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TRANSIENT MHD THERMALLY RADIATING FREE CONVECTIVE FLOW PAST AN EXPONENTIALLY MOVING AND ROTATING ISOTHERMAL VERTICAL PLATE WITH HEAT GENERATION, REACTING SPECIES AND FLUCTUATING MASS DIFFUSION EFFECTS

W.I.A. Okuyade[1]*, C.O. Aminobiren[1] and T.M. Abbey[2]

[1]Department of Mathematics/Statistics, Federal Polytechnic of Oil and Gas, Bonny, Island, Nigeria. E-mail: wiaokuyade@gmail.com, charlesamino@yahoo.com

[2]Applied Mathematics and Theoretical Physics Group, Department of Physics, University of Port Harcourt, Port Harcourt, Nigeria. Email: tamunoimi.abbey@uniport.edu.ng

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ABSTRACT: *The problem of transient MHD thermally radiating free convective flow past an exponentially moving and rotating isothermal vertical plate in the presence of heat generation reacting species, and fluctuating mass diffusion is examined. The flow is governed by a set of non-linear partial differential equations of the Boussinesq approximation type. In particular, the momentum equations are simplified using the 2-D fluid flow analysis in the complex plane, and the governing equations are solved using the time-dependent Homotopy Perturbation Method. Expressions for the concentration, temperature, and velocity are obtained and presented graphically. The results amidst others depict that an increase in the chemical reaction rate decreases the concentration, but causes fluctuation in the flow velocity structure; an increase in the Heat generation/absorption parameter increases the fluid temperature but causes fluctuation in the flow velocity structure.*

KEYWORDS: *Heat generation/absorption, Isothermal plate, Mass diffusion, MHD, Reacting species, Thermal radiation*

MSC: 76/R

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INTRODUCTION

Natural convective flow with the magnetic field, heat, and mass transfer effects has applications in science and engineering. Specifically, they are relevant in MHD pumps and generators, magnetic control of molten iron flow in the steel industry, and the like.

Convective flow past vertical plates has attracted the attention of many research scientists in the past decades. It has been studied in different dimensions using different approaches. Therefore, a lot of literature exists on the flow in this domain of study.

Heat is generated when waves are absorbed by matter, as when electric currents flow through resistors in electric circuits, and in chemically reacting exothermic systems. At the absorption of generated heat, an object/system becomes hot, and this is a dissipative heat transfer. Additionally, the conduction of internally generated heat in a fluid system is very important. The conduction of internally generated heat is more pronounced in low Prandtl number fluids, as in liquid metals such as Bismuth (with $0.001 \leq \Pr \leq 0.1$) and Mercury (with $\Pr \leq 0.23$), which have high thermal conductivity, a potency for transferring heat even when the existing temperature difference between the surface and the fluid is small. On the effects of heat generation/absorption, Sahin [1], and Crepeau and Clarkson [2] studied the flow over a vertical plate with exponentially decaying heat generation; Watanabe and Pop [3], and Chamkha and Khaled [4] worked on the free convective flow over an inclined plate in the presence of internal heat generation/absorption, and transverse magnetic field; Chen [5] investigated the effects of variable wall temperature and concentration on the MHD flow past an inclined plate. Zueco et al. [6] examined the thermo-phoresis MHD flow over a vertical plate with lateral mass flux, heat source, Ohmic heating, and thermal conductivity effects using the network simulation method; Sehra [7] considered chemical reaction and exponential heating effects on the hydro-magnetic flow past a vertical plate.

