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ON SOME PROPERTIES AND APPLICATIONS OF THE TOPP LEONE EXPONENTIATED INVERTED KUMARASWAMY DISTRIBUTION

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ABSTRACT: *In recent years, researchers in the area of distribution theory are proposing and developing new models by generalizing the existing ones so as to make them more flexible and fit in the recent trend in data. In this research, we propose a new distribution called the Topp Leone exponentiated inverse distribution with four positive parameters, which extends the inverse Kumaraswamy distribution with two shape parameters. We derive some mathematical properties of the proposed model including explicit expressions for the quantile function, moments, generating function, probability generating function, survival, hazard rate, reversed hazard rate, cumulative hazard rate and odd functions. The method of maximum likelihood is used to estimate the parameters of the distribution. We illustrate its potentiality with applications to two real data sets which show that the extended generalized inverse exponential model provides a better fit than other models considered.*

KEYWORDS: Central moments, Cumulative hazard rate function, Generalized inverse Kumaraswamy, Linear representation, Million revolutions, Odds functions.

INTRODUCTION

In the area of distribution theory, the development of new and generalized statistical models is a prominent research. The literature is filled with such distributions that are very worthwhile in predicting and modeling real world phenomena. A number of classical distributions have been used comprehensively over the past decades for modeling data in several applied areas including bio-medical analysis, reliability engineering, economics, forecasting, astronomy, demography and insurance.

The addition of parameters has been proved useful in exploring skewness and tail properties, and also for improving the goodness-of-fit of the generated family. Some of the recent families of distributions appearing in the literature are: Topp–Leone exponentiated generalized-G class of distributions by Rasheed (2020), Topp-Leone generated family distribution by Aryuyuen (2018), The Marshall-Olkin Odd Lindley-G Family of Distributions by Jamal (2019), The Exponentiated Kumaraswamy-G family of distributions by Silva et al., (2019), Fréchet Topp Leone G Family of Distributions by Reyad et al., (2019a), Power Lindley-G Family of Distributions by Hassan and Nassr (2019), Modi family of continuous probability distributions by Modi et al., (2020), The Topp Leone Exponentiated-G family of distributions by Ibrahim et al., (2020a), The Topp Leone Kumaraswamy-G family of distributions by Ibrahim et al., (2020b), Odd Chen-G family of distributions by Anzagra et al., (2020).

In the last few few years, the researchers have been proposing many inverted distributions due to its flexibility and wide applications. Al-Fattah et al. (2017) introduced the inverted Kumaraswamy (IKw) distribution with two positive shape parameters $\alpha > 0$ and $\beta > 0$, where the cumulative distribution function (cdf) and pdf are given respectively by

$$H(x;\alpha,\beta) = [1-(1+x)^{-\alpha}]^{\beta} \tag{1}$$

and

$$h(x;\alpha,\beta) = \alpha\beta(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{\beta-1} \tag{2}$$

As shown by the author, the curves of the pdf exhibit a long right tail and produces optimistic predictions of rare events occurring in the right tail of the distribution as compared with other distributions. Reyad et al., (2019b) proposed a four-parameter generalized inverted Kumaraswamy distribution using the Topp Leone-G family of distribution proposed by Al-Shomrani et al., (2016).

This research proposed a new generalized distribution by using the family of distributions proposed by Ibrahim et al., (2020a). Let $h(x;\varphi)$ and $H(x;\varphi)$ denote the pdf and cdf of a baseline distribution with parameter vector φ, then Ibrahim et al., (2020a) defined the Topp-Leone-Exponentiated-G class of distributions with cdf given as

$$F(x;\lambda,\theta,\varphi) = \{1-[1-H(x,\varphi)^{\lambda}]^{2}\}^{\theta} \tag{3}$$

and its pdf given as

$$f(x;\lambda,\theta,\varphi) = 2\lambda\theta h(x;\ \varphi)H(x;\ \varphi)^{\lambda-1}[1-H(x;\ \varphi)^{\lambda}]\{1-[1-H(x;\ \varphi)^{\lambda}]^{2}\}^{\theta-1} \tag{4}$$

$$x > 0, \qquad \lambda, \qquad \theta, \qquad \varphi > 0$$

respectively.

Where $h(x;\varphi) = \frac{dH(x;\varphi)}{dx}$ is the baseline pdf, λ and θ are positive shape parameters and φ is the vector of parameters.

