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INVESTIGATING THE RATE OF RETURN FROM PORTFOLIO MANAGEMENT STRATEGIES

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ABSTRACT: *This work investigates the rate of return from two portfolio management strategies. We first examine the return from total investment which includes both investment in the risky stock and investment in the risk-free asset. Secondly, we examine the return from investment in the risky stock only. We derive some optimality properties for the two portfolio management strategies. We show that the limiting behaviour of the rate of return on total investment is determined by the limiting behaviour of a related diffusion process.*

KEYWORDS: Rate of return; Portfolio management strategies; Geometric Brownian motion; Ito's formula; Gamma distribution.

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INTRODUCTION

The rate of return on investment is defined here as the net gain divided by the cumulative investment. In other words, it is the ratio of money gained or lost whether realized or unrealized on an investment relative to the amount of money invested. Portfolio strategies on the other hand are investment tools relating to active investment approaches. An investor can select and invest based on the conditions of the market. In finance, constant proportions investment strategies play a major role in the theory of portfolio management. Under these policies, an investor continuously rebalances his portfolio to enable him allocate fixed constant proportions of his wealth across the investment opportunities. The above strategy is quite widely used and is sometimes referred to as constant mix, or continuously rebalanced strategy [1]. It is of interest to know the stochastic behaviour of the rate of return on total investment for such policies given the fundamental nature of such policies in theoretical portfolio practice.

Merton [2] introduced the setting for the continuous time financial model which was used in Black and Scholes [3]. Ethier and Tavare [4] studied the return on investment in a discrete-time gambling model, where the return on the individual gambles was assumed to follow a random walk. They showed that the asymptotic distribution of the return is a gamma distribution as the mean increment in the random walk goes to zero. Kelly [5] studied the relationship between the logarithm of wealth and expected asymptotic rate at which wealth compounds. Breiman [6] established that the policy that maximizes the logarithm of wealth is asymptotically optimal for the objective of minimizing the expected time. The relationship is however exact only in continuous-time. Thorp [7], Hakansson [8], Finkelstein and Whitely [9] contain deep analysis of optimality properties in discrete-time of constant proportional investment policies. Browne and Whitt [10] studied the optimal growth policy in both discrete and continuous-time of a Bayesian version. Ling et al. [11] examined buy-and-hold investment strategy in four Asian markets. They described it as passive conservative investment strategy in which an investor keeps the stocks for a longer period of time. Mattei and Mattei [12] defined strategies of asset allocation as a device that helps an investor in diversifying his portfolio and reducing the associated risk. They made an analysis on rebalancing strategies. Zunera and Ahmad [13] examined four strategies namely buy and hold strategy, dynamic asset allocation, strategic asset allocation and tactical asset allocation, along with their dimensions.

This work investigates the rate of return from two portfolio management strategies. We first examine the return from total investment which includes both investment in the risky stock and investment in the risk-free asset. Secondly, we examine the return from investment in the risky stock alone, which is the excess return above the risk-free rate.

Basic Tools and Preliminaries

Consider a complete market with constant coefficients having n risky stocks generated by n independent Brownian motions. Let the stocks be generated by the process, $S_t^i : i = 1, \cdots, n$. It is assumed that the price evolves according to

$$dS_t^{(i)} = \mu_i S_t^{(i)} dt + \sum_{j=1}^{n} \sigma_{ij} S_t^{(i)} dW_t^{(j)}, \tag{1}$$

for $i = 1, \cdots, n$. μ_i and σ_{ij} are constants and $W_t^{(j)}$ denotes a standard independent Brownian motion. Let the bond $\{B_t, t \geq 0\}$ be an available risk-free security for investment. The price of the bond evolves according to

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$$dB_t = rB_t dt, \tag{2}$$

where r is a positive constant. The investor is allowed to invest its surplus in the risky stock. Hence, we denote the total amount of money invested in the risky stock at time t under an investment policy π as $\pi_t: t \geq 0$, where π_t is a suitable admissible adapted control process, that is, π_t is a non anticipative function and satisfies for any T,

$$\int_0^T \pi_t^2 dt < \infty. \tag{3}$$

Under the policy π, the investor's wealth process, Y_t^π evolves according to the stochastic differential equation

$$dY_t^\pi = Y_t^\pi \left(\sum_{i=1}^n \pi_t \frac{dS_t^{(i)}}{S_t^{(i)}}\right) + Y_t^\pi \sum_{i=1}^n (1-\pi_t)\frac{dB_t}{B_t}. \tag{4}$$

