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IS SINGLE FORECAST METHOD BETTER THAN COMBINED FORECAST METHOD?

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ABSTRACT: *Many studies have been done to prove that combining forecast methods gives a better predictive performance relative to individual forecasts. This paper compared the single forecast method and the combined methods in predicting time series data. The study used annual oil revenue for the period 1981–2019 from the Central Bank of Nigeria (CBN), which were divided into two sets: the Training Set (TS) which covered the period 1981–2010 and the Test Set (VS) which covered 2011–2019. The study adopted autoregressive integrated moving average (ARIMA), simple exponential smoothing (SES), and Holt's linear trend (Holt) as the individual forecast methods; it also adopted outperformance of forecasts (OPF) and weighted mean (WM) as weight selection methods. The forecast methods were applied to the Training Set after which they were combined. Two combined methods CM1 (ARIMA + SES) and CM2 (ARIMA + SES + Holt) were obtained. The result of this study showed that simple exponential smoothing (SES) as an individual forecast method is better and less risky than the combined methods for forecasting time series.*

KEYWORDS: Outperformance method; weighted mean method; ARIMA; Holt linear trend; simple exponential smoothing; combining forecasts.

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INTRODUCTION

When some events involve certainty or uncertainty and better decisions are to be made by organisations, investors, governments, entrepreneurs or individuals, a better way to obtain this is by employing a probabilistic or deterministic method. But when we are faced with some events that are indexed with time, and a better decision on the future event is required, the best approach is to investigate some forecast methods. However, sometimes, combining some of these individual forecast methods gives a better predictive performance relative to the individual forecasts [1-5], but not necessarily all the time. Moreover, improving the forecast accuracies has been the major concern of researchers all over the world [6].

There are many combination strategies that have been developed for linear combination of forecast methods. The most popular of these strategies is the weighted linear combinations, that involve assigning weights to the individual forecast methods. The weights are selected and assigned through some selection criteria such as simple average, outperformance, median error-based, and variance-based pooling method [1,7,8]. One issue with the linear combination technique is that, in the combination process, the relationship between the individual forecast methods involved are often ignored, and this however poses a negative effect on the forecast performance of the combination.

This paper compares the individual forecast method and the combined methods in order to ascertain which of the methods predicts time series better and with less risk. The specific objectives in this study are, first, to fit the individual forecast models to the time series data; second, to combine the forecast methods using (a) outperformance method and (b) weighted mean method; and third, to compare the individual forecast methods and the combined forecast methods using three measures of forecast performance.

MATERIALS AND METHODS

Data Analysis

The data used in this study are annual oil revenue for the period 1981–2019 collected from the Central Bank of Nigeria (CBN) Statistical Bulletin. Three forecast methods—autoregressive integrated moving average (ARIMA), simple exponential smoothing (SES), and Holt's linear trend (Holt) methods—are applied to the Training Set (TS) of the annual oil revenue series, y_t. Weights are assigned to the individual forecast methods when combining them.

Linear Combination of Forecast Methods

Let the forecast methods applied to the time series y_t be $f_{1,t+k}(t), \cdots, f_{p,t+k}(t)$ for the future period $t+k$ at the period t. And let the weights assigned to the forecast methods be $w_1, \cdots, w_p$ respectively, and the combined forecast method be $F_{t+k}^{(c)}$. The forecast methods are independent of each other. The combined forecast method is written as:

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$$F_{t+k}^{(c)}(t) = w_1\left(f_{1,t+k}(t)\right) + w_2\left(f_{2,t+k}(t)\right) + \cdots + +w_p\left(f_{2,t+k}(t)\right) \quad (1)$$

Weight Selection Method

This study adopts two selection methods: weighted mean (WM) and outperformance of forecast methods.

