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MULTIVARIATE GARCH MODELS COMPARISON IN TERMS OF THE SYMMETRIC AND ASYMMETRIC MODELS

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ABSTRACT: *Modelling the Multivariate Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model included both symmetric and asymmetric processes. The information includes monthly data for the Consumer Price Index, Crude Oil Price, Exchange Rate, and Inflation Rate from January 2005 to December 2021. For the analysis, E-view Statistical software was employed. The aforementioned four macroeconomic variables show a tendency for volatility to cluster across time. Both symmetrical and asymmetrical processes had the volatility condition. The residual recursive plot was used to analyse the structural fluctuation in the series. The plot showed continual movement in the inflation rate as well as a downward and upward movement in the consumer price index, exchange rate, and crude oil price. The Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion were chosen as the best models based on information criteria (SIC). Symmetric and asymmetric modelling techniques were used to create the Economic Variables Multivariate GARCH (M-GARCH) models. To calculate the covariance and correlation between the four variables, M-GARCH models were utilised. The main finding of the estimation of all M-GARCH models is that the symmetric models (Diagonal BEKK and Constant Conditional Correlation, or "CCC") have the lowest values of the model information criteria compared to the asymmetric models (Diagonal BEKK and CCC), while the asymmetric models (Diagonal VECH) have the lowest values of the model information criteria compared to the symmetric models (Diagonal VECH). Based on the findings, it was discovered that while analysing the interaction between the four economic variables returns series, the Symmetric Diagonal BEKK and CCC model outperformed the Asymmetric Diagonal VECH model.*

KEYWORDS: Multivariate GARCH (M-GARCH) models, Economic (macroeconomic) Variables, structural fluctuation, Hannan-Quinn Information Criterion (HQIC), Schwarz Information Criterion (SIC)

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INTRODUCTION

This study examines the volatility of macroeconomic series and multivariate generalised autoregressive conditional heteroscedasticity (GARCH), with a particular emphasis on patterns of symmetric and asymmetric volatility clustering. There has been a lot of discussion about the GARCH model's various features and excessive volatility in global markets. These variables change constantly, so it would be unreasonable to expect a univariate GARCH model to simply account for these various behaviours in such data. Model identification, estimate, diagnosis, and forecasting are all included in the GARCH model-building process (Daniel & Fola, 2015).The Multivariate Generalised Autoregressive Conditional Heteroscedasticity (M-GARCH) model generalizes the Univariate GARCH model into a Multivariate GARCH model that models variance, covariance, and interactions among the series. There are multiple equations and variables in it. Various exogenous and endogenous variables may also be present, according to Metsileng et al (2020). Although it might not be as flexible as linear models, the Multivariate GARCH model is intended to represent some robust patterns in the data set. However, the Multivariate GARCH model included numerous structural equations that can describe the behaviour of time series in various. This model can capture more complicated dynamic patterns since it allows for interaction between each variable. In more detail, the Multivariate GARCH attribute controls how the state variable's current value is determined by its most recent values. As a result, when switching occurs, one structure may be dominant for a seemingly random amount of time before being replaced by another.

Symmetric and asymmetric multivariate GARCH models have been the subject of extensive research. The Generalised Autoregressive Heteroskedasticity models that Hentschel (1995) presents are all parametric (GARCH). Ijomah and Enewari (2020) looked at the volatility transmission between the price of oil and the exchange rate using multivariate GARCH modelling. BRICS (Brazil, Russia, India, China, and South Africa) currency rate volatility was examined by Metsileng et al. (2020) using Multivariate GARCH modelsFrancq and Zokoian (2015) examined various Multivariate volatility models' estimation methods. Bala and Takimoto (2017) investigated stock return volatility spill-overs in emerging and developed markets using the Multivariate GARCH (M-GARCH) model and its variations. BEKK-GARCH models were used by Chen and Zapata (2015) to simulate volatility and spillover. For the sample period of 2002:M1 to 2009, Mohamed and Abdelkader's (2011) study used the BEKK-GARCH model to construct conditional variances of monthly stock exchange prices, exchange rates, and interest rates for Turkey. Bunnag (2015) examined the correlations and spillovers between the volatility of crude oil, gasoline, heating oil, and natural gas futures. Minovic (2008) evaluated the theoretical and empirical work for multivariate volatility processes diagnostic checking. The model's suitability was tested in the study by Minovic (2017) using the Ljung-Box statistics (Q-stat) of standardised residuals, those of its squared, as well as of the cross product of standardised residuals. The performance of the Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC), and Asymmetric DCC (ADCC) models in estimating the portfolio at risk in the BRICS countries was studied by Bonga-Bonga and Nleya (2016). Nortey et al. (2015) looked into the volatility and conditional relationship between the inflation rate, exchange rate, and interest rates as well as building a multivariate GARCH DCC and BEKK model. To evaluate the interconnectedness and dynamics of volatility in the corn, wheat, and soybean markets in the United States on a daily, weekly, and monthly basis encompassing 1998 to 2012, Gardebroek et al. (2013) used a Multivariate GARCH technique. Hartman and Sedlak (2013) evaluated the performance of the

