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ON ABID ET AL (2016) RATIO ESTIMATORS FOR FINITE POPULATION MEAN USING NON-CONVENTIONAL LOCATION PARAMETER

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ABSTRACT: *Ratio estimator is most effective for estimating population mean when there is a linear relationship between study variable and auxiliary variable when have positive correlation. Abid et al proposed some new modified ratio estimators in simple random sampling, and improved ratio estimator. The first six ratio estimators of Abid et al (2016) for finite population mean in simple random sampling using Tri-mean and mid-range with correlation coefficient and coefficient of variation as supplementary information was used in this work. The aim is to use data simulated from some distributions with varying sample sizes to determine which of the six estimators is more efficient than others. Friedman test was used to rank the bias and the mean square error (MSE) of each of the distribution with varying sample sizes. The sixth estimator have the minimum bias and minimum MSE than other estimators in seven distributions out of the eight distributions computed and that made it to be considered as the best estimator. Only one distribution is odd among the eight distributions and it has the third estimator as the best.*

KEYWORDS: Finite Population Mean, Bias, Square Error, Tri-Mean, Mid-Range, Ratio Estimator, Mean Square Error (MSE)

INTRODUCTION

Abid et al (2016) proposed some ratio estimators for finite population mean in simple random samples using tri-mean, mid-range and hodges lehman with correlation coefficient and coefficient of variation as supplementary information. In this work, we are interested in comparing the efficiency of the first six ratio estimators of Abid et al (2016) population mean using tri-mean and mid-range with correlation coefficient and coefficient of variation as supplementary information. This comparison is done using data simulated from some distributions; Chi-square, Normal, Gamma, Exponential, Binomial, Poisson, Geometric and Negative binomial distributions with varying sample sizes.

Estimation refers to the process by which one makes inferences about a population based on information obtained from a sample.

An estimator is a statistic that estimates some facts about the population. The sample mean is an estimator for the population mean. An estimator is also defined as the formula or statistic which has been chosen to provide an estimate of population value.

Ratio Estimation is a method of estimating sample data. In ratio estimation, an auxiliary variable x_i, correlated with y_i is obtained for each unit in the sample. The population total X for the x_i must be known. The goal is to obtain increase precision by taking advantage of the correlation between y_i and x_i. The ratio estimates of Y, which is the population total is where y and x are sample totals of y_i and x_i respectively.

Ratio Estimator is a statistical parameter and is defined to be the ratio of means of two random variables. Ratio Estimator is an estimation design that makes use of an auxiliary variable that are correlated to target variable.

The ratio estimator is most effective for estimating population mean when there is a linear relationship between study variable and auxiliary variable and they have a positive correlation. Ratio estimator involves the use of known population totals for auxiliary variables to improve the weighting from sample values to population estimates of interest.

Researchers have worked on estimating population means by using auxiliary information. Auxiliary information is obtained from auxiliary variable which is highly positively or negatively correlated with study variable.

The use of auxiliary information in sample surveys is widely studied in the books written by Yates (1960). Chanu and Singn (2014) studied the use of auxiliary information under different sampling designs for improving several estimators. Cochran (1940) was the first to give classical ratio type estimator for estimating population mean based on some prior information of the population of an auxiliary variable.

The auxiliary variable is a variable that is known for every unit of the population. It is not a variable of direct interest but instead employed to improve the sampling plan or to enhance estimator of the variable of interest.

The auxiliary information is commonly associated with the use of ratio product and regression estimation methods and to improve the efficiency of the estimators in survey sampling. The availability of auxiliary information enhances the efficiency of the estimators and increases the precision of an estimator when study variable Y is highly correlated with auxiliary variable X. Auxiliary information may be used either at the planning (designing) stage, selection or estimation stage.

Planning (Designing) Stage; here, auxiliary information may be used for stratification or to form clusters. This stratification based on social status, sex, size, vegetation type etc represents a use of auxiliary information in the design.

At the selection stage; sampling with replacement, with selection probability proportional to size, that is unequal probability sampling represents the use of auxiliary information at the selection stage.

At estimation Stage; Ratio and Regression estimators are example of the use of information auxiliary in estimations. Here the relationship between y_i and x_i are exploited to produce a more precise estimate. In some situation, the x values may be known for the entire population while in other situations, the x values are known only for units included in the sample. The proper use of auxiliary information may result in appreciable gain in precision while indiscriminate use might yield to a loss in precision.

