# COMPARISON OF ADOMIAN DECOMPOSITION METHOD WITH DIFFERENTIAL TRANSFORMATION METHOD FOR UNSTEADY MHD FLOW AND HEAT TRANSFER OVER A STRETCHING/SHRINKING PERMEABLE SHEET WITH OHMIC HEATING

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**ABSTRACT:** *In this paper, two semi-analytical techniques were implemented to solve a two-dimensional unsteady MHD fluid flow and heat transfer through a stretching/shrinking permeable sheet with ohmic heating and viscous dissipation. The governing flow equations in PDE form were reduced to ordinary differential equations using appropriate similarity transformation. We obtained approximate expressions for the velocity and temperature profiles. Comparative results obtained employing Adomian decomposition method and differential transformation method were benchmarked against a numerical solution using Keller box scheme. Our findings revealed that the approximate analytical solution obtained using DTM is more dependable with fast convergence, highly accurate with minimal calculations and computationally convenient. However, the requirement of Adomian polynomials to tackle the nonlinear terms in ADM makes its execution sometimes cumbersome and difficult.*

**Keywords**: Unsteady MHD, Stretching/Shrinking, Permeable Sheet, Ohmic Heating, Stagnation point flow.

## INTRODUCTION

Magnetohydrodynamics (MHD) refers to the branch of physics that studies magnetic properties and behavior of conducting fluid such as plasma which produces electricity when subjected to a magnetic field. The underlying principle behind this phenomenon is that current is induced in a moving conducting fluid by a magnetic field which results in the polarization of the fluid and a change in the behavior of the magnetic field. This concept of magnetohydrodynamic flows has stimulated considerable research interest because it finds useful applications in several fields of endeavor. For instance, MHD is significant in the fields of solar physics, meteorology, astrophysical and geophysical problems, plasma in fusion reactors, fluid dynamics, electromagnetic casting, liquid-metal coiling, and earth's core [1-2]. Many researchers worked on MHD and its applications in different discipline fields of study. Anwar [3] examined magnetohydrodynamics free convection flow through a vertical porous plate. Sahoo et al. [4] investigated MHD unsteady free convection flow through an infinite vertical plate in the presence of constant suction and heat sink. The study reported dual solutions thus exist for the suction case only. Unsteady MHD flow coupled with heat transfer past a porous plate in a rotating system was explored by Jana et al. [5]. Investigation was carried out to unravel the influence of Soret, Hall and Joules heating on magnetohydrodynamics rotating mixed convective flow through an infinite vertical porous plate by Swarnalathamma et al. [6]. Chamkha et al. [7] considered the heat and mass transfer on unsteady magnetohydrodynamic oscillatory flow of second-grade fluid via a porous medium between two vertical plates in the presence of fluctuating heat source/sink and chemical reaction. Veera and Reddy [8] analyzed unsteady MHD reactive flow of second grade fluid through porous medium in a rotating parallel plate channel. Moniem and Hassanin [9] looked at the analytical solution of MHD flow past a vertical porous plate through a porous medium in the presence of oscillatory suction. Venkatalakshmi et al. [10] studied unsteady MHD free convective fluid flow through a vertical porous plate with Ohmic heating under the influence of suction or injection. On the other hand, the heat transfer and flows which impinge on solid surfaces technically, called stagnation point flows, over a stretching or shrinking sheet have equally been extensively studied because of their important applications in aircraft and submarines. Khuzaimah et al. [11] conducted a stability analysis of an unsteady MHD flow and heat transfer over a shrinking sheet with Ohmic heating.

The study showed that a dual solution exists for the shrinking sheet and both magnetic field and unsteadiness parameters significantly impacted the flow, accelerating the skin friction and rate of heat transfer in the system. Trail-blazing studies on stretching sheets were first carried out by Crane [12]. Ever since Crane's pioneering studies, many authors have researched on studies involving stretching sheets. Hatami et al. [13] did a comprehensive analysis of the flow and heat transfer of a nanofluid over an unsteady stretching flat plate. An investigative solution for a two-dimensional stagnation point flow was obtained by Hiemenz [14]. An unsteady stagnation point flow of a nanofluid through a stretching sheet with slip effects was implemented by Malvandi and Hedayati [15]. Similarly, combined effects of thermal radiation and slip on MHD stagnation point flow of nanofluid over a stretching sheet has been examined by Nadeem et al. [16]. Ishak et al. [17] studied the effect of unsteadiness on mixed convection boundary layer stagnation point flow over a vertical flat surface embedded in a porous medium. Many other studies on stagnation point flows on different bodies have been comprehensively investigated [17-27].


