African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 6, Issue 4, 2023 (pp. 68-76)
www.abjournals.org

SYMMETRY AND ASYMMETRY MULTIVARIATE GARCH MODELING OF CONSUMER PRICES INDEX, CRUDE OIL PRICE, INFLATION RATE AND EXCHANGE RATE

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Cite this article:

Wiri L., Archibong M.E. (2023), Symmetry and Asymmetry Multivariate Garch Modeling of Consumer Prices Index, Crude Oil Price, Inflation Rate and Exchange Rate. African Journal of Mathematics and Statistics Studies 6(4), 68-76. DOI: 10.52589/AJMSS-6IYLHM4Z

Manuscript History

Received: Aug July 2023

Accepted: 31 Sept 2023

Published: 14 Oct 2023

ABSTRACT: *The study looked at changes in Nigeria's exchange rate, inflation rate, consumer price index, and price of crude oil. Monthly data from January 2004 to December 2020 were utilized in this analysis, and they were taken from the statistical bulletin of the Central Bank of Nigeria (CBN). The data's time graphic showed the trend series' present state. For the analysis, E-view 12 statistical software was employed. Modeling employed both symmetric and asymmetric processes. Using both symmetric and asymmetric modeling techniques, the Multivariate Generalized Autoregressive Conditional Heteroscedasticity (M-GARCH) model was developed. To estimate three models for multivariate GARCH, a constant conditional correlation, a diagonal VECH, and a diagonal BEKK. The conditional variance and conditional covariance were estimated using these models. Every variance and covariance model had a significance level of 5%.*

KEYWORDS: Symmetric, Asymmetric, Multivariate – GARCH, Diagonal VECH, Diagonal BEKK.

Article DOI: 10.52589/AJMSS-6IYLHM4Z
DOI URL: https://doi.org/10.52589/AJMSS-6IYLHM4Z

INTRODUCTION

One of the most challenging problems nowadays for econometricians, time series analysts, and policymakers is modeling the interaction between microeconomic variables, which has become a significant source of worry in the financial markets. This has been the subject of all research for a very long time. Ijomah and Enewari (2020) affirm that researcher may encounter volatility modeling of microeconomic variables like the price of crude oil, the exchange rate, the inflation rate, and the consumer price index when analyzing financial time-series data in global markets with very persistent volatility. Latent volatility is a problem that results from the high frequency of microeconomic variables. Volatility is an important component to take into account when evaluating investment options for portfolio pricing, building, hedging, and risk management even if it has been said that volatility is not the same as risk, especially when it is perceived as a certainty. Volatility can be defined as the rate of variation in movement between the four variables, namely the exchange rate for crude oil prices, the inflation rate, and the consumer price index. It is presented as a percentage shift in the monthly variances of the series. The extent of the shift determines how much change there will be, and the higher the change, the higher the volatility. The risk of other factors increases with the degree of exchange rate volatility on the global market, and rising crude oil prices result in higher inflation rates across the board. Exchange rates that fluctuate seem riskier than fixed rates since they can be altered at any time (Wiri & Sibeate, 2021). Research on multivariate generalized autoregressive conditional heteroscedasticity and volatility of microeconomic series has also concentrated on a number of series return-related features, such as time-varying volatility clustering, symmetric and asymmetric patterns, and so on. There has been a lot of discussion about the Garch model's various characteristics' excessive volatility in global markets. The rise in financial volatility and conditional mean in recent years has raised investors' and traders' anxiety of trading on international markets, which has been well-documented in financial literature. These four variables oscillate between two different levels. It would be unreasonable to anticipate that a Univariate Garch model will adequately capture these unique patterns for such data. Applying linear time series models to analyze the dynamic performance of economic variables is quite prevalent (Madse, 2008).

LITERATURE REVIEW

The idea behind GARCH models refers to a scenario in which series are utilized to simulate the variance of a process that mimics the characteristics of volatile persistent trend behavior over time. This idea has been connected to the volatility, also known as the conditional standard deviation of the underlying price return level.

Ijomah and Enewari (2020) looked into the volatility transmission between the price of oil and the exchange rate using Multivariate GARCH modeling. Between January 2009 and December 2018, the transmission of volatility between series using the BEKK, DCC, and CCC models was investigated. Similar behaviors were shown by the model estimation process for variances and covariances. The crude oil price and the Nigerian exchange rate have a tenuous negative relationship, as shown by the conditional correlation using the DCC model, which is very weak and negative. This shows that there is a tendency for the volatility of the exchange rate and the volatility of the price of oil to move in different directions. The results demonstrate significant

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temporal fluctuations in conditional covariances for both the exchange rate and the price of crude oil.