Substituting equations (1) and (2), the stochastic differential equation for the wealth process of the investor reduces to

$$dY_t^\pi = Y_t^\pi\big(r + \pi_t(\mu - r)\big)dt + Y_t^\pi \sum_{i=1}^n \sum_{j=1}^n \pi_t \sigma_{ij} dW_t^{(j)}. \tag{5}$$

We note that π_t may be negative, in which case the investor is short selling a stock. Let g be a fixed constant vector, where g_i denotes the percentage of the investor's wealth invested in risky security i for $i = 1, \cdots, n$. Under this policy, the investor's wealth is denoted as Y_t^g and it evolves as

$$dY_t^g = Y_t^g \left(r + g'\tilde{\mu} - \frac{1}{2} g' \Sigma \quad g\right) t + \sum_{i=1}^n \sum_{j=1}^n g_i \sigma_{ij} W_t^{(j)}. \tag{6}$$

Hence, Y_t^g is the geometric Brownian motion.

$$Y_t^g = Y_0 \, exp\, exp \left\{\left(r + g'\tilde{\mu} - \frac{1}{2} g' \Sigma \quad g\right) t + \sum_{i=1}^n \sum_{j=1}^n g_i \sigma_{ij} W_t^{(j)}\right\}, \tag{7}$$

where $\tilde{\mu} = \mu - r$ denotes the excess return of the risky stock over the return from the risk free bond. We allow $g < 0$ as well as $g > 1$. In the first instance, the investor is selling the stock short while in the second instance, the investor borrows money to invest long in the stock.

THE MODEL

Rate of return from investment in risky stock and risk-free asset

Our interest here is to investigate the rate of return from total investment comprising investment in risky stock and risk-free bond. We define the rate of return from total investment $\{\rho_g(t): t \geq 0\}$ as the ratio of the net gain to the cumulative investment. Therefore,

$$\rho_g(t) = \frac{Y_t^g - Y_0}{\int_0^t Y_s^g ds}, t \geq 0 \tag{8}$$

where $\rho_g(t)$ is a measure of the wealth it takes to finance a gain. If $\rho_g(t)$ is large, the investor is accumulating gains at a faster rate than if it is small. If we divide the numerator and the

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denominator by t in equation (8), we also interpret $\rho_g(t)$ as the average net gain over the average wealth level. But

$$Y_t^g = Y_0\, exp\, exp\left\{(r + g\tilde{\mu})dt + g\sigma Y_t^g dW_t\right\}, \tag{9}$$

and

$$\int_0^t Y_s^g ds = Y_0 \int_0^t exp\, exp\left\{(r + g\tilde{\mu})dt + g\sigma Y_s^g dW_s\right\}ds\,. \tag{10}$$

From geometric Brownian motion,

$$Y_t^g = Y_0\, exp\, exp\left\{\left(r + g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)t + g\sigma W_t\right\}, \tag{11}$$

and

$$\int_0^t Y_s^g ds = Y_0 \int_0^t exp\, exp\left\{\left(r + g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)s + g\sigma W_s\right\}\, ds. \tag{12}$$

Substituting equations (11) and (12) in equation (8) gives

$$\rho_g(t) = \frac{Y_0 expexp\left\{\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right\} - Y_0}{Y_0 \int_0^t expexp\left\{\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)s+g\sigma W_s\right\}\, ds}. \tag{13}$$

That is,

$$\rho_g(t) = \frac{Y_0\left\{expexp\left[\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right]-1\right\}}{Y_0 \int_0^t expexp\left\{\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)s+g\sigma W_s\right\}\, ds}. \tag{14}$$

Simplifying equation (14) gives

$$\rho_g(t) = \frac{expexp\left[\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right]-1}{\int_0^t expexp\left\{\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)s+g\sigma W_s\right\}\, ds}. \tag{15}$$

