker, 2008).

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Gilbert and Weaver (2010) acknowledge that sleep loss was due to late bedtime more generally which corresponded to a consistent worsening of morning fatigues. A total of 557 male and female psychology students participated in the Gilbert and Weaver quantitative study by completing a demographic survey, the Gilbert depression inventory, and the Pittsburg sleep quality index.

Gilbert and Weaver (2010) found that adequate sleep is essential to support satisfactory awareness and performance of cognitive function, however, dynamics such as family distractions and ambition temperature within the online. Students' surroundings may contribute to poor sleep quality.

The function of sleep, at least so far as the brain is concerned, has to do with learning and memory (Abel et al., 2013; Ribeiro, 2012; Wang et al., 2011).

Ming et al. (2018). Asserted that individuals spend a third of their lives sleeping, as such, sleep deprivation may contribute to concerns such as daytime sleepiness and reduced mental awareness. The social change implications of the results from this study will be used by online universities to create programs that may improve sleep patterns for students. The outcome can also be used to revive the approach of diagnosing and analyzing students' sleep patterns and provide information to reduce and prevent sleep deprivation.

Fatigue is a feeling of tiredness that may also occur with other symptoms, such as lack of motivation and energy. It can be the result of various conditions. Daniel (2017). Tired all the time is a popular complaint, tiredness and fatigue are common problems often, it is not a medical issue but one that can be reversed by a change of lifestyle and Tiredness is also known as fatigue. Daniel (2017). Tiredness can negatively impact performance at work, family life, academic, and social relationships. Dinges (2005).

Fatigue and workplace sleepiness are consequences of modern industrial society. Fatigue is a complex biological phenomenon that occurs as a function of time awake, time of the day, workload, health, and off-duty lifestyle. Fatigue is a function of two major biological factors – the homeostatic drive for sleep and the circadian rhythm of sleepiness Caldwell et al. (2019).

Krausman et al. (2002) provide a detailed account of the effects of physical fatigue on cognition. Mental and physical fatigue result from different conditions and have different symptoms, and it is important to distinguish between the two (Lieberman, 2011).

Laura et al. (2023) stated that fatigue and sleep deprivation undermined the benefits of shift at work, as it impacted the ability to enjoy social and family events by the workers.

Giulio and Chiara (2013) stated that sleep is the price the brain pays for plasticity and without sleep, the body becomes tired and irritable, and the brain functions less well.

Total sleep deprivation and chronic sleep restriction increase the homeostatic sleep drive and diminish waking neurobehavioral functioning, producing deficits in attention, memory and cognitive speed, increases in sleepiness and fatigue, and unstable wakefulness (Goel, 2017).

Weiss and Donlea (2022); stated that while the precise functions of sleep remain poorly understood, a large body of research has examined the negative consequences of sleep loss on neural and behavioral plasticity. While sleep disruption generally results in degraded neural

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

plasticity and cognitive function, the impact of sleep loss can vary widely with age, between individuals, and across physiological contexts.

METHODOLOGY

Model Formulation

In this research, we consider five (5) compartmental models for morning fatigue which include: (B, D, F, P, E) where;

B = population of the students who go to bed

D = population of the students who delay in going to bed.

F = population of the students who wake up tired as a result of sleeping late.

P = population of the students who perform poorly as a result of morning fatigue.

E = population of students who sleep early, wake up strong and perform excellently

N = Total population

μ = The rate at which students who experience morning fatigue die.

λ: proportion of the population who sleep at night.

α: the rate at which individuals who go to bed are exposed to late sleep (Delay).

γ: the rate at which individuals who sleep late wake up tired.

β = rate at which tired students perform poorly academically.

Z = rate at which students who perform excellently drawback.

τ = rate at which poorly performed students recovered and began to perform excellently.

θ: the rate at which students who initially gain strength lose it.

1. In this research, we assumed that the students who go to bed are recruited by birth at a level of λ.
2. We assume that the students who go to bed sometimes delay going to bed and become exposed to late sleep at level α. The population of students who go to bed is also increased due to the coming in of the students who sleep early, wake up strong, and perform excellently at the level of θ. This class is decreased by natural death denoted by μ and also as a result of progression of the students who go to bed (B) become delayed (D) because of failure to hearken to control measures to avoid being delayed and lastly as a result of the acceptance of the enlightenment control measure to go to bed earlier.
3. The students who delay (D) in going to bed increase in population by the progression of students from those who go to bed and by the progression of some individuals who

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

through carelessness fail to go to bed in time; hence become delayed at a level α. This population reduces as a result of the progression of the students who delay going to bed to the students who wake up tired as a result of sleeping late at the rate γ and as a result of natural death at the rate of μ.

4. The number of students who wake up tired as a result of sleeping late (F) increases due to the progression of the natural death (μ). Also, as a result of those that are being corrected which is subject to the rate at which the students who wake up tired as a result of sleeping late accept the control measure to go to bed on time.

5. The population of students who perform poorly as a result of morning fatigue reduces due to the progression of students who wake up tired as a result of sleeping late at the rate (β). This class also increased due to the coming in of the students who at first performed excellently but later drawback at the rate Z, it reduces by the natural death (μ). Finally, this class reduces as a result of the students who were performing poorly as a result of morning fatigue becomes excellent as the control measure is being accepted and practiced at the rate τ.

6. The population of students who sleep early, wake up strong, and perform excellently (E) increases as a result of the progression of students who perform poorly at first but later accept the control measure to become excellent at the rate τ. This class reduces due to the progress of students who go to bed early, wake up strong and perform excellently to the population of student who goes to bed at a rate θ and finally, it reduces by the natural death (μ).

