COMPARATIVE STUDY OF THE GOMPERTZ AND LOGISTIC GROWTH MODELS ON THE PREVALENCE AND FATALITY OF COVID-19 PANDEMIC IN NIGERIA

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ABSTRACT: *This study models the prevalence and fatality of the Covid-19 pandemic in Nigeria from February 2020 to July 2022. It is a comparative study of two prominent models: The Gompertz and Logistic population growth models. The data for this study was obtained from the website of Our World in Data, OWID (https//www.ourworldindata.org). The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were employed to compare the performance of the models, and the number of iterations before convergence and convergence tolerance for each model was also put into consideration. The study revealed that the Gompertz population growth model provides a better fit compared to the logistic growth in modelling the cumulative covid-19 cases and cumulative covid-19-related deaths in Nigeria. From the models, we obtained important features of the pandemic, such as the growth rate and asymptotes.*

KEYWORDS: Covid-19, Gompertz, Logistic, Model, Basic Reproduction Rate.

INTRODUCTION

The coronavirus (COVID-19) pandemic is a major global health crisis of recent times and, perhaps, the greatest challenge the world has faced since World War II. Since its emergence in China in late 2019, the virus has spread to virtually every continent of the world. Cases of the pandemic are rising daily in Africa, America, and Europe (Samuel O. A., Muhammad A. B., Samuel O. O., Haruna U. Y., and Rafiu O. A., 2020). Countries are fighting to slow down the spread of the virus by testing and treating patients, carrying out contact tracing, limiting travel, quarantining citizens, and restricting large gatherings such as sporting events, concerts, etc. The pandemic is moving like a wave, and fatal cases are being reported worldwide daily. According to World Health Organisation, WHO (2020), COVID-19 is much more than a health crisis because of its ability to cause serious damage to every country it touches; it has the potential to create devastating social, economic, and political crises that will leave deep scars.

The first case of the Coronavirus Disease 2019 (COVID-19) in Nigeria was recorded on the 27th day of February 2020. The virus was brought into the country by an Italian citizen who had landed at Lagos airport two days earlier on a flight from northern Italy and had subsequently travelled from Lagos to Ogun State, western Nigeria, where he became ill and was promptly isolated (Nigeria Centre for Disease Control, 2020). He was treated for mild symptoms of COVID-19 in a hospital in Lagos. Upon identifying the index case, National Emergency Operations Centres were immediately activated to trace his contacts. By March 9, 2020, 27 suspected cases had been identified across five states (Edo, Lagos, Ogun, Federal Capital Territory, and Kano), of which two were confirmed to be positive (i.e., the index case and a contact), with no deaths. 1,216 contacts were linked to the index case, 136 of whom are closely followed up (Samuel O. A. et al., 2020).

The coronavirus pandemic (COVID-19) outbreak has stimulated several responses from individuals, government, and researchers from various fields. Statisticians have also carried out a number of studies using various statistical methods to study various aspects of the pandemic. Ahmad, Aminu, Usman, and Suleiman (2021) estimated the case fatality rate of the covid-19 pandemic in Nigeria using both simple and polynomial regression models. The study was carried out using the disease's epidemiological data from 44 days since the first reported case of death. It was reported that as of 30th April 2020, the fatality rate stood at 3.0% with a 95% confidence interval from 2.23% to 3.42% (Ahmad et al., 2021). The study equally revealed that the polynomial regression model provides a better fit with a coefficient of determination (R^2) of 93% and the linear regression model with a coefficient of determination (R^2) of 90%. The recorded cases of the incidence of this disease were also modelled using some selected count data regression models (Samuel, Muhammad, Samuel, Haruna, and Rafiu, 2020). The study employed Poisson Regression, Negative Binomial and the Generalised Poisson Regression models to examine the relationship between the daily laboratory-confirmed cases, cumulative lab-confirmed cases and recorded covid-19 related deaths in the country. The study found out that among the three count Regression models examined, the Generalised Poisson Regression proved to be the best model for the daily cumulative confirmed cases, active cases and critical COVID-19 cases in Nigeria when over-dispersion is present in the predictor variables. Their study also revealed that active and critical cases have a positive and significant effect on the number of COVID-19-related deaths in Nigeria (Samuel et al., 2020). Rauf and Hannah (2020) also forecasted the spread of covid 19 in Nigeria using Box-Jenkins modelling procedures. They employed the Autoregressive Integrated Moving Average (ARIMA) model to study the covid-19 data from February 27 to April 26, 2020, and made a