Weighted Mean (WM) Method: In this method, the mean absolute error (MAE), mean absolute scaled error (MASE), and root mean square error (RMSE) metrics on the test set are used to obtain the weights of each forecast method [9]. Let MASE or MAE or RMSE be the error value (e_j) of a particular forecast method on a test set and its weight w_j. The weights can be obtained using equation (2):

$$w_j = \frac{Adjusted\ e_j}{\sum_{j=1}^{p} \ Adjusted\ e_j} = \frac{\left(1 - average\ e_j\right)}{\sum_{j=1}^{p} \ \left(1 - average\ e_j\right)}\ average\ e_j = \frac{e_j}{\sum_{j=1}^{p} \ e_j} \ \} \quad (2)$$

Outperformance (OP) Method: In this method, the weights are assigned to the forecast methods based on their variances, where the forecast method with the least variance is assigned w_1 [1, 10-11]. If the two forecasts are unbiased, then the sum of the two weights will be equal to 1, such that the resultant combined forecast is also unbiased. This is written as:

$$w_1 + w_2 = 1, \ \ w_2 = 1 - w_1 \quad (3)$$

The weight w_1 can be obtained using equation (4)

$$w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (4)$$

where σ_1^2 and σ_2^2 are the variances of the two forecast methods, and ρ is the correlation coefficient between the forecast errors of the two forecast methods, while σ_1 and σ_2 are the standard deviations of the forecast errors of the two forecast methods.

Decision Criteria

Decision for the Best Forecasts under Outperformance Method

if the $Min\ var\left(e_{t+k}^{(c)}(t)\right) \leq Min\ (\sigma_1^2, \sigma_2^2)$, then we can conclude that the combined forecasts performs better. The $Min\ var\left(e_{t+k}^{(c)}(t)\right)$ can be obtained by obtaining the variance of the forecast errors first, and then minimizing it. It is obtained as follows:

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$$\left.\begin{aligned} var\left(e_{t+k}^{(c)}(t)\right) &= var\left(y_{t+k} - F_{t+k}^{(c)}(t)\right) \\ &= var\left(w_1 e_{1,t+k}(t) + (1-w_1)e_{2,t+k}(t)\right) \\ &= w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho\sigma_1\sigma_2 \end{aligned}\right\} \quad (5)$$

where $e_{1,t+k}(t)$ and $e_{2\,,t+k}(t)$ are the forecast errors from the two forecast methods in period $t+k$, and $e_{t+k}^{(c)}(t)$ is the forecast errors from the combined forecast method.

The minimum variance of the combined forecast errors $Min\ var\left(e_{t+k}^{(c)}(t)\right)$ is obtained using equation (6) and it is written as:

$$Min\ var\left(e_{t+k}^{(c)}(t)\right) = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (6)$$

Decision for the Best Forecasts under Weighted Mean (WM) Method

If the combined forecast method has the lowest mean absolute error (MAE), mean absolute scaled error (MASE), and/or root mean square error (RMSE) compared to the individual forecast method, then we can conclude that the combined forecast performs better. MAE, MASE, and RMSE are obtained as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |y_t - f_t|\ MASE = \frac{1}{n}\sum_{t=1}^{n} \frac{|y_t - f_t|}{\frac{1}{n-1}\sum_{t=2}^{n} |y_t - y_{t-1}|}\ RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (y_t - f_t)^2}\ \} \quad (7)$$

where n is the total time series observations, f_t is the predicted values, y_t are time series observations.

Individual Forecast Methods

Simple Exponential Smoothing (SES): This is a linear forecast method that is applied to time series data when there is no evidence of trend or seasonal pattern in the time series. Forecasts are computed using the weighted averages and the weights decrease exponentially. The SES is defined as:

$$f_{1,t+k}(t) = \alpha y_t + \alpha(1-\alpha)y_{t-1} \quad (8)$$

where the weight is given by the smoothing parameter, α which lies between zero and one, i.e., $0 < \alpha < 1$.

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Autoregressive Integrated Moving Average (ARIMA): This is a linear forecast method that is applied to time series data involving both past and current values. It is the most used forecast method and is very simple and appropriate for forecasting. The major aim of ARIMA is to describe the autocorrelations in the data. In fitting ARIMA model to time series data, we use the method of Box-Jenkins, which involves four steps: identifying the model, estimating the parameters, model diagnostic, and forecasting. If the time series data are non-stationary, then we have to transform or difference the data to attain stationarity. An $ARIMA(p, d, q)$ method is given as:

$$f_{2,t+k}(t) = \delta + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \cdots - \theta_q \varepsilon_{t-q} \qquad (9)$$

where $\phi_1, \cdots, \phi_p$ are parameters of the autoregressive process, $\theta_1, \cdots \theta_q$ are parameters of the moving average process, $y_{t-1}, \cdots, y_{t-p}$ are the observed value at period $t-1, \cdots, t-p$, and $\varepsilon_{t-1}, \cdots, \varepsilon_{t-q}$ are the errors at period $t-1, \cdots, t-p$, and δ is the drift.