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Motivated by the above extensive literature, we consider a comparative study using two semi-analytical techniques to look at unsteady MHD flow and heat transfer over a stretching/shrinking permeable sheet under the influence of Ohmic heating and viscous dissipation. The specific objective of this study is to compare the analytical approximate solutions obtained using Adomian decomposition method and differential transformation method. Armenian-American mathematician George Adomian pioneered the Adomian decomposition method (ADM). This method since its conception has been applied to several problems in the field of sciences, engineering, and social sciences. It involves writing a given equation in operator form. Similarly, the concept of differential transformation method (DTM) was proposed by Zhou while working on his doctoral thesis. This novel technique is based on Taylor's series expansion in which the given equation under consideration is reduced to a recurrence relation where the iterative approximate solution is obtained through series in polynomial. For exhaustive application of these approximation methods to diverse problems, see [28-48]. The study is organized as follows: In the next section, a survey of related literature on unsteady MHD flows and heat transfer past a stretching and shrinking permeable sheet is presented. Section 2 gives the governing flow equations and their reduction through similarity transformation. The solution of the problem using ADM, DTM and their operational laws is contained in Sections (3-4). Section 5 presents the comparison between the approximate analytical solutions using the solution techniques and the accompanying errors relative to the numerical scheme. The conclusion of the study highlighting the major findings is given in Section 6.

## MATHEMATICAL FORMULATION

Figure 1 depicts the flow and heat transfer of a permeable stretching/shrinking sheet over an unsteady two-dimensional stagnation point flow of a viscous and electrically conducting fluid, where x and y are the cartesian coordinates measured along the sheet's surface and normal to it, respectively. When a sheet is stretching or shrinking, its velocity is given by $u_w = \lambda u_w(x,t)$, where $u_e(x,t)$ represents the flow's speed away from the sheet's surface and is a constant corresponding to either stretching or shrinking. The temperature of the sheet is taken to be $T_w(x,t)$, while the temperature of the surrounding fluid is taken to be $T_\infty$. Additionally, it is believed that a transverse magnetic field is present in the flow. The governing unsteady boundary layer equations for continuity, momentum, and energy following Khuzaimah et al. [11] are given as:


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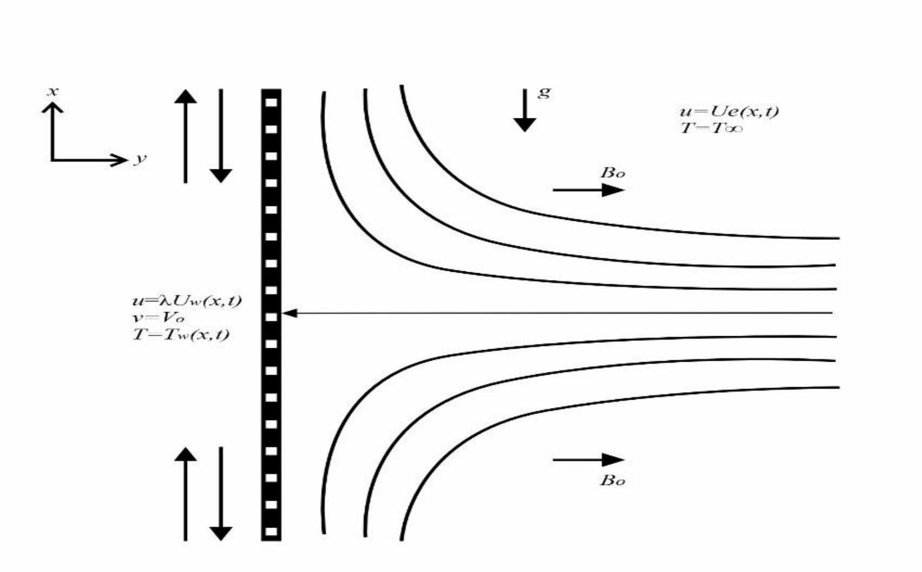