In reality, Symmetric and Asymmetric GARCH models are included in this classification (Wiri & Sibeate, 2020). This symmetric model solely considers risk related to volatility, but the asymmetric model captures risk related to leverage impact and ensures non-negativity of the estimate of returns on microeconomic data.

Using the Unrestricted Vector Autoregressive (UVAR) model, Tuaneh and Wiri (2018) investigated the relationship between crude oil prices, currency exchange rates, and inflation rates. Granger causality among the interactions is present in their investigation. The outcome demonstrated that, at order one 1(1), all variables were stationary. The vector autoregressive model's inverse root revealed a stable VAR model. One lag length was chosen using the lag selection criteria.

Maryam and Ramanathan (2012) further revealed that under the Autoregressive conditional Heteroscedasticity framework, it is generally assumed that large shocks tend to follow large shocks vice-versa; the small stocks tend to follow small shocks, a phenomenon which is known as volatility clustering. According to Maryam and Ramanathan (2012), using long memory models in modelling the volatility of a time series the main target is to improve the efficiency in parameter estimation and the accuracy of the forecast. Financial time series and other financial related literature models which attempts to explain the changes in conditional standard deviation are generally referred to as conditional Heteroscedastic models

Bunnay (2015) examined the correlations and spillovers between the volatility of crude oil, gasoline, heating oil, and natural gas futures. Daily data collection served as the basis for this inquiry. Multivariate GARCH models included the VAR (1)-diagonal VECH, VAR (1)-diagonal BEKK, and VAR (1)-CCC. The empirical findings revealed that, with the exception of GASOLINE with RNG, the estimations of the Multivariate GARCH parameters were statistically significant in almost all situations. Accordingly, RCRUDE with GASOLINE had the best long-run shock persistence on the conditional correlations, whereas CRUDE with RHEATOIL had the best short-run shock persistence.

METHODOLOGY

SYMMETRY MULTIVARIATE GARCH MODEL

3.1.1 The General Case of VECH Model

The VECH model was introduced by Bollerslev, Engle ay Wooldridge (1988). It is much more general compared to the subsequent formulations. In the VECH model, the conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns. The unrestricted VECH model can be expressed below:

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$$VECH(H_t) = A + \sum_{i=1}^{p} B_i VECH(\mu_{t-i}\mu_{t-1}^{/}) + \sum_{i=1}^{q} C_i VECH(H_{t-i}) \quad \textbf{(1)}$$

where $VECH(H_t)$ is an operator that stacks the columns of the upper and lower triangular part of its square matrix, N presents the number of variables, t is the index of the t^{th} observation, c is an $\frac{N(N+1)}{2} \times 1$ vector, B_i and C_i are $\frac{N(N+1)}{2} \times \frac{N(N+!)}{2}$ parameter matrices and μ_{t-1} in an $N \times 1$ vector. The condition for H_t is to be positive definite for all t is unrestrictive**.** The VECH operator takes the upper triangular portion of the matrix and stack each element into a vector with a single column i.

The diagonal VECH model is the restricted kind of VECH model which was proposed by Bollerslev et al. (1988). It assumes the parameter A_i and B_i upper or lower diagonal matrices which makes it possible for H_t to be positive definite for all *t*. the parameter estimation process are more smooth compared to the unrestricted VECH model. The restriction implies that there are no direct volatility spillovers from one series to another.

THE GENERAL BEKK MODELS

The BEKK model (simply an acronym BEKK representing Baba, Engle, Kraft and Kroner, which was a preliminary version of Engle and Kroner (1995) addresses the difficultly with VECH of ensuring that the H_t matrix is always positive definite the BEKK models is represented as follow

$$H_t = AA^{/} + B^{/}\varepsilon_{t-1}\varepsilon_{t-1}^{/}B + C^{/}h_{t-1}C \quad (2)$$

In the Diagonal BEKK model, B and C are $N \times N$ matrix and A is a lower or upper triangular matrix of the parameter. The positive definiteness of the covariance matrix is ensure

The Constant Correlation Model

The constant correlation model was introduced by Bollerslev (1986) to mainly model the condition covariance matrix indirectly by estimating the conditional correlation matrix. The conditional correlation is expected to be constant while the conditional variances are changing. Consider the CCC model.

