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EVALUATION OF SOME ALGEBRAIC STRUCTURES IN BALANCED INCOMPLETE BLOCK DESIGN

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ABSTRACT: *Balanced incomplete block design is an incomplete block design in which any two varieties appear together an equal number of times. In algebra, the existence of block design is closely related to balanced incomplete block design. To ascertain the claim, this research aim to employ some algebraic structures to examine whether or not balanced incomplete block design is related to the above statement. The methods adopted are finite group, ring and field algebra. The result shows that balanced incomplete block design (BIBD) cannot form a finite group under multiplication binary operation, but it is for additive case. It is also revealed that balanced incomplete block design is not a field algebra in both binary operations no matter the size of the design, but it is a ring in all cases. In conclusion, BIBD of the form (X, B) is a semigroup, commutative group, semiring, commutative ring and subfield in both binary operations. Several theorems with proofs have been established in harmony with the algebraic structure mentioned above.*

KEYWORDS: Algebra, Group, Field, Ring, Block design, Balanced and Incomplete Block design.

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INTRODUCTION

Block design is a branch of combinatorial mathematics which underlines the study of existence, construction and properties of finite sets whose arrangements satisfy some concepts of balance and symmetry BIBD [8]. The generation of block designs is a well-known combinatorial problem, which is very hard to solve [2]. Block design is a type of combinatorial design defined as a pair (V, B) such that V is a finite set and B is a collection of nonempty subsets of V, the elements in V are called points while subsets in B are called blocks. This design has wider applications from many areas, including experimental design, finite geometry, physical chemistry, software testing, cryptography and algebraic units [10]. A regular block design with V- varieties and b blocks is balanced and is called (v, b, r, k, λ) - design or (v, k, λ) - design if each pair of elements x_i and $x_j$has the same covalence called index of the design. An incomplete block design (k < t) is said to be a balanced incomplete block design (t, b, r, k, λ) or (t, k, λ) - BIBD is a design (X, A) if the following postulates are satisfied;

(i) $|X| = t$

(ii) Each block contains exactly k points

(iii) Every pair of distinct contained points exactlyλblocks

A BIBD is specified by five parameters(v, b, r, k, λ). The five parameters defined as(v, b, r, k, λ) - BIBD are related and satisfy the following two relations: $bk = vr$and $\lambda(v - 1) = r(k - 1)$. Clearly, these relations restrict the set of admissible parameters for a BIBD. However, a balanced incomplete block design is an incomplete block design in which any two varieties appear together an equal number of times. This design can be constructed by taking $\binom{v}{k}$blocks and assigning a different combination of varieties to each block [5]. [9] developed balanced incomplete block design using Hadamard rhotrices. [4] Applied Galois field to construct balanced incomplete block design. [1] constructed balanced incomplete block design using the concept of finite Euclidean geometry of N – dimensional space. One of the most powerful techniques for determining the existence of combinatorial designs is the idea of partial balanced design (PBD) – closure [3]. [11] Introduced three types of new combinatorial designs, external difference families (EDF), external BIBDs (EBIBD) and splitting BIBDs and showed their applications to splitting authentication codes and secret sharing schemes secure against cheaters. An EDF can be considered as an extension of different sets and different families. An EBIBD is a generalization of a balanced incomplete block design (BIBD). Two of these combinatorial designs, external difference families (EDF) and external BIBDs (EBIBD), are to show that EDF is equivalent to EBIBD with a particular automorphism. As one of the fundamental discrete structures, combinatorial designs are used in fields as diverse as error-correcting codes, statistical design of experiments, cryptography and information security, mobile and wireless communications, group testing algorithms in DNA screening, software and hardware testing, and interconnection networks [6]. Combinatorial design is one of the fastest growing areas of modern mathematics focusing on a major part of introduction to Combinatorial Designs which provides a solid foundation in the classical areas of design theory as well as in more contemporary designs based on applications in a variety of fields [12]. Suggestions have been made by several authors that balanced incomplete block design as a combinatorial design is related to algebra concept. [7] apply methods of modern algebra to

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construct a BIBD. Then, the objective of this research is to examine whether BIBD using algebraic approach can form a finite group, rings and field algebra.