Holt's Linear Trend Method (Holt): This method is the extension of the SES. It is used in forecasting when there is a trend in the data and there is no seasonal pattern. The BLT method is defined as:

$$f_t^{(j)} = l_t + T_t \qquad (10)$$

where

$$\left.\begin{aligned} l_t &= \alpha y_t + (1-\alpha)(l_{t-1} - T_{t-1}) = l_{t-1} + T_{t-1} + \alpha e_t; \; and \; e_t \\ &= y_t - l_{t-1} - T_{t-1} \; T_t = \alpha(l_t - l_{t-1}) + (1-\beta)\, T_{t-1} \\ &= T_{t-1} + \alpha\beta e_t \end{aligned}\right\} \qquad (11)$$

where e_t is the error correction, and the trend is given as:

where α is the smoothing factor of data series $(0 < \alpha < 1)$ and β is the trend smoothing factor $(0 < \beta < 1)$, l_t is the estimate of the level of the series at time t, T_t is the estimate of the trend of the series at time t.

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RESULTS/FINDINGS

Figure 1 shows the time plot for the annual oil revenue in Nigeria, for the period 1980—2020. An outlier is observed in 2009 causing the time series data to be non-stationary.

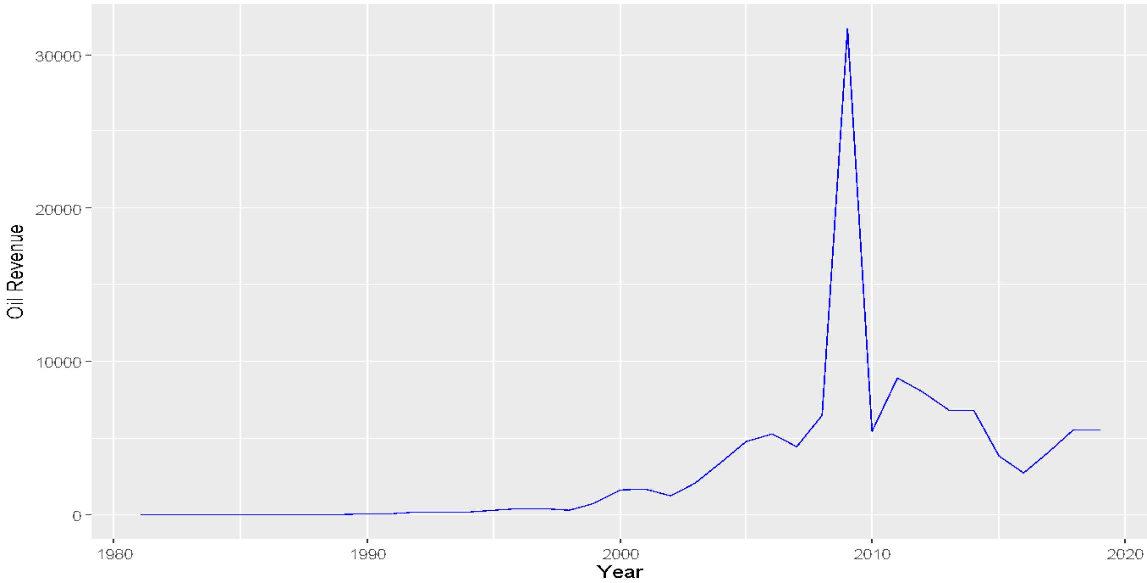

Figure 1: Time plot of annual oil revenue for the period 1981–2019

Table 1: Measures of forecast performances under training and test set for the forecast methods

Model	Training Set (TS)			Test Set (VS)		
	1981–2010			2011-2019		
	RMSE	MAE	MASE	RMSE	MAE	MASE
ARIMA	4633.028	1216.041	0.580933	19578.389	19475.836	9.3040894
SES	5205.145	1662.101	0.794027	5037.035	4659.342	2.2258833
HLT	4820.024	1420.509	0.6786124	22271.588	20475.871	9.7818305