Let $g = 0$, equation (15) becomes

$$\rho_0(t) = \frac{e^{rt}-1}{\int_0^t e^{rs}ds}. \tag{16}$$

But

$$\int_0^t e^{rs}ds = \frac{1}{r}(e^{rt} - 1). \tag{17}$$

Substituting equation (17) in equation (16) gives

$$\rho_0(t) = \frac{e^{rt}-1}{\frac{1}{r}(e^{rt}-1)} = r. \tag{18}$$

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Considering the case when $g \neq 0$ in equation (15):

$$\int_0^t \quad exp\ exp\ \left\{\left(r + g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)s + g\sigma W_s\right\}\ ds = \frac{expexp\left[\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right]}{r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2+g\sigma dW_t} - 1. \qquad (19)$$

Substituting equation (19) in equation (15) gives

$$\rho_g(t) = \frac{\left\{expexp\left[\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right]-1\right\}\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2+g\sigma dW_t\right)}{expexp\left[\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right]-\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2+g\sigma dW_t\right)}. \qquad (20)$$

For $g \neq 0$, $\{\rho_g(t), t > 0\}$ does not yield to a simple direct analysis. Since

$$r + g\tilde{\mu} - \frac{1}{2}g^2\sigma^2 > 0, \qquad (21)$$

the process $\{\rho_g(t), t > 0\}$ admits a unique limiting distribution, which is a gamma distribution. That is,

$$\rho_g(t)\ d \to\ \rho_g \sim gamma\left(\frac{2(r+g\tilde{\mu})}{\sigma^2 g^2} - 1, \frac{2}{\sigma^2 g^2}\right), \qquad (22)$$

where $d \to$ stands for convergence in distribution. Hence,

$$\begin{aligned} E(\rho_g) &= \left(\frac{2(r+g\tilde{\mu})}{\sigma^2 g^2} - 1\right)\left(\frac{\sigma^2 g^2}{2}\right) \\ &= \frac{2(r+g\tilde{\mu})-\sigma^2 g^2}{2} \qquad (23) \\ &= r + g\tilde{\mu} - \frac{1}{2}\sigma^2 g^2 > 0. \end{aligned}$$

To obtain the ratio of the expected gain to the expected total investment for $t > 0$, we have

$$\frac{E(total\ gain)}{E(total\ investment)} = \frac{E\left(Y_t^g - Y_0\right)}{E\left(\int_0^t \ Y_s^g ds\right)} \qquad (24)$$

$$\frac{E(total\ gain)}{E(total\ investment)} = \frac{Y_0 e^{(r+g\tilde{\mu})t} - Y_0}{E\left(\int_0^t \ Y_s^g ds\right)}. \qquad (25)$$

But

$$\begin{aligned} \int_0^t \ Y_s^g ds &= Y_0 \int_0^t \quad exp\ exp\ \left\{\left(r + g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)s + g\sigma W_s\right\}\ ds \\ &= Y_0\left\{\frac{expexp\left\{\left(r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2\right)t+g\sigma W_t\right\}}{r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2+g\sigma dW_t} - \frac{1}{r+g\tilde{\mu}-\frac{1}{2}g^2\sigma^2}\right\} \qquad (26) \\ &= Y_0\left\{\frac{1}{r+g\tilde{\mu}}\left(e^{(r+g\tilde{\mu})t} - 1\right)\right\}, for\ g \neq 0. \end{aligned}$$

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Substituting equation (26) in equation (25) gives

$$\frac{E(total\ gain)}{E(total\ investment)} = \frac{Y_0\left(e^{(r+g\tilde{\mu})t}-1\right)}{Y_0\frac{1}{r+g\tilde{\mu}}\left(e^{(r+g\tilde{\mu})t}-1\right)} = r + g\tilde{\mu}. \tag{27}$$

To show that equation (15) converges in distribution to gamma distribution for $g \neq 0$, we first show that $\rho_g(t) \to Q_g$ where

$$Q_g(t) = \frac{Y_t^g}{Y_0 + \int_0^t Y_s^g ds}, \tag{28}$$

with $Q_g(0) = 1$.