THE TOPP LEONE EXPONENTIATED INVERTED KUMARASWAMY (TLExIK) DISTRIBUTION

In this section, we define a new model called Topp Leone exponentiated inverted Kumaraswamy distribution to obtain more flexibility and accuracy in fitting data than some well- known distributions and study some of its mathematical properties. The new model is derived by inserting (1) into (3) to obtain

$$F(x;\lambda,\theta,\alpha,\beta) = \{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^{2}\}^{\theta} \tag{5}$$

and

$$f(x;\lambda,\theta,\alpha,\beta) = 2\lambda\theta\beta\alpha(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{\beta\lambda-1}[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta-1} \quad (6)$$

Where $\lambda,\theta,\alpha,\beta > 0$ are the positive shape parameters.

The plots of the pdf showing the shape of the distribution with different parameter values are presented in figure 1.

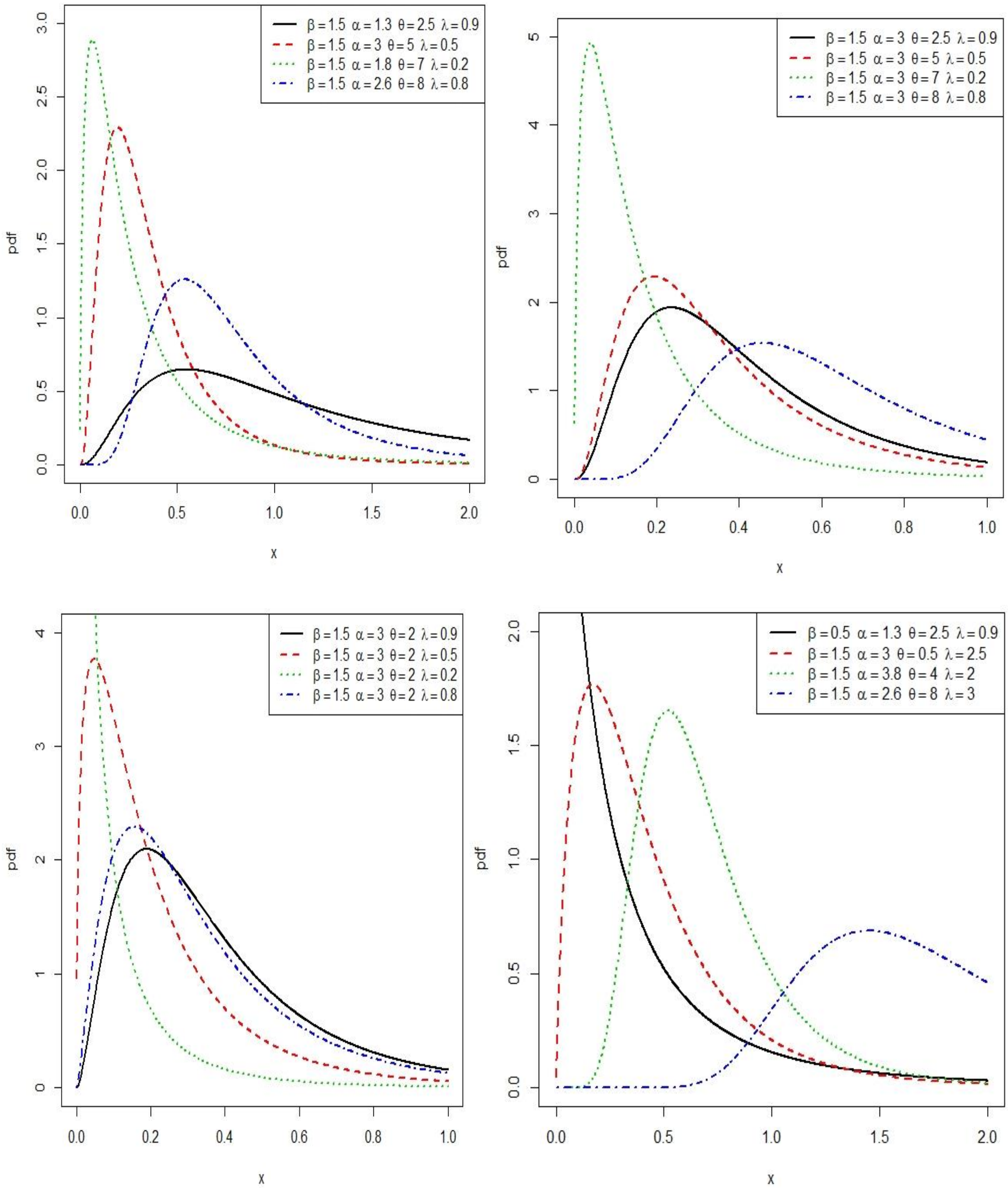

Figure 1: Plots of the TLExIK pdf with different parameter values

LINEAR REPRESENTATION

Here, the infinite mixture representations for the pdf of the TLExIK distribution is using the following series expansion

$$(1-y)^{b-1} = \sum_{i=0}^{\infty} \frac{(-1)^i \Gamma(b)}{i!\Gamma(b-i)} y^i \quad (7)$$

