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MULTIVARIATE VOLATILITY MODELING OF NIGERIAN BANK SHARE PRICES

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ABSTRACT: This study aims at finding the optimal Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) model among Diag-BEKK, Scalar-BEKK and CCC that captures the dynamics of returns in Nigerian bank share prices, using the data of daily share prices of two highly capitalized banks in Nigeria listed on the platform of Nigerian Stock Exchange (NSE) which span from 2nd January, 2009 to 28th December, 2019. Multivariate Normal and Multivariate student-t log-likelihood functions were simplified using the BHHH and Marquardt algorithm and the optimal solution was obtained using the information criteria. The BHHH and Marquardt algorithm was implemented in the GARCH 7 software of Laurent. Finally, the findings of this study showed that the Diag-Bekk (1,1) with Multivariate student-t distribution was the overall best model out of the six model combinations. Multivariate volatility modeling is therefore, recommended for bank share prices in Nigeria, and even in other banks from emerging economies.

KEYWORDS: CCC, BHHH, Scalar-BEKK, Share prices, MGARCH

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INTRODUCTION

A very good application of time series is found in finance, where the behaviour of stocks is investigated and future predictions are made. Modelling volatility in stock prices gives a signal about the intrinsic value of securities, thereby, making it easier for firms to raise funds in the market. Also, detection of stock return volatility trends would provide insight for designing investment strategies and for portfolio management.

The financial model is mostly volatility models (see GARCH model of Bollerslev (1986); Exponential GARCH (EGARCH) model of Nelson (1991); Asymmetric Power ARCH (APARCH) model of Engle and Paton (2001); etc). More complex methods, such as varieties of multivariate generalized autoregressive conditional Heteroskedasticity (MGARCH), have been extensively investigated in the econometric literature and are used by a few sophisticated practitioners.

The MGARCH models are used to study the common events among volatility series. These models will be estimated based on the following GARCH distributions: the Multivariate Gaussian, and the Multivariate Student –t. The assumption of conditional normality can be quite restrictive because the symmetry imposed under normality is difficult to justify, and the tails of conditional distributions often seem father than that of Normal distributions for modelling the tail behaviour of residuals of these volatility models the Multivariate Student –t and the Multivariate Skew student-t distributions are the most commonly used distribution.

The Volatility Model

Modeling volatility plays an important role in controlling and forecasting risks in various financial operations pricing equity, risk management and portfolio management modeling volatility will improve the use of stock prices as a signal about the intrinsic value of securities, thereby, making it easier for firms to raise fund in the market. Also, detection of stock returns volatility-trend would provide insight for designing investment strategies and for portfolio management.

The studies of Mandelbrot (1963) and Black (1976) highlight volatility clustering, leptokurtosis and leverage characteristics of stock returns. Engle (1982) introduced the autoregressive conditional Heteroscedasticity (ARCH) to model volatility by the conditional variance of the disturbance term to the linear combination of the Squared disturbances in the recent past Bollerslev (1986) generalized the ARCH model by modelling the conditional variance to depend on its lagged values as well as Squared lagged values of disturbance since the works of Engle (1982) and Bollerslev (1986)various variants of GARCH Model have been developed to model volatility.

Some of the models include EGARCH originally proposed by Nelson (1991), the GJR-GARCH model introduced by Glosten, Jagannathan and Runkle (1993) and Threshold GARCH (TGARCH) model.

Concerning the effectiveness of the ARCH family models in capturing the volatility of financial times series, Bellerslev (1986) found that GARCH (1,1) model worked well to capture most of the stochastic dependencies in the time series. Therefore, it became the bedrock of the dynamic volatility models, see Engle (2003). The advantage of these models

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was that they were practically easy to estimate in addition they allow us to perform diagnostic tests, see Christie (1982).

However, GARCH (1,1) only captured some of the Skewness and leptokur-tosis (fat tails relative to the normal distribution) in the financial data. Bollerslev (1986), and Nelson (1991) also found that if the observed conditional densities were non-normal, it was higher than what could be forecasted by normal GARCH (1,1). Therefore, more researchers explored alternative distributional functions for the error term in order to supply a better explanation of the data. Consequently, numerous non-normal condition densities had been introduced in the GARCH framework. In particular, Bollerslev (1986) presented the students t GARCH that had also been captured by GARCH models (Chou, 1988). These developments in GARCH models were obviously crucial for modelling volatility variation. If the conditional variance did not follow the normal distribution, the normal GARCH model could not explain the entire leptokurtosis in the sample data and it was better to apply the non-normal distributions, such as student t, normal-lag normal distribution or the exponential GARCH model to capture higher conditional moments, see Alexander and Lazar (2006). Further generalizations have been proposed by many researchers. There is now an alphabet soup of ARCH models that include: AARCH, APARCH, FIGARCH, FIE- GARCH, STARCH, SWARCH, GJR-GARCH, TARCH, MARCH, NARCH, SNPARCH, SPARCH, SQGARCH, CESGARCH, component ARCH, TAYLOR-SCHWERT, GED-ARCH, and many others that we have regrettably overlooked. Many of these models were surveyed in Bollerslev (1986), Engle (2003), and Engle and Paton (2001).