More so, the phenomenon of isothermal flow exists where the fluid temperature is constant, both at the origin and destination. This phenomenon is seen where the temperature difference between the fluid ambient temperature and that of the plate over which the fluid flows is infinitesimally small, and as such, the temperature of the moving plate is said to be constant at the wall. Considering the convective flow past a moving isothermal vertical plate, Ostrach [8] studied the heat generation effects; Joshi and Gebhart [9] examined the pressure work and viscous dissipation effects; Raptis [10] discussed free convection and mass transfer effects on the oscillatory flow with constant suction and heat sources effects; Mahanti and Gaur [11] looked into the heat source effects; Singh et al., [12] studied the effect of heat generation/absorption; Philiph et al., [13] examined the time, hydro-magnetic, Prandtl, Schmidt, rotation and fluctuating mass diffusion effects when the plate is exponentially accelerating; Megaraju and Shekar [14] considered the Hall and chemical reaction effects. Additionally, convective flow over an isothermal inclined plate was considered. For example, Alam et al. [15] considered the effects of viscous dissipation and Joule heating on the steady flow, and Noor et al. [16] investigated the effects of radiation, heat source/sink, and thermo-phoresis on the flow.

Furthermore, among the factors that affect the flow of fluids is the rotation of a flow channel or the plate over which the fluid flows. During rotation, waves are generated in the fluid, and this may lead to the emergence of angular velocity and wave number in the problem. The MHD convective flow over a moving vertical plate in the presence of rotation had been

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investigated by many research workers. For instance, Biswal et al. [17] studied the viscoelastic fluid flow when the plate lagged against heat; Rajput and Kumar [18] investigated the effects of rotation, radiation, and variable temperature when the plate is impulsively started; Narayana et al [19] studied the Hall and radiation effects on the flow of a micro-polar fluid; Venkateswarlu et al. [20] examined the effects of Hall current, Soret, heat absorption, and radiation on the flow when the plate is suddenly started; Umamaheswar et al. [21] looked into the double diffusion and chemical reaction effects on the flow. Other reports on the flow with rotation can be are seen in Singh and Singh [22], Singh et al. [23], Singh et al. [24], and Das and Jana [25].

On a similar note, thermal radiation, as a process of heat propagation using electromagnetic waves occurs due to high-temperature differences. It is a heat transfer comparable to convective heat transfer. Neglecting the effects of MHD and rotation on the flow over a vertical plate, Hossain et al. [26] studied the radiation effects, Chamkha [27] investigated the effects of heat absorption and thermal radiation; Seddeek [28] examined the effect of radiation and variable fluid viscosity; Abd El-Naby et al. [29] looked at the effects of radiation and variable surface temperature; Ali et al. [30] investigated the effect of radiation and viscous dissipation using finite difference method; Mamtha et al. [31] considered radiation and mass transfer effects; Devi et al. [32] examined the thermal radiation and chemical reaction effects. Other reports exist on the MHD convective flow over a vertical plate with thermal radiation but in the absence of rotation are found in Ahmed [33], Kumar et al. [34], and Akhter et al. [35].

Based on the above studies, this paper aims to examine analytically the significant factors affecting the transient MHD thermally radiating free convective flow past an exponentially moving and rotating isothermal vertical plate with heat generation, reacting species, and fluctuating mass diffusion effects using the time-dependent Homotopy Perturbation Method.