METHODOLOGY

Finite group in balanced incomplete block design (BIBD)

A finite group is a group of which the underlying set contains a finite number of elements. In other words, a group of finite number of elements is called a finite group. A group is a design$< X,*>$, where X is a non – empty set in BIBD with a binary operation (*) that satisfied the following axioms. For any $a, b, c \in X$

(i) $a * (b * c) = (a * b) * c$

(ii) $\exists I \in X s.t a * I = I * a = a, \forall a \in X$

(iii) for each $a \in X \exists a^{-1} \in X s.t a * a^{-1} = a^{-1} * a = I$

The design X is commutative or abelian group if, $\forall a, b \in X, a * b = b * a$.

A group is always a semigroup while the converse is not true in general.

Ring algebra in balanced incomplete block design (BIBD)

A BIBD defined as $< X, +,*>$ is a ring iff the following properties hold. For any $a, b, c \in X$

(i) $a + b = b + a$

(ii) $(a + b) + c = a + (b + c)$

(iii) $\exists 0 \in X s.t a + 0 = 0 + a = a, \forall a \in X$

(iv) For each $a \in X \exists - a \in X s.t a + (-a) = (-a) + a = 0$

(v) $(a * b) * c = a * (b * c)$

(vi) $a * (b + c) = a * b + a * c$

$(b * c) * a = b * a + c * a$

A design X in which $ab = ba$ for every$a, b \in X$is a commutative ring.

The postulates R1 – R4 show that a ring R is an abelian additive group $< R, +>$. R5 show that a ring R is associative law under semigroup$< R,\bullet>$. A ring R in which $ab = ba$ for every$a, b \in R$; is a commutative ring. In other words, a ring $< R, +,\bullet>$ is commutative if $< R,\bullet>$ is a commutative semigroup. An element e of a ring R is called unity (or an identity) of R if $ae = ea = a$for every $a \in R$. A non - empty subset$S$of a ring $< R, +,\bullet>$ is called a subring of $< R, +,\bullet>$ if$< S, +,\bullet>$ is also a ring. In general, a ring may or may not have a unity, however it

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can be easily shown that if a ring R has an element$es.tae = ea \forall a \in R$, then e is unique and this e is called the unity or the identity of R. the unity of a ring R is denoted by I.

Field algebra in balanced incomplete block design (BIBD)

A commutative division ring is called a field. A ring with unity in which all non – zero elements form a group under multiplication is called a division ring. A design X from BIBD of the form $(X, +, *)$ is a finite field algebra with two binary operations which satisfies the following postulates:

$X_1: (X, +)$ is an abelian (additive) group. The axioms of a ring (i) – (iv) is an abelian additive group.

$X_2: (X - \{0\}, \bullet)$is an abelian (multiplicative) group defined as:

(i) $a * b = b * a$, for every$a, b, c \in X$

(ii) $(a * b) * c = a * (b * c)$

(iii) $\exists I \in Xs.ta * I = I * a = a, \forall a \in X$

(iv) $\forall a \neq 0 \in X \exists a^{-1} \in Xs.ta * a^{-1} = a^{-1} * a = I,$

$X_3: \forall a, b, c \in X, a * (b * c) = a * b + a * c$

A non – void subset $S$of a field $< F >$is said to be a sub – field of $< F >$ if;

(a) $a \in S, b \in S \Rightarrow a + b \in S, ab \in S$

(b) $S$is a field under the induced $< +, \bullet >$operations

Any subset $S$of a field $< F >$, containing at least two elements is a subfield of $< F >$iff;

(i) $a \in S, b \in S \Rightarrow a - b \in S$

(ii) $a \in S, b \in S, b \neq 0 \Rightarrow (ab)^{-1} \in S$

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RESULTS

Illustration 1

Let (X, B) be a BIBD where $(X = Z_{15})$ and B is a subset of X, then$\{X\}_1^{15}$ is a finite group, semigroup and abelian group under additive and multiplicative operations.