Sampling is a process used in statistical analysis in which a predetermined number of observations are taken from a larger population. It helps to make statistical inference about the population. Sampling has the sole objective of selecting a fraction subset of the population that will be representative of the entire population.

A random sample is a sample selected in such a way that each unit in the population has an equal chance of being selected. A random sample is obtained by using methods such as random numbers which can be generated from calculators, computers or tables.

The use of random numbers is preferred way of selecting a random sample. The theory behind random numbers is that each digit, 0 through 9, has an equal probability of occurring. To obtain a sample by using random numbers, number the elements of the population sequentially and then select each item using random numbers until the required number of samples are selected. Another method of obtaining random sample is to number each element of the population and then place the number on cards, mix them thoroughly and then select the sample by drawing cards using lottery method until the required sample size is obtained.

One important characteristics of random sampling is that each item in the population has equal probability of being selected and selection of one will not hinder the selection of the other. Random sampling is the simplest and provides the best techniques of sampling.

However, for a researcher to make valid inferences about population characteristics, the sample must be random.

Sampling and Simulation are two techniques that enable the researcher to gain information that might otherwise be unobtainable. Many real-life problems can be solved by employing simulation techniques. A simulation technique uses a probability experiment to mimic a real-life situation.

Mathematical simulation techniques use probability and random numbers to create conditions similar to those of real-life problems. Computers have played an important role in simulation techniques, since they can generate random numbers, perform experiments, tally the outcomes, and compute the probabilities much faster than human beings.

The purpose of simulation is to duplicate situations that are too costly or too time-consuming to study in real-life. Most simulation techniques can be done on the computer or calculator, since they can rapidly generate random numbers, count the outcomes and perform the necessary computations.

In this work, a simulation technique was used to generate data from different distributions and different sample sizes.

Several authors have worked on ratio estimators.

Kadilar and Cingi (2006) worked on new ratio estimators using correlation coefficient. They proposed a class of ratio estimation of population mean adopting the estimators in Singh and Tailor (2003). They obtained mean square error for all the proposed estimators found theoretical conditions that make each proposed estimator more efficient than the traditional estimators. The result shows that the proposed estimators have a smaller mean square error (MSE) than the traditional estimators.

Mir et al (2017) did a research on an improved class of ratio estimators for estimating population mean using auxiliary information in survey sampling. They proposed some new estimators by using the auxiliary information of deciles, mean, second quartile and quartile deviation with other measures of population such as skewness, coefficient of correlation, and coefficient of variation of the concomitant variable. The performance linked among the anticipated estimators are determined by mean square error (MSE) and Bias and compare by means of used ratio estimator by Cochran (1940) and with existing estimators proposed via Abid et al (2016a,2016b). With this evaluation, they initiated that anticipated estimators are proficient set of estimators than the ratio estimator by Cochran (1940) and the existing estimator via Abid et al (2016a, 2016b).

Research carried out by Singh et al (2012) on ratio estimators in simple random sampling using information on auxiliary attribute where some ratio estimators for estimating the population mean of the variable under study, which make use of information regarding the population proportion possessing certain attribute are proposed. Under simple random sampling without replacement (SRSWOR) scheme, the expressions of bias and mean squared error (MSE) up to the first order of approximation are derived.

Kazeem et al (2012) worked on efficiency of ratio estimator based on Linear-combination of median, coefficient of skewness and kurtosis. In their work an estimator which is robust in the present of outlier and skewness is proposed. This is achieved by incorporating median, a good measure of location in this regard to the modified ratio estimator developed. Using well analyzed data to illustrate the procedure for the ratio estimator, its mean square error was observed. The minimum of the existing estimators considered. The proposed modified estimator is uniformly better than other estimators and thus most preferred over the existing modified ratio estimators for the use in practical applications for certain population with peculiar characteristics.

In the research carried out by Etebong (2012), he introduced a new family of exponential ratio estimators of population variance in stratified random sampling. From numerical and analytical studies carried out by him, the results show that under certain prescribed conditions, the new estimator has equal optimal efficiency with the regression estimator of population variance but always fares better than the classical ratio estimator of population variance and every identified existing estimator of its family.