Physics of the Problem and Mathematical Formulation

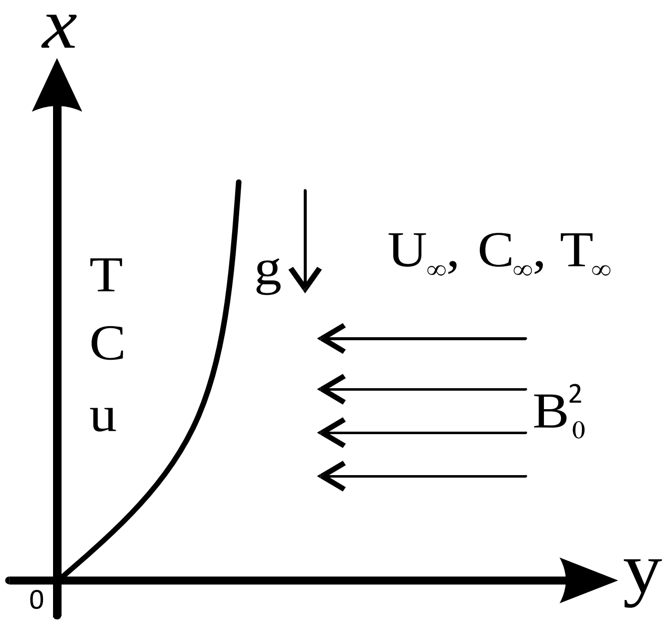

Fig.1 The model of a vertically accelerating plate in a fluid

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The transient hydro-magnetic MHD thermally radiating free convective flow past an exponentially moving and rotating isothermal vertical plate with heat generation, reacting species, and fluctuating mass diffusion effects are considered. The schematic of the problem is given in Fig. 1. The problem is formulated on the following premises: the fluid is viscous, incompressible, electrically conducting, thermally radiative, magnetically permissive, and chemically reactive; the plate is porous with infinite length, isothermal**,** and heated to a high-temperature regime such that radiant rays are emitted**. A** magnetic field force of constant strength is applied. Assumedly, the magnetic field has negligible induction such that the magnetic Reynolds number becomes insignificant; the Hall Effect is also negligible; Let the $x'-$ axis be along the plate in the vertical direction, and $y'-$ axis normal to the plate; $y'=0$ the edge of the plate, also taken as the origin, and $y'=\infty$ the end of the plate. The fluid pressure is hydrostatic such that $p'(x)=0$. In the spatial $x',y'-$coordinates, let (u',v') be the velocity components, T_∞ and C_∞ the ambient temperature and concentration at $t'\leq 0$; T_w and C_w the temperature and concentration at the wall of the plate, T' and C' the fluid temperature and concentration at $t'>0$. Similarly, the plate accelerates exponentially at $t'>0$. Now, since the length of the plate is infinite, all physical quantities are functions of y' and t' only, such that by Boussinesq's approximations, the mass balance, momentum, energy, and diffusion equations governing the flow are:

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\rho(\frac{\partial u'}{\partial t'}-2\Omega' v')=\mu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta_t(T'-T_w)+\rho g\beta_c(C'-C_w)-\left(\sigma_e B_0^2+\frac{\mu}{\kappa}\right)u' \tag{2}$$

$$\rho(\frac{\partial v'}{\partial t'}+2\Omega' u')=\mu\frac{\partial^2 v'}{\partial y'^2}-\sigma_e B_0^2 v'-\frac{\mu}{\kappa}v' \tag{3}$$

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial y'^2}+Q(T'-T_\infty)-\frac{\partial q_r'}{\partial y'} \tag{4}$$

$$\frac{\partial C'}{\partial t'}=D_m\frac{\partial^2 C'}{\partial y'^2}-k_r(C'-C_\infty) \tag{5}$$

(where ρ is the fluid density, μ is the fluid viscosity, g is the acceleration due to gravity, β_t is the volumetric expansion coefficient of the fluid due to temperature change, β_c is the volumetric expansion coefficient of the fluid due to concentration change, σ_e is the electrical conductivity of the fluid, B_o^2 is the magnetic field flux, κ is the permittivity of the plate, C_p is the heat capacity of the fluid at constant pressure, k is the heat diffusion coefficient of the fluid, Q is the heat generation/absorption parameter, D_m is the mass diffusion coefficient, k_r is the chemical reaction parameter)

with the boundary conditions

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$t' \le 0 : u = 0, v' = 0, T' = T_\infty, C' = C_\infty$ for all y' (everywhere) (6)

$t' > 0 : u' = \exp(a't'), v' = 0, T' = T_W, C' = C_\infty + (C_W - C_\infty)t'$ at $y' = 0$ (7)

$u' \to 0, v' \to 0, T' \to 0, C' \to 0$ at $y' \to 0$ (8)

A radiative heat transfer is comparable to a convective heat transfer. In a fluid, it is described terms of the optical depth of penetration α (the distance a photon travels in the fluid before something happens to it). Based on photon penetration, fluids are classified into optically thick, and thin (Vincent and Krugger [36], Spegiel and Howel [37]). For the optically thin, $\alpha << 1$, and the fluid density is relatively low to allow long photon travel, as in grey gas; the fluid is slowly radiating and its Raleigh number is (Ra < 1) (Alagoa et al. [38]. For the optically thick fluid, the fluid is non-transparent and its density is high enough to allow short photon travel. Importantly, optically thick fluids emit and absorb radiations at the boundaries. In this work, the fluid is taken to be optically thin such that by the Roseland approximation,

$$\frac{\partial q_r'}{\partial y'} = -4\alpha\sigma \frac{\partial^2}{\partial y'^2}(T'^4 - T_\infty^4) \quad (9)$$

where σ is the Stefan-Boltzman constant.