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two Multivariate GARCH models: BEKK and DCC, using 10 years' worth of exchange rate data. The Multivariate GARCH model was used in Tastan's (2006) study to analyse the relationship between exchange rates and stock market performance. Efimova and Serletis (2014) contrasted the Univariate and Multivariate GARCH models when analysing the empirical characteristics of the volatility of the prices of electricity, natural gas, and crude oil. Chen and Zapata (2015) used data from the sample period of June 1996 to December 2013 to model volatility and spill-over effects using the BEKK-GARCH model. To explore and compare the effects of two financial crises, Chen and Zapata's (2015) study looked at both the short-run and long-run links between equities markets in China and the US (Asian 1997 Financial Crisis and the Subprime 2007 to 2010 Financial Crisis). In order to simultaneously disclose the return transmission and volatility spillover between market return series, Yi et al. (2009) enhanced the fractionally integrated Vector Error Correction Model (VECM) model with the Multivariate GARCH model. According to Baybogan (2013), the volatility in financial time series econometrics was calculated. He also looked into the practical use of estimating applications in the theoretical framework of GARCH models. Modelling the returns of the expanding pension funds was done by Kvasnakova (2009) using both the Copula and Multivariate GARCH models.

According to the review, while modelling stock market volatility, alternative GARCH model versions and error distributions should be explored for the robustness of results.

The peculiar volatility in Nigeria's exchange rate, inflation rate, and consumer price index is the issue that led to this research investigation. Simulating the conditional variance and covariance of the investigated variables will simulate the volatility between macroeconomic variables utilizing Multivariate GARCH Symmetric and Asymmetric processes. This will be done by;

Identifying any structural jumps that may be present in the series.

Using symmetric and asymmetric models of diagonal VECH, diagonal BEKK, and constant conditional correlation (CCC) to estimate the M-GARCH model

Evaluating and contrasting the performance of the symmetric and asymmetric M-GARCH models.

Finding the optimum model for forecasting the interaction between the four economic variables returns series involves using parameter estimates and information criteria.

MATERIALS AND METHODS

The symmetric and asymmetric models for multivariate GARCH modelling and the model selection criteria are included in the research methods for this paper. The selection of data does not imply that other variables are ineffectual; rather, some variables seem to interact especially.

This study spans sixteen (16) years, from (January 2005-December 2021). Secondary data from the National Bureau of Statistics and the Central Bank of Nigeria (http://www.centralbank.org) were used for the study. The information includes the consumer price index, exchange rate, inflation rate, and price of crude oil.

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Research Design

Through the use of diagonal VECH, diagonal BEKK, and constant conditional correlation (CCC) models created for econometrics, this work adopted symmetric and asymmetric modelling processes for the Multivariate Generalised Autoregressive Conditional Heteroscedasticity (M-GARCH) models. Maximum Likelihood Estimation (MLE) and model evaluation can be done with the help of these statistical techniques.

Multivariate GARCH (M-GARCH) Model

To model the variance, covariance, ARCH, and interactions among the study's variables, multivariate GARCH models are a robust extension of univariate GARCH models. Some of these traits can be found using a robust process produced by a more trustworthy model that can deconstruct univariate models into discrete parts. The first thing to think about is what limitations the Multivariate GARCH model should have to allow for dynamic flexibility of the conditional variances and covariance. The specification is challenging to reduce the model or estimate the parameter of the models because the unrestricted Multivariate GARCH model has a large number of parameters that grow rapidly along with the model's dimension.

However, if the process is restricted, the number of parameters may drop, making it impossible to capture circumstances that are important to the dynamics of the covariance matrix. So when creating the M-GARCH model definition, it is crucial to strike a balance between limitation and flexibility. The covariance matrix must be positive definite, which is another requirement for M-GARCH models. The Vector Error Conditional Heteroscedastic (VECH), Baba, Engle, Kraft and Kroner (BEKK), and Constant Conditional Correlation (CCC) models are among the most widely used M-GARCH model formulations that have been developed in the literature. Below, each of these is briefly covered; for a more thorough treatment (see Kroner & Ng, 1998).

The General Case of VECH model

By Bollerslev, Engle, and Wooldridge, the VECH model was first presented (1988). Comparatively to the next formulations, it is significantly more generic. In the VECH model, the conditional variance and covariance are functions of the lagged squared returns as well as all conditional variances and covariances.

Unrestricted VECH

The unrestricted VECH model can be expressed below:

$$VECH(H_t) = A + \sum_{i=1}^{p} B_i VECH(\mu_{t-i}\mu'_{t-1}) + \sum_{i=1}^{q} C_i VECH(H_{t-i}) \quad (1)$$

where $VECH(H_t)$ is an operator that stacks the columns of the upper and lower triangular part of its square matrix, N presents the number of variables, t is the index of the t^{th} observation, c is an $\frac{N(N+1)}{2} \times 1$ vector, B_i and C_i are $\frac{N(N+1)}{2} \times \frac{N(N+!)}{2}$ parameter matrices and μ_{t-1} in an $N \times 1$ vector. The condition for H_t is to be positive definite for all t is unrestrictive

$H_t = [h_{11t}\ h_{12t}\ h_{21t}\ h_{22t}\ ],$

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$A = [a_{11}\ a_{12}\ a_{13}\ a_{21}\ a_{22}\ a_{23}\ a_{31}\ a_{32}\ a_{33}\ ]$,