Then the series expansion for the TLExIK distribution is given as

$$f(x;\lambda,\theta,\alpha,\beta) = 2\lambda\theta\beta\alpha \sum_{i,j,k=0}^{\infty} \frac{(-1)^{i+j+k}\Gamma(\theta)\Gamma(2(i+1))\Gamma(\beta\lambda(j+1))}{i!j!k!\Gamma(\theta-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)} (1+x)^{-\alpha(k+1)-1} \quad (8)$$

This expansion will be used in deriving some the mathematical properties of the TLExIK distribution.

PROPERTIES OF TLEXIK DISTRIBUTION

Moments

Moments function is used to study many important properties of distribution such as dispersion, tendency, skewness and kurtosis. The r^{th} moments of the Topp Leone exponentiated inverted Kumaraswamy distribution is obtained as follow:

$$E(X^r) = \int_0^{\infty} x^r f(x)dx \quad (9)$$

The moments of the TLExIK is given as

$$E(X^r) = 2\lambda\theta\beta\alpha \sum_{i,j,k=0}^{\infty} \frac{(-1)^{i+j+k}\Gamma(\theta)\Gamma\big(2(i+1)\big)\Gamma\big(\beta\lambda(j+1)\big)}{i!j!k!\Gamma(\theta-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)} B(\alpha(k+1)-r, r+1) \quad (10)$$

The mean is obtained by setting $r = 1$ in (10)

n^th Central Moment

$$\mu_n = \sum_{r=0}^{n} \binom{n}{r} (-\mu_1')^{n-r} E(X^r) \quad (11)$$

$$\mu_n = 2\lambda\theta\beta\alpha \sum_{i,j,k=0}^{\infty} \sum_{r=0}^{n} \binom{n}{r} (-\mu_1')^{n-r} \frac{(-1)^{i+j+k}\Gamma(\theta)\Gamma\big(2(i+1)\big)\Gamma\big(\beta\lambda(j+1)\big)}{i!j!k!\Gamma(\theta-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)} B(\alpha(k+1)-r, r+1) \quad (12)$$

Momoemt Generating Function (MGF)

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f(x)dx \quad (13)$$

The MGF of TLExIK is given as

$$e^{tx} = \sum_{m=0}^{\infty} \frac{t^m x^m}{m!} \quad (14)$$

$$M_X(t) = 2\lambda\theta\beta\alpha \sum_{i,j,k=0}^{\infty} \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{(-1)^{i+j+k}\Gamma(\theta)\Gamma(2(i+1))\Gamma(\beta\lambda(j+1))}{i!j!k!\Gamma(\theta-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)} B(\alpha(k+1)-r, r+1) \quad (15)$$

Probability Generating Function (PGF)

$$P_X(t) = 2\lambda\theta\beta\alpha \sum_{i,j,k=0}^{\infty} \sum_{r=0}^{\infty} \frac{(lnt)^r}{r!} \frac{(-1)^{i+j+k}\Gamma(\theta)\Gamma(2(i+1))\Gamma(\beta\lambda(j+1))}{i!j!k!\Gamma(\theta-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)} B(\alpha(k+1)-r, r+1) \quad (16)$$

Quantile Function

The Quantile function is given by;

$$Q(u) = F^{-1}(u) \quad (17)$$

Therefore, the corresponding quantile function for the TLExIK model is given by;

$$x = \left[1 - \left[1 - \left[1 - u^{\frac{1}{\theta}}\right]^{\frac{1}{2}}\right]^{\frac{1}{\lambda\beta}}\right]^{\frac{1}{-\alpha}} - 1 \quad (18)$$

and the median is obtained by setting $u = 0.5$ in (18) to obtain

$$x_{median} = \left[1 - \left[1 - \left[1 - 0.5^{\frac{1}{\theta}}\right]^{\frac{1}{2}}\right]^{\frac{1}{\lambda\beta}}\right]^{\frac{1}{-\alpha}} - 1 \quad (19)$$

