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APPROXIMATING LINEAR PROGRAMMING BY GEOMETRIC PROGRAMMING AND ITS APPLICATION TO URBAN PLANNING

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ABSTRACT: *In this study, we approximated linear programming by geometric programming; the developed method converts linear programming to geometric programming. We applied the developed method to Neighbourhood planning, a vital aspect of urban planning, and obtained the optimal cost of Neighbourhood designs. The method demonstrated that geometric programming is a robust non-linear optimization model that can be extended to approximate linear optimization problems. This method has obvious advantages in the sense that it allows every decision variable to contribute to the optimal objective function. This is not the case with the known regular Simplex method and the Interior Point Algorithm of solution to linear programming which assign zeros to some variables when the matrix of the non-basic variables is rectangular or when some of the non-basic variables did not enter the basis. The developed method was used to find the global optimal solution, optimal primal and dual decision variables. The solution was better compared to the linear programming method via Simplex method or Interior Point Algorithm because it achieved the global optimal solution. We observed that in addition to achieving the global optimal solution, we obtained the optimal dual decision variables which was absent in the other methods and all the primal decision variables have value against the other methods that assigned some of the variables with zeroes.*

KEYWORDS: Linear Programming, Geometric Programming, Urban Planning, Optimal Objective Function, Optimal Dual Decision Variables, Optimal Primal Decision Variables.

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INTRODUCTION

Geometric programming is a non-linear optimization technique that is robust, very flexible and can be used to approximate linear optimization problems. Geometric programming has a complex look and because of the presence degrees of difficulties, Kochenberger et al. (1973) approximated a greater than zero degrees of difficulty geometric programming problem with separable programming. The result served a very important purpose of finding a solution to such complex problems as at that time. Recent developments in geometric programming, according to Amuji et al. (2020a), provide solutions to greater than zero degrees of difficulty, and as such, geometric programming problems can be solved directly. In some cases, where the original optimization problem is a terminal one or in extreme complex cases, geometric programming can be used to approximate such a problem.

Geometric programming has found a way into linear programming approximation with efficient and global optimal solution. The solution shows that it is robust and can be applied in some cases where linear programming cannot be applied. In linear programming, we seek to attain both feasibility and optimality condition to attain optimal solution. For maximization problem in linear programming, the solution is said to be optimal if the coefficients of the non-basic variables in the z-row (equation) are at least zeros and non-negative for feasibility to be attained. Again, for minimization problem, the condition of feasibility is the same but the coefficients of non-basic variables at the z-row (equation) are at most zeros. If a problem is feasible but non-optimal, we recover optimality by the Simplex method, but if, on the contrary, a problem is optimal and infeasible, we recover the feasibility by the dual simplex method. In the case where the problem is both infeasible and non-optimal, we recover the feasibility first. Also, in a case where the optimal solution is infeasible, we say that the problem has no feasible solution or we regard the problem as improperly formulated. Hence, for optimality and feasibility condition to be attained, which is the necessary and sufficient condition for optimality, the solution must be feasible. In the same vein, the necessary and sufficient condition for optimality are the orthogonality and normality conditions in geometric programming. In addition, the optimal dual decision variables must be strictly positive.

The number of constraint equations corresponds to the number of slack or surplus variables to be augmented in the linear programming. These slack or surplus variables form the basic starting solution to the linear programming problems. If the number of constraint equations are exactly the same as the number of slack variables or surplus variables, the resulting matrix inverse at the optimal and final tableau will be a square, and in this case, the non-basic variables will all enter the basis thereby making each of the variables contribute to the optimal solution. But in the case where the resulting matrix is rectangular, one or more of the non-basic variables that did not enter the basis will assume zero. That means that they did not contribute to the optimal objective function. We have discovered that the reason for the zero contribution was because of the resulting matrix of the problem, but if we can find a better solution to the resulting rectangular matrix without allowing any of the value to assume zero, then all the decision variables (non-basic variables) will contribute to the optimal objective function. This is a limitation with linear programming via Simplex method or Interior point algorithm. At this point, geometric programming will be extended and used to solve such a problem and it gives each of the non-basic variables (decision variables) a sense of belonging in contributing to the optimality of the problem.