Let$B_1, B_2, \ldots, B_{15} \in X$, then selecting any three elements in X, say $a = 3, b = 6$ and $c = 7$. It follows that:

(i) $a * b \in X, \forall a, b \in Z_{15}$ (ii) $(a * b) * c = a * (b * c), \forall a, b, c \in Z_{15}$ (iii) $b * c = c * b, \forall b, c \in Z_{15}$

In Table 1, the identity element does not exist in some rows and some integers occur more than once in a column. Therefore, not all integers have an inverse. Hence $(X,*)$ is not a group. But from (ii) and (iii), X form a semigroup and abelian group.

Table 1: Group table for$(Z_{15},*)$ design

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	0	2	4	6	8	10	12	14	1	3	5	7	9	11	13
3	0	3	6	9	12	0	3	6	9	12	0	3	6	9	12
4	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11
5	0	5	10	0	5	10	0	5	10	0	5	10	0	5	10
6	0	6	12	3	9	0	6	12	3	9	0	6	12	3	9
7	0	7	14	6	13	5	7	4	11	3	10	2	9	1	8
8	0	8	1	9	2	10	3	11	4	12	5	13	6	14	7
9	0	9	3	12	6	0	9	3	12	6	0	9	3	12	6
10	0	10	5	0	10	5	0	10	5	0	10	5	0	10	5
11	0	11	7	3	14	10	6	2	13	9	5	1	12	8	4
12	0	12	9	6	3	0	12	9	6	3	0	12	9	6	3
13	0	13	11	9	7	5	3	1	14	12	10	8	6	4	2
14	0	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Illustration 2

Let a system$(X, +,*)$ be a design with two binary compositions defined on the varieties$(X = R_{15})$, then$(X, B's)$ is a ring algebra and a commutative ring.

To show that $(X, B's)$ is a ring algebra and a commutative ring, the design must be abelian additive group and semigroup defined on $(X, +,*)$. Also, the left and right distributive law must hold. For the design$(X, +,*)$, let a, b, cbe any three real number in X, say $a = 4, b = 10$ and $c = 13$, then for any $a, b, c \in X$:

(i) $(a + b) + c = a + (b + c) \Rightarrow 12\, mod\, 15 \therefore 12 \in X$

(ii) $a + b = b + a \Rightarrow 14\, mod\, 15 \Rightarrow 14 \in X$

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(iii) $(a * b) * c = a * (b * c) \Rightarrow 10\, mod\, 15 \Rightarrow 10 \in X$

(iv) $a * (b + c) = a * b + a * c \Rightarrow 2\, mod\, 15 \Rightarrow 2 \in X$

$(b + c) * a = b * a + c * a \Rightarrow 2\, mod\, 15 \Rightarrow 2 \in X$

(v) $a * b = b * a \Rightarrow 10\, mod\, 15 \Rightarrow 10 \in X$

Table 2 illustrates the existence of inverse and identity elements. The table shows that X is closed under ⊕ and zero (0) is the identity element. The inverse of the elements are; -0 = 0, -1 = 14, -2 = 13, -3 = 12, -4 = 11, -5 = 10, -6 = 9, -7 = 8, -8 = 7, -9 = 6, -10 = 5, -11 = 4, -12 = 3, -13 = 2, -14 = 1. Since all the axioms of ring algebra are satisfied, therefore the design $(X, +, *)$ is a ring and a commutative ring.