Similarly, we take the temperature differential between the fluid ambient temperature and that at the wall $(T_W - T_\infty)$ to be small enough such that T'^4 is assumed a linear function of T_∞, then T'^4 can be presented in the Taylor series as $T'^4 = 4T_\infty^3 T' - 3T_\infty^4$

Invoking this into equation (4), we have

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + Q(T' - T_\infty) + 16\alpha\sigma \frac{\partial^2 T'}{\partial y'^2} \quad (10)$$

Introducing the following non-dimensionalized quantities

$$u = \frac{u'}{u_o}, v = \frac{v'}{u_o}, t = \frac{t'u_o^2}{\upsilon}, y = \frac{y'u_o}{\upsilon}, x = \frac{x'u_o}{\upsilon}, \quad \theta = \frac{T' - T_\infty}{T_W - T_\infty}, \phi = \frac{C' - C_\infty}{C_W - C_\infty}, Gr = \frac{g\beta_t (T_W - T_\infty)}{u_o^3},$$

$$Gc = \frac{g\beta_c (C_W - C_\infty)}{u_o^3}, \Pr = \frac{\mu C_p}{k}, Sc = \frac{\upsilon}{D}, \Omega = \frac{\Omega'\upsilon}{u_o^2}, a = \frac{a'\upsilon}{u_o^2}, H_m = \frac{\sigma B_0^2 \upsilon}{\rho u_o^2}, \delta = \frac{k_r}{D_m}, \chi = \frac{\upsilon}{\kappa}, N = \frac{Q\upsilon^2}{k} \quad (11)$$

(where θ is the dimensionless temperature, ϕ is the dimensionless concentration, Gr is the Grashof number due to temperature differential, Gc is the Grashof number due to concentration differential, Pr is the Prandtl number, Sc is the Schmidt number, Ω is rotation parameter, a is a constant, Hm is the Hartmann number, Ra is the Raleigh number, δ is the chemical reaction rate parameter, N is the heat generation parameter) in equations (1) – (3), (5)-(8) and (10), we have

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{12}$$

$$\frac{\partial u}{\partial t}-2\Omega v=\frac{\partial^2 u}{\partial y^2}-M_1 u+Gr\theta+Gc\phi \tag{13}$$

$$\frac{\partial u}{\partial t}+2\Omega u=\frac{\partial^2 v}{\partial y^2}-M_1 v \tag{14}$$

$$\frac{\partial \theta}{\partial t}=\lambda\frac{\partial^2 \theta}{\partial y^2}+N\theta \tag{15}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-\delta\phi \tag{16}$$

where $M_1=Ha+\chi, \lambda=\frac{1+Ra}{\Pr}$

with the boundary conditions

$t\le 0: u=0, v=0, \theta=0, \phi=0$ for all y (everywhere) (17)

$t>0: u=\exp(at), v=0, \theta=1, \phi=t$ at $y=0$ (18)

$u\to 0, v\to 0, \theta\to 0, \phi\to 0$ at $y\to\infty$ (19)

Two-Dimensional Fluid Flow Analysis in a Complex Plane

To solve equations (13) and (14), a complex velocity potential is introduced.

At any point, (x, y), fluid flows at a certain velocity determined by its magnitude and direction. This velocity can be represented by a complex variable

$$U=u+iv$$

If Γ is a boundary curve (or closed curve) of a connected domain D, then by the Green theorem, the circulation can be represented by a double integral

$$\int_\Gamma (udx+vdy)=\iint_D \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy$$

More so, if Γ is a circle of radius r, multiplying the circulation by $\frac{1}{2\pi r}$ gives the mean velocity of the fluid

$$U=\frac{1}{2\pi r}\iint_D \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy$$

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Also, dividing the mean velocity by r yields the angular velocity

$$\omega = \frac{1}{\pi r^2} \iint_D \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy$$

with the rotation defined as

$$\Omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

and the velocity = 2ω.