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In this paper, we approximate linear programming by geometric programming with special interest in urban planning. The method produces a global and better optimal solution compared to linear programming via Simplex method. The method has an added advantage of not only producing the dual decision variables but also giving every primal decision variable equal opportunity to contribute to the optimal objective function.

LITERATURE REVIEW

Avriel and Williams (1970) and Ecker (1980) observed that posynomials are concave functions; however, the conversion of geometric programming to a convex program is based on logarithmic transformation of both objective and constraint functions. But instead of the original decision variables, x_i, replace it with $y_i = \log x_i \Rightarrow x_i = e^{y_i}$, and instead of minimizing $f_0(x)$, minimize $\log f_0(x)$. Then the inequality $f_i(x) \leq 1$ with $\log f_i(x) \leq 0$, and $g_i(x) = 1$ with $\log g_i(x) = 0$ (see Boyd et al., 2007). Duffin (1970) and Ben-Tal and Ben-Israel (1976) transformed primal geometric programming function to linear programming. The author made a Logarithmic transformation and it became a linear function which was solved using linear programming code. They stated and proved the optimality theorem of the transformed problem in terms of linear programs. Ecker and Zorack (1976) observed that if posynomials consist of a single term, then the log- linear transformation of it was equivalent to a linear program in the Log of the variables. Their work was mainly on posy-binomial program. We discovered that no work, to our best knowledge, has been done on approximating linear programming by geometric programming; rather, the reverse was the case from the literature. In this paper, we have approximated linear programming by geometric programming. This method has some advantages: It makes geometric programming wider in scope and applications. It does not give room for zero contribution of the decision variables to the optimal objective function due to the resulting matrix in the final tableau. A rectangular matrix or non-basic variable that did not enter the solution does not contribute to the optimal solution of the program. Our method, approximating linear programming by geometric programming, has taken care of such problem. In addition, our method achieves a global optimal solution.

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METHODOLOGY

Development of the Approximating Model

The standard form of linear programming (LP) problem with m constraint and n variables are given as

$$Max\ (Min)\ \ Z = \sum_{j=1}^{n} C_j X_j \tag{1}$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} X_j \le b_i \tag{2}$$

$$X_j \ge 0 \quad ; \quad b_j \ge 0$$

where C_j are the cost coefficients, X_j are the decision variables, a_{ij} are the available resources, b_j are the constraints due to limited resources.

From Equation (1), we have

$$Min\ \ Z = C_1 x_1 + C_2 x_2 + \ldots + \mathrm{C_n} x_n \tag{3}$$

Taking the ln of both sides, we have

$$\ln(Min\ \ Z) = \ln(C_1 * C_2 * \ldots \mathrm{C_n} * x_1 x_2 * \ldots * x_n) \tag{4}$$

$$\ln(Min\ \ Z) = \ln\left(\sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}}\right) \tag{5}$$

Taking the exponential of both sides of Equation (5), we have

$$Min\ \ f(x) = \left(\sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}}\right) \tag{6}$$

Equation (6) is the same as the unconstrained geometric programming problem, where $\{a_{ij}\}$ is the exponent matrix obtained from the powers of the primal decision variables, x_i and Z = f(x).

From Equation (2), the constraint equation, we have

$$\min g(x) = \frac{1}{b_i}\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} X_j \le 1 \tag{7}$$

$$\min g(x) = -\frac{1}{b_i}\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} X_j \ge 1 \tag{8}$$

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Equation (8) is in the standard form and a reversed form of constraint equation (see Abrahams & Bunting, 1974); our interest is not on the constant ($-1/b_i$); hence, it is ignored. We write Equation (8) as (9);

$$\min g(x) = a_1 x_1 + a_2 x_2 + \ldots + \mathrm{a}_{\mathrm{n}} x_n \geq 1 \qquad (9)$$

Taking ln of both sides, we have

$$\ln(\min g(x) = \ln(a_1 a_2 \ldots a_n * x_1 x_2 \ldots x_n) \geq 0 \qquad (10)$$

$$\ln(\min g(x)) = \ln\left(\sum_{j=1}^{n} a_i^{+} \prod_{i=1}^{m} x_i^{aij}\right) \geq 0 \qquad (11)$$

where a_i^+ is the same as the cost coefficients (C_j).