Survival Function

The survival function, which is the probability of an item not failing prior to some time. It can be defined as

$$S(x) = 1 - F(x) \quad (20)$$

$$S(x) = 1 - \{1 - [1 - [1 - (1 + x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta} \quad (21)$$

Hazard Rate Function

The hazard rate function is given as

$$\tau(x)=\frac{f(x)}{1-F(x)}=\frac{f(x)}{S(x)} \quad (21)$$

$$\tau(x)=\frac{2\lambda\theta\beta\alpha(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{\beta\lambda-1}[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^{2}\}^{\theta-1}}{1-\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^{2}\}^{\theta}} \quad (22)$$

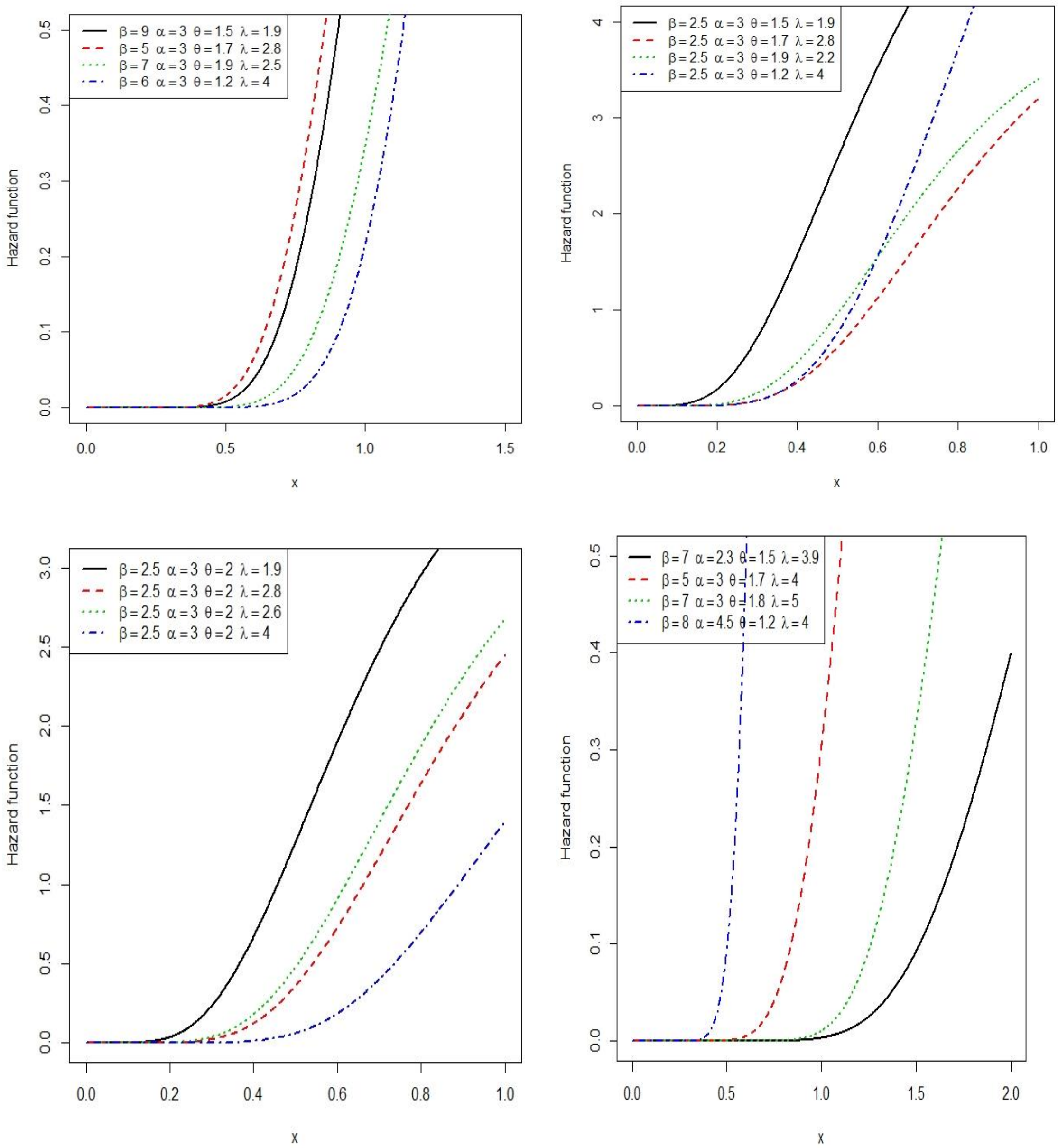

Figure 2: Plots of hazard rate function of the TLExIK with different parameter values