For an irrotational flow (i.e. $\omega = 0$),

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

and for an incompressible fluid (where ρ is constant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

in every region free of sources and sinks. Here, at all points, the fluid disappears.

Additionally, by the criteria for exactness and independence of the path in a domain, the line integral

$$\int_\Gamma (udx + vdy)$$

is independent of the path in the domain D in space. Therefore, integrating from a fixed point (a,b) to a variable point (x, y) in D, and the integral becomes a function of the point (x, y), say $\varphi(x, y)$ i.e.

$$\varphi(x, y) = \int_{(a,b)}^{(x,y)} (udx + vdy) =$$

where the function $\varphi(x, y)$ is called the velocity potential of the motion. Since the integral is independent of the path, then $u\partial x + v\partial y$ is an exact differential of $\varphi(x, y)$, that is

$$u\partial x + v\partial y = \frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy$$

Hence,

$$u = \frac{\partial \varphi}{\partial x}, \quad v = \frac{\partial \varphi}{\partial y}$$

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And, the velocity vector becomes

$$U = u + iv = \frac{\partial \varphi}{\partial x} + i\frac{\partial \varphi}{\partial y} \qquad (20)$$

The curves at which $\varphi(x, y)$ = constant are called equipotential lines (see Kreyzig [39])

Combining equations (13) and (14), and substituting equation (20) into the result, we have

$$\frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial x} + i\frac{\partial \varphi}{\partial y}) + (2\Omega(\frac{\partial \varphi}{\partial x} + i\frac{\partial \varphi}{\partial y})) = \frac{\partial^2}{\partial y^2}(\frac{\partial \varphi}{\partial x} + i\frac{\partial \varphi}{\partial y}) + Gr\theta + Gc\phi - M(\frac{\partial \varphi}{\partial x} + i\frac{\partial \varphi}{\partial y}) \qquad (21)$$

yielding

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} - M_2 U + Gr\theta + Gc\phi \qquad (22)$$

where $M_2 = Ha + \chi + 2\Omega$

Now, the governing equations reduce to equations (15) – (19) and (22). Rewriting equations (15), (16), and (22), we have

$$\frac{\partial U}{\partial t} - \left(\frac{\partial^2 U}{\partial y^2} - M_2 U + Gr\theta + Gc\phi \right) = 0 \qquad (23)$$

$$\frac{\partial \theta}{\partial t} - \left(\lambda \frac{\partial^2 \theta}{\partial y^2} + N\theta \right) = 0 \qquad (24)$$

$$\frac{\partial \phi}{\partial t} - \left(\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \delta\phi \right) = 0 \qquad (25)$$

Similarly, combining the velocity components in the boundary conditions as in the complex plane, equations (17) - (19), give

$t \le 0 : U = 0, \theta = 0, \phi = 0$ for all y (everywhere) (26)

$t \ge 0 : U = e^{at}, \theta = 1, \phi = t$ at $y = 0$ (27)

$U \to 0, \theta \to 0, \phi \to 0$ at $y \to \infty$ (28)

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METHODS OF SOLUTION

Equations (23) - (28) are solved using the Homotopy Perturbation Method:

$$L(u)+N(u)-f(t)=0 \tag{29}$$

where $L(u)$ is the linear operator, $N(u)$ is the nonlinear operator, and $f(t)$ is a known analytic function