$B = [b_{11}\ b_{12}\ b_{13}\ b_{21}\ b_{22}\ b_{23}\ b_{31}\ b_{32}\ b_{33}\ ]$,

$H_{t-i} = [h_{11t-i}\ h_{12t-i}\ h_{21t-i}\ h_{22t-i}\ ]$,

$C = [c_{11}\ c_{12}\ c_{13}\ ]$

$\varepsilon_{t-i} = [\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ], \varepsilon^I_{t-i} = [\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ]$

The VECH operator take the upper triangular portion of the matrix and stack each element into a vector with a single column i

$$[h_{11t}\ h_{12t}\ h_{22t}\ ] = [c_{11}\ c_{12}\ c_{13}\ ] + [a_{11}\ a_{12}\ a_{13}\ a_{21}\ a_{22}\ a_{23}\ a_{31}\ a_{32}\ a_{33}\ ]VECH[\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ][\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ] + [b_{11}\ b_{12}\ b_{13}\ b_{21}\ b_{22}\ b_{23}\ b_{31}\ b_{32}\ b_{33}\ ]VECH[h_{11t-i}\ h_{12t-i}\ h_{21t-i}\ h_{22t-i}\ ] \quad (2)$$

$$[h_{11t}\ h_{12t}\ h_{22t}\ ] = [c_{11}\ c_{12}\ c_{13}\ ] + [a_{11}\ a_{12}\ a_{13}\ a_{21}\ a_{22}\ a_{23}\ a_{31}\ a_{32}\ a_{33}\ ][\varepsilon^2_{1t-1}\ \varepsilon^2_{2t-1}\ \varepsilon_{1t-1}\varepsilon_{2t-1}\ ] + [b_{11}\ b_{12}\ b_{13}\ b_{21}\ b_{22}\ b_{23}\ b_{31}\ b_{32}\ b_{33}\ ][h_{11t-1}\ h_{12t-1}\ h_{22t-1}\ ] \quad (3)$$

Restricted VECH

The diagonal VECH model is the restricted kind of VECH models which was proposed by Bollerslev *et al.* (1988). It assumes the parameter A_i and B_i upper or lower diagonal matrices which makes it possible for H_t to be positive definite for all t. The parameter estimation process are more smooth compared to the unrestricted VECH model. The restriction implies that there are no direct volatility spillovers from one series to another. This reduces the number of parameters to be estimated. The diagonal VECH is represented as,

$$h_{ijt} = w_{il} + A_{ij}\mu_{it-i}\mu^I_{jt-1} + B_{ij}h_{ijt-1} \quad (4)$$

If N=2

$h_{ijt} = [h_{11t}\ h_{12t}\ h_{21t}\ h_{22t}\ ]$, if

$h_{12t} = h_{21t}$

$h_{ijt} = VECH[h_{11t}\ h_{12t}\ 0\ h_{22t}\ ]$, the upper diagonal matrix

$A_{ij} = [a_{11}\ a_{12}\ 0\ a_{22}\ ]$,

$b_{ij} = [b_{11}\ b_{12}\ 0\ b_{22}\ ]$,

$\mu_{t-i}[\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ]$,

$\mu^I_{t-i} = [\varepsilon_{1t-i}\ \varepsilon_{2t-i}\ ]$,

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$h_{ijt} = VECH[h_{11t-1}\ h_{12t-1}\ 0\ h_{22t-1}\ ]$,

$w = [c_{11}\ c_{12}\ c_{13}\ ]$

$[h_{11t}\ h_{12t}\ h_{22t}\ ] = [c_{11}\ c_{12}\ c_{13}\ ] + [a_{11}\ a_{12}\ 0\ a_{22}\ ][\varepsilon_{1t-1}^2\ \varepsilon_{2t-1}^2\ \varepsilon_{1t-1}\varepsilon_{2t-1}\ ] + [b_{11}\ b_{12}\ 0\ b_{22}\ ][h_{11t-1}\ h_{12t-1}\ 0\ h_{22t-1}\ ]$ (5)

Unrestricted BEKK Models

The BEKK model, which is just an abbreviation for Baba, Engle, Kraft, and Kroner, handles the challenge of ensuring that the H_t matrix is always positive definite using VECH (Vector Error Correction Heteroscedastic). The BEKK model is represented as follows:

$$H_t = AA' + B'^{\varepsilon_{t-1}\varepsilon'_{t-1}}B + C'h_{t-1}C \quad (6)$$

$H_t[h_{11,t}\ h_{12,t}\ h_{13,t}\ h_{21,t}\ h_{22,t}\ h_{23,t}\ h_{31,t}\ h_{32,t}\ h_{33,t}\ ]$

H_t is reduces a matrix of 6 * 1 parameter 3 variance and 3 co-variance

$H_t = [h_{11,t}\ h_{22,t}\ h_{33,t}\ \ h_{12,t}\ h_{23,t}\ h_{13,t}\ ]$6□1 matrix of variances and co-variance

$$h_{t-1} = [h_{11,t-1}\ h_{12,t-1}\ h_{13,t-1}\ h_{21,t-1}\ h_{22,t-1}\ h_{23,t-1}\ h_{31,t-1}\ h_{32,t-1}\ h_{33,t-1}\ ], C = [C_{11}\ C_{12}\ C_{13}\ C_{21}\ C_{22}\ C_{23}\ C_{31}\ C_{32}\ C_{33}\ ], C' = [C_{11}\ C_{21}\ C_{31}\ C_{12}\ C_{22}\ C_{32}\ C_{13}\ C_{23}\ C_{33}\ ]$$