Table 2: Group table for $(X, +)$ design

⊕	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	0
2	2	3	4	5	6	7	8	9	10	11	12	13	14	0	1
3	3	4	5	6	7	8	9	10	11	12	13	14	0	1	2
4	4	5	6	7	8	9	10	11	12	13	14	0	1	2	3
5	5	6	7	8	9	10	11	12	13	14	0	1	2	3	4
6	6	7	8	9	10	11	12	13	14	0	1	2	3	4	5
7	7	8	9	10	11	12	13	14	0	1	2	3	4	5	6
8	8	9	10	11	12	13	14	0	1	2	3	4	5	6	7
9	9	10	11	12	13	14	0	1	2	3	4	5	6	7	8
10	10	11	12	13	14	0	1	2	3	4	5	6	7	8	9
11	11	12	13	14	0	1	2	3	4	5	6	7	8	9	10
12	12	13	14	0	1	2	3	4	5	6	7	8	9	10	11
13	13	14	0	1	2	3	4	5	6	7	8	9	10	11	12
14	14	0	1	2	3	4	5	6	7	8	9	10	11	12	13

Illustration 3

Let$(X = Z_{15})$ be BIBD, then $(X, +, \bullet)$ is not a finite field.

Based on the axioms of field and from table 3, $(Z_{15}, +)$ is an abelian group. Consider$(Z_{15} - \{0\}, \bullet)$, then $Z_{15} - \{0\}$is closed under$(\bullet)$. Associativity, commutative and distributive law follow from those of integers. One (1) is the multiplicative identity. But from Table 1, some integers have no inverse. Therefore, $Z_{15} - \{0\}$does not satisfy the second properties of a field. Hence $(X, +, \bullet)$ is not a finite field.

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Theorem 1

Suppose $B = \{v_1, v_2, \ldots, v_t\}$be a semigroup and X be a non – void design $s.t(X,B) = \{\sum_i^n a_i v_i / a_i \in X, v_i \in B\}$then $(X, B, +,*)$ is a ring.

Proof

For any $a, b, c \in X$and $v_i \in B$, then $(+,*)$defined on (X, B) as;

(i) $\sum_i^t a_i v_i + \sum_i^t b_i v_i = \sum_i^t (a_i + b_i) v_i$

(ii) $(\sum_i^t a_i v_i)(\sum_i^t b_i v_i) = \sum_i^t c_k v_k$where$\sum_i^t a_i b_i = c_k$and $v_i v_j = v_k$

Then

(a) $(\sum_i^t a_i v_i + \sum_i^t b_i v_i) + \sum_i^t c_i v_i = \sum_i^t (a_i + b_i) v_i + \sum_i^t c_i v_i = \sum_i^t [(a_i + b_i) + c_i] v_i$

$= \sum_i^t [a_i(b_i + c_i)] v_i = \sum_i^t a_i v_i + \sum_i^t (b_i + c_i) v_i = \sum_i^t a_i v_i + (\sum_i^t b_i v_i + \sum c_i v_i)$

$\Rightarrow (\sum_i^t a_i v_i + \sum_i^t b_i v_i) + \sum_i^t c_i v_i = \sum_i^t a_i v_i + (\sum_i^t b_i v_i + \sum c_i v_i)$

(b) $\sum_i^t a_i v_i + \sum_i^t b_i v_i = \sum_i^t (a_i + b_i) v_i = \sum_i^t (b_i + a_i) v_i = \sum_i^t b_i v_i + \sum_i^t a_i v_i$

(c) Let $\sum_i^t f_i v_i = 0$where $f_i = 0, \forall i$, then $\sum_i^t a_i v_i + \sum_i^t f_i v_i = \sum_i^t (a_i + f_i) v_i = \sum_i^t a_i v_i$as $f_i = 0, \forall i$

(d) Also$\sum(-a_i) v_i + \sum a_i v_i = \sum(-a_i + a_i) v_i = 0, \forall i$, Hence$(X, B, +)$ is an abelian group.