Kadilar and Cingi (2004) did a work on improvement in estimating the population mean in simple random sampling and the result shows that all proposed estimators are always more efficient than ratio estimators. This result is also supported numerically.

In sample surveys, it is usual to make use of auxiliary information to increase the precision in estimating the population parameters. In a research work conducted by Mir et al (2017) on class of improved ratio estimators for population mean using conventional location parameters, they proposed a new class of improved ratio type estimator in simple random sampling without replacement for estimating finite population using the linear combinations of population deciles, median and correlation coefficient, coefficient of variation of the auxiliary variable, obtain their mean square error (MSE), bias and compare with the existing estimators. It was found that their proposed estimators are more efficient than the existing estimators, as their mean square error and bias are lower than the existing estimators.

A generalized modified ratio estimator was proposed by Subramani (2013) for estimating the population mean using the known population parameters. The simple random sampling without replacement sample mean, the usual ratio estimator, the linear regression estimator and all the existing modified ratio estimators are the particular cases of the proposed estimator. The performance of the proposed estimator was assessed with that of the existing estimators for certain natural populations.

Kadrilar and Cingi (2006) carried a research on improvement in variance estimation in simple random sampling. They developed a new population variance estimator whose mean square error (MSE) is smaller than the mean square error (MSE) of traditional ratio and regression estimators. This theoretical inference is also supported by the result of an application with original data presented in Kadilar and Cingi (2004).

Housila et al (2010) proposed two ratio and product- type estimators using transformation based, known minimum and maximum value of auxiliary variable. The biases and mean squared errors (MSE) of the suggested estimators were obtained under large sample approximation. The superiority of the proposed estimators is also established through some natural population data sets.

Etebong (2016) in his research suggests an improved ratio estimator for population mean in stratified random sampling. Analytically, the bias and mean square error (MSE) expressions for the proposed estimator are obtained. Efficiency comparisons are made to evaluate the relative performance of the proposed estimator. In addition, an empirical study is provided to support the analytical study. Analytical and numerical results showed that the proposed estimator is more efficient than the estimator under study. It is also observed that the proposed estimator is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates.

Abid et al (2016) proposed six ratio estimators for finite population mean in simple random samples using tri-mean and mid-range with correlation coefficient and coefficient of variation as supplementary information and the choice of using one or all of them is not certain. This work is set out to make a choice among the six estimators by simulating data from eight distributions, namely, Chi-square, Normal, Gamma, Exponential, Binomial, Poisson, Geometric and Negative binomial distributions and varying sample sizes (small, moderate and large sample sizes).

The aim of this work is to compare the efficiency of each of the six (6) methods of Abid et al (2016) ratio estimators by comparing the estimators based on some distribution and sample sizes.

METHODOLOGY

The population considered in this research is 305, that is N =305 and sample sizes are; n_1= 15 for small sample size, n_2 = 55 for moderate sample size and n_3 = 125 for large sample size.

The six (6) Abid et al (2016) ratio estimators for finite population mean are

$$\hat{\bar{Y}}_{N1} = \frac{\bar{y}_n + b\left(\bar{X}_N - \bar{x}_n\right)}{\left(\bar{x}_n + TM_N\right)}\left(\bar{X}_N + TM_N\right) \tag{2.1}$$

$$\hat{\bar{Y}}_{N2} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{X_n} + TM_N)}(\bar{X}_N C_{X_n} + TM_N) \tag{2.2}$$

$$\hat{\bar{Y}}_{N3} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n \rho_{xy} + TM_N)}(\bar{X}_N \rho_{xy} + TM_N) \tag{2.3}$$

$$\hat{\bar{Y}}_{N4} = \frac{\bar{y}_n + b(\bar{X}_N - x_n)}{(\bar{x}_n + MR_N)}(\bar{X}_N + MR_N) \tag{2.4}$$

$$\hat{\bar{Y}}_{N5} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{X_n} + MR_N)}(\bar{X}_N C_{X_n} + MR_N) \tag{2.5}$$

$$\hat{\bar{Y}}_{N6} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n \rho_{xy} + MR_N)}(\bar{X}_N \rho_{xy} + MR_N) \tag{2.6}$$