$B = [B_{11}\ B_{12}\ B_{13}\ B_{21}\ B_{22}\ B_{23}\ B_{31}\ B_{32}\ B_{33}\ ]$

$B' = [B_{11}\ B_{21}\ B_{31}\ B_{12}\ B_{22}\ B_{32}\ B_{13}\ B_{23}\ B_{33}\ ]$

$\varepsilon_{t-1} = [\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ ]$

$\varepsilon'_{t,t-1} = [\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ \ ]$

$A = [A_{11}\ A_{21}\ A_{31}\ ]$

$A' = [A_{11}\ A_{21}\ A_{31}\ ]$

In general unrestricted BEKK model can be represented mathematical as follow:

$$[h_{11,t}\ h_{12,t}\ h_{13,t}\ h_{21,t}\ h_{22,t}\ h_{23,t}\ h_{31,t}\ h_{32,t}\ h_{33,t}\ ] = [A_{11}\ A_{21}\ A_{31}\ ][A_{11}\ A_{21}\ A_{31}\ ] + [B_{11}\ B_{21}\ B_{31}\ B_{12}\ B_{22}\ B_{32}\ B_{13}\ B_{23}\ B_{33}\ ]\ [\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ ]\ [\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ \ ] [B_{11}\ B_{12}\ B_{13}\ B_{21}\ B_{22}\ B_{23}\ B_{31}\ B_{32}\ B_{33}\ ] + [C_{11}\ C_{21}\ C_{31}\ C_{12}\ C_{22}\ C_{32}\ C_{13}\ C_{23}\ C_{33}\ ] [h_{11,t-1}\ h_{12,t-1}\ h_{13,t-1}\ h_{21,t-1}\ h_{22,t-1}\ h_{23,t-1}\ h_{31,t-1}\ h_{32,t-1}\ h_{33,t-1}\ ] [C_{11}\ C_{12}\ C_{13}\ C_{21}\ C_{22}\ C_{23}\ C_{31}\ C_{32}\ C_{33}\ ] \quad (7)$$

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Restricted BEKK Model

In the Diagonal BEKK model, B and C are N□N matrix and A is a lower or upper triangular matrix of the parameter. The positive definiteness of the covariance matrix is ensured.

$[B_{11}\ 0\ 0\ 0\ B_{22}\ 0\ 0\ 0\ B_{33}\ ] = B' = B,$

$[C_{11}\ 0\ 0\ 0\ C_{22}\ 0\ 0\ 0\ C_{33}\ ] = C' = C$

In general, the diagonal BEKK models is represented as follows:

$$\left[h_{11,t}\ h_{22,t}\ h_{33,t}\ \ h_{12,t}\ h_{23,t}\ h_{13,t}\ \right] = [A_{11}\ A_{22}\ A_{33}\ \ A_{12}\ A_{23}\ A_{13}\ ] +$$
$$[B_{11}\ 0\ 0\ 0\ B_{22}\ 0\ 0\ 0\ B_{33}\ ]\left[\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ \right]\left[\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ \right][B_{11}\ 0\ 0\ 0\ B_{22}\ 0\ 0\ 0\ B_{33}\ ] +$$

$$[C_{11}\ 0\ 0\ 0\ C_{22}\ 0\ 0\ 0\ C_{33}\ ]\left[h_{11,t-1}\ h_{12,t-1}\ h_{13,t-1}\ h_{21,t-1}\ h_{22,t-1}\ h_{23,t-1}\ h_{31,t-1}\ h_{32,t-1}\ h_{33,t-1}\ \right][C_{11}\ 0\ 0\ 0\ C_{22}$$
(8)

$h_{13,t-1} = h_{32,t-1} = h_{12,t-1} = h_{23,t-1} = h_{31,t-1} = h_{32,t-1}$ their respective co-variances of the return series

$\left[\varepsilon_{1,t-1}\ \varepsilon_{2,t-1}\ \varepsilon_{3,t-1}\ \right]$ = are the error of the three variable.

The Constant Conditional Correlation (CCC) Model

Bollerslev (1986) first proposed the Constant Conditional Correlation (CCC) model, which primarily models the condition covariance matrix indirectly by estimating the conditional correlation matrix. While the conditional variances are projected to fluctuate, the conditional correlation is predicted to remain constant. Think about the CCC model.

$$h_{iit} = C_i + a_1\varepsilon_{it-i}^2 + b_i h_{iit-t} \tag{9}$$

The off Diagonal element of $H_t H_{ij}(i \neq j)$ is defined indirectly via the correlation denoted p_{ij}

$$h_{ijt} = p_{ij} h_{iit}^{1/2} h_{jjt}^{1/2} j, i = 1,2 \dots . N i < j \tag{10}$$

Asymmetry M-GARCH Model

When conditional variance and covariance are allowed to respond differentially to both positive and negative shocks of equal size, the asymmetric model has proven increasingly popular in empirical applications. The following diagram illustrates how the asymmetric M-GARCH model of differences works:

Asymmetric Diagonal VECH Models

$$h_{ijt} = w_{il} + A_{ij}\mu_{it-i}\mu_{jt-1}^{I}A'_{ij} + B_{ij}h_{ijt-1} + D_{ij}\epsilon_{it-i}\epsilon_{jt-1}^{I}D'_{ij} \tag{11}$$

Asymmetric Diagonal BEKK Models

$$H_t = AA' + B'^{\varepsilon_{t-1}\varepsilon'_{t-1}}B + C'^{h_{t-1}}C + D'\mu_{t-1}\mu'_{t-1}D \tag{12}$$

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Asymmetric CCC Models

$$h_{iit} = C_i + a_i \varepsilon_{it-1}^2 + b_i h_{iit-t} + \theta_i \mu_{it-i}^2 \qquad (13)$$

Models Estimation of M-GARCH

When estimating M-GARCH models, the Maximum Likelihood Estimation (MLE) is used. The approach for estimating the process parameter is highly challenging. Therefore, Engle is in favour of a two-stage estimate method in which each variable in the system is first individually modelled as a Univariate GARCH process. It is possible to create a joint log-likelihood function at this stage that is simply the sum of the log-likelihoods for each GARCH model. The conditional likelihood is then maximised with regard to any unidentified parameters in the correlation matrix in the second stage. The second stage estimation's log-likelihood function is as follows: $l\left(\frac{\theta_1}{\theta_2}\right) = \sum_{t=1}^{T} \quad \langle log\ log\ |R_t| + \mu_t^I R_t^{-1} \mu_t \ \rangle \qquad \theta_2$

(14)

where "θ_1 " stands for all the parameters that were unknown but were estimated in the first stage, and "θ_2 " stands for all the parameters that will be estimated in the second stage. Due to the fact that any parameter uncertainty from the first stage is carried over to the second, estimation utilising this two-step technique will be consistent and ineffective.

Model Estimation of Optimal Lag Length

The number of observations in a time series used to interpret the lag length of the M-GARCH model is known as the lag length selection. When choosing the lag length, using fewer lags might lead to auto-correlated errors while using too many lags can overfit the model, increasing the mean-square-forecast errors. To reduce the information criteria, the lag duration p is used (Tuaneh & Wiri, 2019). Model selection criteria can be used to calculate the GARCH model's lag length.

Model Selection Criteria

The commonly used information criteria are: (i) Akaike Information Criterion (AIC), (ii) Hannan-Quinn Information Criterion (HQIC), and (iii) Schwarz Information Criterion (SIC). The Information Criteria and their formula are shown below:

- AIC $= ln\ ln\ |\quad _r| + \frac{2}{T}\ MK^2$ (15)
- HQIC $= ln\ ln\ |\quad _r| + \frac{2 lnln\ T}{T}\ MK^2$ (16)
- SIC $= ln\ ln\ |\quad _r| + \frac{lnln\ T}{T}\ MK^2$
  (17)

T is the number of observations (after accounting for lags) $ln\ ln\ |\quad _r|$

M is the total number of parameters, including the constant that is calculated in each equation of the unrestricted system. The natural log of the determinant of the residuals' covariance matrix

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for the restricted system is $ln\, ln\,|\quad _r|$. In each case, MK^2 is the number of M-GARCH parameters in a model with order M (Chris, 2008).