ten-day forecast of the covid-19 confirmed cases using ARIMA (1,1,0). Other studies also incorporated analysing and monitoring the trend and prevalence by states of the pandemic using statistical quality control techniques (Edike, Braimah and Agbedeyi, 2020). Some studies also examined the empirical links between covid-19 situation report and available data in Nigeria. According to Yusuf (2020), no Nigerian state has up to 12 laboratory-confirmed cases of covid-19 per 10,000 population. This implies that the number of confirmed COVID-19 cases is less than 0.15% of the population of people across each state in Nigeria. The study further revealed that, compared to the population, the proportion of samples tested for COVID-19 is low, with percentages ranging from less than 0.1% to a maximum of 0.7% of each state population in which 23 states out of the 37 states are within the less than 0.1%. The study further shows that there is a significant correlation between the confirmed cases and covid-19 testing, while no significant correlation was found between the confirmed cases with the average weather conditions of temperature or rainfall (Yusuf, 2020). The dynamics of the spread of covid-19 pandemic in the first wave was studied for 23 Asian, European and American countries using two-parameter Gompertz and logistic models. The study showed that the Gompertz model better fit the official statistics compared to the logistic model (Pelinovsky, Kokoulina, Epifanova, Kurkin, Kurkina, Tang, Macau and Kirillin, 2021). The study leveraged on the relationship between the daily confirmed cases and the cumulative daily confirmed cases. Covid-19-related deaths were not taken into consideration, none of the African countries were included in the studies, and only the first wave of the pandemic was accounted for in their studies.

This study was therefore carried out to model the prevalence and fatality of covid-19 pandemic in Nigeria using the three-parameter Gompertz and logistic growth models. The data used in this study is primarily the cumulative laboratory-confirmed covid-19 cases and the cumulative covid-19 deaths.

MATERIALS AND METHODS

Data Collection

The data for this study was obtained from the website of Our World In Data (OWID) (https://www.owid.org). It is a reliable database that monitored and recorded the global spread of the Covid-19 pandemic. The Covid-19 data for Nigeria was obtained, cleaned and used for the purpose of this research work. The features of interest are the cumulative covid-19 lab-confirmed cases and the cumulative covid-19 related death cases from February 2020 to July 2022.

The Gompertz function

Consider the differential equation for population growth

$$\frac{dp}{dt} = c \ln \ln \left(\frac{k}{p}\right) p \tag{1}$$

Where c is a constant

k is the carrying capacity

using the variable separable on equation (1), we have

$$\frac{dp}{lnln\left(\frac{k}{p}\right)p} = cdt$$
(2)

Integrating, we have

$$\int \frac{dp}{lnln\left(\frac{k}{p}\right)p} = \int cdt$$
(3)

Now, letting $u = ln\,ln\left(\frac{k}{p}\right) = ln\,ln\,k - ln\,ln\,p$, we differentiate u with respect to p and obtain

$$\frac{du}{dp} = 0 - \frac{1}{p} = -\frac{1}{p}$$
(4)

$$\therefore du = -\frac{dp}{p}$$

$$dp = -pdu$$

Substituting for dp in equation (2)

$$\int -\frac{pdu}{up} = -\int \frac{du}{u} = \int cdt$$

$$-ln\,ln\,|u| + c_1 = ct + c_2 \tag{5}$$

$$-ln\,ln\,|u| = ct + C$$

Where $C = c_2 - c_1 = constant$

$$ln\,ln\,|u| = -ct - C \tag{6}$$

To obtain u, we exponentiate equation (6) to obtain

$$e^{lnln\,|u|} = e^{-ct-C} \tag{7}$$

$$|u| = e^{-C}e^{-ct}$$

$$u = \pm\, e^{-C}e^{-ct}$$

$$u = De^{-ct} \tag{8}$$

Where $D = \pm\, e^{-C}$ (constant)

Substituting u

$$ln\,ln\left(\frac{k}{p}\right) = De^{-ct} \tag{9}$$

To solve for D

Let the initial population at time $t = 0$ be $P(0) = P_0$

$$ln\, ln\left(\frac{k}{P_0}\right) = De^0$$

$$D = ln\, ln\left(\frac{k}{P_0}\right) = De^0$$
(10)

Hence, substituting for D in equation (9), we have

$$ln\, ln\left(\frac{k}{p}\right) = ln\, ln\left(\frac{k}{P_0}\right) e^{-ct}$$
(11)