Table 2: Measures of variability from the forecast methods under training set

Model	Standard deviation	Variance
ARIMA	4842.942	23454090.00
SES	4841.017	23435445.59
HLT	4940.631	24409834.68

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(a) Combining ARIMA and SES Using the Outperformance Strategy

In Table 2, the SES has the least variance $\sigma_2^2 = 23435446$ and correlation between ARIMA (0,1,1) and SES is $\rho = 0.9996$, then the weight w_1 is assigned to SES, where $f_{1,t+k}(t)$ is the estimated SES method and $f_{2,t+k}(t)$ is the estimated ARIMA method. The weight w_1 is obtained as:

$$w_1 = \frac{23435446 - 0.9996(4842.94)(4841.02)}{23454090 + 23435446 - 2(0.9996)(4842.9423)(4841.0170)} = 0.003 \quad (12)$$

$$w_2 = 1 - w_1 = 1 - 0.003 = 0.997 \quad (13)$$

Then the combined forecast method of ARIMA and SES is given as:

$$CM1 = F_{t+k}^{(c)}(t) = 0.003\left(f_{1,t+k}(t)\right) + 0.997\left(f_{2,t+k}(t)\right)$$

The minimized variance of the combined forecast errors is obtained as:

$$\left.\begin{aligned} Min\ var\left(e_{t+k}^{(c)}(t)\right) &= \frac{(23454090)(23435446)(1 - 0.9996^2)}{23454090 + 23435446 - 2(0.9996)(4842.9423)(4841.0170)} \\ &= 21080190.52 \end{aligned}\right\} \quad (14)$$

Since the $Min\ var\left(e_{t+k}^{(c)}(t)\right) = 21080190.52 < Min\ (\sigma_1^2, \sigma_2^2) = 23435446$, then we could say that combining ARIMA method and SES method gives better forecasts than when using them individually on the time series (oil revenue).

(b) Combining ARIMA, SES, and Holt using weighted mean (WM) strategy

The weights to be assigned to ARIMA, SES, and Holt under the weighted mean strategy is given as:

$$w_1 = \frac{0.419067}{0.946428} = 0.44 \qquad w_2 = \frac{0.205973}{0.946428} = 0.22 \qquad w_3 = \frac{0.205973}{0.946428} = 0.34 \quad (15)$$

The combined forecast method is given as:

$$CM2 = F_{t+k}^{(c)}(t) = 0.44\left(f_{1,t+k}(t)\right) + 0.22\left(f_{2,t+k}(t)\right) + 0.34\left(f_{3,t+k}(t)\right) \quad (16)$$

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Table 3: Comparing the forecast methods

Model	MASE	MAE	RMSE
ARIMA	0.912395	1758.363	4717.130
SES	0.906612	1747.219	4715.255
Holt	1.002026	1931.099	4680.413
$CM1 = ARIMA + SES$	0.912376	1758.327	4717.117
$CM2 = ARIMA + SES + Holt$	0.919356	1772.876	4688.166

In Table 3, simple exponential smoothing (SES) has the lowest MASE and MAE compared to the other methods; the first combined method CM1(ARIMA + SES) has the lowest MASE and MAE compared to ARIMA, Holt and the second combined method CM2 (ARIMA + SES + Holt).

Figure 3 shows the box plot of the individual forecast and combined forecast methods. In the plot, the single forecast method autoregressive integrated moving average (ARIMA) method and the combined forecast method CM1 (ARIMA + SES) still contain outliers which could affect their measures of performances. Figure 2 shows the time plot for the actual time series, the individual forecast methods, and combined forecast methods.