Similarly, let $\sum_{(v_i v_j) v_k = v_u} a_i b_j c_k$and $\sum_{(v_i v_j) v_k = v_p} a_i b_j c_k$represent the elements $\alpha_u = \sum_{(v_i v_j) v_k = v_u} a_i b_j c_k$and $\beta_v = \sum_{(v_i v_j) v_k = v_u} a_i b_j c_k$of the ring X, where the two summations run over triples $a_i, b_j, c_k s.t$

$(v_i v_j) v_k = v_u$and$v_i(v_j v_k) = v_p$respectively. Now

(e) $[(\sum a_i v_i)(\sum b_j v_j)](\sum c_l v_l) = (\sum_k \alpha_k v_k)(\sum_l c_l v_l)$, Where$\alpha_k = \sum_{v_i v_{j = v_k}} a_i b_j = \sum_u \beta_u v_u$

But $\beta_u = \sum_{v_k v_i = v_u} \alpha_k c_i = \sum_{v_k v_i = v_u} \left(\sum_{v_i v_j = v_k} a_i b_j\right) c_l = \sum_{(v_i v_j) v_i = v_u} a_i b_j c_l$

Again $(\sum a_i v_i)[(\sum b_j v_j)(\sum c_l v_l)] = (\sum_i \alpha_i v_i) \sum_k \beta_k v_k$where $\beta_k = \sum_{v_i v_{j = v_k}} b_j c_l = \sum_p \gamma_v v_p$

But $\gamma_v = \sum_{v_i v_k = v_p} \alpha_i \beta_k = \sum_{v_i v_k = v_p} a_i \sum_{v_j v_l = v_k} b_j c_l = \sum_{v_i (v_j v_k) = v_p} a_i b_j c_l$

Since β is a semi – group under multiplication, $v_u = v_p, \forall u, p = 1,2, \ldots\ldots, t$. Hence

$[(\sum a_i v_i)(\sum b_j v_j)](\sum c_l v_l) = (\sum a_i v_i)[(\sum b_j v_j)(\sum c_l v_l)]$. Thus $(X, B,*)$ is a semi – group.

(f) $\sum_i^t a_i v_i \left(\sum_j^t b_j v_j + \sum_i^t c_l v_l\right) = \sum_i^t a_i v_i \left(\sum_j (b_j + c_l) v_j\right) \Rightarrow \sum_i^t a_i v_i \left(\sum_j m_j v_j\right)$where $m_j = b_j + c_{l,} \forall j = 1,2, \ldots, t \Rightarrow \sum_k^t \alpha_k v_k$, where $\alpha_k = \sum_{v_i v_j = v_k} a_i m_j = \sum_{v_i v_j = v_k} a_i (b_j + c_l)$

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$\sum_{v_i v_j = v_k} a_i b_j + \sum_{v_i v_j = v_k} a_i c_l = \beta_k + \gamma_k$ where $\beta_k = \sum_{v_i v_j = v_k} a_i b_j$ and $\gamma_k = \sum_{v_i v_j = v_k} a_i c_l$

Also $(\sum_i a_i v_i)(\sum_j b_j v_i) + (\sum_i a_i v_i)(\sum_l c_l v_l) = \sum_k \beta_k v_k + \sum_k \gamma_k v_k$

$= \sum_k (\beta_k + \gamma_k) v_k = \sum_k w_k v_k$ where $w_k = \beta_k + \gamma_k$

Hence $\sum_i^t a_i v_i \left(\sum_j^t b_j v_j + \sum_i^t c_l v_l\right) = (\sum_i a_i v_i)(\sum_j b_j v_j) + (\sum_i a_i v_i)(\sum_l c_j v_j)$

Also

$$\left(\sum_j^t b_j v_j + \sum_i^t c_j v_j\right)\sum_i^t a_i v_i = \left(\sum_j^t (b_j + c_j) v_j\right)\sum_i a_i v_i$$

$(\sum_j n_j v_j) \sum_i a_i v_i$ where $n_j = b_j + c_j, \forall j = 1,2,3, \ldots \ldots, t$

$= \sum_k z_k v_k$ where $z_k = \sum_{v_i v_j = v_k} n_j a_l = \sum_{v_i v_{j=v_k}} (b_j + c_j) a_i$

