African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)
www.abjournals.org

GARCH MODELS COMPARISON WITH SYMMETRIC AND ASYMMETRIC PROCESS FOR UNIVARIATE ECONOMETRIC SERIES

Ockiya Atto Kennedy[1], Orumie Ukamaka Cynthia[2] and Emmanuel Oyinebifun[2]

[1]Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Rivers State, Nigeria

[2]Department of Mathematics and Statistics, University of Port Harcourt, Choba, Rivers State, Nigeria

Cite this article:

Ockiya A.K., Orumie U.C., Emmanuel O. (2023), Garch Models Comparison with Symmetric and Asymmetric Process for Univariate Econometric Series. African Journal of Mathematics and Statistics Studies 6(2), 1-23. DOI: 10.52589/AJMSS-JDZ6ZOXG

Manuscript History

Received: 2 Feb 2023

Accepted: 28 Feb 2023

Published: 18 March 2023

ABSTRACT: *The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was modeled using both symmetric and asymmetric processes. Secondary data from January 2005 to December 2021 on the Consumer Price Index, Exchange Rate, Crude Oil Price, and Inflation Rate were used for this study. The research was conducted using the statistical software packages Minitab and E-view. The aforementioned four macroeconomic variables show a tendency for volatility to cluster across time. In both symmetric and asymmetric processes, the volatility condition and leverage impact coefficients were present. By contrasting the symmetric models (ARCH, GARCH, and GARCH-M) and the asymmetric models, the best model was chosen using Akaike Information Criteria (E-GARCH, T-GARCH and APARCH). For the investigated univariate economic variables, the results indicated that the found asymmetric model GARCH models outperformed the symmetric model GARCH models. Therefore, these models can be applied to the forecasting of these series of economic indicators. Models include the Asymmetric E-GARCH (1, 1) Model for Consumer Price Index, Crude Oil Price, and Inflation Rate Series and the Asymmetric T-GARCH (1, 1) Model for Exchange Rate Series.*

KEYWORDS: Univariate GARCH (M-GARCH) models, Information criteria, Symmetric and asymmetric process, Univariate economic variables, Leverage effect.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

INTRODUCTION

One of the most challenging problems nowadays for econometrics academics, time series analysts, and policy makers is modeling the variance that happens in an econometric series, which has become a significant source of worry in the financial markets. This has been the subject of all research for a very long time. Four economic variables were employed in this study: inflation rate, exchange rate, price of crude oil, and consumer price index. Volatility clustering is a characteristic that these variables display. When variables continuously rise and decrease over time, this phenomenon is known as volatility clustering.

Ijomah and Enewari (2020) claim that the variables under study lack an appropriate model formulation known as the volatility model of macroeconomic variables in the financial market. The researcher may encounter volatility modeling of macroeconomic variables like the price of crude oil, the exchange rate, the inflation rate, and the consumer price index when analyzing financial time-series data in international markets with very persistent volatility. The problem of latent volatility is another issue brought on by the increasing frequency of macroeconomic variables. On the other hand, it can be difficult to model changes in the Crude Oil Price (COP), Exchange Rate (EXCH), Inflation Rate (INF), and Consumer Price Index (CPI). The nation's economy is directly impacted by the cost of crude oil on the world market. Macroeconomic indices like the exchange rate, inflation rates, and the consumer price index are known to be significantly impacted by changes in the price of crude oil.

Volatility can be defined as the rate of variation in movement between the four variables discussed above in this study. The risk of other factors increases with the degree of exchange rate volatility on the global market, and rising crude oil prices result in higher inflation rates across the board. Because they might vary at any time, variable exchange rates first seem riskier than stable rates (Wiri & Essi, 2018).

Asymmetric Models (ARCH, GARCH, and GARCH-M) and Symmetric Models have both been the subject of several studies (E-GARCH, T-GARCH and APARCH). The Generalized Autoregressive Conditional Heteroscedasticity in Mean (GARCH-M) model proposed by Engle and others in 1987, the Exponential Generalized Autoregressive Heteroscedasticity (E-GARCH) model proposed by Bollerslev, the Autoregressive Conditional Heteroscedasticity (ARCH) model developed by Engle in 1982, the Generalized Autoregressive Conditional Heteroscedasticity (GAR (1994). The Spline T-GARCH model, which is proposed by Elena and Shen (2018), examines Asymmetric Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models in the Threshold GARCH (T-GARCH) family and captures high-frequency return volatility, low-frequency macroeconomic volatility, and an asymmetric response to prior negative news in both ARCH and GARCH terms. A number of parametric Generalized Autoregressive Heteroskedasticity models are proposed by Hentschel (1995). (GARCH). Ching and Siok (2013) employed empirical data and various GARCH models to evaluate the volatility of the Malaysian stock market (Symmetric and Asymmetric). Other researchers include Maryam and Ramanathan (2012), Ojo (1998), Osabuohien and Edokpa (2013), Omorogbe and Ucheoma (2017), Minovic, Bollerslev (1986), Chris, 2008), Robert (2013), Musa et al. (2014), Osabuohien and Edokpa (2013), Suliman (2012), Cyprian et al. (2017), and Wiri and Sibeate (2020). (2017).

This study's objective is to identify Univariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models in modeling the causal relationship among monthly data

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)
www.abjournals.org

sets like the price of crude oil, the exchange rate, the inflation rate, and the Consumer Prices Index in Nigeria from January 2005 to December 2021. This will be accomplished by determining the lag length using the information criteria (AIC, HQIC, and SIC), comparing the performance of the symmetric and asymmetric Univariate GARCH models using parameter estimates, and using the information criteria.

MATERIALS AND METHODS

The time plot of the original series, the unit roots test, the model definition, the symmetric and asymmetric models, the general case of univariate GARCH modeling, and the models selection criteria make up the research technique for this study (or model information criteria).

Research Design and Source of Data

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, created for econometrics and time series modeling of the univariate series, was used in this study in both symmetric and asymmetric modeling processes. These methods offer statistical instruments for model evaluation and maximum likelihood estimation (MLE). Secondary data on the rate of inflation, the price of crude oil, the consumer price index, and the exchange rate were gathered from the Central Bank of Nigeria's statistical bulletins, which covered the period from January 2005 to December 2021.

Time Plot

When working with time-series data, the initial stage in the study is usually to make a time plot of the data and analyze it using a straightforward descriptive measure of the series' essential characteristics. In order to show the overall movement of the original data over time and to determine if the pattern is consistent or changes over time, the graph of the inflation rate, crude oil price, consumer price index, and exchange rate is plotted against time. The following is also depicted in this graph: Temporal trend movement (either upward or downward), seasonal variation, recurring variations, and structural jumps. The series' primary characteristics might then be used to interpret the series' essential qualities.

Test for Stationarity

Time series data must be distinguished in order to take into consideration non-stationarity that appears as a pattern since time series data are frequently non-stationary. Using non-stationary data can result in inaccurate regression since a series' stationarity can have a big impact on how it behaves. A stationary time series' statistical characteristics, such as mean zero and variance, remain constant over time. The time series variables in the model have to be stationary in order to run a joint relevant test on the lags of the variables (Gujarati, 2003). Stationarity is tested using the Dickey Fuller, Philip Peron, correlograms, autocorrelation functions (ACF), and partial autocorrelation functions (PACF). However, the Augmented Dickey-Fuller test was used in this study.