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Reversed Hazard Rate Function

The reverse hazard rate function is defined as

$$\emptyset(x) = \frac{f(x)}{F(x)} \tag{23}$$

$$\emptyset(x) = \frac{2\lambda\theta\beta\alpha(1+x)^{-(\alpha+1)}[1-(1+x)^{-\alpha}]^{\beta\lambda-1}[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta-1}}{\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta}} \tag{24}$$

Odds Function

$$O(x) = \frac{F(x)}{S(x)} \tag{25}$$

$$O(x) = \frac{\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta}}{1-\{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta}} \tag{26}$$

Cumulative Hazard Function

$$C(x) = -\ln\left(S(x)\right) \tag{27}$$

Then, the cumulative hazard function of the TLExIK distributions is given as

$$C(x) = -\ln\left(1 - \{1-[1-[1-(1+x)^{-\alpha}]^{\beta\lambda}]^2\}^{\theta}\right) \tag{28}$$

DISTRIBUTION OF ORDER STATISTICS

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample and its ordered values are denoted as $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$. The pdf of order statistics is obtained using the below function

$$f_{r:n}(x) = \frac{1}{B(r,n-r+1)} f(x)[F(x)]^{r-1}[1-F(x)]^{n-r} \tag{29}$$

Minimum Order Statistics

The minimum order statistics is obtained by setting $r = 1$ in (29) as

$$f_{1:n}(x) = nf(x)[1-F(x)]^{n-1} \tag{30}$$

Then the minimum order statistics of the TLExIK distribution is given as

$$f_{1:n}(x) = 2n\alpha\theta\beta\lambda \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^{i+j+k+l}\Gamma(n)\Gamma(\theta(i+1))\Gamma(2(j+1))\Gamma(\beta\lambda(k+1))}{i!j!k!l!\Gamma(n-i)\Gamma(\theta(i+1)-j)\Gamma(2(j+1)-k)\Gamma(\beta\lambda(k+1)-l)}(1+x)^{-\alpha(l+1)-1} \tag{31}$$

Maximum Order Statistics

The maximum order statistics is obtained by setting $r = n$ in (29) as

$$f_{n:n}(x) = nf(x)[F(x)]^{n-1} \tag{32}$$

Then the maximum order statistics of the TLExIK distribution is given as

$$f_{n:n}(x) = 2n\alpha\theta\beta\lambda \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-1)^{i+j+k}\Gamma(\theta n)\Gamma(2(i+1))\Gamma(\beta\lambda(j+1))}{i!j!k!\Gamma(\theta n-i)\Gamma(2(i+1)-j)\Gamma(\beta\lambda(j+1)-k)}(1+x)^{-\alpha(l+1-1)} \tag{33}$$

MAXIMUM LIKELIHOOD ESTIMATES

Let $X_1, X_2, X_3, \dots, X_n$ be random variables of TLExIK distribution of size n. Then sample likelihood function of TLExIK distribution is obtained as

$$l = nlog2 + nlog\alpha + nlog\beta + nlog\lambda + nlog\theta - (\alpha+1)\sum_{i=1}^{n}\log(1+x_i) + (\lambda\beta - 1)\sum_{i=1}^{n}\log(1-(1+x_i)^{-\alpha}) + \sum_{i=1}^{n}\log\left(1-(1-(1+x_i)^{-\alpha})^{\lambda\beta}\right) + (\theta-1)\sum_{i=1}^{n}\log\left(1-[1-[1-(1+x_i)^{-\alpha}]^{\beta\lambda}]^2\right) \tag{34}$$

Therefore, The MLE's of parameters $\alpha, \beta, \theta, \lambda$ which maximize the above log-likelihood function must satisfy the normal equations. We take the first derivative of the above log-likelihood equation with respect to each parameter and equate to zero respectively.