$$\begin{aligned} \rho_g(t) &= Q_g(t)\left(\frac{Y_t^g - Y_0}{Y_t^g}\right)\left(\frac{Y_0 + \int_0^t Y_s^g ds}{\int_0^t Y_s^g ds}\right) \\ &= Q_g(t)\left(1 - \frac{Y_0}{Y_t^g}\right)\left(\frac{Y_0}{\int_0^t Y_s^g ds} + 1\right) \\ &= Q_g(t)(1 - e^{-B_t})\left(1 + \frac{1}{\int_0^t e^{B_s} ds}\right), \end{aligned} \tag{29}$$

where $\{B_s, s \geq 0\}$ is the linear Brownian motion defined thus: $B_s = \left(r + g\tilde{\mu} - \frac{g^2\sigma^2}{2}\right)s + g\sigma W_s$, with $B_0 = 0$.

If equation (21) holds, then $e^{-B_t} = 0$ and $\left(\int_0^t e^{B_s} ds\right)^{-1} = 0$. Hence, from equation (29), $Q_g = \rho_g$. The process $Q_g(t)$ follows the stochastic differential equation

$$dQ_g(t) = \{(r + g\tilde{\mu})Q_g(t) - Q_g^2(t)\}dt + g\sigma Q_g(t)dW_t. \tag{30}$$

Let $\alpha_t = \int_0^t Y_s^g ds$. Hence, equation (28) becomes

$$Q_g(t) = \frac{Y_t^g}{Y_0 + \alpha_t}. \tag{31}$$

Applying Ito's rule on equation (31) gives

$$\begin{aligned} d\left(\frac{Y_t^g}{Y_0+\alpha_t}\right) &= \frac{(Y_0+\alpha_t)dY_t^g - d(Y_0+\alpha_t)Y_t^g}{(Y_0+\alpha_t)^2} \\ &= \frac{dY_t^g}{Y_0+\alpha_t} - \frac{Y_t^g}{(Y_0+\alpha_t)^2}d\alpha_t. \end{aligned} \tag{32}$$

But $dY_t^g = Y_t^g(r + g\tilde{\mu})dt + Y_t^g g\sigma dW_t$ and $d\alpha_t = Y_t^g dt$. Equation (32) becomes

$$\begin{aligned} d\left(\frac{Y_t^g}{Y_0+\alpha_t}\right) &= \frac{Y_t^g(r+g\tilde{\mu})dt + Y_t^g g\sigma dW_t}{Y_0+\alpha_t} - \frac{Y_t^g}{(Y_0+\alpha_t)^2}d\alpha_t \\ &= \frac{Y_t^g}{Y_0+\alpha_t}(r + g\tilde{\mu})dt + \frac{Y_t^g}{Y_0+\alpha_t}g\sigma dW_t - \left(\frac{Y_t^g}{Y_0+\alpha_t}\right)^2 dt, \end{aligned} \tag{33}$$

which is equivalent to equation (30).

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Rate of return from investment in risky stock

Here, we focus on the excess return above the risk-free rate, that is, the gain and return from the investment in the risky stock alone. Hence, the excess gain in wealth above what could have been obtained by investing in the risk-free asset is the discounted or present value of the gain $e^{-rt}Y_t^g - Y_0$. If $g = 0$, it means that all the wealth is always invested in the risk-free asset, then the quantity becomes zero. Since g is the proportion of wealth invested in the risky stock, the total amount invested at time t is gY_t^g. Hence, the cumulative amount of money invested in the risky stock until t is $g\int_0^t \quad Y_s^g ds$. Similarly, $g\int_0^t \quad e^{-rs}Y_s^g ds$ becomes the discounted or present value of the cumulative amount of money invested in the risky stock until time t.

Let $\tilde{\rho}_g(t)$ denote the return on the risky investment which can be defined as the discounted gain divided by the discounted cumulative investment in the risky stock. Therefore,

$$\tilde{\rho}_g(t) = \frac{e^{-rt}Y_t^g - Y_0}{g\int_0^t \quad e^{-rs}Y_s^g ds}. \tag{34}$$