A time series, X t, is tested for the unit root using the enhanced Dickey-Fuller test. Equations 1 and 2 mathematically present the Augmented Dickey-Fuller test.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)
www.abjournals.org

$$\Delta COP_t = \beta_0 + \beta_1 COP_{t-1} + \sum_{T=1}^{m} pi\Delta COP_{t-1} + \varepsilon_t \tag{1}$$

$$\Delta INF_t = \beta_0 + \beta_1 INF_{t-1} + \sum_{T=1}^{m} pi\Delta INF_{t-1} + \varepsilon_t \tag{2}$$

$$\Delta EXCH_t = \beta_0 + \beta_1 EXCH_{t-1} + \sum_{T=1}^{m} pi + \varepsilon_t \tag{3}$$

$$\Delta CPI_t = \beta_0 + \beta_1 CPI_{t-1} + \sum_{T=1}^{m} pi\Delta CPI_{t-1} + \varepsilon_t \tag{4}$$

Where: ε_t = Random terms

Δ = Difference operator

ρi = Coefficient of the previous observations

$t-1$ = Past observation

m = Number of lags and β is the parameter to be estimated

In the Augmented Dickey-Fuller (ADF) autoregressive equations, the lagged dependent variables' function is to guarantee that ε_t is white noise. If β is determined to be more negative than statistical significance or less than statistically significant, the null hypothesis claims that the series have unit roots at the initial difference. The t-statistic is then compared to the critical value of the tabulated t-statistic (Green, 2012).

Data Transformation

The monthly data for the Nigerian Naira/US Dollar exchange rate, inflation rate, crude oil price, consumer price index, and other variables are utilized from January 2004 to December 2020. For the purposes of fitting the model to logarithm returns, this results in a total of 204 data, which are transformed. Let the series (X_t) denoted series. Then, the returns series Y_t is

$$Y_t = \left(\frac{X_t}{X_{t-1}}\right) = \left(\frac{Series_t}{Series_{t-1}}\right) \tag{5}$$

where X_t represent the each of four series time t and X_{t-1} represent the series at time $t-1$.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Conditional Mean Model

ARMA (p, q) Model

An Autoregressive (AR) and Moving Average (MA) process is a component of the movement of the Crude Oil Price, Inflation Rate, Consumer Price Index, and Exchange Rate. The conditional mean in a series is modeled using the Autoregressive Moving Average (ARMA). Both the AR model and the MA model include lagged factors that reflect the series' historical values. We reach the ARMA model by including both lagged terms. Thus ARMA (*p*,*q*), where p is the order of Autoregressive term and q is the order of the moving-average term, can generally be represented as

$$y_t = \sum_{i=1}^{p} \alpha_i y_{t-1} + \varepsilon_t + \sum_{j=1}^{q} \beta_j \varepsilon_{t-1} \tag{6}$$

A series$\{y_t\}$ is said to follow an Autoregressive Moving Average model of orders p and q, designated ARMA (p, q), where β_j and α_i are constants such that the model is stationary as well as invertible and $\{\varepsilon_t\}$is a white noise process.

Volatility Models

Symmetric and asymmetric models are the two basic categories into which modeling methodologies for volatilities are classified. While the conditional variance in the symmetric model solely depends on the asset's size and not its sign, in the asymmetric model, the impacts of a negative and positive size shock on future volatility are different (Chris, 2008).

Symmetric Models

Autoregressive Conditional Heteroscedastic (ARCH) Model

The Autoregressive Conditional Heteroscedastic (ARCH) model is used to model the error of the conditional variance of a series. Mathematically ARCH (q) model is represented as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \tag{7}$$

where α_0 is the constant coefficient, α_i are parameter estimates and u_{t-i} are the error variance for y_t.

Suppose we are modeling the error variance of a series y_t the ARCH (1) model for the error variance for y_t is the condition on Y_{t-1} at time t (it implies q=1).

Mathematically ARCH (1) model is represented as follow:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{8}$$

We impose the constraints that $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ to avoid negative variance.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Generalized Autoregressive Conditional Heteroscedastic: GARCH (p, q) Model

The past squared observation value and past variance are used by the GARCH model to model the variance at time t. The conditional variance is allowed to depend on prior lags by the model. The models gauge how much a volatility shock from today will affect volatility in the coming term. It gauges how quickly this effect has subsided over time. The definition of GARCH (p, q) model is

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{9}$$

where α_0 and α_i are defined in Equation (7)and β_i are the conditional variance for y_t.

An example of the GARCH (1, 1) model is below

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{10}$$

This is a GARCH (1, 1) model? Is known as the conditional variance since it is a one-period ahead estimate for the variance.

GARCH IN MEAN [GARCH-M (p, q)] Model

High risk in the financial market is anticipated to result in a high return. One might take the GARCH IN MEAN model into account in these circumstances. The model permits a sequence's conditional variance to affect its conditional mean.

The GARCH-Model is as follows:

$$\gamma_t = \mu + \lambda\sigma_t^2 + y_t \tag{11}$$

$$y_t = \sigma_t \varepsilon_t,\ \varepsilon_t \sim (0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{12}$$

Where □ and μ are constant, if □ is positive the return is also positively related to volatility (Wiri & Sibeate, 2020).

Asymmetric Models

There is a need to discuss the Asymmetric GARCH model because bad news (negative shocks) tend to have a greater impact on volatility than good news (positive shocks). We therefore limited our research to the more widely used Asymmetric GARCH models, such as E-GARCH, T-GARCH, and APGARCH.

Exponential GARCH [E-GARCH(p,q)] Model

The Exponential GARCH (E-GARCH) model is superior to the plain GARCH specification in a number of ways. (σ_i^2) will be positive even if the parameters are negative since the $\log(\sigma_i^2)$ is modeled. Second, if the connection between volatility and returns is negative γ, asymmetries are permitted under the E-GARCH formulation. The model is represented as follows:

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

$$log\ log\ h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \frac{|\varepsilon_{t-1}| + \gamma_i \varepsilon_{t-1}}{h_{t-1}^{1/2}} + \sum_{j=1}^{q} \beta_j\ log\ log\ h_{t-j} \quad (13)$$

where γ is leverage effect coefficient (If $\gamma > 0$ it indicates the presence of leverage effect) and $\sigma_t^2 = h_t$.

Note that when ε is positive there is good news, when ε is negative, there is bad news.

Threshold GARCH[T-GARCH(p,q)] Model

The Threshold GARCH (TGARCH) model, which has the following form, is another GARCH technique that can model leverage effects.

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \gamma_1 \varepsilon_{t-1}^2 s_{t-1} + \sum_{j=1}^{q} \beta_j h_{t-j} \quad (14)$$

Where$s_{t-1} = \{1\ if \varepsilon_{i-1<0}\ 0\ if \varepsilon_{i-1\geq 0}$ (15)

where γ is leverage effects coefficient (if$\gamma > 0$ it indicates the presence of leverage effect). That is depending on whether ε is above or below the threshold value of zero, ε_t^2 has different effects on conditional variance h_t when ε_{t-1} is positive (Wiri & Sibeate, 2020).