Similarly, taking the exponential of both sides, we have

$$\min g(x) = \sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}} \geq 1 \qquad (12)$$

Since Equation (12) has a reversed constraint, we have

$$\min g(x) = \sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{-a_{ij}} \leq 1 \qquad (13)$$

Equation (13) is a standard form of geometric programming constraint equation. Hence, the approximated geometric programming (Gp) problem from Equations (6) and (13) becomes

$$Min\ \mathrm{f(x)} = \left(\sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{a_{ij}}\right) \qquad (14)$$

Subject $g(x) = \sum_{j=1}^{n} C_j \prod_{i=1}^{m} x_i^{-a_{ij}} \leq 1$ (15)

Subject to $Ay = B$ (16)

Equations (14) and (15) are standard constrained geometric programming model subject to the orthogonality and normality condition of Equation (16).

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DATA PRESENTATION/APPLICATION OF THE METHOD

Table 1: Cost of Neighbourhood Design for Three Layouts (Million Naira)

Layout	Street planning	Street Maintenance	Structure mapping
1	3	-1	2
2	-2	4	0
3	-4	3	0

Source: Survey Research (2023)

Table 2: Resources Available for Each of the Three Layouts

Neighbourhood design	Layout 1	Layout 2	Layout 3
Available Resources (million naira)	7	12	10

Source: Survey Research (2023)

Table 3: Cost of Planning for Each of the Three Layouts

Neighbourhood design	Layout 1	Layout 2	Layout 3
Cost of Planning (million naira)	1	-3	-2

Source: Survey Research (2023)

Formulating the Neighbourhood planning problem presented in Tables 1-3, let x_1 = cost of street planning, x_2 = cost of maintenance and x_3 = cost of mapping/structure location. Our objective is to the cost of neighbourhood planning. Putting the problem in a linear programming form and applying our developed approximation method, we have

$$\text{Minimize}\,(z) = x_1 - 3x_2 - 2x_3$$

$$\begin{aligned} \text{subjecto}: 3x_1 - x_2 + 2x_3 &\leq 7 \\ -2x_1 + 4x_2 &\leq 12 \\ -4x_1 + 3x_2 &\leq 10 \end{aligned}$$

Applying the procedure from Equation (1) to (16), we have:

$$\text{Minimize}\,(z) = x_1 - 3x_2 - 2x_3 \qquad (17)$$

$$\text{subjecto}: 3x_1 - x_2 + 2x_3 \leq 7 \qquad (18a)$$

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$$-2x_1 + 4x_2 \leq 12 \quad (18b)$$

(18c)

$$-4x_1 + 3x_2 \leq 10$$

Now,

Taking the ln of both sides of Equation (17), we have

$$\ln(\mathrm{Min}(z)) = \ln(x_1 - 3x_2 - 2x_3)$$
$$\ln(Min(z)) = \ln(x_1 . 3x_2^{-1} . 2x_3^{-1})$$
$$\ln(Min(z)) = \ln(6x_1 x_2^{-1} x_3^{-1})$$

Taking the exponential of both sides, we have

$$e^{\ln(Min(z))} = e^{\ln(6x_1 x_2^{-1} x_3^{-1})}$$

$$Min(z) = 6x_1 x_2^{-1} x_3^{-1} \quad (19)$$

From Equation (18a), we have

$$\frac{3}{7}x_1 - \frac{1}{7}x_2 + \frac{2}{7}x_3 \leq 1$$

$$\ln(\frac{3}{7}x_1^{1} . \frac{1}{7}x_2^{-1} . \frac{2}{7}x_3^{1}) \leq \ln(1)$$