The constant for the six (6) ratio estimators Abid et al (2016) are as follows

$$R_1 = \frac{\bar{Y}_N}{(\bar{X}_N + TM_N)} \tag{2.7}$$

$$R_2 = \frac{\bar{Y}_N C_{X_n}}{(\bar{X}_N C_{X_N} + TM_N)} \tag{2.8}$$

$$R3 = \frac{\bar{Y}_N \rho_{xy}}{(\bar{X}_N \rho_{xy} + TM_N)} \tag{2.9}$$

$$R_4 = \frac{\bar{Y}_N}{(\bar{X}_N + MR_N)} \tag{2.10}$$

$$R_5 = \frac{\bar{Y}_N C_{X_n}}{(\bar{X}_N C_{X_N} + MR_N)} \tag{2.11}$$

$$R_6 = \frac{\bar{Y}_N \rho_{xy}}{(\bar{X}_N \rho_{xy} + MR_N)} \tag{2.12}$$

The biases for the six (6) ratio estimators Abid et al (2016) are as follows

$$B\left(\hat{\bar{Y}}_{N1}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_1^2 \tag{2.13}$$

$$B\left(\hat{\bar{Y}}_{N2}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_2^2 \tag{2.14}$$

$$B\left(\hat{\bar{Y}}_{N3}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_3^2 \tag{2.15}$$

$$B\left(\hat{\bar{Y}}_{N4}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_{51}^2 \tag{2.16}$$

$$B\left(\hat{\bar{Y}}_{N5}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_5^2 \tag{2.17}$$

$$B\left(\hat{\bar{Y}}_{N6}\right) = \frac{(1-f_n)}{n_n}\frac{S^2{x_n}}{\bar{Y}_N}R_6^2 \tag{2.18}$$

The mean square error (MSE) for the six (6) ratio estimators Abid et al (2016) are as follows

$$MSE\left(\hat{\bar{Y}}_{N1}\right) = \frac{(1-f_n)}{n_n}\left[R_1^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.19}$$

$$MSE\left(\hat{\bar{Y}}_{N2}\right) = \frac{(1-f_n)}{n_n}\left[R_2^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.20}$$

$$MSE\left(\hat{\bar{Y}}_{N3}\right) = \frac{(1-f_n)}{n_n}\left[R_3^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.21}$$

$$MSE\left(\hat{\bar{Y}}_{N4}\right) = \frac{(1-f_n)}{n_n}\left[R_4^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.22}$$

$$MSE\left(\hat{\bar{Y}}_{N5}\right) = \frac{(1-f_n)}{n_n}\left[R_5^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.23}$$

$$MSE\left(\hat{\bar{Y}}_{N6}\right) = \frac{(1-f_n)}{n_n}\left[R_6^2S_{X_n}^2 + S_{y_n}^2\left(1-\rho^2{}_{xy}\right)\right] \tag{2.24}$$

The parameters in equation (2.1) to (2.24) are defined as follows:

N_N	Size of the population
n_n	Size of the sample
$f_n = {}^{n_n}/_{N_N}$	Fraction of sampling
Y_N	Response variable
X_N	Auxiliary variable
$\bar{Y}_N$	Population mean of the Response
$\bar{X}_N$	Population mean of the Auxiliary variable
$\bar{Y}_n$	Sample means of the Response
$\bar{x}_n$	Sample means of the Auxiliary variable
Y	Sample totals of the Response
X	Sample totals of the Auxiliary variable
S_{y_n}	Population standard deviations of the Response
S_{x_n}	Population standard deviations of the Auxiliary variables
C_{x_n}	Population coefficient of variation of the Auxiliary variable
ρ_{xy}	Population correlation coefficient
B(.)	Bias of the estimator
MSE (.)	Mean square error of the estimator
$\hat{\bar{Y}}_{Ni}$	Existing modified ratio estimator of $\bar{Y}_{Ni}$
$MR_N = (X_{NI} + X_{NN})/2$	Population mid-range
$TM_N = (Q_{N1} + 2Q_{N2} + Q_{N3})/4$	Population tri-mean

The data used in this work were simulated using the following distributions and parameters;