Asymmetric Power ARCH[APARCH (p,q)] Model

Asymmetric effect and power transformation of the variance are supported by the Asymmetric Power ARCH (APARCH) model. The conditional variance is specified as follows:

$$\sigma_t^2 = \alpha_0 z_t + \sum_{i=1}^{q} \alpha_i (|u_t| - \gamma_t u_t) + \sum_{j=1}^{p} \beta_j \sigma_{t-1}^2 \quad (16)$$

where$\sigma_t = \sqrt{h_t}$, the parameter γ (assumed positive and ranging between 1 and 2)

The Volatility Jumps Models

The first jump volatility model was proposed by Harvey and Chakravarty (2008) and was created by rewriting GARCH(1,1) in Equation (10) as

$$\sigma_t^2 = w + \alpha_1 z_{t-1}^2 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (17)$$

$$\sigma_t^2 = w + \alpha_1 (z_{t-1}^2 - 1)\sigma_{t-1}^2 + (\alpha_1 + \beta_1)\sigma_{t-1}^2 \quad (18)$$

Which is finally written as:

$$\sigma_t^2 = w + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \varphi_1 \sigma_{t-1}^2 \quad (19)$$

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Where$\varphi_1 = \alpha_1 + \beta_1 u_t = z_{t-1}^2 - 1$ and is proportional to the score of the conditional distribution of $\varepsilon_1$concerns σ_{t-1}^2. This is the Beta-GARCH model because $\frac{(u_t+1)}{(v+1)}$ has a Beta distribution and the innovations u_t are given as:

$u_t = z_t^2 - 1$ for Normal distribution, $u_t \approx N(0,1)$ (20)

$u_t = \frac{(v+1)z_t^2}{v-2+z_t^2} - 1$ for Student-t distribution, $z_t \approx T(0,1,v)$ (21)

$u_t = 0.5v|z_t|^v/\lambda_v^v - 1$ for Generalized Error Distribution (GED), $z_t \approx GED(0,1,v)$ and $u_t = \frac{(v+1)z_t z_t^*}{(v-2)g_t \quad 1} - 1$ for Skewed Student-t distribution, $z_t \approx skT(0,1,,v)$

$where\, z_t^* = sz_t + m, I_t = sign\,(z_t^*) = I(z_t^* \geq 0) - (z_t^* < 0), g_t = 1 + \frac{z_t^{*2}}{(v-2) \quad 2I_t}$,

$m = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(- \frac{1}{\ }\right)$ and $s = \sqrt{\left(\quad ^2 + \frac{1}{\ ^2} - 1\right) - m^2}$

Methods of Estimation of GARCH Models

Maximum Likelihood Function (MLF)

The maximum likelihood estimator is the technique used to estimate the GARCH model. The technique is used to determine the parameter value that is most likely given the actual series. The GARCH model is estimated in the following phase.

(i) Specify the mean and variance equation, example (AR(1) and GARCH(l,1) model)

$y_t = \mu + \theta y_{t-1} + \mu_t \quad \mu \sim (0, \sigma_t^2)$ (22)

$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ (23)

(ii) Estimate the likelihood function to maximize the normality assumption of disturbance terms.

$$log\, log\, L = -\frac{T}{2} log\, log\, (2\pi) - \frac{1}{2}\sum_{t=1}^{T} \quad log\, log\, (\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T} \quad \frac{\mu_t^2}{\sigma_t^2} \qquad (24)$$

Model Selection Criteria

The Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion are the most popular information criteria (SIC). Below is a list of the information criteria and their formula:

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)
www.abjournals.org

AIC $= ln\, ln\,|\quad_{r}| + \frac{2}{T}\, MK^2$ (25)

➢ HQIC $= ln\, ln\,|\quad_{r}| + \frac{2 ln ln\, T}{T}\, MK^2$
(26)

➢ SIC $= ln\, ln\,|\quad_{r}| + \frac{ln ln\, T}{T}\, MK^2$
(27)

T is the number of observations (after accounting for lags)

M is the total number of parameters, including the constant, that are calculated in each equation of the unrestricted system. $ln\, ln\,|\quad_{r}|$ is the natural log of the determinant of the covariance matrix of residuals of the restricted system. In each case, MK^2 is the number of M-Garch parameters in a model with order M (Chris, 2008).

RESULT AND DISCUSSIONS

Descriptive Measures of the Time Series Variables

The monthly data on the price of crude oil (in US dollars), the exchange rate between the naira and the dollar, the consumer price index (in naira), and the inflation rate (in naira) in Nigeria from January 2005 to December 2021 (204 observations) were utilized to achieve the study's aims. The raw data of these series were obtained from the website http:www.centralbank.com.

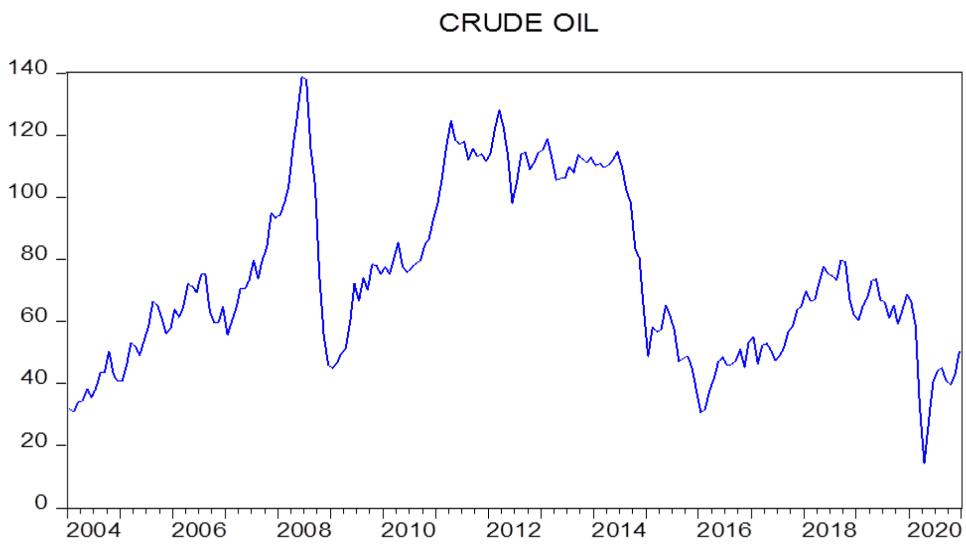

Figure 1: Time Plot of Crude Oil Price Series (US Dollars) at Level

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Figure 1 shows a temporal plot of the level price of crude oil in Nigeria, with the horizontal axis representing time and the vertical axis representing series (months). The time plot of crude oil prices at various levels revealed erratic movement, which is proof that the Crude Oil Price series is subject to a non-stationary process. Over the course of the observations, the series demonstrates constant fluctuation (volatility).

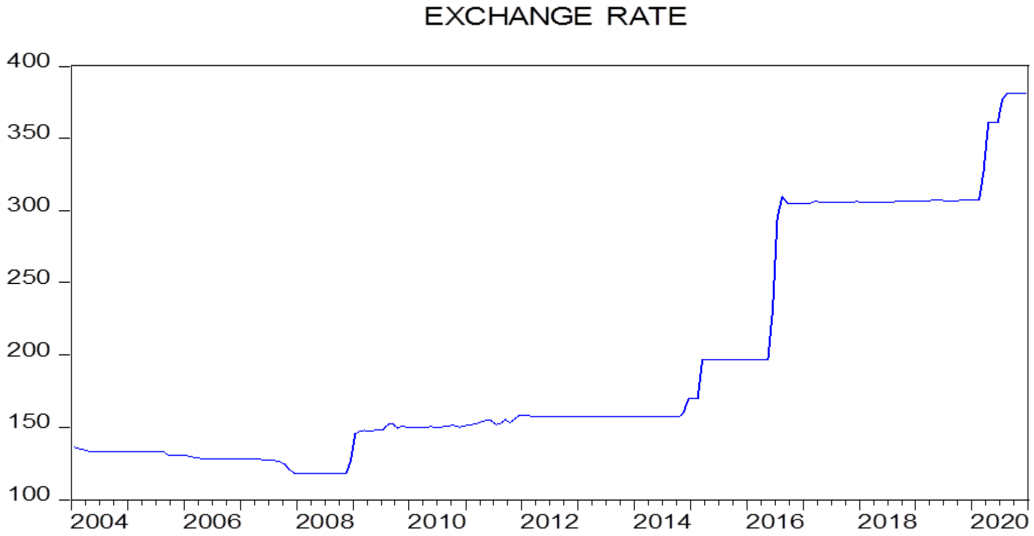

Figure 2: Time Plot of Exchange Rate Series (Naira\US Dollars) at Level

The level time plot of the exchange rate series is shown in Figure 2. While the horizontal axis denotes time, the vertical axis shows series (months). The upward (positive) trend that the exchange rate time plot displayed at the levels is evidence of the non-stationary process that underlies the exchange rate series. This shows significant volatility levels and a consistent upward movement during the course of the observations.