We then show that the limiting distribution of equation (34) is gamma distribution. For any constant g such that

$$0 < g < \frac{2\tilde{\mu}}{\sigma^2}, \tag{35}$$

equation (34) converges as $t \to \infty$ to a gamma distribution, that is,

$$\tilde{\rho}_g(t)\ d \to\ \tilde{\rho}_g \sim gamma\left(\frac{2\tilde{\mu}}{g\sigma^2} - 1, \frac{2}{g\sigma^2}\right) \quad , \tag{36}$$

where $d \to$ stands for convergence in distribution. Hence,

$$\begin{aligned} E(\tilde{\rho}_g) &= \left(\frac{2\tilde{\mu}}{g\sigma^2} - 1\right)\left(\frac{g\sigma^2}{2}\right) \\ &= \frac{2\tilde{\mu} - g\sigma^2}{2} \\ &= \tilde{\mu} - \frac{1}{2}g\sigma^2. \end{aligned} \tag{37}$$

To obtain the ratio of the expected values of the discounted gain to the discounted cumulative investment, for $t > 0$, we have

$$\begin{aligned} \frac{E(discounted\ gain)}{E(discounted\ cumulative\ risky\ investment)} &\equiv \frac{E\left(e^{-rt}Y_t^g - Y_0\right)}{gE\int_0^t \quad e^{-rs}Y_s^g ds} \\ &= \frac{E\left(e^{-rt}Y_t^g\right) - E(Y_0)}{gE\int_0^t \quad e^{-rs}Y_s^g ds}. \end{aligned} \tag{38}$$

But $E\left(e^{-rt}Y_t^g\right) = Y_0 e^{g\tilde{\mu}t}$ and

$$E\int_0^t \quad e^{-rs}Y_s^g ds = Y_0\int_0^t \quad exp\ exp\ \left(\left(g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)s + g\sigma W_s\right) ds$$

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$$= Y_0 \left\{ \frac{expexp\left[\left(g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)t + g\sigma W_t\right]}{g\tilde{\mu} - \frac{1}{2}g^2\sigma^2 + g\sigma dW_t} - \frac{1}{g\tilde{\mu} - \frac{1}{2}g^2\sigma^2} \right\} \tag{39}$$

$$= Y_0 \frac{1}{g\tilde{\mu}}\left(e^{(g\tilde{\mu}t)} - 1\right).$$

Substituting equation (39) in equation (38) gives

$$\frac{E(discounted\ gain)}{E(discounted\ cumulative\ risky\ investment)} \equiv \frac{Y_0 e^{g\tilde{\mu}t} - Y_0}{gY_0 \frac{1}{g\tilde{\mu}}\left(e^{g\tilde{\mu}t} - 1\right)}$$

$$= \frac{Y_0\left(e^{g\tilde{\mu}t} - 1\right)}{gY_0 \frac{1}{g\tilde{\mu}}\left(e^{g\tilde{\mu}t} - 1\right)}$$

$$= \left(\frac{g}{g\tilde{\mu}}\right)^{-1} = \tilde{\mu}, \tag{40}$$

for any value of $g > 0$ and $t > 0$.

To study the limiting distribution of $\tilde{\rho}_g(t)$, we first define the process

$$\tilde{Q}_g(t) = \frac{e^{-rt}Y_t^g}{g\left(Y_0 + \int_0^t \ e^{-rs}Y_s^g ds\right)}. \tag{41}$$

For any constant g satisfying equation (35) as

$$t \to \infty, \tilde{\rho}_g(t)\ d \to \ \tilde{Q}_g, \tag{42}$$

we first define the discounted wealth process $X_t^g = e^{-rt}Y_t^g$.

Applying Ito's formula gives

$$dX_t^g = g\tilde{\mu}X_t^g dt + \sigma g X_t^g dW_t. \tag{43}$$

Therefore,

$$X_t^g = Y_0\ exp\ exp\ \left\{\left(g\tilde{\mu} - \frac{1}{2}g^2\sigma^2\right)t + g\sigma W_t\right\}, \tag{44}$$

and from equation (34),

$$\tilde{\rho}_g(t) = \frac{X_t^g - X_0}{g\int_0^t \ X_s^g ds}. \tag{45}$$

We can then write

$$\tilde{\rho}_g(t) = \tilde{Q}_g(t)\left(\frac{X_t^g - X_0}{X_t^g}\right)\left(\frac{X_0 + \int_0^t \ X_s^g ds}{\int_0^t \ X_s^g ds}\right)$$

$$= \tilde{Q}_g(t)\left(1 - \frac{X_0}{X_t^g}\right)\left(1 + \frac{X_0}{\int_0^t \ X_s^g ds}\right). \tag{46}$$