RESULT AND DISCUSSIONS

Plot and Correlation Coefficient Measures of the Time Series Variables

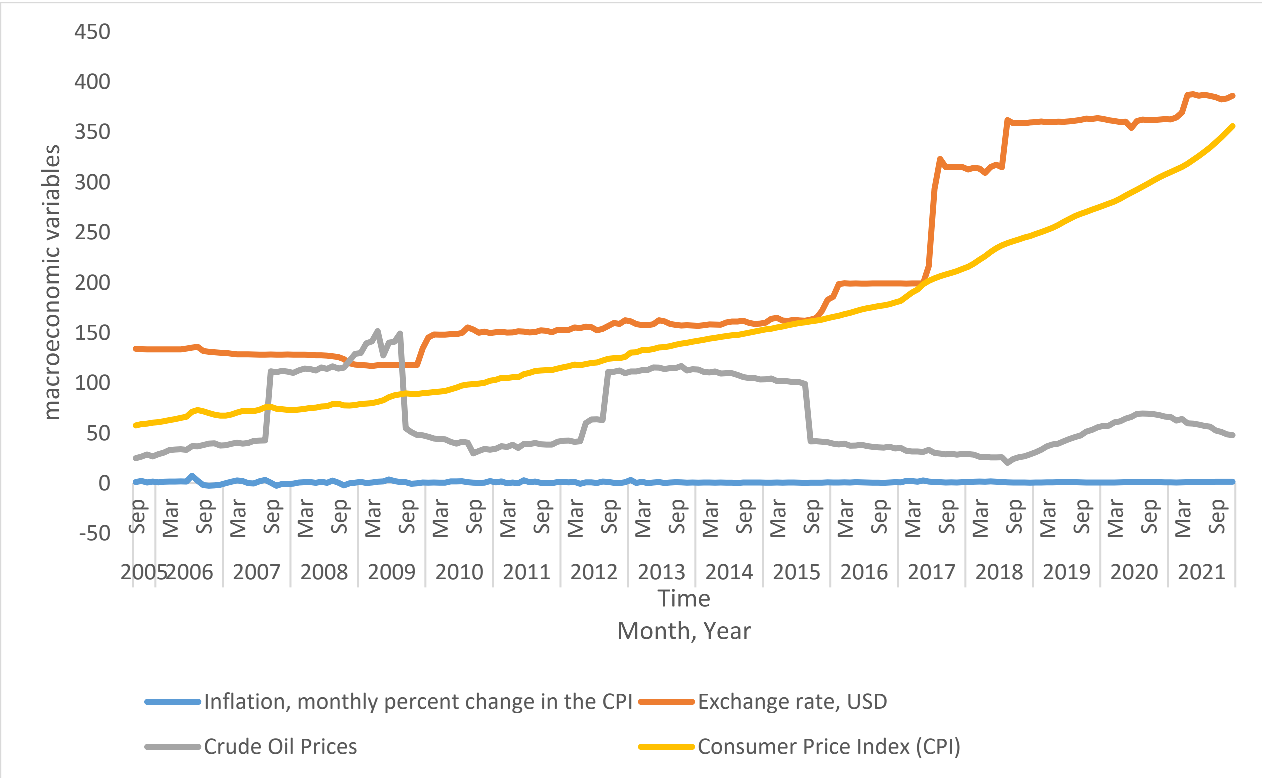

Figure 1: Time Plot of the macroeconomic variables: Crude Oil Price Series (US Dollars), Exchange Rate Series (Naira\US Dollars), Inflation Rate Series (Naira) and Consumer Price Index Series (Naira) at Time

The time plot of macroeconomic variables over time (at level) is shown in Figure 1, where the horizontal axis denotes time and the vertical axis denotes series (months, years). The crude oil price time plot at such levels displayed erratic movement, which is evidence of the non-stationary process present in the Crude Oil Price series. Throughout the observational period, the series displays constant fluctuation (volatility). Over the course of the observations, the exchange rate series' time plot shows a consistently increasing trend (high volatility levels). The plot of the inflation rate series revealed erratic motion and a consistent trend. The continuous fluctuation in the series is a sign of the non-stationary nature of the operation. The Consumer Price Index series' time plot, however, displayed a pattern distinct from that of other

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series. Beginning with a downward fluctuation up to the year 2013, the series progressively began to fluctuate upward up to the year 2021.

Table 1: Correlation Coefficients Statistics of the four variables

	Inflation, monthly per cent change in the CPI	*Exchange rate, USD*	*Crude Oil Prices*	*Consumer Price Index (CPI)*
Inflation, monthly per cent change in the CPI	1	0.077	-0.133	0.074
Exchange rate, USD	0.077	1	-0.345	0.955
Crude Oil Prices	-0.133	-0.345	1	-0.231
Consumer Price Index (CPI)	0.074	0.955	-0.231	1

According to Table 1's correlation coefficient statistics, there is an almost perfect positive association between the exchange rate and the consumer price index, which suggests that they have a positive mean return. Additionally, the link between crude oil prices, inflation, and the exchange rate is generally negative (i.e. -0.133 and -0.345). This only means that among all the financial time series data taken into account, the macroeconomic variables will exhibit volatility clustering tendencies over time. Since the study variables are non-stationary at levels, the next step is to use a recursive residuals plot to check the structural breakpoint of the series in Figures 2 to 5.

Structural Jump Process

The structural breakpoint of the series is shown in the figure below using the recursive residuals plot.

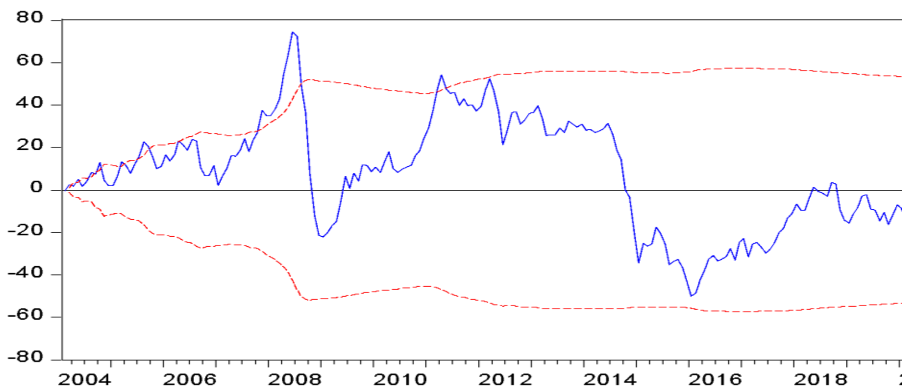

Figure 2: The Structural Break Point in Crude Oil Price ($)

Figure 2's recursive residuals plot displays a breakpoint in 2008 that is both downward and upward, as well as a spike in that same year. The second break occurs in the year 2014 and is a downward break below the conventional line of zero to a negative sign, indicating weak

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demand for the goods. It is evident that the price of crude oil has two structural breakpoints (January 2004 to March 2014 and April 2014 to December 2020).