For $f(t)=0,$ presenting equations (23)-(25) in HPM form, we have

$$(1-p)\frac{\partial U}{\partial t}+p\left(-(\frac{\partial^2 U}{\partial y^2}-M_2U+Gr\theta+Gc\phi)\right)=0 \tag{30}$$

$$(1-p)\frac{\partial \theta}{\partial t}+p\left(-(\lambda\frac{\partial^2 \theta}{\partial y^2}-N\theta)\right)=0 \tag{31}$$

$$(1-p)\frac{\partial \phi}{\partial t}+p\left(-(\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}+\delta\phi)\right)=0 \tag{32}$$

yielding

$$\frac{\partial U}{\partial t}=p\left(\frac{\partial U}{\partial t}+\frac{\partial^2 U}{\partial y^2}+M_2U-Gr\theta-Gc\phi\right) \tag{33}$$

$$\frac{\partial \theta}{\partial t}=p\left(\frac{\partial \theta}{\partial t}+\lambda\frac{\partial^2 \theta}{\partial y^2}+N\theta\right) \tag{34}$$

$$\frac{\partial \phi}{\partial t}=p\left(\frac{\partial \phi}{\partial t}+\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-\delta\phi\right) \tag{35}$$

More so, presenting the dependent variables as

$$U=U_o+pU_1+p^2U_2+\dots \tag{36}$$

$$\theta=\theta_o+p\theta_1+p^2\theta_2+\dots \tag{37}$$

$$\phi=\phi_o+p\phi_1+p^2\phi_2+\dots \tag{38}$$

where $p<1$, and substituting into equations (33) - (35), and (26) - (28), and equating the coefficients of the powers of p, we have

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$$p^o : \frac{\partial U(y,0)}{\partial t} = 0 \tag{39}$$

$$\frac{\partial \theta(y,0)}{\partial t} = 0 \tag{40}$$

$$\frac{\partial \phi(y,0)}{\partial t} = 0 \tag{41}$$

$$p^1 : \frac{\partial U_1}{\partial t} = \frac{\partial U_o}{\partial t} + \frac{\partial^2 U_o}{\partial y^2} + M_2 U_o - Gr\theta_o - Gr\phi_o \tag{42}$$

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_o}{\partial t} + \lambda \frac{\partial^2 \theta_o}{\partial y^2} + N\theta_o \tag{43}$$

$$\frac{\partial \phi_1}{\partial t} = \frac{\partial \phi_o}{\partial t} + \frac{1}{Sc}\frac{\partial^2 \phi_o}{\partial y^2} - \delta\phi_o \tag{44}$$

$$p^2 : \frac{\partial U_2}{\partial t} = \frac{\partial U_1}{\partial t} + \frac{\partial^2 U_1}{\partial y^2} + M_2 U_1 - Gr\theta_1 - Gr\phi_1 \tag{45}$$

$$\frac{\partial \theta_2}{\partial t} = \frac{\partial \theta_1}{\partial t} + \lambda \frac{\partial^2 \theta_1}{\partial y^2} + N\theta_1 \tag{46}$$

$$\frac{\partial \phi_2}{\partial t} = \frac{\partial \phi_1}{\partial t} + \frac{1}{Sc}\frac{\partial^2 \phi_1}{\partial y^2} - \delta\phi_1 \tag{47}$$

with boundary conditions

$t \le 0 : U_o = 0, \theta_o = 0, \phi_o = 0,\ U_1 = 0, \theta_1 = 0, \phi_1 = 0,\ U_2 = 0, \theta_2 = 0, \phi_2 = 0$ for all y (everywhere) (48)

$t \ge 0 : U_o = \exp(at), \theta_o = 1, \phi_o = t,\ U_1 = 0, \theta_1 = 0, \phi_1 = 0,\ U_2 = 0, \theta_2 = 0, \phi_2 = 0$ at $y = 0$ (49)

$U_o = 0, \theta_o = 0, \phi_o = 0,\ U_1 = 0, \theta_1 = 0, \phi_1 = 0,\ U_2 = 0, \theta_2 = 0, \phi_2 = 0$ at $y = \infty$ (50)

The solutions of equations (39) - (41) are determined from equation (48), equations (42) - (44) from equation (49), and equations (45) - (47) from the solutions of U_1, θ_1 and ϕ_1.