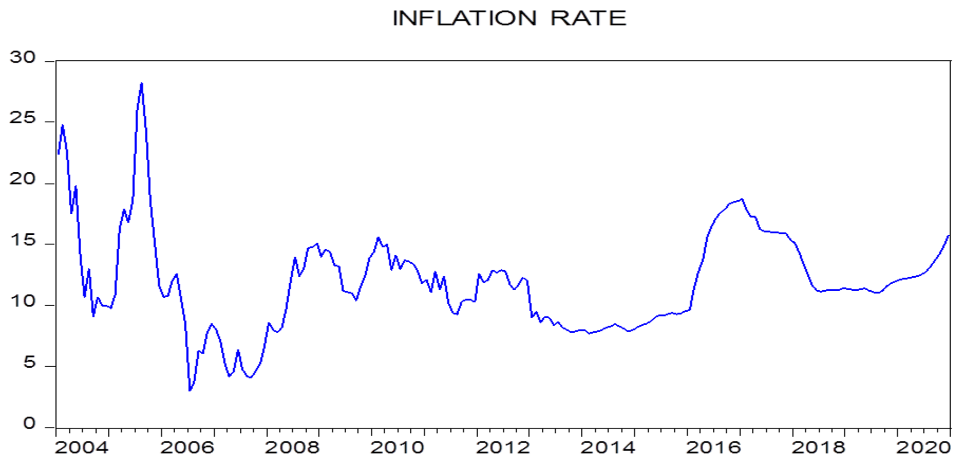

Figure 3: Time Plot of Inflation Rate Series (Naira) at Level

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)

The horizontal axis in Figure 3's Inflation Rate Series Plot shows time, and the vertical axis denotes series (months). At certain levels, the inflation rate exhibited erratic movement and a consistent tendency. A sign of the non-stationary process that exists in the process is the series' consistent fluctuation.

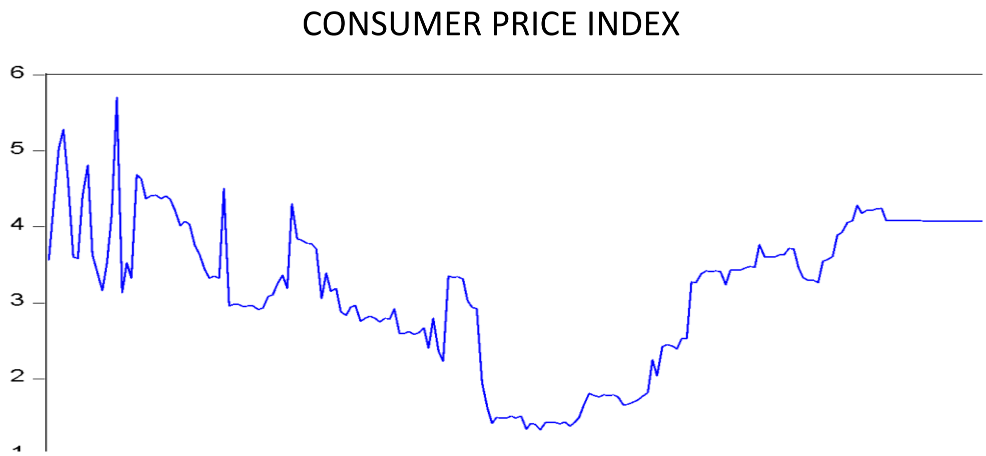

Figure 4: Time Plot of Consumer Price Index Series (Naira) at Level

Figure 4's time plot of the Consumer Price Index series revealed a pattern distinct from that of other series. Beginning to fluctuate downward till the year 2012, the series progressively began to fluctuate higher until the year 2020. While the horizontal axis denotes time, the vertical axis shows series (months). The levels of the series provided evidence of the non-stationary nature of the underlying data.

Since the research variables are non-stationary at levels, the first differences between the four series are shown in Figures 5 to 8. Differentiation is required to achieve stationarity in the process. Figures 5 to 8 display the four variables' returns (first differences). Figures 5 to 8 all show series that were volatile at zero, indicating a stationary process.