$\sum_{v_i v_j = v_k} b_j a_l + \sum_{v_i v_{j=v_k}} c_j a_i = T_k + S_k$ where $T_k = \sum_{v_i v_j = v_k} b_j a_l$ and

$$S_k = \sum_{v_i v_{j=v_k}} c_j a_i$$

Also $(\sum_j b_j v_j)(\sum_i a_i v_i) + (\sum_l c_j v_j)(\sum_i a_i v_i) = \sum_k T_k v_k + \sum_k S_k v_k$

$$= \sum_k (T_k + S_k) v_k = \sum_k Z_k v_k$$

Hence $\left(\sum_j b_j v_j + \sum_i c_j v_j\right)\sum_i a_i v_i =$ $(\sum_j b_j v_j)(\sum_i a_i v_i) + (\sum_l c_j v_j)(\sum_i a_i v_i)$

Therefore $< X, B, +, * >$ is a ring.

Proposition 1

Let a design (X, B) be (t, b, r, k, λ) - BIBD, then $B = \{b_1, b_2, b_3, \ldots, b_v\}$ is multiplicative abelian group $(B - \{0\}, \bullet)$.

Theorem 2

A design $(X, B, +, \bullet)$ is a field $iff\, (X, +)$ is an abelian group.

Proof

For any $x, y, z \in X$, then the binary operation (+) defines on X as; $\varphi(x) + \varphi(y) = \varphi(x + y)$, where φ denote the number of each variety in X. Given $x, y, z \in X$, then;

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(i) $(\varphi(x) + \varphi(y)) + \varphi(z) = \varphi(x + y) + \varphi(z) = \varphi((x + y) + z)$

$= \varphi(x + (y + z)) = \varphi(x) + \varphi(x + z) = \varphi(x) + (\varphi(y) + \varphi(z))$

Hence $(\varphi(x) + \varphi(y)) + \varphi(z) = \varphi(x) + (\varphi(y) + \varphi(z))$. X is associative under (+)

(ii) $\varphi(x) + \varphi(y) = \varphi(x + y) = \varphi(y + x) = \varphi(y) + \varphi(x)$

Therefore X is commutative under (+)

(iii) $\forall x \in X \; \exists \; \varphi(-x) \in X \; s.t \; \varphi(-x) + \varphi(x) = \varphi(-x + x) = 0$

Hence the inverse exists in X

(iv) Let $\varphi(t) = 0$, where $t = 0, 0 \in X, \; s.t \; \varphi(x) + \varphi(t) = \varphi(x + t) = \varphi(x) \; as \; t = 0$

Then $x \in X$, hence $\varphi(x)$ is an identity element in X

The results (i) – (iv) shows that the design $(X, +)$ under the additive binary operation is an abelian group. And since $B$is an abelian multiplicative group $(B - \{0\}, \bullet)$, then the design $(X, B, +, \bullet)$ is a field.

DISCUSSION OF RESULTS

From the above results, X is used as a variety and B as blocks which are represented as(X, B). The result shows that the design (X, B) form a group, semigroup under additive binary operation but failed under the multiplicative binary operation. The design X satisfied all the postulates of the ring algebra. Therefore, the design X form a ring. Based on the axioms of a field, the varieties(X) form an abelian group $(X, +)$ but $(X - \{0\}, \bullet)$do not satisfy the second properties of a field as an abelian multiplicative group. Hence the variety (X) with two binary operations $(X, +, \bullet)$ cannot form a field.

CONCLUSION

Based on the findings, it is observed that irrespective of the block sizes and number of varieties the design(X, B) cannot form a group and a field under multiplicative binary operation but it is under additive binary operation. However, the design (X, B) form a ring algebra in all cases. Hence, balanced incomplete block design of the form (X, B) is a semigroup, commutative group, semiring, commutative ring and subfield.

CONFLICT OF INTEREST

We declare that there is no conflict of interest.

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