Model Assumptions

The development of the model is based on the following assumptions;

1. We assume that morning fatigue is attributed to late-night sleep or inability to sleep at all.

2. We also assume that students who perform poorly as a result of morning fatigue do not possess lasting immunity against morning fatigue, this is because the moment the students who are performing poorly due to morning fatigue stop delaying going to bed and having enough sleep, he/she goes back to the population of students who sleep early, wake up strong and perform excellently.

3. We assume that unchecked morning fatigue can lead to mental illness or even death.

4. We also assume that natural death can occur in all the compartments.

5. All the variables and parameters are assumed to be positive.

Putting these assumptions together we have a model flow diagram as shown in the diagram above.

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Mathematical Model Equations

The model equation for the impact of morning fatigue on academic performance is shown below:

$$\frac{dB}{dt} = \lambda - (1-\delta)\pi\beta F - \alpha B - \mu_B + \theta E \quad (1)$$

$$\frac{dD}{dt} = \alpha B - \gamma D - \mu_D \quad (2)$$

$$\frac{dF}{dt} = \gamma D + (1-\delta)\pi\beta F - \beta F - \mu_F \quad (3)$$

$$\frac{dP}{dt} = zE - \tau P + \beta F - \mu_P \quad (4)$$

$$\frac{dE}{dt} = \tau P - zE - \theta E - \mu_E \quad (5)$$

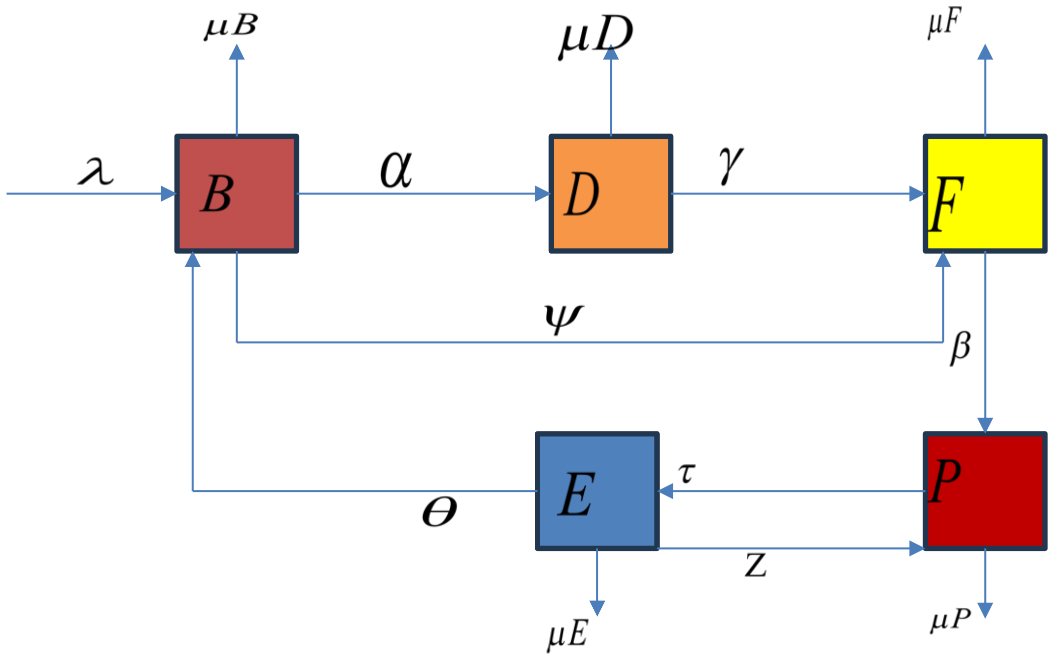

Figure 1: Flow chart of the formulated model

EXISTENCE AND UNIQUENESS

Theorem 1: (As used by Ogwumu and Ibrahim, 2017)

Model Equation from the flow Diagram

With the parameters in the table retaining their meanings as defined the corresponding model equation is as given above:

Considering the system of equations,

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

$x' > f(t,x),\ t_0 = x_0,$

If D' denotes the region $|t-t_0| \le a,\quad |x-x_0| \le b,\ x=(x_1,x_2,\ldots x_n),\ x_0=(x_{10},x_{20},\ldots x_{n0}).$ And suppose that $f(t,x_1)$ satisfies the Lipshitz Condition $|f(t,x_1)-f(t,x_2)| \le k|x_1-x_2|$ whenever the pair $f(t,x_1)$ and $f(t,x_2)$ belong to D' where k is a positive constant, then there exists a constant $\delta > 0$ such that there exists a unique continuous vector solution X(t) of the system x' = f(t, x), as defined above, in the interval $|t-t_0| \le \delta$. It is important to note that the condition is satisfied by the requirement that $\frac{\partial f_i}{\partial x_j},\ i,j=1,2,\ldots$ be continuous and bonded in D'.

Considering our model defined in equation (*) below, we are interested in the region $0 \le \alpha \le R.$ We look for a bounded solution in the region whose partial derivatives satisfy $f \le \alpha \le 0,$ where α and δ are positive constants.

Theorem 2

Let D denote the region $0 \le \alpha \le R,$ then each of the equations of the system (*) has a unique solution which is bounded in D.

We are saddled with the responsibility to show that $\frac{\partial f_i}{\partial x_j},\ i,j=1,2,3,4,5$ are continuous and bounded in D in order to be able to prove the existence and uniqueness of the solution for the system of equations obtainable in our model.