**Figure 1: Physical configuration of the model problem**

## Governing Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{\partial u_e}{\partial t}-u_e\frac{\partial u_e}{\partial x}-v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u_e-u)=0 \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho C_\rho}(u_e-u)^2 \tag{3}$$

subject to the initial and boundary conditions given by

$$t<0: u=v=0, T=T_\infty \text{ for any } x, y$$

$$t\geq 0: u=\lambda u_w(x,t), v=V_0, T=T_w(x,t) \text{ at } y=0 \tag{4}$$

$$u\longrightarrow u_e(x,t), T\longrightarrow T_\infty \text{ as } y\longrightarrow\infty$$

where $(u,v)$ represents the velocity in the $x,y$ axes, $v$ denotes the kinematic viscosity of the fluid, $\sigma$ represents electrical conductivity, $\rho$ represents the density of the fluid, $T$ denotes temperature of the fluid, $\alpha$ represents the thermal diffusivity of fluid, $C_p$ is the specific heat capacity of the fluid at constant pressure and $V_0$ is the mass flux of the fluid at the surface.

For the model equations and their associated initial and boundary conditions to admit similarity, we express the terms, $u_w(x,t), u_e(x,t), T_w(x,t)$ and $B_0^2(t)$ in the form.

$$u_w(x,t)=\frac{ax}{1-\beta t}, u_e(x,t)=\frac{ax}{1-\beta t}, B_0^2(t)=\frac{B_0^2}{1-\beta t}, T_w(x,t)=T_\infty+\frac{bx^2}{(1-\beta t)^2} \tag{5}$$

where $a,b>0$ are constants, $\beta$ is the volume expansivity of the fluid, $B_0$ is constant applied magnetic field.


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To simplify and solve the set of equations in Eqs. (1-3) subject to (4), it is necessary to non-dimensionalize the physical parameters and equations. We introduce the following similarity transformations given as

$$\psi(x,y,t) = \sqrt{\frac{av}{1-\beta t}}xf(\eta),\ \eta = \sqrt{\frac{a}{v(1-\beta t)}}y,\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty},u = \frac{\partial\psi}{\partial y},v = -\frac{\partial\psi}{\partial x} \quad (6)$$

where $\eta$ is the dimensionless similarity variable, $\theta$ is the dimensionless temperature and $\psi(x,y)$ is the stream function of the fluid.

Plugging Eq. (6) into Eqs. (1-4) leads to the nonlinear system of ordinary differential equation.

$$f'''(\eta) + f(\eta)f''(\eta) + 1 - f'^2(\eta) - A\left(f'(\eta) - 1 + \frac{\eta}{2}f''(\eta)\right) + M\big(1 - f'(\eta)\big) = 0$$
(8)

$$\frac{1}{Pr}\theta''(\eta) + f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta) - A\left(2\theta(\eta) + \frac{1}{2}\eta\theta'(\eta)\right) + Ec\left[M\big(1 - f'(\eta)\big)^2 + f''(\eta)\right] = 0 \quad (9)$$

subject to the appropriate boundary conditions given as

$$f(0) = S, f'(0) = \lambda, \theta(0) = 1$$

$$f'(\eta) \longrightarrow 1, \theta(\eta) \longrightarrow 0 \text{ as } \eta \longrightarrow \infty \quad (10)$$

where the prime represent differentiation with respect to $\eta$, $S > 0$ is the suction parameter whereas $S < 0$ is the injection parameter, $M$ is the constant magnetic parameter, $Ec$ is the Eckert number and $Pr$ denotes Prandtl number.

The non-dimensional variables are defined as

$$A = \frac{\beta}{a},M = \frac{\sigma B_0^2}{a\rho},Ec = \frac{u_w^2}{C_p(T_w-T_\infty)},Pr = \frac{v}{\alpha} \quad (11)$$

## ANALYTICAL SOLUTION USING DTM

Let $u(t)$ be a given analytic function in the given domain D and let $x = x_o$ be an initial point of the function.

Then the $k$th derivative of $u(t)$ about point $t = t_0$ is defined as follows:

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=t_0} \quad (12)$$

where u(t) is the original function and $U(k)$ is the transformed function.