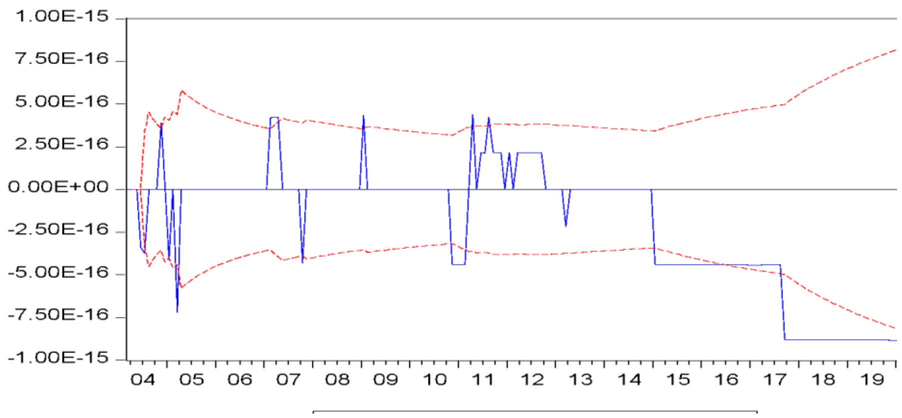

Figure 3: The Structural Break Point in Consumer Price Index (#)

A structural breakpoint is visible in 2004, 2007, 2010, and 2015 on the plot of the recursive residual of the consumer price index in Figure 4.3. The declining trend suggests that some commodities in the nation are not being negatively demanded by consumers. The series' structural leaps occur in the ensuing years: 2005, 2006, 2009, and 2012, respectively. This demonstrates how keen consumers are to buy more products domestically.

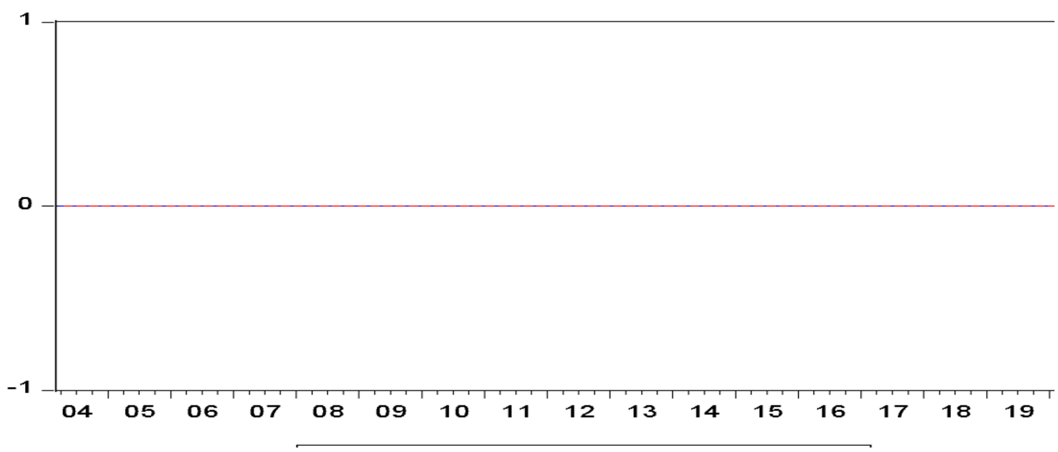

Figure 4: The Structural Break Point in Inflation Rate (#)

The plot of the recursive residual of the inflation rate in Figure 4.4 depicts a steady movement, indicating that the country's average price of products does not have any structural breakpoints. The plot also shows that the building either collapses or rises at a steady rate.

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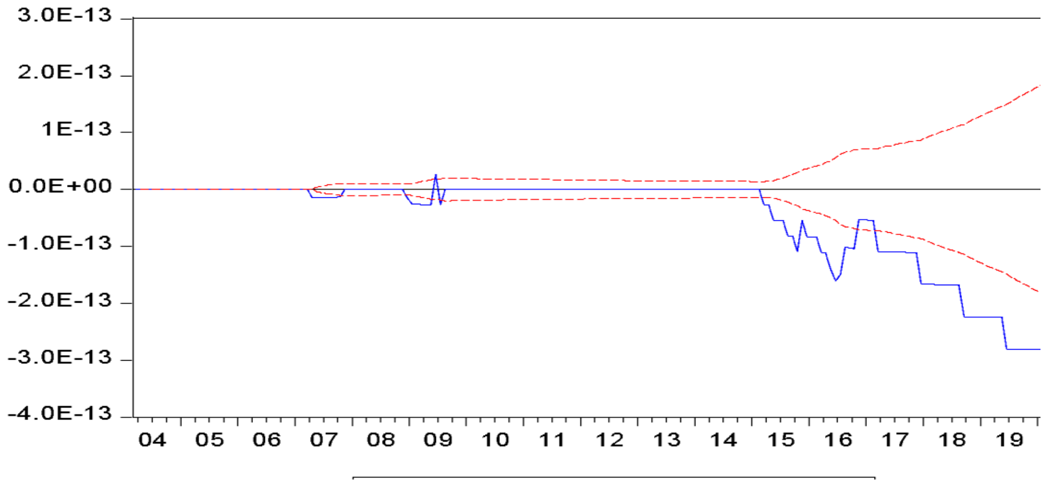

Figure 5: The Structural Break Point in Exchange Rate (#\$)

A structural breakpoint and a non-structural jump are visible in Figure 5's representation of the recursive residual of the naira to dollar exchange rate. In other words, the naira is losing value against the dollar.