From our model formulation above, we recall that, from (*) setting:

$$\frac{dB}{dt} = \lambda - (1-\delta)\pi BF - \alpha B - \mu_B + \theta E \qquad B_{(0)} = B_0$$

$$\frac{dD}{dt} = \alpha B - \gamma D - \mu_D \qquad D_{(0)} = D_0$$

$$\frac{dF}{dt} = \gamma D - (1-\delta)\pi BF - \beta F - \mu_F \qquad F_{(0)} = F_0 \qquad \ldots\ldots.(*)$$

$$\frac{dP}{dt} = zE - \tau P + \beta F - \mu_P \qquad P_{(0)} = P_0$$

$$\frac{dE}{dt} = \tau P - zE - \theta E - \mu_E \qquad E_{(0)} = E_0$$

Let M = $\{(B,D,F,P,E), |B-B_0| \le a, |D-D_0| < b, |F-F_0| \le c, |P-P_0| \le d, |E-E_0| \le e. |t| \le f\}$

The problem (*) has a unique solution.

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Proof:

$$\frac{dB}{dt} = f_1\,(B,D,F,P,E) = \lambda - \big((1-\delta)\pi F - (\alpha+\mu)\big)B\ + \theta E\ \equiv \lambda - KB\ + \theta E$$

$$\frac{dD}{dt} = f_2(B,D,F,P,E) = \alpha B - (\gamma+\mu)D\ = \alpha B\ - ND$$

$$\frac{dF}{dt} = f_3(B,D,F,P,E) = \gamma D - (B + \mu - \pi B + \pi\delta B)F\ = \gamma D\ - QF$$

$$\frac{dP}{dt} = ZE\ + \beta F - (\tau+\mu)P = ZE\ + \beta F - RP$$

$$\frac{dE}{dt} = \tau P - (Z+\theta+\mu)E = \tau P - WE$$

$$\frac{\delta f_1}{\delta B}|_{B,D,F,P,E} = -(1-\delta)\pi F + \mu)\,, \qquad \left|\frac{\delta f_1}{\delta B}\right| = (1-\delta)\pi F + \mu)$$

$$\frac{\delta f_2}{\delta D}|_{B,D,F,P,E} = -\gamma - \mu, \left|\frac{\delta f_2}{\delta D}\right| = \gamma + \mu$$

$$\frac{\delta f_3}{\delta F}|_{B,D,F,P,E} = -(\beta + \mu - \pi B + \pi\delta B)\,, \qquad \left|\frac{\delta f_3}{\delta F}\right| = \beta + \mu$$

$$\frac{\delta f_4}{\delta P}|_{B,D,F,P,E} = -\tau - \mu, \qquad \left|\frac{\delta f_4}{\delta P}\right| = \tau + \mu$$

$$\frac{\delta f_5}{\delta E}|_{B,D,F,P,E} = -z - \theta - \mu, \qquad \left|\frac{\delta f_5}{\delta E}\right| = z + \theta + \mu$$

Since $\frac{\delta f_i}{\delta x_i}|_{i,j,k,l,m} = 1, 2, 3, 4, 5$ are continuous and bounded, the model has a unique solution and the model (1)…….(5) is mathematically and epidemiologically well-posed

Morning Fatigue-free Equilibrium Point

Fatigue-free equilibrium points (DFE) are steady-state solutions where there is no morning fatigue in the life of students at the Federal University, Wukari.

Let the morning fatigue-free equilibrium be denoted by E_0 then at equilibrium.

$$\frac{dB}{dt} = \frac{dD}{dt} = \frac{dF}{dt} = \frac{dP}{dt} = \frac{dE}{dt} = 0.$$

$$\lambda - (1-\delta)\pi BF - \alpha B\ - \mu_B\ + \theta E\ = 0 \qquad i$$

$$\alpha B - \gamma D\ - \mu_D\ = 0 \qquad ii$$

$$\gamma D + (1-\delta)\pi BF - \beta F\ - \mu_F = 0 \qquad iii$$

$$zE\ - \tau P\ + \beta F - \mu_P = 0 \qquad iv$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

African Journal of Health, Nursing and Midwifery
ISSN: 2689-9418
Volume 7, Issue 4, 2024 (pp. 11-30)
www.abjournals.org

$$\tau P - zE - \theta E - \mu_E = 0 \qquad v$$

At fatigue free equilibrium D = F = P = E = 0 then equation (i) becomes

$$\lambda - (1-\delta)\pi BF - \alpha B - \mu_B + \theta E = 0$$

$$\lambda - (1-\delta)\pi BF - \alpha B - \mu_B = 0$$

$\lambda = ((1-\delta)\pi F + \alpha + \mu)\beta$. Divide through by $((1-\delta)\pi F + \alpha + \mu)$

$$E_0 = (B_0, D_0, F_{0,}P_{0,}E_0) = (\frac{\lambda}{(\alpha+\mu)}, 0, 0, 0, 0)$$

Endemic Equilibrium Points

An outbreak is endemic when it is consistently present but limited to a particular region. This makes the spread and rate predictable.