The inverse transforms of $U(k)$ in Eq. (1) is given as

$$u(t) = \sum_{k=0}^{\infty}(t-t_0)^k U(k) \quad (13)$$


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Combining Eqs. (12) and (13), the original function, $u(t)$ can be rewritten as a finite series of the form:

$$u(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^k}{k!} \left[\frac{d^k u(t)}{dt^k}\right]_{t=t_0} \quad (14)$$

**Fundamental Theorems of the Differential Transformation Method (DTM)**

Theorem 1. If $y(t) = \alpha f(t) \pm \beta g(t)$, then $Y(k) = \alpha F(k) \pm \beta G(k)$

Theorem 2. If $y(t) = \alpha f(t)$, then $Y(k) = \alpha F(k)$

Theorem 3. If $y(t) = \frac{df(t)}{dt}$, then $Y(k) = (k+1)F(k+1)$

Theorem 4. If $y(t) = \frac{d^2 f(t)}{dt^2}$, then $Y(k) = (k+1)(k+2)F(k+2)$

Theorem 5. If $y(t) = \frac{d^r f(t)}{dt^r}$, then $Y(k) = \frac{(k+r)!F(k+r)}{k!}$

Theorem 6. If $y(t) = f(t)g(t)$, then $Y(k) = \sum_{r=0}^{k} F(k)G(k-r)$

Theorem 7. If $y(t) = t^r$, then $Y(k) = \delta(k-r) = \begin{cases} 1, if\ k = r \\ 0, if\ k \neq r \end{cases}$

Theorem 8. If $y(t) = f^3(t)$, then $Y(k) = \sum_{k_1}^{k} \sum_{r=0}^{k_1} F(k)F(k_1 - r)F(k-r)$

Theorem 9. If $y(t) = f'(t)g'(t)$, then $Y(k) = \sum_{r=0}^{k}(k+1)(k-r+1)F(r+1)G(k-r+1)$

Theorem 10. If $y(t) = e^{\lambda t}$, then $Y(k) = \frac{\lambda^k}{k!}$

Theorem 11. If $y(t) = f(t)g'(t)$, then $Y(k) = \sum_{r=0}^{k} F(k)(k-r+1)G(k-r+1)$

Theorem 12. If $y(t) = (1+t)^m$, then $Y(k) = \frac{m(m-1)(m-2)....(m-k+1)}{k!}$

Theorem 13. If $y(t) = \sin(nt+\alpha)$, then $Y(k) = \frac{n^k}{k!}\sin\left(\frac{nk}{2}+\alpha\right)$

Theorem 14. If $y(t) = \sin(nt+\alpha)$, then $Y(k) = \frac{n^k}{k!}\sin\left(\frac{nk}{2}+\alpha\right)$

Rearranging Eqs. (8) and (9) in an equivalent expression, we have:

$$f'''(\eta) = f'^2 + A\left(f' - 1 + \frac{1}{2}\eta f''\right) - ff'' - M(1-f') - 1 \quad (15)$$

$$\theta''(\eta) = Pr\left[2f'\theta + A\left(2\theta + \frac{1}{2}\eta\theta'\right) - f\theta' - Ec[M(1-f')^2 + f''^2]\right] \quad (16)$$


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Applying DTM to both sides of Eqs. (15) and (16), we obtain the form:

$$(k+1)(k+2)(k+3)F(k+3) = \sum_{r=0}^{k}(k+1)(k-r+1)F(k+1)F(k-r+1) - \sum_{r=0}^{k}F(r)(k-r+1)(k-r+2)F(k-r+2) - M\big(\delta(k)-(k+1)F(k+1)\big) - \delta(k) - A\left\{(k+1)F(k+1) - \delta(k) + \frac{1}{2}\eta(k+1)(k+2)F(k+2)\right\} \quad (17)$$

$$(k+1)(k+2)\Theta(k+2) = Pr\,[2\sum_{r=0}^{k}\Theta(k)(k-r+1)F(k-r+1) + A\left(2\Theta(k) + \frac{1}{2}\eta(k+1)\Theta(k+1)\right) + \sum_{r=0}^{k}F(r)(k-r+1)\Theta(k-r+1) - Ec\left[M\big(1-(k+1)F(k+1)\big)^2\right] + \sum_{r=0}^{k}(k-r+1)(k-r+2)F(k-r+2)(k+1)(k+2)F(k+2) \quad (18)$$

where $F(k)$ and $\Theta(k)$ are the differential transforms of $f(\eta)$ and $\theta(\eta)$. The systems in Eqs. (17) and (18) are subject to the following boundary conditions:

$$F(0) = S, F(1) = \lambda, F(2) = \delta_1,\ \Theta(0) = 1, \Theta(1) = \delta_2$$

$$F'(\infty) \longrightarrow 1, \Theta(\infty) \longrightarrow 0 \quad (19)$$

Rearranging Eqs. (17) and (18), we have the iterative schemes as follows:

$$F(k+3) = \frac{1}{(k+1)(k+2)(k+3)}\left[\sum_{r=0}^{k}(k+1)(k-r+1)F(k+1)F(k-r+1) - \sum_{r=0}^{k}F(r)(k-r+1)(k-r+2)F(k-r+2) - M\big(\delta(k)-(k+1)F(k+1)\big) - \delta(k) - A\left\{(k+1)F(k+1) - \delta(k) + \frac{1}{2}\eta(k+1)(k+2)F(k+2)\right\}\right] \quad (20)$$

$$\Theta(k+2) = \frac{Pr}{(k+1)(k+2)}\,[2\sum_{r=0}^{k}\Theta(k)(k-r+1)F(k-r+1) + A\left(2\Theta(k) + \frac{1}{2}\eta(k+1)\Theta(k+1)\right) + \sum_{r=0}^{k}F(r)(k-r+1)\Theta(k-r+1) - Ec\left[M\big(1-(k+1)F(k+1)\big)^2\right] + \sum_{r=0}^{k}(k-r+1)\,(k-r+2)F(k-r+2)(k+1)(k+2)F(k+2) \quad (21)$$


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Putting the values of $k = 0,1,2..$ into Eq. (20) and (21), we have the following iterative algorithm:

$F(\eta) = \sum_{r=0}^{k} F(k)\eta^k$ and $\Theta(\eta) = \sum_{r=0}^{k} \Theta(k)\eta^k$ (22)

where the inverse differential transform of the solution is given by the expression

$$f(\eta) = F(0) + F(1)\eta + F(2)\eta^2 + F(3)\eta^3 + \cdots$$

$$\theta(\eta) = \Theta(0) + \Theta(1)\eta + \Theta(2)\eta^2 + \Theta(3)\eta^3 + \cdots$$

Using Eq. (22), the approximate solution of the flow gradients gives the following expression:

$f(\eta) = S + \lambda\eta + \delta_1\eta^2 + \frac{1}{6}[\lambda^2 + (A+M)(\lambda-1) + (A\eta - 2S)\delta_1 - 1]\eta^3 + \cdots$ (23)

$\theta(\eta) = 1 + \delta_2\eta - \frac{Pr}{2}\left[EcM\lambda^2 - 2(1-EcM)\lambda - \left(\frac{A}{2}\eta - S\right)\delta_2 + 4Ec\delta_1^2 - 2A + EcM\right]\eta^2 + \cdots$ (24)

## ANALYTICAL SOLUTION USING ADM

In this section, we give a brief outline of the Adomian decomposition method by considering a general nonlinear differential equation of the form.

$L\big(y(x)\big) + R\big(y(x)\big) + N\big(y(x)\big) = g(x)$ (25)

$L\big(y(x)\big) = g(x) - R\big(y(x)\big) - N\big(y(x)\big)$ (26)

while $N(y)$ is a nonlinear term, $g$ is the source term and $R$ is the remainder of the linear term.

Suppose the inverse operator, $L^{-1}(.) = \int_0^x \int_0^x (.)dxdx$ exists and on application to Eq. (26) gives the expression.

$L^{-1}(Ly(x)) = L^{-1}\big(g(x)\big) - L^{-1}(Ry(x)) - L^{-1}(Ny(x))$ (27)

$y(x) = \varphi_0(x) + g(x) - L^{-1}R(y(x)) - L^{-1}N(y(x)$ (28)

where g$(x)$ is the term obtained from integrating of the source term and $\varphi_0$ is the given conditions. Now rewriting the solution and nonlinear terms as decomposition series of the form

$y(x) = \sum_{n=0}^{\infty} y_n(x)$ and $N\big(y(x)\big) = \sum_{n=0}^{\infty} A_n(x)$ (29)

where the $A_n'^s$ are the Adomian polynomials obtained using the formula

$A_k = \frac{1}{k!}\frac{\partial^k}{\partial\lambda^k}[N(\sum_{n=0}^{\infty} y_{n\lambda^n})]_{\lambda=0}, k = 0,1,2 \ldots$ (30)

Putting Eq. (28) into Eq. (27), we obtain the solution in the form a decomposition series.