$$h_{iit} = C_i + a_i\varepsilon_{it-i}^2 + b_i h_{iit-t} \quad (3.29)$$

The off Diagonal element of H_t H_{ij} $(i \neq j)$ is defined indirectly via the correlation denoted

p_{ij}

$$h_{ijt} = p_{ij} h_{iit}^{\frac{1}{2}} h_{jjt}^{\frac{1}{2}} \text{ j,i=1,2…………..N } i < j \quad (3.30)$$

ASYMMETRY MULTIVARIATE GARCH MODEL

The asymmetric model have become very popular in empirical application when the conditional variance and covariance are permitted to react differently to positive and negative shock of the same size. The asymmetric multivariate Garch model of differences Garch model is represented as follow.

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Asymmetric Diagonal Vech Models

$$h_{ijt}= w_{il}+A_{ij}\mu_{it-i}\mu^{I}_{jt-1}A^{/}_{ij}+B_{ij}h_{ijt-1}+ D_{ij}\epsilon_{it-i}\epsilon^{I}_{jt-1}D^{/}_{ij} \quad (3.4)$$

Asymmetric Diagonal BEKK Models

$$H_t= AA^{/}+ B^{/}\varepsilon_{t-1}\varepsilon^{/}_{t-1}B+ C^{/}h_{t-1}C + D^{/}\mu_{t-1}\mu^{/}_{t-1}D \quad (3.5)$$

Asymmetric Constant Correlation Model.

$$h_{iit}= C_i + a_i\varepsilon^2_{it-i} +b_i h_{iit-t}+\theta_i\mu^2_{it-i} \quad (3.6)$$

Model Estimation of Optimal Lag Length

The Multivariate Garch model lag length selection is the number of observations in a time series that will be used to interpret the lag length of the R-Garch model. In selecting the lag length, using few lags can result in auto-correlated errors whereas using too many lags results in over-fitting, causing an increase in mean-square-forecast errors of the model. The lag length p is selected in such a way as to minimize the information criteria (Tuaneh & Wiri, 2019). The lag length for the Garch model can be determined using model selection criteria. The commonly used information criteria are: Akaike information criterion (AIC), Hannau-Quinn information criterion

(HQ), and Schwarz information criterion (SIC).

RESULT

Monthly series on crude oil prices in US dollars, the exchange rate in US dollars, the consumer price index, and the inflation rate in Nigeria from 2004 to 2020 (204 observations) were utilized to achieve the study's aims. These series' raw data can be seen on the website http://www.centralbank.com.

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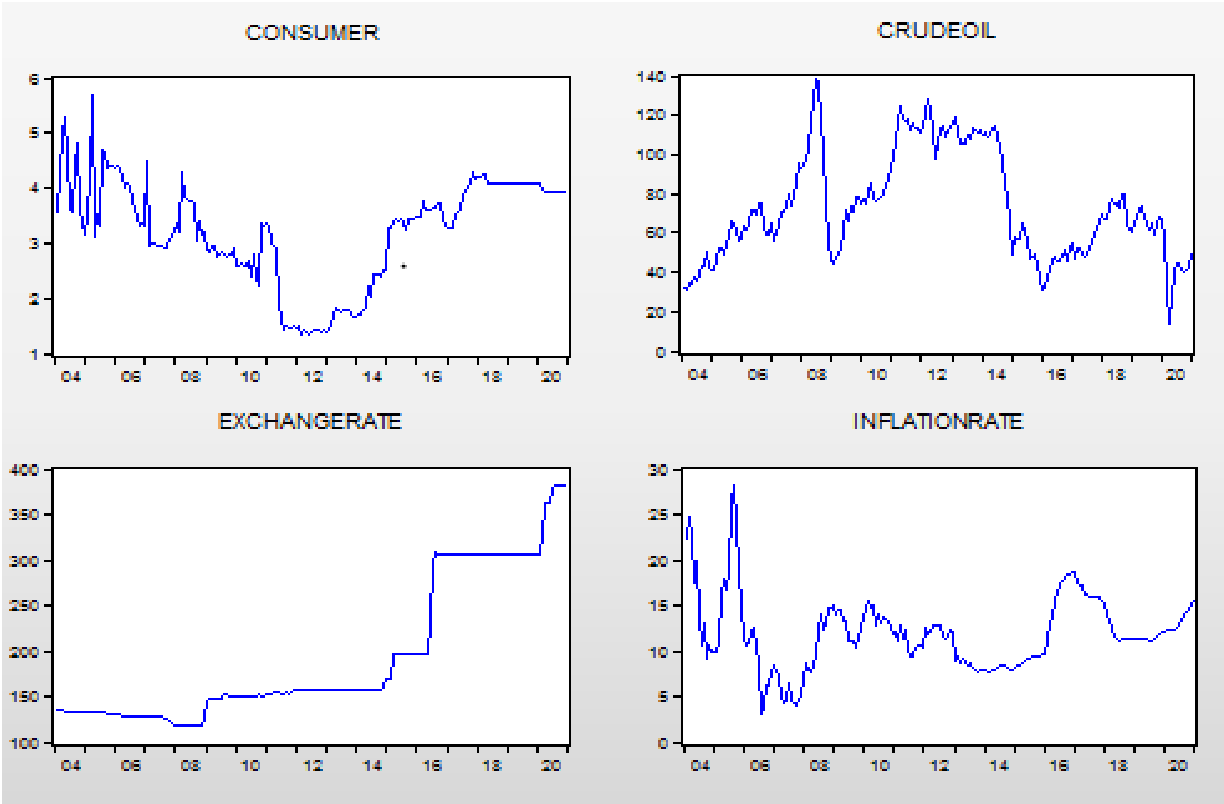