Let the endemic equilibrium be E_1 then at the equilibrium.

$$\frac{dB}{dt} = \frac{dD}{dt} = \frac{dF}{dt} = \frac{dP}{dt} = \frac{dE}{dt} = 0.$$

$$\lambda - (1-\delta)\pi F - \alpha B - \mu_B + \theta E = 0 \qquad 1$$

$$\alpha B - \gamma D - \mu_D = 0 \qquad 2$$

$$\gamma D + (1-\delta)\pi F - \beta F - \mu_F = 0 \qquad 3$$

$$zE - \tau P + \beta F - \mu_P = 0 \qquad 4$$

$$\tau P - zE - \theta E - \mu_E = 0 \qquad 5$$

From the equation (1)

$$\lambda + \theta E - (1-\delta)\pi F + \alpha + \mu)B = 0$$

$$B = \frac{\lambda + \theta E}{(1-\delta)\pi F + \alpha + \mu} \qquad a$$

From equation (2)

$$\alpha B - (\gamma + \mu)D = 0$$

$$\alpha B = (\gamma + \mu)D$$

$$B = \frac{(\gamma + \mu)D}{\alpha} \qquad b$$

Equating (a) and (b)

$\frac{\lambda+\theta E}{(1-\delta)\pi F+\alpha+\mu} = \frac{(\gamma+\mu)D}{\alpha}$ cross multiplying this gives

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

$$\alpha(\lambda+\theta E)=(1-\delta)\pi F+\alpha+\mu(\gamma+\mu)D$$

Make D the subject of the formula

$$D=\frac{\alpha(\lambda+\theta E)}{(1-\delta)\pi F+\alpha+\mu(\gamma+\mu)}$$

From equation (3)

$$\gamma D-(\beta+\mu-\pi B+\pi\delta B)F=0.\Rightarrow F$$

$$=\frac{\gamma D}{\beta+\mu-\pi B+\pi\delta B} \qquad (*)$$

But $D=\frac{\alpha(\lambda+\theta E)}{(\pi F-\pi\delta F+\alpha+\mu)(\gamma+\mu)}$

Substitute this in $(*)$ above

$$F=\frac{\gamma}{(\beta+\mu-\pi B+\pi\delta B)}\frac{\alpha(\lambda+\theta E)}{(\pi F-\pi\delta F+\alpha+\mu)(\gamma+\mu)}$$

$Let\ A1=\ \beta+\mu-\pi B+\pi\delta B$ and $A2=\pi F-\pi\delta F+\alpha+\mu$

F $=\frac{\gamma(\alpha\lambda+\alpha\theta E)}{A1A2(\gamma+\mu)}$ $\qquad d$

From equation 4

$$zE+\beta F-\tau P-\mu P=0$$

$$zE+\beta F-(\tau+\mu)P=0$$

Making P the subject gives,

$P=\frac{zE+\beta F}{(\tau+\mu)}$ $\qquad (**)$

But F $=\frac{\gamma(\alpha\lambda+\alpha\theta E)}{A1A2(\gamma+\mu)}$

Substituting F in the equation $(**)$ we have

$$P=\frac{ZE}{(\tau+\mu)}+\frac{\beta}{(\tau+\mu)}\frac{\gamma(\alpha\lambda+\alpha\theta E)}{A1A2(\gamma+\mu)}$$

$$P=\frac{ZEA1A2(\gamma+\mu)+\beta\gamma\alpha(\lambda+\theta E)}{A1A2(\tau+\mu)(\gamma+\mu)} \qquad (***)$$

From equation (5)

$$\tau P-(z+\theta+\mu)E=0.\Rightarrow P=\frac{(z+\theta+\mu)E}{\tau} \qquad (****)$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Equating $(***)$ and $(****)$

$$\frac{(z+\theta+\mu)E}{\tau}=\frac{ZEA1A2(\gamma+\mu)+\beta\gamma\alpha(\lambda+\theta E)}{A1A2(\tau+\mu)(\gamma+\mu)}$$

Let $A_3 = Z+\theta+\mu$ and $A4=(\tau+\mu)(\beta+\mu)$.

$$\frac{A_3E}{\tau}=\frac{(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)E+\beta\gamma\alpha\lambda}{A1A2A4}$$

Cross multiplying

$$A1A2A3A4E=\tau(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)E+\beta\gamma\alpha\lambda)$$

$$A1A2A3A4-(\tau(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)E=\beta\gamma\alpha\tau\lambda$$

$$E=\frac{\beta\gamma\alpha\tau\lambda}{A1A2A3A4-(ZA1A2(\gamma+\mu+)\tau+\beta\gamma\alpha\theta\tau)} \qquad f$$

Putting equation (f) in (d)

$$F=\frac{\gamma\alpha\lambda}{(\gamma+\mu)A1A2}+\frac{\gamma\alpha\theta}{(\gamma+\mu)A1A2}\left[\frac{\beta\gamma\alpha\tau\lambda}{A1A2A3A4-(\tau(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)}\right]$$

$$F=\frac{\gamma\alpha\lambda[A1A2A3A4-A1A2A3A4-(\tau(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)]+\beta\gamma\alpha\tau\lambda\tau}{(\gamma+\mu)A1A2[A1A2A3A4-(\tau(ZEA1A2(\gamma+\mu)+\beta\gamma\alpha\theta)]} \qquad g$$

Putting the (f) into (c) also yields;

$$D=\frac{\alpha\lambda(A1A2A3A4-(ZA1A2(\gamma+\mu)\tau+\beta\gamma\alpha^2\theta\lambda\tau)}{A2(\gamma+\mu)(A1A2A3A4-\left(ZA1A2(\gamma+\mu)\tau+\beta\gamma\alpha\theta\tau\right)} \qquad h$$

Consider $P=\frac{(Z+\theta+\mu)E}{\tau}=\frac{A3E}{\tau}$ from the equation $(****)$ substitute (f) in it

$$P=\frac{A3\beta\alpha\gamma\lambda}{A1A2A3A4-(ZA1A2(\gamma+\mu)\tau+\beta\gamma\alpha\theta\tau)} \qquad i$$

Finally, by putting (h) in (b)

$$B=\left[\frac{\alpha\lambda(A1A2A3A4-(ZA1A2(\gamma+\mu)\tau+\beta\gamma\alpha^2\theta\lambda\tau)}{A2(\gamma+\mu)(A1A2A3A4-(ZA1A2(\gamma+\mu)\tau+\beta\gamma\alpha\theta\tau))}\right]$$

The endemic equilibrium E_1 is given as

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Basic Reproduction Number (R_0)

The basic reproduction number is a threshold for stability of Morning fatigue-free equilibrium and is related to the peak and final size of an epidemic. The epidemic interpretation of this threshold parameter is connected to the local and global stability of the morning fatigue-free equilibrium

To calculate the Basic Reproduction number.