On the other hand, many authors (Christie, 1982 and Nelson, 1991) had pointed out the evidence of asymmetric responses, suggesting the leverage effect and differential financial risk depending on the direction of price change movements. In response to the weakness of symmetric assumption, Nelson (1991) brought exponential GARCH (EGARCH) models with a conditional variance formulation that successfully capture asymmetric response in the conditional variance. EGARCH models had been demonstrated to be superior compared to other competing asymmetric conditional variance in many studies.

Aim and Objectives

This study aims at finding the optimal Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) model among Diag-BEKK, Scalar-BEKK and CCC that captures the dynamics of returns in Nigerian bank share prices, using the data of daily share prices of two highly capitalized banks in Nigeria listed on the platform of Nigerian Stock Exchange (NSE) which span from 2nd January 2009 to 28th December 2019

We achieve our aim in this paper by:

1. Giving time plots of share prices of banks to show the trend and dynamics over time.
2. Estimating the vector GARCH estimates that is the MGARCH model for the series.
3. Finding the optimal MGARCH model between Diag-BEKK, Scalar-BEKK and CCC models that captures the dynamics of returns in Nigerian bank share prices.

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METHODOLOGY

Time plot is needed to study the trend in the share prices and volatility in the log returns. The quasi Maximum Likelihood Estimation (QMLE) approach is employed. The GARCH assumed distributions applied are Multivariate Normal and Multivariate Student-t. The QMLE does not have a close solution, therefore algorithms like BHHH (Berndt, Hall, Hall and Hausman, 1974), Marquardt, or MaxSa algorithm of Goffe, Ferries and Rogers (1994) are used to get the optimal solution to the MGARCH estimates.

Distribution Assumptions

In our empirical analysis, two conditional distributions for the standardized residuals of returns innovations will be considered: Multivariate Gaussian, and Multivariate student's-t.

Parameter vectors $\theta = (\omega, \alpha, \beta, \gamma, \rho, \delta, \phi$ and $\xi)$ are obtained from the maximization of the log-likelihood function:

$$\log L = \sum_{t=1}^{T} l_t = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log \sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\frac{\mu_t^2}{\sigma_t^2} \tag{1}$$

where T is the sample size, and

$$l_t = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma_t^2) - \frac{1}{2}\frac{(y_t - x'_{t-1}\gamma)^2}{\sigma_t^2}$$

For student's t–distribution and log-likelihood distributions are assumed to be of the form:

$$l_t = -\frac{1}{2}\log\left[\frac{\pi(\nu-2)\Gamma\left(\frac{\nu}{2}\right)^2}{\Gamma\left(\frac{(\nu+1)}{2}\right)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \frac{(\nu+1)}{2}\log\left[1 + \frac{(y_t - x'_t\gamma)^2}{\sigma_t^2(\nu-2)}\right] \tag{2}$$

Where σ_t^2 is the variance at time t, and the degree of freedom $\nu > 2$ controls the tail behaviour. The t-distribution approaches the normal as $\nu \to \infty$

The MGARCH model

The general GARCH (q,p) model is,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i \varepsilon_{t-i} + \sum_{j=1}^{p}\beta_j \sigma_{t-j}^2 \text{..........................} 2.1$$

where ω,α and β_j are parameters and y_i is the return series defined as

$$y_t = \varepsilon_t, \varepsilon_t = \sigma_t z_t \text{ and } z_t \approx N(0,1) \text{ or } z_t \approx t \text{ or } z_t \approx GED \text{................} 2.2$$

The Vec-GARCH (p,q),

$$Vech(H_t) = \omega + \sum_{i=1}^{q} A^{(i)} Vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B^{(j)} Vech(H_{t-j})$$

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where $\omega, \mathrm{A}^{(i)}, B^{(j)}$ are parameter vector and

$\varepsilon_t = \mathrm{H}_t^{1/2} n_t$ and H_t is positive definite matrix

ω is a vector of size

$\left[\mathrm{m(m+1)}/2 \right] \times 1$ and $\mathrm{A}^{(i)}$ and $\mathrm{B}^{(j)}$ are matrices of dimension $\mathrm{m(m+1)}/2 \times m(m+1)/2$