Taking exponential of both sides, we have

$$\frac{6}{345}x_1^{1} x_2^{-1} x_3^{1} \leq 1 \quad (20)$$

From Equation (18b), we have

$$-\frac{1}{6}x_1 + \frac{1}{3}x_2 \leq 1$$

$$\ln(\frac{1}{6}x_1^{-1} . \frac{1}{3}x_2^{1}) \leq \ln(1)$$

$$\ln(\frac{1}{18}x_1^{-1} x_2^{1}) \leq \ln(1)$$

Taking exponential of both sides, we have

$$\frac{1}{18}x_1^{-1} x_2^{1} \leq 1 \quad (21)$$

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From Equation (18c), we have

$$-\frac{2}{5}x_1 + \frac{3}{10}x_2 \le 1$$

$$\ln(\frac{2}{5}x_1^{-1}.\frac{3}{10}x_2^{1}) \le \ln(1)$$

Taking exponential of both sides, we have

$$\frac{3}{25}x_1^{-1}x_2^{1} \le 1 \qquad (22)$$

Putting Equations (19) to (22) together, we have a standard constrained geometric programming problem as follows

$$\text{Min } f(x) = 6x_1x_2^{-1}x_3^{-1} \qquad (23)$$

We split Equation (23) to obtain equation (24), see Amuji et al. (2020b)

$$\text{Min } f(x) = 4x_1x_2^{-1} + 2x_3^{-1} \qquad (24)$$

Subject to:

$$g_1(x) = \frac{6}{345}x_1^{1}x_2^{-1}x_3^{1} \le 1 \qquad (25a)$$

$$g_2(x) = \frac{1}{18}x_1^{-1}x_2^{1} \le 1 \qquad (25b)$$

$$g_3(x) = \frac{3}{25}x_1^{-1}x_2^{1} \le 1 \qquad (25c)$$

Equation (24) is the objective function of a geometric program in standard form constrained by three constraint equations (25a–25c) in standard geometric programming form.

Solving the above geometric programming problem, we have

Solution:

$$\textit{Minimize } f_0(x) = \sum_{j=1}^{N_0} C_{0j} \prod_{i=0}^{m_0} x_i^{a_{0ij}}$$

$$\textit{Subject to} \quad g_k(x) = \sum_{j=1}^{N_k} C_{kj} \prod_{i=0}^{m_k} x_i^{a_{kij}} \le 1$$

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The degree of difficulty of the problem is: $K = N - (m + 1)$

In this problem, $n = 3$; $N_o = 2$, $N_1 = 1$, $N_2 = 1$, $N_3 = 1$, $N = 5$, $m = 3$. Hence, $K = 5 - 4 = 1$, so the problem has one degree of difficulty. Since the problem has no unique solution, we maximize the dual objective function subject to linear constraint (see Boyd et al., 2007). This gives rise to the dual geometric program given as

$$Max\ f(y) = \prod_{k=0}^{m} \prod_{j=1}^{N_k} \left(\frac{C_{kj}}{y_{kj}} \sum_{i=1}^{N_k} y_{kl} \right)^{y_{kj}}$$

Subject to the normality and orthogonality conditions of Equation (16)

$$Ay = B$$

Forming the orthogonality and normality condition for the dual decision variables, we have

$y_1 + 0y_2 + y_3 - y_4 - y_5 = 0$

$-y_1 + 0y_2 - y_3 + y_4 + y_5 = 0$

$0y_1 - y_2 + y_3 + 0y_4 + 0y_5 = 0$

$y_1 + y_2 + 0y_3 + 0y_4 + 0y_5 = 1$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & -1 \\ -1 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}_{A(4\times 6)} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}_{Y(5\times 1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{B(4\times 1)}$$

Solving for y_i using MATLAB, we have

```
A = [1, 0, 1, -1, -1; -1, 0, -1, 1, 1; 0, -1, 1, 0, 0; 1, 1, 0, 0, 0];

B = [0; 0; 0; 1];

y* = Pinv(A)*B
```

$$y^* = \begin{bmatrix} 0.6667 \\ 0.3333 \\ 0.3333 \\ 0.5000 \\ 0.5000 \end{bmatrix} > 0$$

The above values of y* are the optimal weights of the dual decision variables and they satisfy the orthogonality and normality conditions of Equation (16)