$\sum_{n=0}^{\infty} y_n(x) = y(x) = \varphi_0(x) + g(x) - L^{-1}R(\sum_{n=0}^{\infty} y_n(x)) - L^{-1}N(\sum_{n=0}^{\infty} A_n(x))$ (31)


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where $y_0(x) = \varphi_0(x) + g(x)$ is the zeroth component of $y_{n+1}(x)$. The subsequent member of the series is obtained recursively as

$$y_{k+1} = -L^{-1}R\big(y_k(x)\big) - L^{-1}(A_k(x)), \qquad k \geq 0 \tag{32}$$

The exact solution of the problem is the limit of the recursive relation.

$$y(x) = \sum_{k=0}^{n} y_k(x) \tag{33}$$

Rewrite Eqs. (8) and (9), we have the following expressions:

$$L_1 f = f'^2 + A\left(f' - 1 + \frac{1}{2}\eta f''\right) - ff'' - M(1 - f') - 1 \tag{34}$$

$$L_2\theta = Pr\left[2f'\theta + A\left(2\theta + \frac{1}{2}\eta\theta'\right) - f\theta' - Ec\{M(1 - f')^2 + f''^2\}\right] \tag{35}$$

where the differential operators $L_1$ and $L_2$ are defined as $L_1 = \frac{d^3}{d\eta^3}$ and $L_2 = \frac{d^2}{d\eta^2}$. Assuming the inverse of these operators, $L_1^{-1}$ and $L_2^{-1}$ exist and are integrated from 0 to $\eta$ as follows:

$$L_1^{-1}(.) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (.)d\eta d\eta d\eta \tag{36}$$

$$L_2^{-1}(.) = \int_0^\eta \int_0^\eta (.)d\eta d\eta \tag{37}$$

Applying the inverse differential operators, $L_1^{-1}$ and $L_2^{-1}$ to both sides of Eqs. (34) and (35), we have the equivalent expressions

$$f(\eta) = f(0) + \eta f'(0) + \frac{\eta^2}{2} f''(0) + L_1^{-1}(N_1 f) \tag{38}$$

$$\theta(\eta) = \theta(0) + \eta\theta'(0) + L_2^{-1}(N_2\theta) \tag{39}$$

where $N_1(u) = f'^2 + A\left(f' - 1 + \frac{1}{2}\eta f''\right) - ff'' - M(1 - f') - 1$

$$N_2(u) = Pr\left[2f'\theta + A\left(2\theta + \frac{1}{2}\eta\theta'\right) - f\theta' - Ec\{M(1 - f')^2 + f''^2\}\right]$$

In view of the standard Adomian decomposition procedure, we introduce the following expression:

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) = f_0(0) + L_1^{-1}(N_1 f) \tag{40}$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) = \theta_0(0) + L_1^{-1}(N_1\theta) \tag{41}$$

To obtain the components of $f_n(\eta)$ and $\theta_n(\eta)$, the initial values of $f_0(\eta)$ and $\theta_0(\eta)$ are determined by invoking the appropriate boundary conditions as follows:

$$f(0) = S, f'(0) = \lambda, f''(0) = \alpha_1, \theta(0) = 1, \theta'(0) = \alpha_2$$

$$f'(\infty) \longrightarrow 1, \theta(\infty) \longrightarrow 0 \tag{42}$$


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Putting the boundary conditions into Eqs. (39) and (40) gives zeroth order components as

$$f_0(\eta) = S + \lambda\eta + \frac{\alpha_1}{2}\eta^2 \,,\ \theta_0(\eta) = 1 + \alpha_2\eta \quad (43)$$