Consider the formulated model equation shown below;

$$\frac{dB}{dt} = \lambda - (1-\delta)\pi BF - \alpha B - \mu_B + \theta E \quad 1$$

$$\frac{dD}{dt} = \alpha B - \gamma D - \mu_D \quad 2$$

$$\frac{dF}{dt} = \gamma D + (1-\delta)\pi BF - \beta F - \mu_F \quad 3$$

$$\frac{dP}{dt} = zE - \tau P + \beta F - \mu_P \quad 4$$

$$\frac{dE}{dt} = \tau P - zE - \theta E - \mu_E \quad 5$$

$$\lambda - (1-\delta)\pi BF - \alpha B - \mu_B + \theta E = 0 \quad i$$

$$\alpha B - \gamma D - \mu_D = 0 \quad ii$$

$$\gamma D + (1-\delta)\pi BF - \beta F - \mu_F = 0 \quad iii$$

$$zE - \tau P + \beta F - \mu_P = 0 \quad iv$$

$$\tau P - zE - \theta E - \mu_E = 0 \quad v$$

$$f = \begin{pmatrix} (1-\delta)\pi BF \\ 0 \\ 0 \end{pmatrix}$$

$$F = \begin{pmatrix} 0 & (1-\delta)\pi B & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$v = \begin{pmatrix} (\gamma+\mu)D - \alpha B \\ (\beta+\mu-\pi B+\pi\delta B)F - \gamma D \\ (\tau+\mu)P - ZE - \beta F \end{pmatrix}$$

$$V = \begin{pmatrix} \gamma+\mu & 0 & 0 \\ -\gamma & \beta+\mu-\pi B+\pi\delta B & 0 \\ 0 & -\beta & \tau+\mu \end{pmatrix}$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

$$V^{-1}=\begin{pmatrix} \frac{1}{\gamma+\mu} & 0 & 0 \\ \frac{\gamma}{(\gamma+\mu)((\beta+\mu-\pi B+\pi\delta B)} & \frac{1}{(\beta+\mu-\pi B+\pi\delta B)} & 0 \\ \frac{\gamma B}{(\gamma+\mu)(\beta+\mu-\pi B+\pi\delta B)(\tau+\mu)} & \frac{B}{(\beta+\mu-\pi B+\pi\delta B)(\tau+\mu)} & \frac{1}{\tau+\mu} \end{pmatrix}$$

$$FV^{-1}=\begin{pmatrix} 0 & (1-\delta)\pi B & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\gamma+\mu} & 0 & 0 \\ \frac{\gamma}{(\gamma+\mu)((\beta+\mu-\pi B+\pi\delta B)} & \frac{1}{(\beta+\mu-\pi B+\pi\delta B)} & 0 \\ \frac{\gamma B}{(\gamma+\mu)(\beta+\mu-\pi B+\pi\delta B)(\tau+\mu)} & \frac{B}{(\beta+\mu-\pi B+\pi\delta B)(\tau+\mu)} & \frac{1}{\tau+\mu} \end{pmatrix}$$

$$FV^{-1}=\begin{pmatrix} \frac{\gamma(1-\delta)\pi B}{(\gamma+\mu)(\beta+\mu-\pi B+\pi\delta B)} & \frac{(1-\delta)\pi B}{(\beta+\mu-\pi B+\pi\delta B)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

But we have $R_0=\rho(FV^{-1})$. Hence, we must find the Eigen value.

$$\rho(FV^{-1})=\begin{pmatrix} \frac{\gamma(1-\delta)\pi B}{(\gamma+\mu)(\beta+\mu-\pi B+\pi\delta B)}-\lambda & \frac{(1-\delta)\pi B}{(\beta+\mu-\pi B+\pi\delta B)} & 0 \\ 0 & 0-\lambda & 0 \\ 0 & 0 & 0-\lambda \end{pmatrix}=0$$

$$\lambda^3+\frac{(-1+\delta)\pi\gamma^2\lambda^2}{\left(\pi\delta\gamma-\pi\gamma+\alpha\beta+\alpha\mu+\beta\mu+\mu^2\right)(\gamma+\mu)}=0$$

$$\lambda 1=\lambda 2=0, \lambda 3=\frac{(-1+\delta)\pi\gamma^2}{\left(\pi\delta\gamma-\pi\gamma+\alpha\beta+\alpha\mu+\beta\mu+\mu^2\right)(\gamma+\mu)}$$

But $B=\frac{\lambda}{\alpha+\mu}$

This shows that our basic reproduction number is

$$\frac{(-1+\delta)\pi\gamma^2}{\left(\pi\delta\gamma-\pi\gamma+\alpha\beta+\alpha\mu+\beta\mu+\mu^2\right)(\gamma+\mu)}$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

Local Stability of the Fatigue-free Equilibrium Point

We investigate the local stability of the fatigue-free equilibrium point of the model. We first linearize the fatigue model by computing its Jacobian Matrix, (J). at the fatigue-free equilibrium point.