Figure 4.1: Time Plot of The Series at Level

The series is shown by the vertical axis, while the months are represented by the horizontal axis. A positive trend was visible in the time plot of the four variables at the levels, which is evidence of the series' non-stationary process. Over the course of the observations, this display has been consistent and positive (the volatility levels are high).

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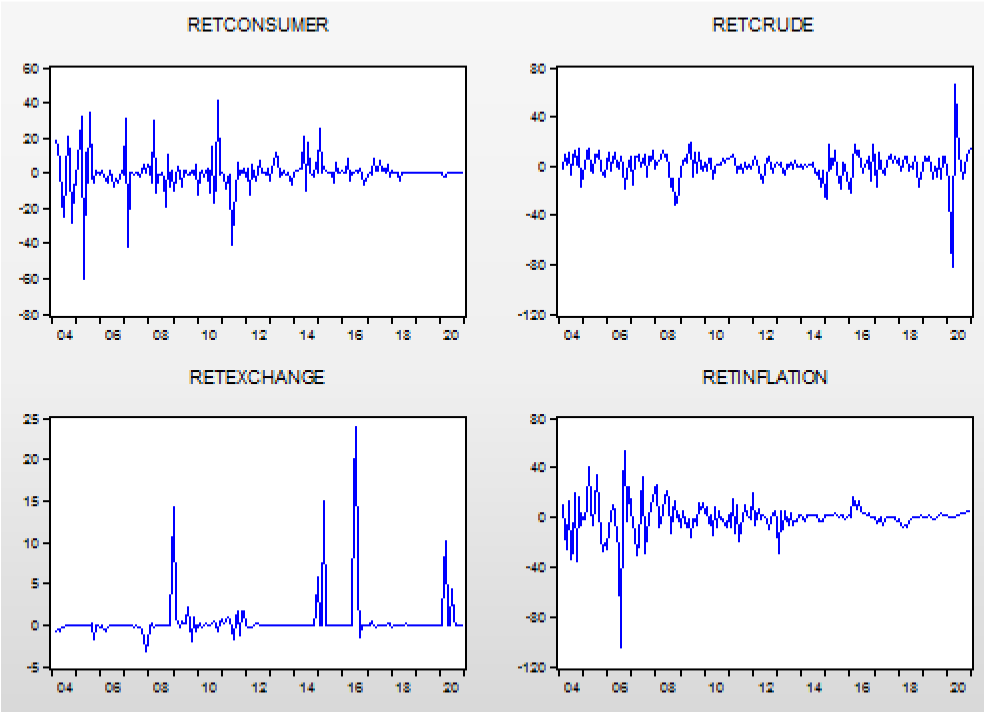

Figure 4.2: Time Plot of the Return Series at First Differences

The study variables require differencing at certain levels in order to achieve stationarity during the process. Figure 4.5 displays the four variables' returns. At 0, all series were volatile, indicating a stagnant process.

Table 4.1: Lag Selection Criteria

The criteria to select exact lag length (p) for a univariate and Multivariateseries is determined using the lag length selection such as Akaike information criterion (AIC), Hannau-Quinn information criterion and Schwarz information criterion (SIC). The best lag length is selected based on minimizing information criteria. Chris brooks, (2008).