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Then the optimal objective function is:

$$f*(x) = \left(\left(\frac{4}{0.6667}\right)^\wedge 0.6667\right) * \left(\left(\frac{2}{0.3333}\right)^\wedge 0.3333\right) * \left(\left(\frac{0.0175}{0.3333}\right)^\wedge 0.3333\right)$$
$$* \left(\left(\frac{0.0556}{0.5}\right)^\wedge 0.5\right) * \left(\left(\frac{0.096}{0.5}\right)^\wedge 0.5\right) * \left((0.3333)^\wedge 0.3333\right) * \left((0.5)^\wedge 0.5\right) * \left((0.5)^\wedge 0.5\right)$$

$$f*(x) = 113{,}800.00$$

The above is the optimal (minimum) cost of carrying out the neighbourhood (urban) planning for the three layouts and this represents the optimal objective function.

Solving for the optimal primal decision variables from the relationship,

$y * f*(x) = C_j \prod x_i^{a_{ij}}$, (see Rao, 2009)

we have:

$$\frac{(0.6667)(0.1138)}{4} = x_1 x_2^{-1}$$

Hence, we have

$$0.0190 = x_1 x_2^{-1}$$

$$0.0190 = x_3^{-1}$$

$$2.1674 = x_1 x_2^{-1} x_3$$

$$1.0234 = x_1^{-1} x_2$$

$$0.5927 = x_1^{-1} x_2$$

The first two rows of the above (by intuition) satisfied the optimal objective function; hence, we restrict ourselves to the two rows.

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Taking the ln of both sides of the two rows, we have:

$$-3.9633 = \ln x_1 - \ln x_2 + 0\ln x_3$$

$$-3.9633 = 0\ln x_1 + 0\ln x_2 - \ln x_3$$

Let $\ln x_i = w_i$, see Amuji et al. (2021).

$$-3.9633 = w_1 - w_2 + 0w_3$$

$$-3.9633 = 0w_1 + 0w_2 - w_3$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -3.9633 \\ -3.9633 \end{bmatrix}$$

A = [1, -1, 0; 0, 0, -1];

B = [-3.9633; -3.9633];

w* = Pinv(A)*B

$$x^* = \begin{bmatrix} 0.1378 \\ 7.2543 \\ 52.6307 \end{bmatrix} > 0$$

The above values of x* in millions are the optimal weights of the primal decision variables which satisfied the optimal objective function.

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SUMMARY AND CONCLUSION

Summary

In this study, we developed a method that converted linear programming to geometric programming. From our method, we demonstrated that geometric programming is a robust non-linear optimization model that can be extended to approximate linear optimization problem. This method has obvious advantages in the sense that it allows every decision variable to contribute to the optimal objective function. The method was applied to a vital aspect of urban planning called neighbourhood planning and optimal cost obtained. Therefore, the method was used to find the global optimal solution, optimal dual and primal decision variables. The solution was better compared to the linear programming method because it has the global minimal (optimal) solution. The optimal solutions from both methods are: Linear programming via Simplex method ($z = 19m$; ($x_1 = 0$, $x_2 = 3$, $x_3 = 5$) millions) after three iterations; Interior point algorithm method ($z = -19m$; ($x_1 = 0$, $x_2 = 3$, $x_3 = 5$) millions) after seven iterations; and Geometric programming method ($f(x) = 113800$; ($y1 = 0.6667$, $y2 = 0.3333$, $y3 = 0.3333$, $y4 = 0.5000$, $y5 = 0.5000$), ($x_1 = 0.1378$, $x_2 = 7.2543$, $x_3 = 52.6307$) millions). We observed that in addition to achieving the global optimal solution, we obtained the optimal dual decision variables which were absent in the other methods. The global optimal (minimal) solution is in agreement with Kochenberger et al. (1973).

Conclusion

We have approximated linear programming problem by geometric programming and obtained a better solution. The geometric programming approximation method has advantages of finding the optimal dual decision variables, optimal primal decision variables and global optimal solution. The method also finds some hidden values of some variables which linear programming assigned zero to because it could not find such values. No matter the nature of matrix that results from the final and optimal tableau of the linear programming (rectangular or square), the developed geometric programming method can comfortably solve it and obtain an appropriate solution to it. Hence, we have extended geometric programming to approximate linear programming problem.

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