S/N	Distributions	Parameters used
1.	Normal	Mean = 35; Standard deviation =4.3
2.	Gamma	Shape = 6; Scale = 4; Threshold = 0.0
3.	Chi-square	Degree of freedom = 50
4.	Exponential	Scale = 35; Threshold 55
5.	Binomial	Number of trials = 10; Event probability = 0.6
6.	Poisson	Mean = 40
7.	Geometric	Event probability = 0.3
8.	Negative Binomial	Event probability = 0.5; Number of event needed = 45

Efficiency Comparisons

The bias and mean square errors for each of the ratio estimators will be calculated. The smaller the mean square error, the more efficient the estimator. To rank the values of the biases and mean square errors, we shall use the Friedman test to first of all test if the values of the bias and mean square error are significant and consequently rank them. If their values are significant, it means some estimators are better than others.

DATA ANALYSIS

Chi-Square Distribution

The values of the parameters used in Chi-square distribution for the six estimators has been calculated and presented in Table 3.1.

Table 3.1: Values of Parameters Used in Chi-Square Distribution

Parameters	Small Sample	Medium Sample	Large Sample
N_n	305	305	305
n_n	15	55	125
$\bar{Y}_N$	50.779	50.779	50.779
$\bar{X}_N$	50.852	50.852	50.852
$\bar{y}_n$	50.73	51.71	51.531
$\bar{x}_n$	50.53	51.59	51.652
$S^2_{Y_n}$	91.043	91.043	91.043
$S^2_{X_n}$	91.911	91.911	91.911
C_{X_n}	18.85	18.85	18.85

ρ_{xy}	0.993	0.993	0.993
b	0.98825	0.98825	0.98825
TM_N	50.637	50.637	50.637
MR_N	53.1405	53.1405	53.1405
f_n	0.04918032787	0.1803278689	0.4098360656

The ranking of the estimators based on the biases are shown by the Friedman result shown in Table 3.2.

Table 3.2: Friedman Test: Bias versus Estimators Blocked by Sample Sizes

S = 15.00, DF = 5, P = 0.010

ESTIMATORS	N	Est Median	Sum of Ranks
1	3	0.00675	12.0
2	3	0.02427	18.0
3	3	0.00671	9.0
4	3	0.00643	6.0
5	3	0.02415	15.0
6	3	0.00639	3.0

Grand median = 0.01245

The corresponding mean square errors are shown in Table 3.3

Table 3.3: Friedman Test: MSe versus Estimators Blocked by Sample Sizes.

S = 15.00, DF = 5, P = 0.010

ESTIMATORS.	N	Est Median	Sum of Ranks
1	3	0.3618	12.0
2	3	1.2511	18.0
3	3	0.3594	9.0
4	3	0.3455	6.0
5	3	1.2450	15.0
6	3	0.3432	3.0

Grand median = 0.6510

The corresponding biases, mean square errors and ranks are shown in Table 3.4

Table 3.4: Constants, Biases, Mean Square Errors and Ranks of Chi-Square

Estimators	Constant for All Sample Size	Bias			Mean Square Error (MSe)			Ranks
		Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 125	
$\hat{\bar{Y}}_{N1}$	0.50034	0.028722	0.006753	0.002139	1.539006	0.361835	0.114629	4
$\hat{\bar{Y}}_{N2}$	0.948461	0.103212	0.024266	0.007687	5.321501	1.251137	0.39636	6
$\hat{\bar{Y}}_{N3}$	0.498586	0.028521	0.006706	0.002124	1.5288	0.359436	0.113869	3
$\hat{\bar{Y}}_{N4}$	0.488295	0.027356	0.006432	0.002038	1.46928	0.345524	0.109462	2
$\hat{\bar{Y}}_{N5}$	0.946114	0.102702	0.024146	0.007649	5.295595	1.245046	0.394431	5
$\hat{\bar{Y}}_{N6}$	0.486542	0.02716	0.006386	0.002023	1.459674	0.343184	0.108721	1

Similar computations have been done for other distributions and presented in the following sections and Tables.

Gamma Distribution

Table 3.2: Constants, Biases, Mean square errors and Rank of Gamma distribution.