$$(B_0, D_0, F_{0,} P_{0,} E_0) = [\frac{\lambda}{\alpha+\mu}, 0, 0, 0, 0]$$

The Jacobian matrix of the system is given as;

$$J_0 = \begin{bmatrix} \frac{\partial B^0}{\partial B} & \frac{\partial B^0}{\partial D} & \frac{\partial B^0}{\partial F} & \frac{\partial B^0}{\partial P} & \frac{\partial B^0}{\partial E} \\ \frac{\partial D^0}{\partial B} & \frac{\partial D^0}{\partial D} & \frac{\partial D^0}{\partial F} & \frac{\partial D^0}{\partial P} & \frac{\partial D^0}{\partial E} \\ \frac{\partial F^0}{\partial B} & \frac{\partial F^0}{\partial D} & \frac{\partial F^0}{\partial F} & \frac{\partial F^0}{\partial P} & \frac{\partial F^0}{\partial E} \\ \frac{\partial P^0}{\partial B} & \frac{\partial P^0}{\partial D} & \frac{\partial P^0}{\partial F} & \frac{\partial P^0}{\partial P} & \frac{\partial P^0}{\partial E} \\ \frac{\partial E^0}{\partial B} & \frac{\partial E^0}{\partial D} & \frac{\partial E^0}{\partial F} & \frac{\partial E^0}{\partial P} & \frac{\partial E^0}{\partial E} \end{bmatrix}$$

Where;

$$\frac{\partial B^0}{\partial t} = B^0, \frac{\partial D^0}{\partial t} = D^0, \frac{\partial F^0}{\partial t} = F^0, \frac{\partial P^0}{\partial t} = P^0, \frac{\partial E^0}{\partial t} = E^0$$

$$J_0 = \begin{bmatrix} -(\alpha+\mu) & 0 & \frac{-\lambda(1-\delta)\pi}{\alpha+\mu} & 0 & \theta \\ \alpha & -(\gamma+\mu) & 0 & 0 & 0 \\ 0 & \gamma & -(\beta+\mu) & 0 & 0 \\ 0 & 0 & \beta & -(\tau+\mu) & \mathrm{Z} \\ 0 & 0 & 0 & \tau & -(\mathrm{Z}+\theta+\mu) \end{bmatrix}$$

To determine the local stability of the fatigue-free equilibrium, we use the Routh-Hurwitz theorem which states that an equilibrium state will be locally and asymptotically stable if and only if the sum of the Trace of the Jacobian is less than zero and the determinant is greater than zero.

From our calculation, the Trace of the Jacobian is $-(\alpha+\mu)+-(\gamma+\mu)+-(\beta-\mu)+-\tau-\mu)+-(\mathrm{Z}-\mu-\theta)$

$$= -\left(\alpha+5\mu+\gamma+\beta+\tau+\mathrm{Z}+\theta\right) =< 0$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

The Determinant:

$$\frac{\lambda\alpha\gamma\tau Z(1-\delta)}{\alpha+\mu}+\alpha\beta\tau\mu+\alpha\beta\mu^2+\beta\tau\mu^2+\beta\mu^3+\alpha\tau\mu^2+\alpha\mu^3+\tau\mu^3+\mu^4+\gamma\beta\theta\tau$$
$$+\tau\beta\theta\mu+\theta\tau\gamma\mu+\theta\tau\mu^2+\alpha\theta\beta+\alpha\tau\mu+\tau\alpha\beta\gamma\theta$$
$$-\begin{bmatrix}\alpha\beta\gamma\tau+\alpha\beta\gamma\mu+\beta\gamma\tau\mu+\beta\gamma\mu^2+\alpha\gamma\tau\mu+\alpha\gamma\mu^2+\gamma\tau\mu^2+\gamma\mu^3+\beta\alpha\gamma Z\\+\beta\alpha\gamma\theta+\beta\alpha\gamma\mu+\alpha\beta Z\tau+\beta Z\tau\mu+\alpha Z\tau\mu+Z\gamma\mu^2+\alpha\gamma\theta\tau+\alpha\gamma\theta\mu\end{bmatrix}>0$$

Provided that $\alpha+\mu\neq 0$

RESULTS AND DISCUSSION

Parameter Estimation

We obtained the values for the parameters involved in the model (1) – (5) which will be used for the analysis and simulation from the questionnaires which we administered to the student

Table 1: Table of the values of the variables and the parameters

S/N	Variables	Meaning	Values	Sources
1	B	population of the students who go to bed	300	Questionnaire
2	D	population of the students who delay in going to bed.	220	Questionnaire
3	F	the population of students who wake up tired as a result of sleeping late.	190	Questionnaire
4	P	the population of students who perform poorly as a result of morning fatigue.	240	Questionnaire
5	E	the population of students who sleep early, wake up strong and perform excellently	145	Questionnaire

S/N	Parameter	Meaning	Values	Sources
1	Λ	Recruitment level of the population of students who go to bed	0.45	Questionnaire
2	Α	The rate at which the students delay going to bed	0.80	Questionnaire
3	Γ	The rate at which the students who delay in going to bed wake up tired.	0.90	Questionnaire
4	Β	The rate at which tired students perform poorly academically.	0.85	Questionnaire

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

5	Z		The rate at which students who performed excellently drown back.	0.03	Questionnaire
6		τ	The rate at which poorly performed students recovered and began to perform excellently	0.40	Questionnaire
7	Θ		Rate at which students who initially gain strength loss it.	0.70	Questionnaire
8		μ	Death rate of the compartments.	0.02	World meter (2023)
9		π	Transmission rate between people who go to bed early and those who wake up tired as a result of sleeping late.	0.03	Questionnaire
10		δ	The proportion of individuals who are protected from morning fatigue	0.006	Questionnaire

Sensitivity analysis

We calculate the sensitivity index of the reproduction number for the control model (R_0) to the parameters involved in the model (1) - (5), this analysis shows us how to reduce the poor performance of students at the Federal University Wukari due to morning fatigue. It helps us to find out those parameters that should be targeted by intervention strategies which also enables us to know how important each parameter is to the impact of morning fatigue on academic performance.