RESULTS AND DISCUSSION

The data used in this study are the daily share prices of 2 highly capitalized banks in Nigeria listed on the platform of the Nigerian Stock Exchange (NSE). They are the: First Bank (FBN) and United Bank for Africa (UBA). Different episodes of bank re-capitalization have taken place and some banks were stopped from operating by CBN, while others merged with those with stronger financial backings. The share prices for each bank, therefore, span from 2nd January 2008 to 28th December 2019 (Pre-covid19 era) and no adjustment was made for non-trading days (weekends and holidays).

The price-log returns transformation

The log returns are obtained by transforming the prices, P_t as

$$r_t = \log P_t - \log P_{t-1} = \log\left(\frac{Pt}{Pt-1}\right)$$

Where P_t is the price at the time t and P_{t-1} is the price at time t-1 and r_t is the log returns series at time t. The log-returns r_t are often obtained with a Microsoft Excel spreadsheet.

Time plots of share prices of banks.

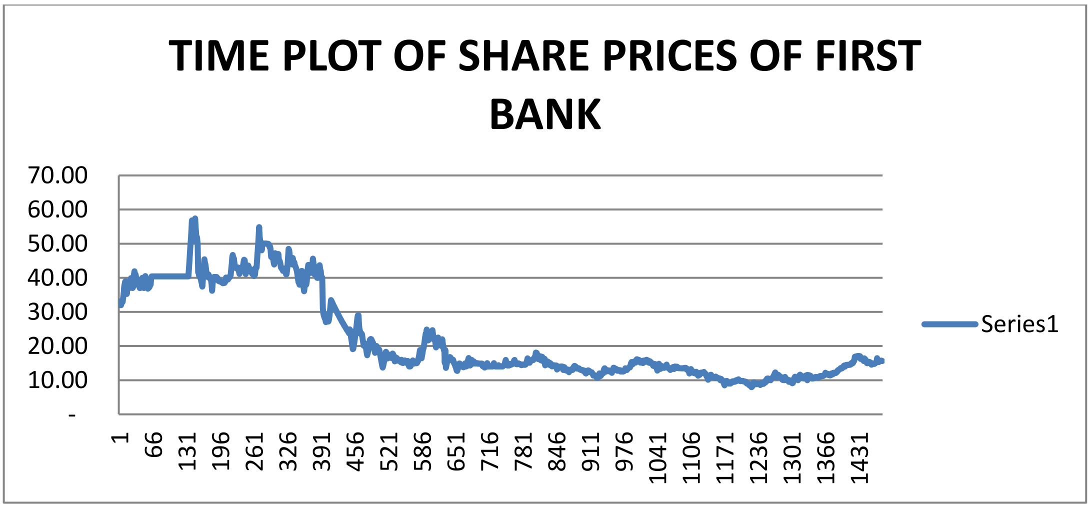

Fig. 1 Time plots of share prices for First Bank; showing variation in prices

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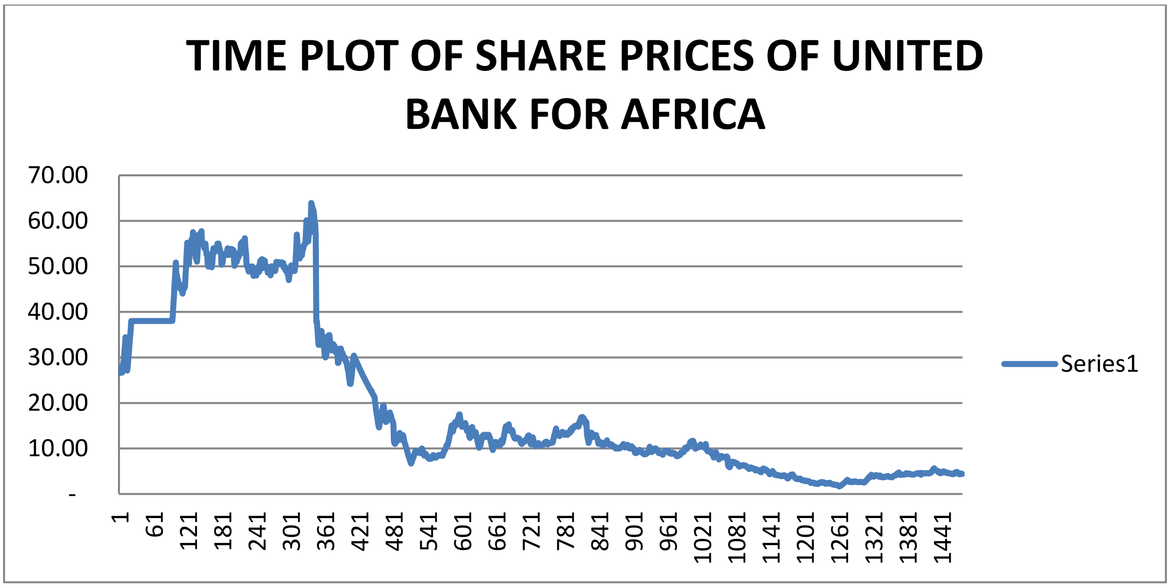