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-2744.595	NA	20567156	28.19071	28.25785*	28.21790
1	**-2707.064**	**73.13644***	**16492636***	**27.96989***	**28.30558**	**28.10581***
2	-2693.296	26.26604	16878132	27.99278	28.59702	28.23743
3	-2684.668	16.10482	18213501	28.06839	28.94119	28.42178
4	-2677.308	13.43662	19920749	28.15701	29.29836	28.61913
5	-2672.359	8.832720	22346743	28.27035	29.68025	28.84120
6	-2659.820	21.86268	23208082	28.30584	29.98431	28.98543
7	-2647.865	20.35321	24269186	28.34734	30.29435	29.13566
8	-2639.148	14.48397	26263086	28.42203	30.63760	29.31909

Based on the Schwarz information criterion (SC), Akaike information criterion (AIC), and Hannan-Quinn information criterion, the table above shows that the lag of order one (p=1) is

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suitable for the univariate and multivariate models of inflation rate, exchange rate, consumer prices index, and crude oil prices. The lag with the shortest informational criteria has the best length.

TABLE 4.2: COMPARING THE RESULT OF SYMMETRIC AND ASYMMETRIC MULTIVARIATE GARCH MODEL USING INFORMATION CRITICAL

Symmetric Multivariate GARCH				**Asymmetric Multivariate GARCH**		
Models	AIC	SC	HQ	AIC	SC	HQ
Diagonal Vech	27.22	27.538	27.35	**26.216**	**26.8**	**26.5**
Diagonal Bekk	**25.61**	**25.87**	**25.72**	26.87	27.366	27.074
CCC	**25.57**	**25.93**	**25.51**	25.346	25.77	25.719

Using information criteria, six models were estimated for the multivariate GARCH model: three for the symmetric multivariate GARCH model and three for the asymmetric multivariate GARCH model. To calculate the covariance and correlation between series, these six models were utilized. These models are comparable to the univariate GARCH model, but multivariate GARCH allows for time-varying variances and covariance. As stated in the table above, we will limit our estimation in this work to Multivariate GARCH models that provide symmetric and asymmetric Diagonal VECH and Diagonal BEKK constant correlation models.

CONCLUSION

The application of several multivariate Garch models to the modeling of crude oil prices, the consumer price index, the exchange rate, and the rate of inflation in Nigeria was the main emphasis of this study. We used natural modeling techniques to simulate the variance and covariance stationary process when estimating the models in order to ascertain the presence of volatility in the research variable. The series' erratic movement was found via a critical analysis of the time plot. In order to prevent the issue of spurious regression, the series were verified for stationarity. To look for unit roots, the Augmented Dickey-Fuller (ADF) test was used. The Akaike information criterion, Hannau-Quinn information criterion, and Schwarz information criterion (SIC) were used to determine the lag duration of the models. The covariance and correlation between four variables were estimated using the Multivariate Garch models. These models are comparable to the univariate Garch model, with the exception that Multivariate-Garch permits the variances and covariance to change over time. However, the CO-VARIANCE parameter from the VECH model was estimated using six equations, and the parameter in the model was extremely volatile. The variance (the part of the GARCH model to measure the presence of return series volatility) was estimated using four models.

Based on information criteria and the matrix satisfying the positive definite condition that cannot be attained in a symmetric process, the asymmetric Diagonal VECH model was chosen. Six equations were calculated for the CO-VARIANCE (to ascertain the interaction between the variables) and four models were generated for variance (the component of the GARCH model to quantify the presence of volatility in the return series). For the symmetric and asymmetric diagonal BEKK model technique, two models were computed. Using the information criteria, the symmetric model fared better than the asymmetric model for the

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diagonal BEKK. Additionally, because the matrix is positive definite, the coefficient of the BEKK models satisfies the necessary and sufficient criterion for asymmetric multivariate GARCH models. This suggests that a rise in the variance return of the variables follows an increase in risk, which is shown by a rise in the conditional variance. However, all models utilizing the two processes had a constant conditional correlation. Based on the minimum information requirements, the symmetric constant condition correlation was the best. According to the model, there is no ongoing association between the return on crude oil prices, the exchange rate, and the inflation rate. Since the relevant time-varying correlation matrices for these correlations are weighted averages of two sets of constant correlations, they always have a positive or negative effect on the conditional variance.

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