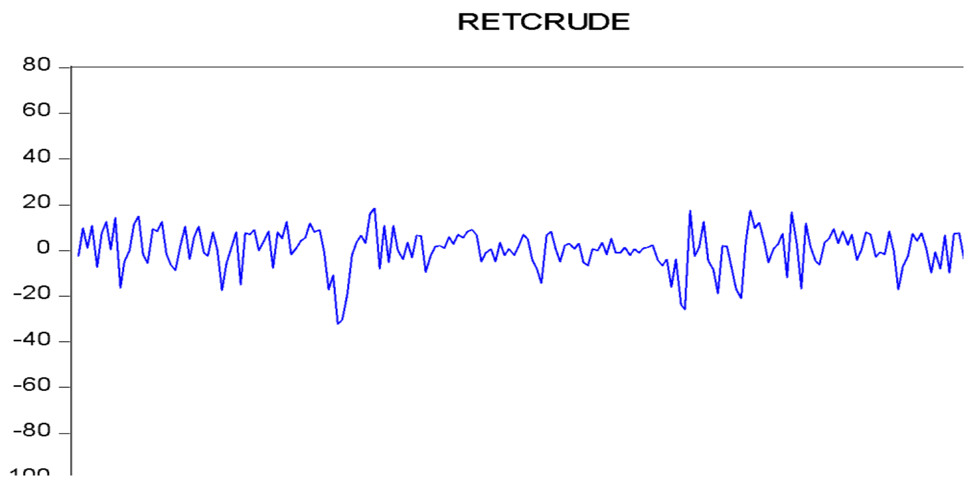

Figure 5: Time Plot of the Return Crude Oil Price Series at First Difference

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

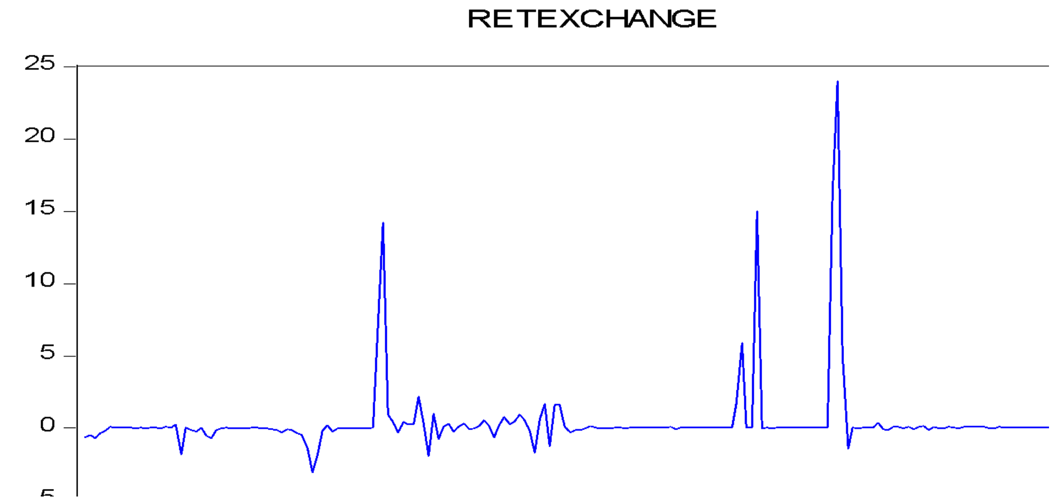

Figure 6: Time Plot of the Return Exchange Rate Series at First Difference

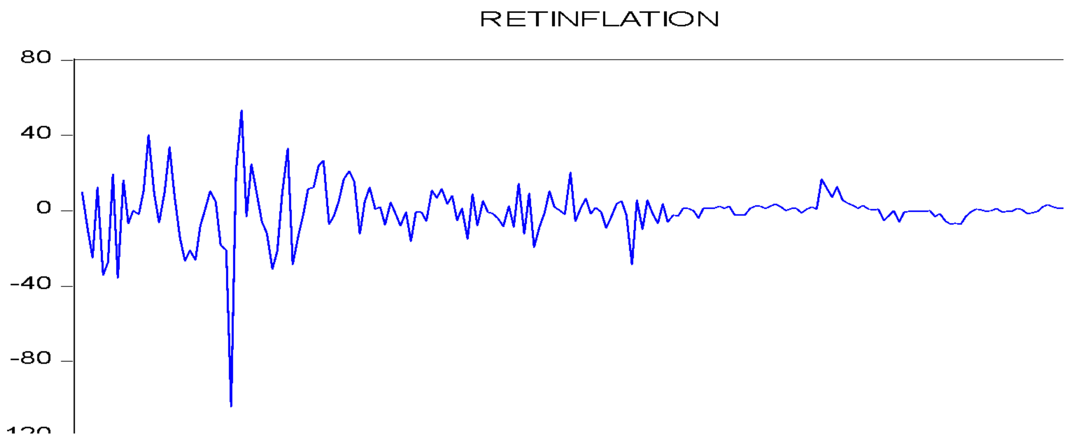

Figure 7: Time Plot of the Return Inflation Rate Series at First Differences

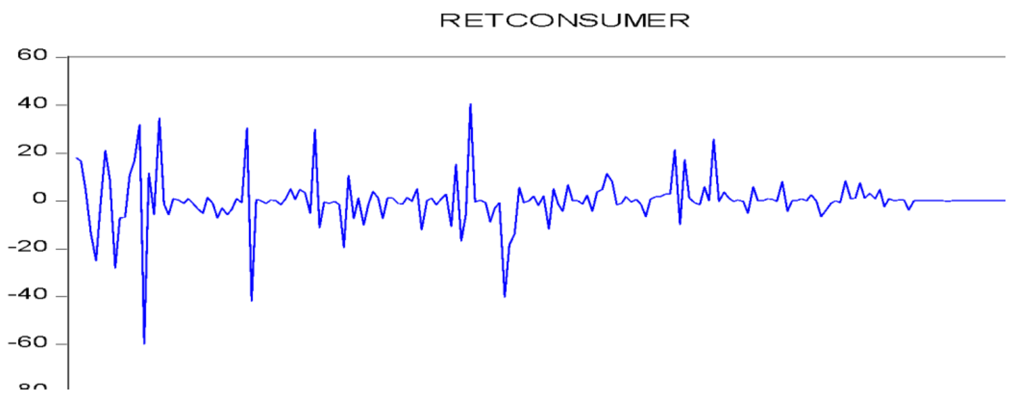

Figure 8: Time Plot of the Return Consumer Price Index Series at first Difference

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)

Stationarity Test

The Augmented Dickey-Fuller (ADF) test was used to examine the four variables in the context of stationery. The results of the (ADF) test at the level and first differences are presented in detail in Table 1, together with the probability values (p-values) in brackets. Except for the inflation rate, some of the variables' probability values (p-values) at a level greater than 0.05 (p-values >0.05) are present. The outcome demonstrated the level's possession of a unit root. The four series were found to be stationary at first when the probability values of all the variables were evaluated. The probability values are (0.000) in Table 1, indicating that the series is stationary at first differences and has zero volatility.

The terms used to denote them are, respectively, REXCHt (Return on the Exchange Rate), RINFt (Return on Inflation Rate), RCOPt (Return on Crude Oil Price), RCPIt (Return on Consumer Prices Index), and RINFt (Return on Exchange Rate) in the Appendices of the EView Statistical Software.

Table 1: Augmented Dickey-Fuller (ADF) Unit Roots Test

Variable	ADF at Levels	ADF at 1st Difference
Crude Oil Price	-2.355 (0.156)	-12.47002(0.000**)
Exchange Rate	-0.394(0.903)	-7.4183(0.000**)
Inflation Rate	-3.35 (0.0100**)	-10.7179 (0.000**)
Consumer Price Index	-2.555(0.256)	-16.79601(0.000**)
Test critical values:		
1% level	-3.46482	
5%level	-2.87659	
10% level	-2.57487	

Footnote: **= Sig. at 5% (or ADF < Test critical values)

Table 2: Descriptive Statistics of the four variables

Statistics	Consumer Price Index	Crude Oil Price	Exchange Rate	Inflation Rate
Mean	3.214853	73.55294	193.4520	11.91152
Median	3.380000	66.84000	157.2900	11.52000
Maximum	5.700000	138.7400	381.0000	28.20000
Minimum	1.330000	14.28000	117.7200	3.000000
Std. Dev.	0.9508 19	27.25655	78.28099	4.045674
Skewness	-0.445565	0.419232	0.995066	0.8455 15
Kurtosis	2.469075	2.133668	2.429747	4.828892
Jarque-Bera	9.145950	12.35520	36.42943	52.73764
Probability	0.010327 **	0.002075**	0.0000**	0.0000**
Observations	204	204	204	204

Footnote: **= Sig. at 5%.

 Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

The descriptive statistics for individual series are shown in Table 2 and include mean, median, maximum, minimum, standard deviation, skewness, excess kurtosis, and the Jarque-Bera test statistic. All of the indicators' sample means are positive, indicating that they are positive mean returning, but the return series' corresponding standard deviation is substantially higher. Additionally, all skewness statistics except for the consumer prices index, which has a negative sign, are positive (-0.445556). This simply means that the left tail of the series is larger than the severe loss of the right tail (extreme gain). One of the frequently occurring features of financial time series data is the occurrence of indicators that are skewed to the left. Additionally, the kurtosis numbers of 3.971, 3.891, and 9.618 indicate the existence of fattail. Jarque-Bera test statistics for individual series p-values (145950, 12.35520, 36.42943, and 52.73764) show that there is evidence that data sets showing the presence of fat tail and they are all statistically significant at levels more than 0.0010, 0.0020, and 0.0000, respectively.

Using lag length selection techniques like the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion, the exact lag length (p) for a Univariate and Multivariate (Robust)series is chosen (SIC). On the basis of criteria for minimizing information, the ideal lag length is chosen (Chris, 2008).

Table 3: Lag Selection Criteria

Lag	LogL	LR	FPE	AIC	SIC	HQIC
0	-2744.595	NA	20567156	28.19071	28.25785 *	28.21790
1	-2707.064	73.13644 *	16492636 *	27.96989 *	28.30558	28.10581 *
2	-2693.296	26.26604	16878132	27.99278	28.59702	28.23743
3	-2684.668	16.10482	18213501	28.06839	28.94119	28.42178
4	-2677.308	13.43662	19920749	28.15701	29.29836	28.61913
5	-2672.359	8.832720	22346743	28,27035	29.68025	28.84120
6	-2659.820	21.86268	23208082	28.30584	29.98431	28.98543

According to Table 3 above, the Univariate and Multivariate (Robust) series model of inflation rate, exchange rate, consumer price index, and crude oil price based on Schwarz Information Criterion (SIC), Akaike Information Criterion (AIC), and Hannan-Quinn Information Criterion, the lag of order one (p=l) is good (HQIC). One is the ideal lag duration because it minimizes the

information criteria.