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\log(1+x_i) + (\lambda\beta-1)\sum_{i=1}^{n}\left[\frac{(1+x_i)^{-\alpha}\log(1+x_i)}{1-(1+x_i)^{-\alpha}}\right] - \sum_{i=1}^{n}\left[\frac{\lambda\beta(1+x_i)^{-\alpha}(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}\log(1+x_i)}{1-(1-(1+x_i)^{-\alpha})^{\lambda\beta}}\right] + (\theta-1)\sum_{i=1}^{n}\left[\frac{2\lambda\beta(1+x_i)^{-\alpha}(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}(1-(1-(1+x_i)^{-\alpha})^{\lambda\beta})\log(1+x_i)}{1-[1-[1-(1+x_i)^{-\alpha}]^{\beta\lambda}]^2}\right] = 0 \tag{35}$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} + (\lambda-1)\sum_{i=1}^{n}\log(1-(1+x_i)^{-\alpha}) - \sum_{i=1}^{n}\left[\frac{\lambda(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}\log(1-(1+x_i)^{-\alpha})}{1-(1-(1+x_i)^{-\alpha})^{\lambda\beta}}\right] - (\theta-1)\sum_{i=1}^{n}\left[\frac{2\lambda(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}(1-(1-(1+x_i)^{-\alpha})^{\lambda\beta})\log(1-(1+x_i)^{-\alpha})}{1-[1-[1-(1+x_i)^{-\alpha}]^{\beta\lambda}]^2}\right] = 0 \tag{36}$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} + (\beta-1)\sum_{i=1}^{n}\log(1-(1+x_i)^{-\alpha}) - \sum_{i=1}^{n}\left[\frac{\beta(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}\log(1-(1+x_i)^{-\alpha})}{1-(1-(1+x_i)^{-\alpha})^{\lambda\beta}}\right] - (\theta-1)\sum_{i=1}^{n}\left[\frac{2\beta(1-(1+x_i)^{-\alpha})^{\lambda\beta-1}(1-(1-(1+x_i)^{-\alpha})^{\lambda\beta})\log(1-(1+x_i)^{-\alpha})}{1-[1-[1-(1+x_i)^{-\alpha}]^{\beta\lambda}]^2}\right] = 0 \tag{37}$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n}\log\left(1-[1-[1-(1+x_i)^{-\alpha}]^{\beta\lambda}]^2\right) \tag{38}$$

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APPLICATION TO REAL-LIFE DATA SET

In this section, we present some applications of the TLExIK distribution using different data sets to demonstrate the flexibility of the distribution to model these real data sets. The data are fitted to the extended generalized inverse exponential (EGenIEx) by Ibrahim et al., (2020c), Topp Leone Exponential (TLEx) by Al-Shomrani et al., (2016), generalized inverse Kumaraswamy (GIK) by Iqbal et al., (2017) and inverse Kumaraswamy (IK) by Al-Fattah et al., (2017) distributions. The test statistics used in testing the flexibility and fit of the model are: Akaike Information Criteria (AIC), Bayessian Information Criteria (BIC), Cramer-vom mises (W), Anderson Darling (A) and Probability value (PV). The smaller the AIC and BIC values, the better the model.

The first data set represents the number of million revolutions before failure for each of twenty-three (23) deep groove ball bearings in the life tests. The data set was given by Lawless (1982) and it has also been used by Shanker et al., (2015). The observations are:

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

The second data set represents the waiting time (mins) of one hundred (100) bank customers before service is being rendered. It has previously been used by Ghitany et al., (2008). The data sets are:

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11, 11, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2,, 18.4, 18.9, 19, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27, 31.6, 33.1, 38.5

Table 1: Goodness of fit statistics from number of million revolutions before failure data.

Model	Estimates	-LL	AIC	BIC	W	A	KS	PV
TLExIK	$\hat{\alpha} = 0.6398$ $\hat{\beta} = 2.5104$ $\hat{\theta} = 2.3233$ $\hat{\lambda} = 3.3758$	122.746	253.491	258.034	0.036	0.267	0.2969	0.0272
EGenIEx	$\hat{\alpha} = 2.7722$ $\hat{\beta} = 2.6594$ $\hat{\theta} = 1.9019$ $\hat{\lambda} = 0.4958$	127.523	263.046	267.588	0.0357	0.2645	0.3692	0.0026
TLEx	$\hat{\beta} = 0.0286$ $\hat{\theta} = 4.7485$	130.534	265.045	267.316	0.0325	0.1896	0.4995	7.9e-6
GIK	$\hat{\alpha} = 04212$ $\hat{\beta} = 4.8460$ $\hat{\lambda} = 1.4038$	136.221	278.441	281.848	0.0345	0.2537	0.4482	9.8e-5
IK	$\hat{\alpha} = 0.3309$ $\hat{\beta} = 1.9103$	144.027	292.731	295.022	0.0325	0.2343	0.4288	0.0002