The covariance and correlation between series were then estimated using six models. Multivariate GARCH permits the variances and covariance to change over time. These models are covered in section 3 above. In this study, we'll limit our estimation to symmetric and asymmetric multivariate GARCH models: Triangles VECH, BEKK, and CCC The tables below exhibit symmetric models as well as asymmetric diagonal VECH, diagonal BEKK, and diagonal CCC models.

Multivariate GARCH Models Comparison in Terms of the Symmetric and Asymmetric Models

In order to analyse the interactions between the four economic variables returns series, this study also used three Multivariate GARCH models. In Tables 2 and 3, respectively, parameter estimates for the three multivariate GARCH models—diagonal VECH, diagonal BEKK, and CCC symmetric and asymmetric models—as well as information criteria (or model selection criteria)—were used to assess how well each model performed.

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Table 2: Symmetric and Asymmetric Multivariate GARCH Models Parameter Estimates

Model	Variables	Symmetric Multivariate GARCH Models Parameter Estimates (p-value)	Asymmetric Multivariate GARCH Models Parameter Estimates (p-value)
Diagonal VECH	RCOP RINF REXCH. RCPI	0.26138 (0.7671) 0.68414 (0.5671) 0.29482 (0.3545) 0.02465 (0.9726)	0.64648 (0.3684) -0.02050 (0.9727) 0.10760 (0.1677) -0.01018 (0.9897)
Diagonal BEKK	RCOP RINF REXCH. RCPI	1.42766 (0.0270**) 0.55920 (0.2339) -0.0727 (0.7758) 0.19668 (0.6597)	0.15676 (0.9116) 0.28676 (0.7181) 0.19343 (0.6918) -0.93528 (0.6789)
CCC	RCOP RINF REXCH. RCPI	1.25370 (0.0574*) 0.30629 (0.6215) 0.12866 (0.6970) -0.21812 (0.6053)	0.94257 (0.1666) 0.39320 (0.5449) 0.19232 (0.5283) -0.31771 (0.2109)

Footnote: **= Sig. at 5%,*= Sig. at 10%.

Table 3: Comparing the Result of Symmetric and Asymmetric Multivariate GARCH Model Using Information Criteria

Models	**Symmetric Multivariate GARCH**			**Asymmetric Multivariate GARCH**		
	AIC	**SIC**	**HQIC**	**AIC**	**SIC**	**HQIC**
Diagonal VECH	27.22	27.53	27.35	**26.216**	**26.8**	**26.50**
Diagonal BEKK	**25.61**	**25.87**	**25.72**	26.87	27.366	27.074
CCC	**25.57**	**25.93**	**25.51**	25.346	25.77	25.719

Footnote: The bolded Model has the least Information Criteria and they are identified as the suitable model.

Diagonal VECH, Diagonal BEKK, and CCC models are three symmetric and asymmetric multivariate GARCH models whose performance is examined in Tables 2 and 3. The performance is evaluated in terms of model parameters and information requirements.

In both the Diagonal BEKK and CCC Symmetric Multivariate GARCH models parameter estimations, Table 2 shows that only the Crude Oil Price parameter p-values are significant at 5% and 10% from other variables, whereas other variables are not significant. Given that the

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four economic variable returns series are not statistically significant at the 5% and 10% levels, these results demonstrate the absence of any causal relationship between them.

Asymmetric models (Diagonal VECH) have the least values model information criteria compared to symmetric models (Diagonal BEKK and CCC), while symmetric models (Diagonal BEKK and CCC) have the least values model information criteria compared to asymmetric models (Diagonal BEKK and CCC) (Diagonal VECH). The Diagonal BEKK and CCC models outperformed the Diagonal VECH model in terms of outcomes for the information criteria, according to Table 3's findings.

SUMMARY AND CONCLUSION

For the analysis, E-view 12 statistical software was employed. Three symmetric and asymmetric multivariate generalised autoregressive conditional heteroscedasticity (GARCH) models were developed using the symmetric and asymmetric process. The information criteria were used to choose the best model. Each macroeconomic variable had a structural breakpoint.

Both symmetric and asymmetric modelling methods were used to create the Multivariate GARCH (M-GARCH) model. The covariance and correlation between four variables were estimated using the M-GARCH models. The main finding from the estimation of all M-GARCH models is that symmetric models (Diagonal BEKK and Constant Conditional Correlation, or "CCC") have less model information than asymmetric models (Diagonal BEKK and CCC), while asymmetric models (Diagonal VECH) have less model information than symmetric models (Diagonal BEKK and CCC) (Diagonal VECH). Based on the findings, it can be concluded that when analysing the interaction between the four economic variables returns series, the Diagonal BEKK and CCC model outperformed the Diagonal VECH model.

CONCLUSION

This study's main objective was to identify the several top M-GARCH models for predicting and modelling Nigeria's inflation rate, exchange rate, consumer price index, and crude oil price.

The Multivariate GARCH models revealed that Asymmetric models (Diagonal VECH) have fewer values for model information criterion than Symmetric models (Diagonal BEKK and CCC), whereas Symmetric models (Diagonal BEKK and CCC) had fewer values for model information criteria (Diagonal VECH). According to the findings, the Diagonal BEKK and CCC models outperformed the Diagonal VECH modelling when analysing the interaction between the four economic variable returns series.

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