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RESULTS AND DISCUSSION

The problem of transient MHD thermally radiating free convective flow past an exponentially moving and rotating isothermal vertical plate in the presence of heat generation/absorption, reacting species, and fluctuating mass diffusion is investigated. The effects of chemical reaction rate, heat generation/absorption, magnetic field, rotation, and convective current are examined, and the results are shown in Fig. 2- Fig. 8. These results are obtained using Mathematica 11.2 computational software. For constant values of $\chi = 0.1, Sc = 0.62, \Pr = 0.71, Ra = 0.3$ and $a = 0.1$, and varying values of $\delta = 0.1,05,1.0,1.5,2.0,$ $N = 0.1,05,1.0,1.5,2.0,$ $\Omega = 0.1,05,1.0,1.5,2.0,$ $Hm = 0.1,05,1.0,1.5,2.0,$ $Gr = 0.1,05,1.0,1.5,2.0,$ the profiles are shown below.

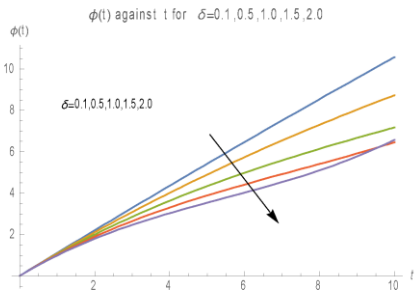

Fig. 2 Concentration-Chemical Reaction Rate Parameter Profiles

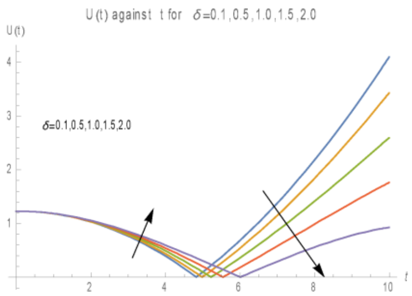

Fig. 3 Velocity-Chemical Reaction Rate Parameter Profiles

The effects of the chemical reaction rate on the flow are shown in Fig. 2 and Fig. 3. They show that an increase in the rate of chemical reaction decreases the concentration, and causes fluctuation in the velocity profiles. The velocity increases as the chemical reaction rate increases in $0 < t \le 4.5$ and drops in $t \ge 4.5$. A chemical reaction occurs when there is a differential in the concentrations of the reacting species. And the strength of the reaction determines the rate, which can be slow, moderate, or abrupt. In reaction, species are used up, and new ones are generated. This may lead to a drop in the concentration of the fluid. And, this is true, as seen in Fig. 2. As a function of concentration, the velocity should have decreased likewise. Therefore, the fluctuation in the velocity structure, as seen in Fig. 3 may be due to the fluctuating mass diffusion effect.

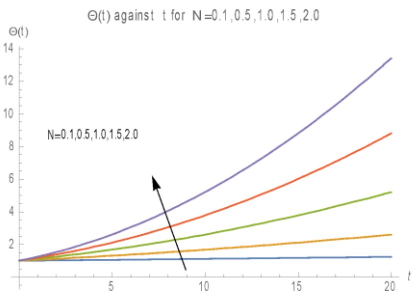

Fig. 4 Temperature-Heat Absorption \Generation Parameter Profiles

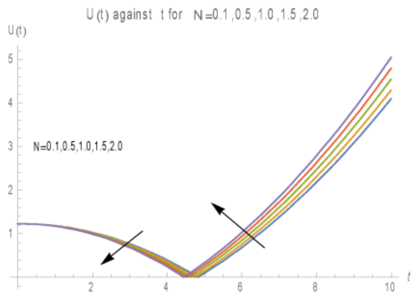

Fig. 5 Velocity-Heat Absorption\ Generation Parameter

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Profiles

The effects of Heat generation/absorption on the flow are seen in Fig. 4 and Fig. 5. They depict that an increase in the Heat generation/absorption parameter decreases the fluid temperature, and causes fluctuation in the velocity structure. The velocity drops in $0 < t \leq 4.5$, but rises in $t \geq 4.5$ as the Heat generation/absorption parameter increases. Heat is generated, or absorbed when waves are absorbed/emitted by matter. Heat is absorbed into the fluid system when the external/environmental heat is at a higher level than the ambient, and is given out when the heat level of the fluid system is higher than that of the external/environment. More so, heat may be generated in a fluid system when an exothermic reaction is present. The temperature of the system is bound to increase in the presence of heat absorption/generation; thus accounting for what is seen in Fig. 4. Similarly, the fluctuation in the velocity structure in Fig.5 may be due to the fluctuating mass diffusion, and other effects.