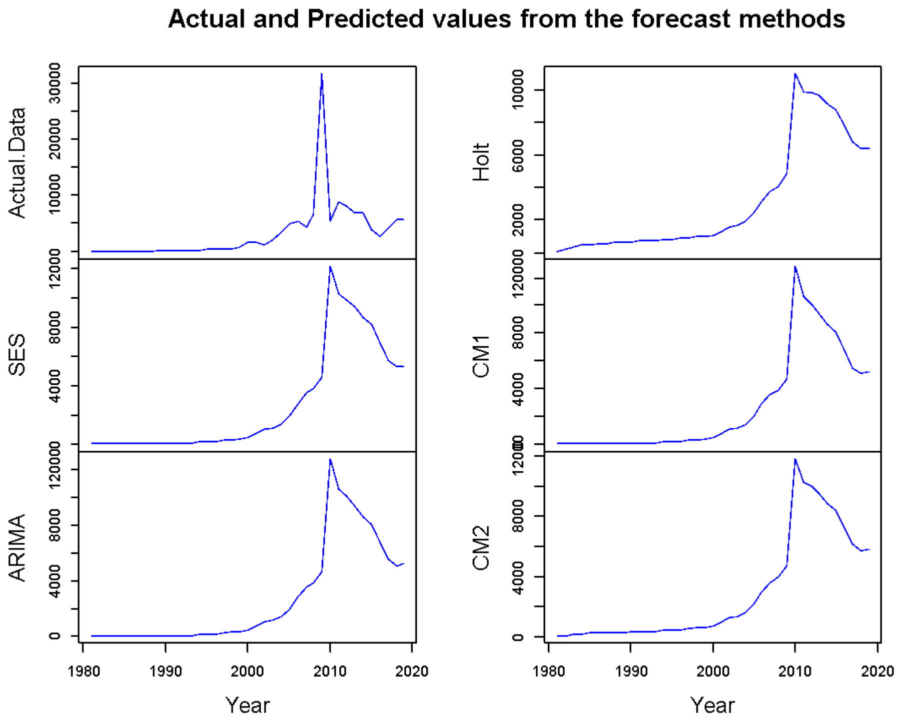

Figure 2: Time plot for the actual data, the individual forecast and combined forecast methods

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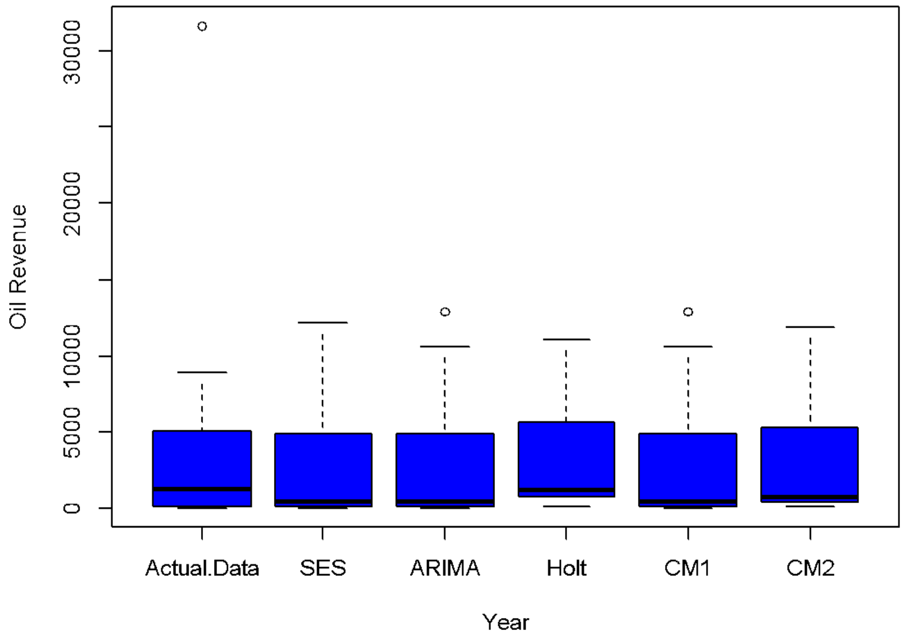

Figure 3. Box plot for the actual data, the individual forecast and combined forecast methods

CONCLUSION

Many works have been done by different researchers in order to prove that combining forecast methods gives a better prediction relative to individual forecast methods. The main intention of this study was to ascertain between single forecast method and the combined forecast methods, which gives a better prediction. The study, however, shows that choosing an individual forecast method out of a set of available forecast methods might be less risky than choosing a combined forecast method, and even gives a better prediction performance. The study also argued in line with Michèle and Theodoros [12], that choosing the combined forecast method instead of choosing individual methods using the selection method may improve the prediction performance.

However, studying the criteria of selecting forecast methods and their combinations is usually a very good lead way in the research environment. Sometimes, it is not always necessary to combine forecasts in order to improve prediction performance, that is, because the individual forecast may provide some better information on how to select among forecast methods and their combinations.

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