Estimators	Constant	Bias			MSe			
	For All Samples	Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 55	Rank
$\hat{\bar{Y}}_{N1}$	0.505819	0.071494	0.061633	0.044375	1.783245	1.537281	1.106842	4
$\hat{\bar{Y}}_{N2}$	0.977735	0.267128	0.230283	0.165804	6.55321	5.649319	4.06751	6
$\hat{\bar{Y}}_{N3}$	0.505068	0.071282	0.06145	0.044244	1.778072	1.53282	1.103631	3
$\hat{\bar{Y}}_{N4}$	0.395622	0.043736	0.037703	0.027146	1.106456	0.953841	0.686766	2
$\hat{\bar{Y}}_{N5}$	0.965512	0.260491	0.224561	0.161684	6.391383	5.509813	3.967065	5
$\hat{\bar{Y}}_{N6}$	0.394904	0.043577	0.037567	0.027048	1.102587	0.950506	0.684365	1

Normal Distribution

Table 3.3: Constants, Biases, and Mean Square Errors and Rank of Normal Distribution

Estimators	Constant for All Sample	Bias			MSe			
		Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 125	Rank
$\hat{\bar{Y}}_{N1}$	0.496594	0.007924	0.001863	0.00059	0.283881	0.066743	0.021144	3
$\hat{\bar{Y}}_{N2}$	0.916548	0.026994	0.006347	0.002011	0.9443	0.222014	0.070334	6
$\hat{\bar{Y}}_{N3}$	0.4956	0.007893	0.001856	0.000588	0.282782	0.243778	0.021062	4
$\hat{\bar{Y}}_{N4}$	0.492593	0.007797	0.001833	0.000581	0.279476	0.065707	0.020816	2
$\hat{\bar{Y}}_{N5}$	0.915405	0.026927	0.006331	0.002006	0.94197	0.221466	0.070161	5
$\hat{\bar{Y}}_{N6}$	0.491598	0.007766	0.001826	0.000578	0.278386	0.065451	0.020735	1

Exponential Distribution

Table 3.4 Constants, Biases, And Mean Square Errors and Rank of Exponential Distribution

Estimators	Constant	Bias			MSe			
	All Sample Size	Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 125	Rank
$\hat{\bar{Y}}_{N1}$	0.525356	0.190449	0.164181	0.11821	18.95674	4.456914	1.41195	4
$\hat{\bar{Y}}_{N2}$	1.02152	0.720055	0.620737	0.446931	68.71845	16.15638	5.11834	6
$\hat{\bar{Y}}_{N3}$	0.524041	0.189497	0.163359	0.117619	18.86721	4.435865	1.405282	3
$\hat{\bar{Y}}_{N4}$	0.406982	0.114294	0.098529	0.070941	11.80115	2.774566	0.878983	2
$\hat{\bar{Y}}_{N5}$	1.005607	0.697796	0.601548	0.433115	66.62702	15.66466	4.962564	5
$\hat{\bar{Y}}_{N6}$	0.405734	0.113594	0.097925	0.070506	11.73536	2.759097	0.874082	1

Binomial Distribution

Table 3.5 Constants, Biases, and Mean Square Errors and Rank of Binomial Distribution

Estimators	Constant All Sample Size	Bias			MSe			
		Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 125	Rank
$\hat{\bar{Y}}_{N1}$	0.500771	0.007259	0.006258	0.004505	0.050863	0.011958	0.003788	3
$\hat{\bar{Y}}_{N2}$	0.968035	0.027125	0.023383	0.016836	0.170905	0.040181	0.012729	5
$\hat{\bar{Y}}_{N3}$	0.494935	0.007091	0.006113	0.004401	0.049847	0.011719	0.003713	1
$\hat{\bar{Y}}_{N4}$	0.502587	0.007311	0.006303	0.004538	0.051182	0.012033	0.003812	4
$\hat{\bar{Y}}_{N5}$	0.96828	0.027138	0.023395	0.016845	0.170988	0.040201	0.012736	6
$\hat{\bar{Y}}_{N6}$	0.496751	0.007143	0.006157	0.004433	0.050162	0.011793	0.003736	2

Poisson Distribution

Table 3.6. Constants, Biases, and Mean Square Errors and Rank of Poisson Distribution