Univariate GARCH Models Comparison (Symmetric and Asymmetric Models)

Then, using the primary conditional mean model for volatility and the symmetric and asymmetric models stated in Section above, the Univariate model volatilities for each individual series were completed. As a result of the ideal lag length being 1, The subsequent conditional mean models were taken into account and constructed with lag 1.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Symmetric:

1) ARCH (1) or GARCH (0,1) Model

From Equation (7); If p=0 and q=1, we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 \tag{28}$$

where α_0 is the constant coefficient and α_1 is the coefficient of the error of conditional variance of the series.

2) GARCH (1,1) Model

From Equation (9); If p=1 and q=1, we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{29}$$

where α_0 and α_1 are defined in Equation (4.1). β_1 is known as the conditional variance since it is a one period ahead estimate for the variance.

3) GARCH-M (1,1) Model

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{30}$$

where $u_{t-1}^2 = \sigma_t e_t$ and $e_t \sim \mathrm{N}\left(0, \sigma_t^2\right)$ is the normal distribution error of the conditional variance of the series. α_0, α_1 and β_1 are defined in Equation (29).

Note that: ARCH (1) model has two parameters estimated, while GARCH (1,1) model and GARCH-M (1,1) model have three parameters estimated.

Asymmetric models:

(1) APARCH (1,1) model

Similarly, from Equation (13); If p=1 and q=1, we have

$$h_t = \alpha_0 + \alpha_1\left(\left|e_{t-1}\right| - \gamma\, e_{t-1}\right) + \beta_1 h_{t-1} \tag{31}$$

and $h_t = \sigma_t^\phi$, where ϕ is the power transformation of the variance. It implies that

$$\sigma_t^\phi = \alpha_0 + \alpha_1\left(\left|e_{t-1}\right| - \gamma\, e_{t-1}\right) + \beta_1 \sigma_{t-1}^\phi$$
$$\sigma_t^\phi = \alpha_0 + \alpha_1\left|e_{t-1}\right| + \left(-\alpha_1\gamma\right)e_{t-1} + \beta_1 \sigma_{t-1}^\phi$$
$$\sigma_t^\phi = \alpha_0 + \alpha_1\left|e_{t-1}\right| + \alpha_2\, e_{t-1} + \beta_1 \sigma_{t-1}^\phi \tag{32}$$

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23)

where $\alpha_2 = -\alpha_1 \gamma$ and γ is leverage effect coefficient.

Note that: APARCH (1, 1) model has five parameters estimated $\alpha_0, \alpha_1, \alpha_2, \beta_1$ and ϕ.

(2) T-GARCH (1,1) model

Likewise, from Equation (14); If p=1 and q=1, we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \gamma e_{t-1}^2 e_{t-1} + \beta_1 \sigma_{t-1}^2$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-1}^2 e_{t-1} + \beta_1 \sigma_{t-1}^2 \quad (33)$$

where $h_t = \sigma_t^2$; $\alpha_2 = \gamma$, γ is leverage effect coefficient.

Note that: T-GARCH (1, 1) model has four parameters estimated $\alpha_0, \alpha_1, \alpha_2$, and β_1.

(3) E-GARCH (1,1)

Likewise, from Equation (3.16); If p=1 and q=1, we have

$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \left(\frac{[e_{t-1}] + \gamma e_{t-1}}{\sigma_{t-1}} \right) + \beta_1 \ln \sigma_{t-1}^2$$
$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \left({|e_{t-1}|}/{\sigma_{t-1}} \right) + \alpha_1 \gamma \left({e_{t-1}}/{\sigma_{t-1}} \right) + \beta_1 \ln \sigma_{t-1}^2$$
$$\ln \sigma_t^2 = \alpha_0 + \alpha_1 \left({|e_{t-1}|}/{\sigma_{t-1}} \right) + \alpha_2 \left({e_{t-1}}/{\sigma_{t-1}} \right) + \beta_1 \ln \sigma_{t-1}^2 \quad (34)$$

where $h_t = \sigma_t^2$; $\alpha_2 = \alpha_1 \gamma$, γ is leverage effect coefficient.

Note that: E-GARCH (1, 1) model has four parameters estimated $\alpha_0, \alpha_1, \alpha_2$, and β_1.

Conditional Mean Model: Symmetric and Asymmetric Models

Equations (28) to (34)'s symmetric and asymmetric models were created using the E-view statistical program in Appendices C and D. Tables 4, 5, and 6 provide summaries of the outcomes of the six models utilizing model parameter estimates and model selection criteria (or Information Criteria).

 Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Table 4a: Symmetric Models Parameter Estimates

Variables	Model Parameters	ARCH (1) Model Estimated Coefficients (p-value)	GARCH (1,1) Model Estimated Coefficients (p-value)	GARCH-M (1,1) Model Estimated Coefficients (p-value)
Crude Oil Price	α_0	53.559(0.0000**)	9.7500 (0.0726*)	13.7557(0.0381**)
	α_1	0.5719(0.0000**)	0.7302 (0.0000**)	0.7163(0.0000**)
	β_1		0.4028(0.0000**)	0.3614(0.0000**)
	$\alpha_1+\beta_1$	0.5719	**1.1320**	**1.0777**
Inflation Rate	α_0	29.0350(0.0000**)	1.8193(0.0000**)	1.92919(0.0000**)
	α_1	1.7002(0.0000**)	0.2731(0.0000**)	0.2725(0.0000**)
	β_1		0.7303(0.0000**)	0.7506(0.0000**)
	$\alpha_1+\beta_1$	**1.7002**	**1.0034**	**1.0231**
Exchange Rate	α_0	3.4933(0.0000**)	1.7684(0.0000**)	1.7479(0.0000**)
	α_1	0.6199(0.0017**)	0.8782(0.0045**)	0.8930(0.0064**)
	β_1		0.3291(0.0000**)	0.3139(0.0000**)
	$\alpha_1+\beta_1$	0.6199	**1.2073**	**1.2069**
Consumer Price Index	α_0	48.2471(0.0000**)	-0.0511(0.0269**)	-0.0527(0.0280**)
	α_1	0.9797(0.0001**)	0.0928(0.0000**)	0.0606(0.0000**)
	β_1		0.9028(0.0000**)	0.9042(0.0000**)
	$\alpha_1+\beta_1$	0.9797	0.9956	0.9648

Footnote: **= Sig. at 5%,*= Sig. at 10%; If $\alpha_1+\beta_1 > 1$, "High level of Volatility".

All of the symmetric models' parameters are significant at 5%, according to Table 4a. As a result, it is possible to predict each individual series using any of the symmetric models, including the ARCH (1) Model, GARCH (1,1) Model, and GARCH-M (1,1) Model.

Table 4b: Symmetric Models level of Volatility

Model /Variables	Parameters	ARCH (1) Model	GARCH (1,1) Model	GARCH-M (1,1) Model
Crude Oil Price		Low level	High level	High level
Inflation Rate		High level	High level	High level
Exchange Rate		Low level	High level	High level
Consumer Price Index		Low level	Low level	Low level

Except for the Consumer Prices Index series, which has a low degree of volatility in all Symmetric models, Table.4b displays the univariate model volatilities for the individual series with high levels of volatility. Table 4b also demonstrates that all symmetric models for the inflation rate series exhibit large levels of volatility. For all individual series with the exception

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

of the return of the Consumer Price Index series, which does not exhibit high levels of negative news (or volatility) in the global market, GARCH (1, 1) and GARCH-M (1,1) symmetric models for the return series satisfy the stationarity condition of $\alpha_1 + \beta_1 < 1$ (low level of volatility).