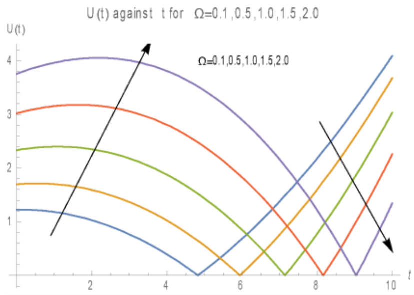

Fig. 6 Velocity-Rotation Parameter Profiles

Furthermore, the effect of the Rotation parameter is seen in Fig. 6. It shows that the velocity structure fluctuates. The velocity increases as the Rotation parameter increases in $0 < t \leq 4.5$ and drops in $t \geq 4.5$. Usually, rotation may lead to the generation of secondary motion in the fluid, and this has the potency for reducing axial and vertical velocity. Therefore, the fluctuation of the structure in Fig. 6 must be due to the fluctuating mass diffusion, and some other factors.

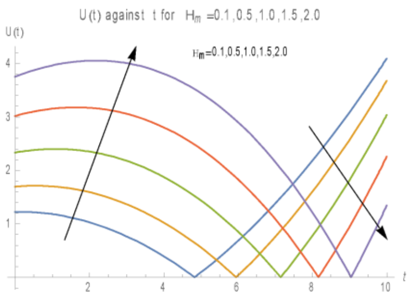

Fig. 7 Velocity-Magnetic Field Parameter Profiles

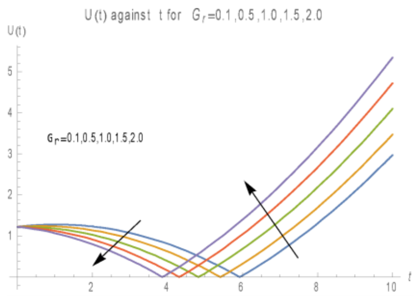

Fig. 8 Velocity-Grashof Number Profiles

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The effect of the Magnetic Field parameter is shown in Fig. 7. It depicts that the velocity structure fluctuates. The velocity rises as the Magnetic field parameter increases in $0 < t \leq 4.5$ and decreases in $t \geq 4.5$. Being chemically reactive, the fluid particles exist as charges, which in the presence of a transverse magnetic field generate electric current. Similarly, the electric current combines with the magnetic field flux to produce the Lorentz force, a mechanical and dissipative force that has a reducing effect on flow velocity. Therefore, the fluctuation in flow velocity may be due to the presence of some other factors.

Additionally, the effect of the Grashop number on the flow is seen in Fig. 8. It shows that an increase in the Grashop number causes fluctuation in the flow velocity structure, which decreases in $0 < t \leq 4.0$, and rises in $t \geq 4.0$. Naturally, convective current/Grashof number arises from the combined effects of gravity, volumetric expansion due to temperature/concentration, and the differential between the ambient and external/environmental temperature/concentration. The combined effects make the fluid to be buoyant, as the grip of the fluid viscosity is reduced. This negates what is seen in Fig. 8. Therefore, the fluctuation observed must be due to the fluctuating mass diffusion, and some other factors.

CONCLUSION

The problem of transient MHD thermally radiating free convective flow past an exponentially moving and rotating isothermal vertical plate in the presence of heat generation, reacting species, and fluctuating mass diffusion is examined. The analysis of results shows that an increase in the

- Chemical reaction rate decreases the concentration, but causes fluctuation in the flow velocity structure;
- Heat generation/absorption parameter increases the fluid temperature, but causes fluctuation in the flow velocity structure;
- Rotation parameter causes fluctuation in the velocity structure;
- Magnetic field parameter causes fluctuation in the velocity structure;
- Grashop number causes fluctuation in the velocity structure.

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