Estimators	Constant For All Sample Sizes	Bias			MSe			Rank
		Sample Size 15	Sample Size 55	Sample Size 125	Sample Size 15	Sample Size 55	Sample Size 125	
$\hat{\bar{Y}}_{N1}$	0.510486	0.01644	0.014177	0.010208	0.724343	0.1703	0.053951	4
$\hat{\bar{Y}}_{N2}$	0.960674	0.058242	0.050208	0.03615	2.435517	0.572614	0.181404	6
$\hat{\bar{Y}}_{N3}$	0.508179	0.016297	0.014049	0.010116	0.718269	0.168872	0.053499	3
$\hat{\bar{Y}}_{N4}$	0.504834	0.016083	0.013865	0.009983	0.709514	0.166814	0.052847	2
$\hat{\bar{Y}}_{N5}$	0.959404	0.058088	0.050076	0.036055	2.429217	0.571133	0.180935	5
$\hat{\bar{Y}}_{N6}$	0.502526	0.015937	0.013739	0.009892	0.703509	0.165402	0.052399	1

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Geometric Distribution

Table 3.7. The Constants, Biases, and Mean Square Errors and Rank of Geometric Distribution

Estimators	Constant for All Sample Size	Bias			Mse			Rank
		Sample size 15	Sample size 55	Sample size 125	Sample size 15	Sample size 55	Sample size 125	
$\hat{\bar{Y}}_{N1}$	0.52727	0.037878	0.032653	0.02351	0.148371	0.034883	0.011051	4
$\hat{\bar{Y}}_{N2}$	0.998538	0.135847	0.117109	0.084319	0.478917	0.112598	0.035671	6
$\hat{\bar{Y}}_{N3}$	0.522177	0.03715	0.032026	0.023058	0.145914	0.034306	0.010868	3
$\hat{\bar{Y}}_{N4}$	0.262752	0.009406	0.008109	0.005838	0.052307	0.012298	0.003896	2
$\hat{\bar{Y}}_{N5}$	0.975464	0.129641	0.111759	0.080467	0.457979	0.107675	0.034112	5
$\hat{\bar{Y}}_{N6}$	0.258844	0.009128	0.007869	0.005666	0.05137	0.012078	0.003826	1

The summary of the ranking of the estimators and the corresponding distributions are shown in Table 3.9.

Table 3.9: Summary of Ranking of the Estimators in Order of Efficiency

Estimators	Distributions	Rank
$\hat{\bar{Y}}_{N1}$	Chi-square, Gamma, Exponential, Poisson, Geometric.	4
$\hat{\bar{Y}}_{N2}$	Chi-square, Gamma, Normal, Exponential, Poisson, Geometric.	6
$\hat{\bar{Y}}_{N3}$	Chi-square, Gamma, Normal, Exponential, Poisson, Geometric, Negative Binomial.	3
$\hat{\bar{Y}}_{N4}$	Chi-square, Gamma, Normal, Exponential, Poisson, Geometric, Negative Binomial.	2
$\hat{\bar{Y}}_{N5}$	Chi-square, Gamma, Normal, Exponential, Poisson, Geometric, Negative Binomial.	5
$\hat{\bar{Y}}_{N6}$	Chi-square, Gamma, Normal, Exponential, Poisson, Geometric, Negative Binomial.	1

DISCUSSION

The statistical analysis to determine the most efficient of the six estimators proposed by Abid et al (2016) for finite population mean using tri-mean and mid-range with correlation coefficient and coefficient of variation as supplementary information has been analyzed using simulated data. Assuming they are affected by distributions and varying sample sizes, using population size of 305 and sample size of 15, 55 and 125, and distributions of Binomial, Geometric, Negative Binomial, Poisson, Normal, Chi-square, Gamma and Exponential, from the analysis made, it was observed that the sixth estimator is the best in seven distributions out of the eight distributions used and as the sample size increases, the bias and mean square error decreases or reduces (becomes more efficient). The third estimator was more efficient in Binomial distribution. Hence, we conclude that the sixth estimator is the best and the most efficient of the first six estimators of Abid et al (2016).

CONCLUSION

From our study, we advise that ratio estimator

$$\hat{\bar{Y}}_{N6} = \frac{\bar{y}_n + b\left(\bar{X}_N - \bar{x}_n\right)}{\left(\bar{x}_n \rho_{xy} + MR_N\right)}\left(\bar{X}_N \rho_{xy} + MR_N\right)$$

should be used while the rest be ignored to save time and possibly cost.

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