Table 5a: Asymmetric Models Parameter Estimates

Variables	Model Parameters	APARCH (1,1) modelEstimated Coefficients (p-value)	T-GARCH (1,1) modelEstimated Coefficients (p-value)	E-GARCH (1, 1) Model Estimated Coefficients (p-value)
Crude Oil Price	α_0	283.8996(0.8501)	47.5924(0.0000*)	0.7183(0.1342)
	α_1	0.4159(0.0487**)	0.0698(0.0078**)	0.8410(0.0000**)
	α_2	0.5449(0.0540*)	0.9723(0.0000**)	-0.2751(0.0497**)
	β_1	0.0407(0.5645)	0.0856(0.3169)	0.6922(0.0000**)
	ϕ	2.8445(0.2676)		
	γ	1.3102	0.9723	0.3271
	$\alpha_1 + \beta_1$	0.4566	0.1554	**1.5332**
Inflation Rate	α_0	0.0393(0.4956)	2.6117(0.0000*)	-0.1088(0.0000**)
	α_1	0.1149(0.0000**)	0.1818(0.0823*)	0.0528(0.0000**)
	α_2	-0.1391(0.2037)	0.2879(0.0017**)	0.0291(0.3577)
	β_1	0.8968(0.9021)	0.7169(0.0000**)	0.9960(0.0000**)
	ϕ	0.4685(0.6114)		
	γ	1.2106	0.2879	0.5511
	$\alpha_1 + \beta_1$	**1.0117**	0.8987	**1.0488**
Exchange Rate	α_0	287.7628(0.9511)	1.7471 (0.0000**)	1.8095(0.0000**)
	α_1	1.7656(0.9840)	1.2673 (0.0181**)	1.3025(0.0575*)
	α_2	-0.4247(0.9388)	-1.3680(0.0165**)	-1.00217(0.4620)
	β_1	0.00094(0.9616)	0.3149(0.0000**)	0.4184(0.0000**)
	ϕ	13.0846(0.7309)		
	γ	0.2405	-1.3680	-0.7694
	$\alpha_1 + \beta_1$	**1.7750**	**1.5822**	**1.7209**
Consumer Price Index	α_0	-0.0243(0.1493)	-0.0739 (0.0026**)	-0.0895(0.0000**)
	α_1	-0.00117(0.6310)	-0.3589 (0.0000**)	-0.0255(0.0575*)
	α_2	0.9497(0.5152)	0.5601(0.0000**)	-0.00217(0.8602)
	β_1	1.0140(0.0000**)	0.9936(0.0000**)	1.0184(0.0000**)
	ϕ	0.20346(0.0048**)		
	γ	811.709	0.5601	39.937
	$\alpha_1 + \beta_1$	**1.0023**	0.6347	0.9929

Footnote: **= Sig. at 5%,*= Sig. at 10%; If $\alpha_1 + \beta_1 > 1$, "High level of Volatility".

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 2, 2023 (pp. 1-23) www.abjournals.org

Note: [APARCH (1, 1) model $\gamma = \frac{\alpha_2}{-\alpha_1}$], [T-GARCH (1, 1) model $\gamma = \alpha_2$] and [E-GARCH (1, 1) model $\gamma = \frac{\alpha_2}{\alpha_1}$] where γ is leverage effect coefficient.

According to Table 4.5a, among all the asymmetric models, the parameter estimates for the T-GARCH (1, 1) and E-GARCH (1, 1) models are respectively significant at 5% and 10% for the individual series. As a result, it appears that the APARCH (1,1) Asymmetric model is inferior to the T-GARCH (1, 1) and E-GARCH (1, 1) Asymmetric models. Hint: You can use the two asymmetric models to predict each unique series. (Crude Oil Price, Inflation Rate, Exchange Rate and Consumer Price Index).

Table 5b: Asymmetric Models level of Volatility

Model Parameters /Variables	APARCH (1,1) model	T-GARCH (1,1) model)	E-GARCH (1, 1) Model
Crude Oil Price	Low level	Low level	High level
Inflation Rate	High level	Low level	High level
Exchange Rate	High level	High level	High level
Consumer Price Index	High level	Low level	Low level

Leverage effect γ is present and statistically significant in Table 4.5b's asymmetric models for the individual return series. Except for the return of the Consumer Prices Index series, the E-GARCH (1, 1) Asymmetric Model for the individual return series satisfy the covariance stationary condition, $\alpha_1 + \beta_1 < 1$ (Crude Oil Price, Inflation Rate, and Exchange Rate), which implies high levels of bad news (or volatility) in the international market.

Table 6: Comparison of the Symmetric and Asymmetric Models Selection Criteria

Variables	Model Selection Criteria	Symmetric Models			Remark	Asymmetric Models			Remark
		ARCH (1) Model	GARCH (1,1) Model	GARCH H-M (1,1) Model		APARCH (1,1) model	T-GARCH (1,1) model	E-GARCH (1,1) Model	
Crude Oil Prices	AIC	7.3454	7.3219	**7.3140**	GARCH H-M (1,1) Model	7.2936	7.2833	**7.2676**	**E-GARCH (1, 1) Model**
	BIC	7.3781	7.3708	**7.3703**		7.3752	7.3486	**7.2239**	
	HQIC	7.3586	7.3416	**7.3404**		7.3266	7.3097	**7.2941**	

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Inflation Rate	AIC	7.4869	**7.2108**	7.2153	GARCH (1,1) Model	7.3419	7.2058	**7.1923**	**E-GARCH (1, 1) Model**
	BIC	7.4396	**7.2598**	7.2808		7.4235	7.2710	**7.2576**	
	HQIC	7.4201	**7.2306**	7.2420		7.3749	7.2322	**7.2187**	
Exchange Rate	AIC	4.2864	4.2615	**4.2537**	GARCH-M (1,1) Model	4.2256	**4.2314**	4.2334	**TS-GARCH (1,1) model)**
	BIC	4.3191	4.3104	**4.3100**		4.3072	**4.2967**	4.2976	
	HQIC	4.2997	4.2813	**4.2801**		4.2586	**4.2578**	4.2582	
Consumer Price Index	AIC	7.2129	**6.7645**	6.7732	GARCH (1,1) Model	6.5867	6.6087	**6.5426**	**E-GARCH (1, 1) Model**
	BIC	7.2456	**6.8135**	6.8385		6.6683	6.6740	**6.6079**	
	HQIC	7.2261	**6.7843**	6.7996		6.6197	6.6351	**6.5690**	

NOTE: **Asymmetric Models** have least values model selection criteria

For all of the return series in Table 6, six models were estimated, three for the symmetric model and three for the asymmetric. By contrasting the symmetric models (ARCH, GARCH, and GARCH-M) and the asymmetric models, the best model was chosen using Akaike Information Criteria (E-GARCH, T-GARCH and APARCH). The outcome showed that for the investigated univariate economic variables, the found asymmetric model GARCH models outperformed the symmetric model GARCH models. As a result, these models can be used to predict this series of economic variables. The models are the Asymmetric T-GARCH (1, 1) model for Exchange rate series and the Asymmetric E-GARCH (1, 1) model for Consumer Price Index, Crude Oil Price, and Inflation Rate Series. By substituting the estimated coefficients, the following models are identified: Crude Oil Price Series: For the asymmetric process in Table 6, the E-GARCH (1, 1) model was chosen.

E-GARCH (1,1) model: $$\ln \hat{\sigma}_t^2 = 0.7173 + 0.8410\left(\frac{|e_{t-1}|}{\sigma_{t-1}}\right) - 0.2762\left(\frac{e_{t-1}}{\sigma_{t-1}}\right) + 0.6920 \ln \sigma_{t-1}^2$$

Inflation Rate Series: The E-GARCH (1, 1) model was selected for the Asymmetric process in Table 6.