To check which of these parameters in the model (1) – (5) has a high impact on R_0 and also to find out key parameter that contributes to the reduction of poor performance of students due to morning fatigue in the population. The normalized forward sensitivity index of the variable to a parameter is the ratio of the relative changes in the parameter.

Maple 2023 programming language was used for this sensitivity analysis.

1. The sensitivity index of *the parameters* with respect to R_0 is given as:

$X_{\pi}^{R_0} = \frac{\partial R_0}{\partial \pi} \times \frac{\pi}{R_0} = 0.3715141254:$

$$X_{\gamma}^{R_0} = \frac{\partial R_0}{\partial \gamma} \times \frac{\gamma}{R_0} = 0.060829555$$

$$X_{\delta}^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0} = -0.006272175596$$

$$X_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0} = -1.013746756$$

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

African Journal of Health, Nursing and Midwifery
ISSN: 2689-9418
Volume 7, Issue 4, 2024 (pp. 11-30)

$$X_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0} = -1.343173572$$

The sensitivity indices are given in the table below, A positive index indicates that the value of R_0 will increase with the increase in the parameter and one with a negative index will decrease the value of R_0 with an increase in the parameter Values

Table 2: Table of the sensitivity index

Parameters	Index	Remark
π	0.3715141254	Positive
γ	0.060829555	Positive
δ	−0.006272175596	Negative
α	−1.013746756	Negative
β	−1.343173572	Negative

We conclude from the sensitivity analysis conducted as given in the table above, that the positive values increase the endemicity of the morning fatigues and the negative values decrease the endemicity of the morning fatigues.

Model Simulation

The values of the variables and the parameters are used to carry out the numerical simulation. The result is presented in the figures below;

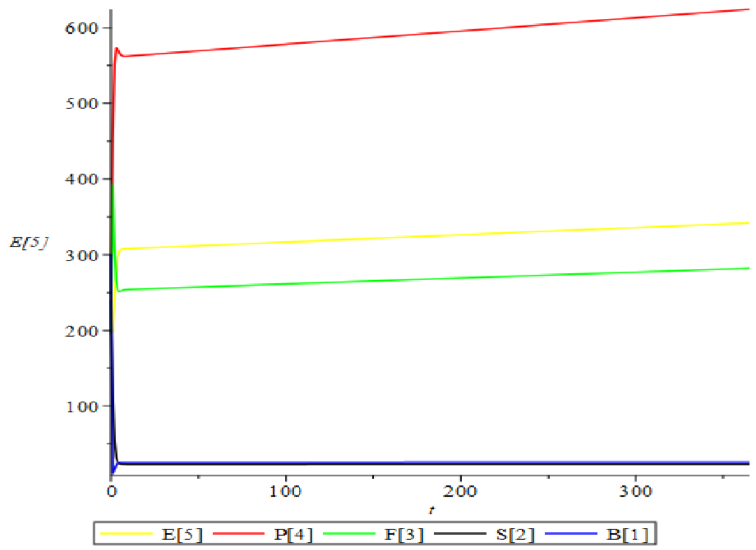

Figure 1: Graph of Students' academic performance in relation to morning fatigue

From this graph, we discovered that students who experienced morning fatigue performed poorly academically.

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

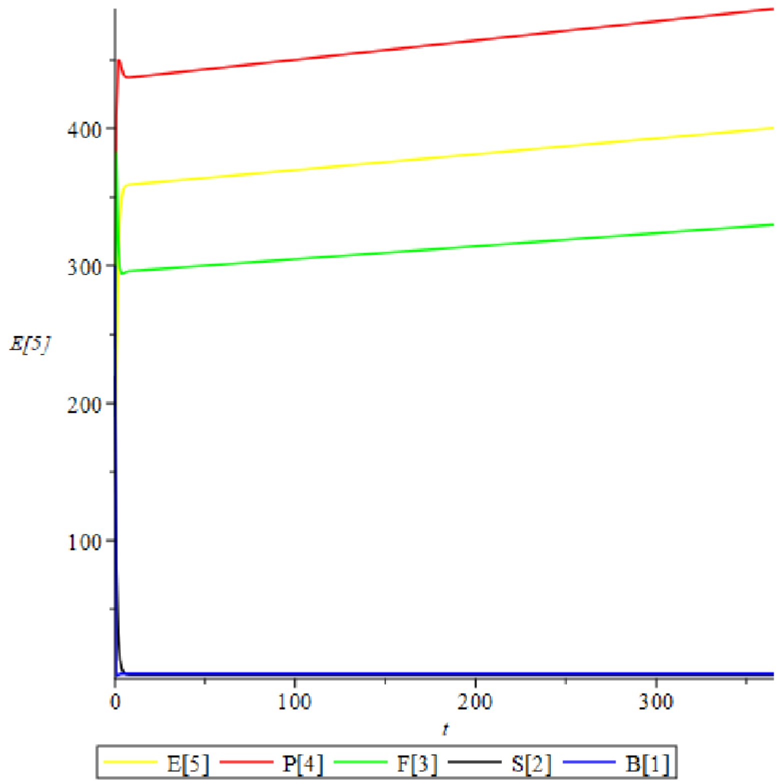

Figure 2: Graph Students' academic performance when some parameters are adjusted

In Figure (2), we discovered that even after adjusting some parameters, the graph still shows that students who experienced morning fatigue are affected academically.