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Since equation (35) holds, from classical results on geometric Brownian motion $inf_{t\geq 0} X_t^g > 0$ and $X_t^g\ a.s. \rightarrow\ \infty$, which implies that $\int_0^t\ X_s^g ds\ a.s. \rightarrow\ \infty$. It therefore follows from equation (46) that if equation (35) holds, we have

$$\left(\frac{X_t^g - X_0}{X_t^g}\right) = 1\ a.s.\ and\ \left(\frac{X_0 + \int_0^t\ X_s^g ds}{\int_0^t\ X_s^g ds}\right) = 1\ a.s., \tag{47}$$

which then implies equation (42). The process $\tilde{Q}_g(t)$ satisfies the stochastic differential equation

$$d\tilde{Q}_g(t) = \left(g\tilde{\mu}\tilde{Q}_g(t) - g\tilde{Q}_g(t)^2\right)dt + g\sigma\tilde{Q}_g(t)dW_t. \tag{48}$$

Let $\tilde{\alpha}_t = g\left(X_0 + \int_0^t\ X_s^g ds\right)$. Hence, equation (41) becomes

$$\tilde{Q}_g(t) = \frac{X_t^g}{\tilde{\alpha}_t}, \tag{49}$$

with $X_t^g = e^{-rt}Y_t^g$. Since $d\tilde{\alpha}_t = gX_t^g dt$, applying Ito's formula on equation (49) gives

$$\begin{aligned} d\left(\frac{X_t^g}{\tilde{\alpha}_t}\right) &= \frac{\tilde{\alpha}_t dX_t^g - X_t^g d\tilde{\alpha}_t}{\tilde{\alpha}_t{}^2} \\ &= \frac{dX_t^g}{\tilde{\alpha}_t} - \frac{X_t^g d\tilde{\alpha}_t}{\tilde{\alpha}_t{}^2} \\ &= \tilde{\alpha}_t{}^{-1} dX_t^g - \left(\frac{X_t^g}{\tilde{\alpha}_t{}^2}\right) d\tilde{\alpha}_t. \end{aligned} \tag{50}$$

But $dX_t^g = g\tilde{\mu}X_t^g dt + \sigma g X_t^g dW_t$ and $d\tilde{\alpha}_t = gX_t^g dt$. Equation (50) becomes

$$\begin{aligned} d\left(\frac{X_t^g}{\tilde{\alpha}_t}\right) &= \frac{g\tilde{\mu}X_t^g dt + \sigma g X_t^g dW_t}{\tilde{\alpha}_t} - \left(\frac{X_t^g}{\tilde{\alpha}_t{}^2}\right)\left(gX_t^g dt\right) \\ &= \left(\frac{X_t^g}{\tilde{\alpha}_t}\right)(g\tilde{\mu}dt + g\sigma dW_t) - g\left(\frac{X_t^g}{\tilde{\alpha}_t}\right)^2 dt, \end{aligned}$$

which is equivalent to equation (48).

CONCLUSION

The quantity in equation (27) is maximized by a strategy that invests as much as possible in the risky asset while the mean of equation (23) is maximized at a finite value. Since $\rho_g(t)$ is a measure of the level of wealth needed to finance a gain, the investor is accumulating gains at a higher rate when $\rho_g(t)$ is large than when it is small. Equation (34) is the present value of the gain from risky investment divided by the present value of the total amount of wealth invested in the risky stock that is needed to obtain the gain. It is therefore a measure of the effectiveness of an investment strategy which signifies a better strategy since it has larger values. The mean in equation (37) is a strictly decreasing function of g which is the proportion invested in the

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risky stock for $g > 0$. Hence, looking at the ratio of the expected values of the discounted gain to the discounted cumulative investment, we have a value that is independent of the proportion invested, for any $t > 0$. If the returns of contingent claim follow geometric Brownian motion, then the resulting distribution is gamma distribution. The mean return on investment is maximized by the strategy that maximizes logarithm utility. It also maximizes the exponential rate at which wealth grows. The return from this policy turns out to have stochastic dominance properties.

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