Accordingly, the recursive scheme for the velocity and temperature distributions takes the form:

$$f_{n+1}(\eta) = S + \lambda\eta + \frac{\alpha_1}{2}\eta^2 + \int_0^\eta \int_0^\eta \left(B_n + A\left(f_n' - 1 + \frac{1}{2}\eta f_n''\right) - C_n - M(1 - f_n') - 1\right) d\eta d\eta d\eta \ (44)$$

$$\theta_{n+1}(\eta) = 1 + \alpha_2\eta + \int_0^\eta \int_0^\eta Pr\left[2D_n + A\left(2\theta_n + \frac{1}{2}\eta\theta_n'\right) - E_n - Ec\{M(1 - f_n')^2 + F_n\}\right] d\eta d\eta \quad (45)$$

where the nonlinear terms are defined as Adomian polynomials in the following expressions:

$$B = f'^2 \Rightarrow B_0 = (f_0')^2, B_1 = 2f_0'f_1',\ \ C = ff'' \Rightarrow C_0 = f_0 f_0'', C_1 = f_0 f_1'' + f_1 f_0''$$

$$D = f'\theta \Rightarrow D_0 = f_0'\theta_0,\ D_1 = f_0'\theta_1 + f_1'\theta_0, E = f\theta' \Rightarrow E_0 = f_0\theta_1' + f_1\theta_0' \quad (46)$$

$$F = f''^2 \Rightarrow F_0 = (f_0'')^2, F_1 = 2f_0''f_1''$$

Hence, the values of $f_n(\eta)$ and $\theta_n(\eta)$ for $n \geq 2$ are determined in the same way.

By the ADM Procedure, we write the unknown functions as a decomposition series of the form:

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta)\,,\theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) \quad (47)$$

Using Eqs. (43) and (44) leads to the three-term approximations for the flow gradients as follows:

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots$$

$$\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \cdots \quad (48)$$

$$f(\eta) = S + \eta\lambda + \frac{\eta^2\alpha_1}{2} + \frac{1}{240}\eta^3\big(40(-1 + \lambda)(1 + A + M + \lambda) - 5\big(8S - 3A\eta - 2\eta(M + \lambda)\big)\alpha_1 + 2\eta^2\alpha_1^2\big) + \cdots \quad (49)$$

$$\theta(\eta) = 1 + \delta_2\eta + \eta^2 Pr\left[-Ec[\alpha_1^2 + M(1 - \lambda - \eta\alpha_1)^2] - \left(S + \eta\lambda + \frac{\eta^2\alpha_1}{2}\right)\alpha_2 + 2(\lambda + \eta\alpha_1)(1 + \eta\alpha_2) + A\left(\frac{\eta\alpha_2}{2} + 2(1 + \eta\alpha_2)\right)\right] + \eta\alpha_2 + \cdots \quad (50)$$

The accuracy of the Adomian decomposition solution increases with increase in the solution terms ($n$). For complete solution of Eqs. (49-50), the constants, $\delta_1$ and $\delta_2$ should be determined using the second boundary condition.


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## COMPARISON OF ADOMIAN DECOMPOSITION AND DIFFERENTIAL TRANSFORM METHODS

In this section, we compare the results for the velocity and temperature distributions using the Adomian decomposition method and differential transform method as well as their errors.

**Table 1: Comparison of numerical, ADM and DTM solutions for velocity profile when $A = 0.1, M = 0.05, S = 2, \lambda = 2,$ (stretching sheet)**

| $\eta$ | $f(\eta)$ | | | | |
|---|---|---|---|---|---|
| | Keller Box | DTM | ADM | Error (DTM) | Error (ADM) |
| 0.00 | 2.10000 | 2.00000 | 2.00000 | 0.1000 | 0.1000 |
| 0.20 | 11.2690 | 11.2680 | 11.2667 | 0.0010 | 0.0023 |
| 0.40 | 66.2000 | 66.1000 | 66.0000 | 0.1000 | 0.2000 |
| 0.60 | 213.9200 | 213.9000 | 213.8000 | 0.0200 | 0.12000 |
| 0.80 | 495.8704 | 495.8690 | 495.8670 | 0.0014 | 0.00340 |
| 1.00 | 947.1100 | 947.1000 | 947.0000 | 0.0100 | 0.11000 |

**Table 2: Comparison of numerical, ADM and DTM solutions for temperature profile when $A = 0.1, M = 0.05, S = 2, \lambda = 2,$ (stretching sheet)**

| $\eta$ | $\theta(\eta)$ | | | | |
|---|---|---|---|---|---|
| | Keller Box | DTM | ADM | Error (DTM) | Error (ADM) |
| 0.00 | 1.2000 | 1.1000 | 1.0000 | 0.10000 | 0.10000 |
| 0.20 | 2.6000 | 2.5400 | 2.50000 | 0.06000 | 0.10000 |
| 0.40 | 10.3000 | 10.2000 | 10.2000 | 0.10000 | 0.10000 |
| 0.60 | 23.0000 | 22.9000 | 22.9000 | 0.10000 | 0.10000 |
| 0.80 | 39.4200 | 39.41000 | 39.4000 | 0.01000 | 0.02000 |
| 1.00 | 58.6100 | 58.6000 | 58.50000 | 0.01000 | 0.11000 |