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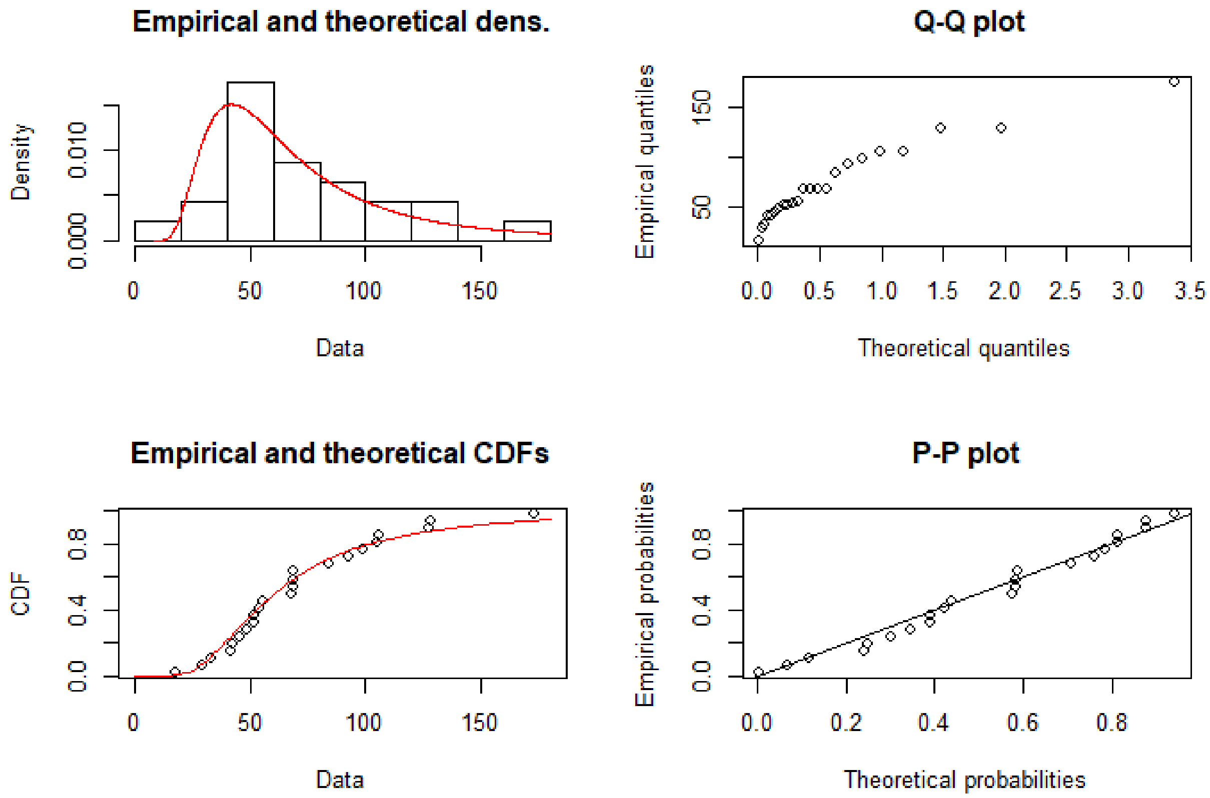

Figur 3: Fitted pdf, cdf,Q-Q plot and P-P plot of the number of million revolutions before failure data.

Table 2: Goodness of fit statistics from waiting time (mins) of one hundred (100) bank customers before service data.

Model	Estimates	-LL	AIC	BIC	W	A	KS	PV
ETLGenExE x	$\hat{\alpha} = 0.7943$ $\hat{\beta} = 1.4618$ $\hat{\theta} = 0.6918$ $\hat{\lambda} = 4.0473$	420.56 9	849.13 8	860.54 6	0.2487	1.6238	0.0993	0.1604
EGenIEx	$\hat{\alpha} = 1.4878$ $\hat{\beta} = 0.7284$ $\hat{\theta} = 1.7990$ $\hat{\lambda} = 0.4484$	450.49 4	908.98 7	920.39 6	0.9821	5.8841	0.1693	0.0013
TLEx	$\hat{\beta} = 0.0570$ $\hat{\theta} = 0.9744$	431.12 5	850.24 9	855.95 4	0.1241	0.7439	0.1048	0.1203
GIK	$\hat{\alpha} = 0.8336$ $\hat{\beta} = 3.3881$ $\hat{\lambda} = 1.1599$	428.11 2	862.22 6	865.51 3	0.4082	2.5879	0.1216	0.0453
IK	$\hat{\hat{\alpha}} = 0.8587$ $\hat{\theta} = 2.8320$	431.69 2	867.38 5	873.08 9	0.3301	2.1201	0.1480	0.0073