Fig. 2 Time plots of share prices for United Bank for Africa; showing variation in prices

Data sources: CBN (2020) NSE (2020)

Time plots of log-returns of banks

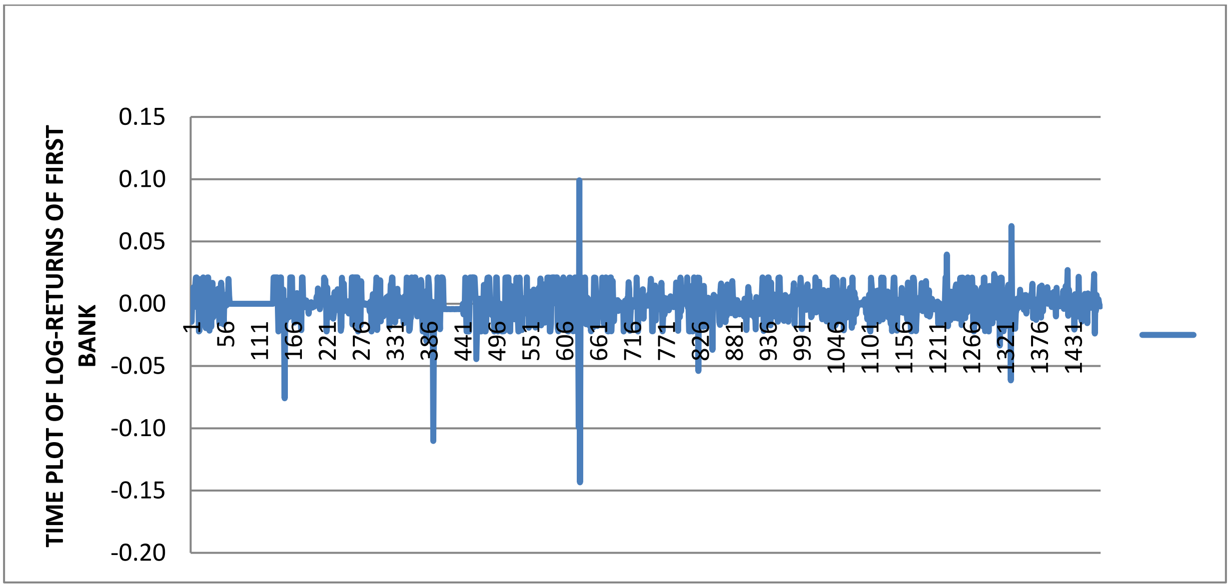

Fig. 3; Time plots of log-returns of First Bank

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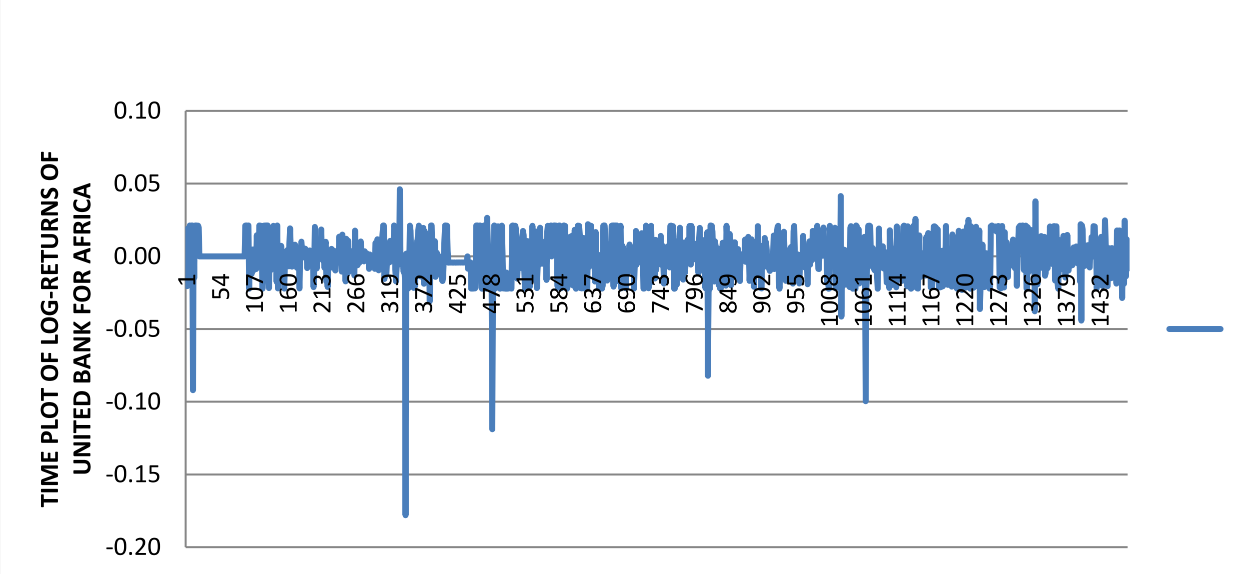