CONCLUSION

Based on the results of the study, we observed that morning fatigue has an adverse effect on the academic performance of students. We therefore concluded that the most effective way to control morning fatigue within a population is to use effective educational enlightenment campaigns on the radio, television, newspaper, in churches, mosques, and even in schools on the need for individuals to avoid late sleep especially those under academic pursuit. A person who is performing poorly in his/her academics should reconsider the time in which he/she goes to bed and target to sleep on time so as to wake up strong. More to that, avoidance of morning fatigue will prevent mental illness and save the lives of students as this can reduce the mortality rate due to the effect of morning fatigue.

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

REFERENCES

Abel, T., Havekes, R., Saletin, J. M., & Walker, M. P. (2013). Sleep, plasticity and memory from molecules to whole-brain networks. Current Biology 23(17), R774–R778. doi:10.1016/j.cub.2013.07.025

Caldwell John A., Caldwell J. Lynn, Thompson Lauren A. , Lieberman Harris R. (2019); Fatigue and its management in the workplace. *Neuroscience and Biobehavioral Reviews*. 96(2019) 272-289 Canadian Centre for Occupational Health and Safety, 2017. OSH Answers Fact Sheet: Fatigue. Updated August 2017; Accessed March 2018. https://ccohs.ca/oshanswers/ psychosocial/fatigue.html.

Dinges, D. F., Rogers, N. L., & Baynard, M. D. (2005). Chronic sleep deprivation.In M. H. Kryger, T. Roth, & W. C. Dement (Eds.) Principles and Practice of Sleep *Medicine* (4th ed., pp. 67-76). Philadelphia, PA: Elsevier Saunders.

Drummond SPA, Brown GG, Gillin JC, Stricker JL, Wong EC, Buxton RB. Altered brain response to verbal learning following sleep deprivation. *Nature.* 2000;403(6770):655–657. [PubMed] [Google Scholar]

Durmer JS, Dinges DF. Neurocognitive consequences of sleep deprivation. *Seminars in Neurology.* 2005;25(1):117–129. [PubMed] [Google Scholar]

Gilbert Steven P. and Weaver Cameron (2010); Sleep Quality and Academic Performance in University Students: A Wake-Up Call for College Psychologists Journal of College Student Psychotherapy 24(4):295-306 48DOI: 10.1080/87568225.2010.509245

Giulio Tononi1, * and Chiara Cirelli1, *(2014); Sleep and the Price of Plasticity:From Synaptic and Cellular Homeostasis to Memory Consolidation and Integration *Neuron* 81, January 8, 2014 ª2014 Elsevier Inc.

Goel, N., 2017. Neurobehavioral effects and biomarkers of sleep loss in healthy adults. *Curr. Neurol. Neurosci. Rep*. 17 (11), 89. https://doi.org/10.1007/s11910-017- 0799-x.

Krausman, A.S., Crowell, H.P., Wilson, R.M., 2002. The Effects of Physical Exertion on Cognitive Performance. Army Research Laboratory Technical Report ARL-TR-2844. Aberdeen Proving Ground. Army Research Laboratory, MD.

Lauren A. Booker1,2 | Jarrah Fitzgerald3 | Jane Mills3 | Melanie Bish3 | Jo Spong2,3 | Melissa Deacon-Crouch3 | Timothy C. Skinner1,4 (2023); Sleep and fatigue management strategies: How nurses, midwives and paramedics cope with their shift work schedules—a qualitative study EMPIRICAL RESEARCH QUALITATIVE Nursing Open. 2024;11:e2099. | 1 of 12 https://doi.org/10.1002/nop2.2099

Lieberman, H.R., 2011. Mental energy and fatigue: science and the consumer. In: Kanarek, R.B., Lieberman, H.R. (Eds.), Diet, Brain, Behavior: Practical Implications. CRC Press, Boca Raton, FL, pp. 1–6.

Lim, J., & Dinges, D. F. (2010). A meta-analysis of the impact of short-term sleep deprivation on cognitive variables. *Psychological Bulletin, 136*(3), 375–389. https://doi.org/10.1037/a0018883

Okano Kana[1] ,Jakub R. Kaczmarzyk[1] , Neha Dave[2] , John D. E. Gabrieli[1] and Jeffrey C. Grossman (2019); Sleep quality, duration, and consistency are associated with better academic performance in college students. npj Science of Learning (2019) 16. Published in partnership with The University of Queensland www.nature.com/npjscilearn

Ribeiro, S. (2012). Sleep and plasticity. Pfügers Archiv-European Journal of Physiology 463, 111–120. doi:10.1007/s00424-011-1031-5 Wang, G., Grone, B., Colas, D., Appelbaum, L., & Mourrain, P. (2011). Synaptic

Article DOI: 10.52589/AJHNM-BUFOHVGK
DOI URL: https://doi.org/10.52589/AJHNM-BUFOHVGK

plasticity in sleep: Learning, homeostasis and disease. Trends in Neuroscience 34(9), 452–463. doi:10.1016/j.tins.2011.07.005

Weiss JT and Donlea JM (2022) Roles for Sleep in Neural and Behavioral Plasticity: Reviewing Variation in the Consequences of Sleep Loss. Front. Behav. Neurosci. 15:777799. doi: 10.3389/fnbeh.2021.777799