**Table 3: Comparison of numerical, ADM and DTM solutions for velocity profile when $A = 0.1, M = 0.05, S = -1, \lambda = 2,$ (shrinking sheet)**

| $\eta$ | $f(\eta)$ | | | | |
|---|---|---|---|---|---|
| | Keller Box | DTM | ADM | Error (DTM) | Error (ADM) |
| 0.00 | -0.20000 | -1.00000 | -1.00000 | 0.85000 | 0.95000 |
| 0.20 | 1.16690 | 0.26680 | 1.26890 | 0.90000 | 0.96200 |
| 0.40 | -0.16000 | -1.11000 | -1.15880 | 0.95000 | 0.95120 |
| 0.60 | -1.10000 | -2.10000 | -0.88000 | 1.000000 | 1.12000 |
| 0.80 | -17.93320 | -19.1332 | -17.9233 | 1.200000 | 1.21000 |
| 1.00 | -54.75000 | -56.0000 | -56.7490 | 1.250000 | 1.251000 |


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**Table 4: Comparison of numerical, ADM and DTM solutions for temperature profile when $A = 0.1, M = 0.05, S = -1, \lambda = 2$, (shrinking sheet)**

| $\eta$ | $\theta(\eta)$ | | | | |
|---|---|---|---|---|---|
| | Keller Box | DTM | ADM | Error (DTM) | Error (ADM) |
| 0.00 | 1.10000 | 1.10000 | 1.00000 | 0.00000 | 0.100000 |
| 0.20 | -2.69000 | -3.78000 | -2.67800 | 1.20000 | 1.21200 |
| 0.40 | -13.4500 | -14.7000 | -13.7000 | 1.25000 | 1.260000 |
| 0.60 | -31.4190 | -32.9700 | -31.21900 | 1.55100 | 1.681000 |
| 0.80 | -58.2500 | -59.9000 | -58.1100 | 1.65000 | 1.690000 |
| 1.00 | -94.8900 | -96.6100 | -94.7080 | 1.72000 | 1.792000 |

## CONCLUSION

In this work, the problem of unsteady MHD flow and heat transfer through a stretching/shrinking sheet using two reliable approximate analytical techniques, Adomian decomposition method (ADM) and differential transformation method (DTM) were investigated. Applying these methods, we obtained approximate analytical solutions for the velocity and temperature profiles from the ordinary differential equations. We observed that the solutions obtained by these two methods are in the form of infinite power series. The velocity and temperature distributions obtained by these methods when compared with benchmarked numerical solutions from the Keller box scheme showed excellent agreement. The accuracy, effectiveness and convergence of both methods are displayed in Tables (1-4). However, the result from DTM is closer to the numerical result with minimal errors as opposed to the ADM result. Nevertheless, due to the drawback in dealing with nonlinear terms using Adomian polynomials, DTM is a preferable choice because it is flexible, convenient and its execution is straightforward without computational encumbrances.

## NOMENCLATURE

| | |
|---|---|
| $u$ | velocity of fluid in $x$ −axis |
| $v$ | fluid velocity in $y$ −axis |
| $\alpha$ | Thermal diffusivity of fluid |
| $\beta$ | Volume expansivity of fluid |
| $B_0$ | Applied magnetic field. |
| $\psi$ | Stream function |
| $S$ | Suction parameter |
| $Ec$ | Eckert number |
| $V_0$ | Mass fluid flux at the surface |
| $c_p$ | Specific heat capacity of the fluid |


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| | |
|---|---|
| $\rho$ | Fluid density |
| $\upsilon$ | Kinematic viscosity of the fluid |
| $\theta$ | Dimensional temperature |
| $T$ | Temperature of the fluid |
| $\eta$ | Dimensionless similarity variable |
| $A$ | Unsteadiness parameter |
| $M$ | constant magnetic field |
| $Pr$ | Prandtl number. |

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