E-GARCH (1,1) model $$\ln \hat{\sigma}_t^2 = -0.1088 - 0.0528\left(\frac{|e_{t-1}|}{\sigma_{t-1}}\right) + 0.0291\left(\frac{e_{t-1}}{\sigma_{t-1}}\right) + 0.9960 \ln \sigma_{t-1}^2$$

Exchange Rate Series: The T-GARCH (1, 1) model was selected for the Asymmetric process in Table 6.

T-GARCH (1, 1) model: $$\hat{\sigma}_t^2 = 1.7471 + 1.2673 e_{t-1}^2 - 1.3680 e_{t-1}^2 e_{t-1} + 0.3149 \sigma_{t-1}^2$$

Consumer Prices Index Series: The E-GARCH (1, 1) model was selected for the Asymmetric process in Table 6.

E-GARCH (1,1) model: $$\ln \hat{\sigma}_t^2 = -0.0895 - 0.0255\left(\frac{|e_{t-1}|}{\sigma_{t-1}}\right) - 0.00217\left(\frac{e_{t-1}}{\sigma_{t-1}}\right) + 1.0184 \ln \sigma_{t-1}^2$$

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Note: Based on the findings in Table 6, it was determined that, for all of the economic variables, the asymmetric models outperformed the symmetric models in terms of the model selection criterion.

SUMMARY AND CONCLUSION

For all of the return series in Table 6, six models were estimated; three for the symmetric model and three for the asymmetric. In light of the fact that all variables were stationary at lag 1, the best model was chosen (or information criteria). The study looked at changes in Nigeria's crude oil price, consumer price index, inflation rate, and univariate exchange rate. Monthly secondary datasets from January 2005 to December 2021 from the Central Bank of Nigeria's (CBN) statistical bulletin and simulated data sets were used in this study.

CONCLUSION

The series' erratic movement was found via a critical analysis of the time plot. A series is said to be stationary if its mean and variance are constant; the presence of a trend will render it non-stationary. Since the variables of the study cannot be employed for analysis unless it is proven that the variables are stationary, the variables used in this study were evaluated for stationarity. To prevent the issue of spurious regression, the stationarity of the data for each series was checked. For each of the variables, the Augmented Dickey-Fuller (ADF) test was used to check for the presence of a unit root. The probability values (p-values) at level were all greater than 0.05 (p-values >0.05) in all the variables, and the result indicated the presence of a unit root because the series is non-stationary. Since the data is non-stationary, the first difference in order is necessary to prevent erroneous regression from occurring. The series was stationary according to the test for stationarity using the p-value for each variable at the first differences. Given that the series is stationary, the probability values are all (0.000**), and the null hypothesis was rejected.

The lag duration of the models was calculated using the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion (SIC). The table demonstrates that the model based on the data is appropriate and the lag of order one (p=l) (AIC, HQIC and SIC). Although they have the minimal information criteria, the Akaike Information Criterion (AIC) and HQIC have the best lag duration of order (1).

Symmetric and asymmetric processes were used in the Univariate GARCH models for each return series. The following information criteria (AIC, SIC, and HQIC) were carefully compared with the process. The outcome showed that for the investigated univariate economic variables, the found asymmetric model GARCH models outperformed the symmetric model GARCH models.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

REFERENCES

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics, 31*(3), 307-327.

Central Bank of Nigeria. www.cbn.gov.ng

Chris, B. (2008). *Introductory econometrics for finance* (3rd Edition). Cambridge University Press. Central Bank of Nigeria (www.cbn.gov.ng)

Cyprian, O., Peter, N. & Anthony, G. (2017). Using conditioner extreme value theory to estimate value-at-risk for daily currency exchange rates. *Journal of Mathematical Finance, 7*(4), 846-870.

Daniel, E. & Fola J. A. (2015). Seasonal component of rainfall in warn town from 2003 to 201 2. *Journal of Geoscience and Environment Protection*, *3*(6), 91-98.

Elena, G. & Shen, X. (2018).*Analysis of asymmetric GARCH volatility models with application to margin measurement*. Bank of Canada Staff Working Paper.

Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory, 11*(1), 122-150.

Engle, R.F. (1982). Autoregressive conditional heteroskedasticity estimates of the variety of UK inflation. *Economical*, *50*(4), 987-1008.

Green, W. H. (2012). *Econometric Analysis* (7th ed). Upper Saddle River, NJ: Pearson Prentice Hall.

Harvey, A.C. & Chakravarty, T. (2008). The econometric analysis of time series. Cambridge working papers in Economics from Faculty of Economics, University of Cambridge.

Hentschel, L. (1995). All in the family nesting symmetric and asymmetric GARCH model. *Journal of Financial Economics, 39*(l), 71-104.

Ijomah, M.A. & Enewari, P. (2020). Modeling volatility transmission between crude oil price and the Nigeria naira exchange rate using multivariate GARCH Models. *International Journal of Innovation, Mathematics, Statistics and Energy Policies*, *8*(2), 1-12.

Maryam, T. & Ramanathan, T. V. (2012). An overview of FIGARCH and related time series mode. *Austrian Journal of Statistics, 41*(3), 175-196.

Mathieu, G. & Anissa, C. (2014). Volatility spillovers between oil prices and stock returns: A focus on frontier markets. *The Journal of Applied Business Research, 30*(2), 509-525.

Minoviá, J. (2017). Application and diagnostic checking of univariate and multivariate GARCH models in Serbian financial market. *Economic Analysis, 41*(1-2), 73-87.

Musa, Y., Tasi'u, M. & Abubakar, B. (2014). Forecasting of exchange rate volatility between Naira and US Dollar using GARCH models. *International Journal of Academic Research in Business and Social Science 4*(7), 369-381.

Ojo, M.O. (1998). "Exchange Rates Developments in Nigeria: A Historical Perspective". Text of a paper delivered at a Seminar on "Exchange Rate Determination and Arithmetic" by Unilag Consult.

Omorogbe, J.A. & Ucheoma, C. E. (2017). An application of asymmetric GARCH models on volatility of bank equity in Nigeria stock market. *CBN Journal of Applied Statistics, 8*(1), 73-99.

Osabuohien-Irabor, D. & Edokpa I. W. (2013). Modeling monthly Inflation rates volatility using Generalized Autoregressive Conditional Heteroscedastic (GARCH) model: Evidence from Nigeria. *Australian Journal of Basic and Applied Sciences, 7*(7), 991-998.

Robert, F.E. (2013). Estimates of the variance of U.S inflation based upon the ARCH model. *Journal of Money, Credit and Banking*, *15*(3), 286-301.

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG

Suliman, Z. (2012). Modeling exchange rate volatility using GARCH models: Empirical evidence from Arab countries. *International Journal of Economics and Finance – Toronto, 4*(3), 216-229.

Wiri, L. & Essi, I. D. (2018). Seasonal Autoregressive Integrated Moving Average (SARIMA) modeling and forecasting of Inflation rate in Nigeria. *International Journal of Applied Science and Mathematical Theory, 4*(1), 48-60.

Wiri, L. & Sibeate, P. (2020). A comparative study of Fourier series models and Seasonal ARIMA model of rainfall data in Port Harcourt. *Asian Journal of Probability and statistics, 10*(31), 36-46.

Zakoian J. M. (1994). Threshold Heteroskedasticity models. *Journal of Economic Dynamics and Control,*

Article DOI: 10.52589/AJMSS-JDZ6ZOXG
DOI URL: https://doi.org/10.52589/AJMSS-JDZ6ZOXG