Fig. 4; Time plots of log-returns of United Bank for Africa

Table 1: Diagonal Bekk (1,1) Multivariate Normal Distribution.

Parameter	Coefficient	Std.Error	t-value	t-prob
Ω_{11}	0.007203	0.0062647	1.150	0.2504
Ω_{12}	0.001043	0.0009355	1.115	0.2650
Ω_{21}	0.003313	0.00097628	3.394	0.0007
Ω_{22}	0.000552	0.00053656	1.029	0.3038
$b_{1.11}$	0.758346	0.42551	1.782	0.0749
$b_{1.22}$	0.943473	0.027815	33.92	0.0000
$a_{1.11}$	0.271274	0.099437	2.728	0.0064
$a_{1.22}$	0.192284	0.045350	4.240	0.0000

No observations : 288 No Parameter: 8 No of Series : 2 Log-likelihood : 35410.798

TESTS

Information criteria (t0 be minimized)

Akaike : -47.889970 Shibata: -47.893971

Schwarz : -47.645930 Hannan-Ouinn : -47.798983

Likewise other distributions tests are obtained for Multivariate normal distribution:

The Scalar Bekk (1,1) Multivariate Normal Distribution and Constant Conditional Correlation (CCC) (1,1) Multivariate Normal Distribution.

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Likewise also, for Multivariate Student-T Distribution; The Diagonal Bekk (1,1) Multivariate Student-T Distribution, Scalar Bekk (1,1) Multivariate Student-T Distribution and Constant Conditional Correlation (CCC) (1,1) Multivariate Student-T Distribution.

Results are shown in Table 2

Selection of optimal MGARCH Model using information criteria and Log-likelihood estimators

Table 2. Comparative Analysis

GARCH ASSUMED DISTRIBUTION	**DIAG-BEKK (1,1)**	**SCALAR-BEKK (1,1)**	**CCC (1,1)**
MULTIVARIATE NORMAL DISTRIBUTION	LL= **35410.798** AIC=**-47.889970** SIC=**-47.893971** SHIC=**-47.645930** HOIC=**-47.798983**	LL=**35149.456** AIC=**-47.554818** SIC=**-47.557372** SHIC=**-47.361022** HOIC=**-47.482564**	LL=**34354.915** AIC=**-46.448395** SIC=**-46.453359** SHIC=**-46.175645** HOIC=**-46.346703**
MULTIVARIATE STUDENT-T DISTRIBUTION	LL=**39839.905** AIC=**-53.890116** SIC=**-53.894232** SHIC=**-53.642488** HOIC=**-53.797791**	LL=**38540.469** AIC=**-52.148331** SIC=**-52.150978** SHIC=**-51.950947** HOIC=**-52.074739**	LL=**36299.721** AIC=**-49.071437** SIC=**-49.077601** SHIC=**-48.766388** HOIC=**-48.957703**

Discussion on the Results

i. From table 2 above, with the assumption of Multivariate normal distribution the DIAG-BEKK (1,1) model has the minimum AIC value therefore this model is selected among the three competing models.

ii. Considering the Multivariate Student-T distribution DIAG-BEKK (1,1) model also proved to be the optimal model for predicting the share prices of the 2 banks among the banks in Nigeria.

CONCLUSION AND RECOMMENDATION

Based on the result, Diag-Bekk (1,1) with Multivariate student's-t distribution is the overall best model out of the six combinations.

We, therefore, recommend this model for Multivariate volatility modeling of bank share prices in Nigeria, and even in other banks from emerging economies.

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