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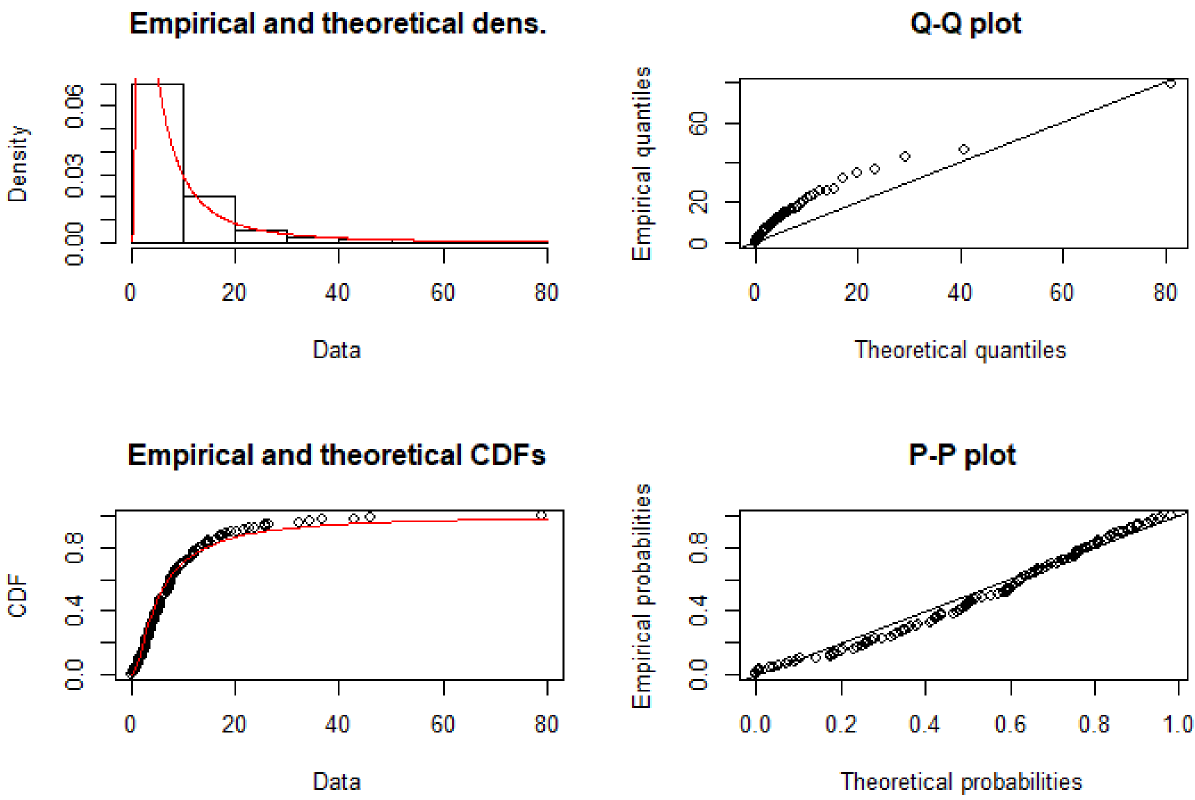

Figure 4: Fitted pdf, cdf,Q-Q plot and P-P plot of the waiting time (mins) of one hundred (100) bank customers before service data.

CONCLUSION

As a result of the study carried out, a new distribution was derived. Some mathematical properties of the new distribution were studied and presented such as moments, moment generating function, probability generating function, quantile function, hazard function, survival function, odd function, cumulative hazard function, reversed hazard rate function and order statistics. Plots of the pdf and hazard rate function of the new model are plotted in figure 1 and figure 2 respectively. The method of maximum likelihood estimate was used to estimate the parameters of the model. The proposed TLExIK distribution was applied to two real life data sets to test the flexibility of the model. The first data set represents the number of million revolutions before failure for each of twenty-three (23) deep groove ball bearings in the life tests and the second data set represents the waiting time (mins) of one hundred (100) bank customers before service is being rendered. From the result shown in table 1 table 2, it can be seen that TLExIK distribution has the lowest AIC and BIC and also has the highest P-Value which makes the proposed model fit better than other competing distributions considered in the study with respect to the data sets used.

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