Exponentiating equation (11) and solving for k, we obtain the following

$$e^{lnln\left(\frac{k}{p}\right)} = e^{lnln\left(\frac{k}{P_0}\right)e^{-ct}}$$

$$\frac{k}{p} = e^{lnln\left(\frac{k}{P_0}\right)e^{-ct}}$$

$$k = Pe^{lnln\left(\frac{k}{P_0}\right)e^{-ct}}$$

$$\therefore P(t) = ke^{-lnln\left(\frac{k}{P_0}\right)e^{-ct}}$$
(12)

Equation (12) which is the Gompertz population growth model

letting $b = ln\, ln\left(\frac{k}{P_0}\right)$ simplifies equation (12) to give

$$f(t) = ke^{-be^{-ct}}$$
(13)

where k is the carrying capacity (asymptote)

b is the displacement along the x-axis

c is the growth rate (y-scaling)

The figure below shows the shape of the logistic curve with parameters k = b = c = 1
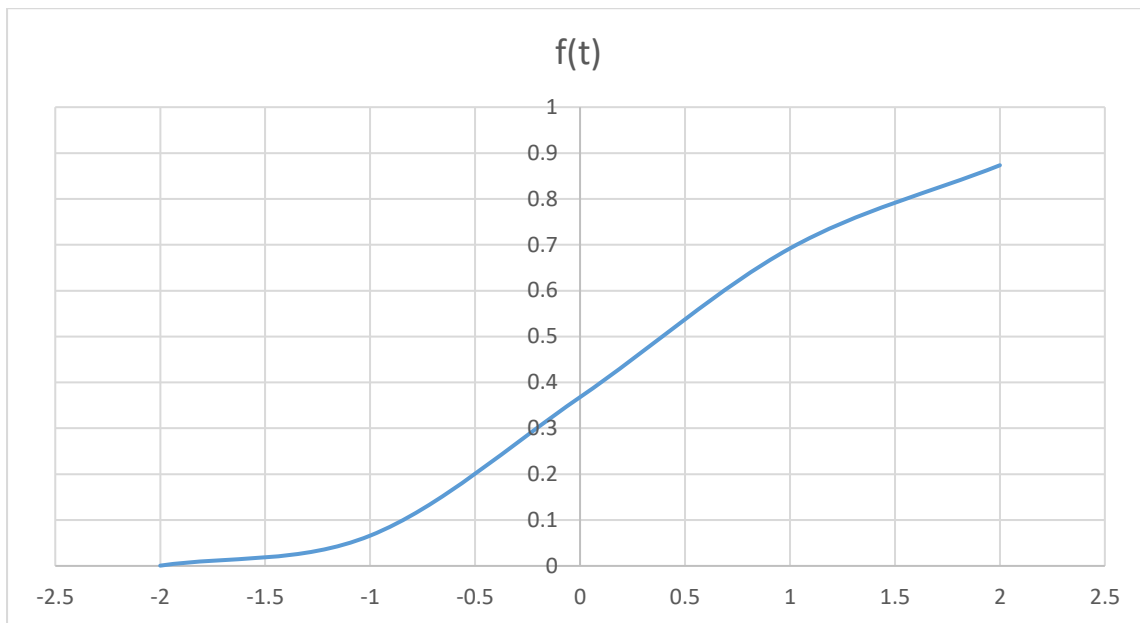
Figure 1: Graph of the Gompertz model

The Logistic model

The logistic model was proposed by Verhulst in 1838. Like the Gompertz model, the logistic model assumes there is a carrying capacity K, such that if the population is above K, the population will decrease but if it is below K, it will increase.

For the logistic model, it is assumed that the rate of change dy/dt of the population y is proportional to the product of the current population y and K – y. Equivalently, we can say that the rate of change dy/dt of the population is proportional to the product y(1 – y/K). This gives the logistic differential equation.

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right)$$
$$(14)$$

Here, r is a positive constant. Note that when $y < K$, $\frac{dy}{dt}$ is positive, so y increases, but when $y > K$, $\frac{dy}{dt}$ is negative, so y decreases. To solve the above differential equation, we first separate the variables to have

$$\frac{1}{y\left(1 - \frac{y}{K}\right)} dy = rdt$$
$$(15)$$

Integrating

$$\int \frac{1}{y\left(1-\frac{y}{K}\right)} dy = \int r\, dt$$
(16)

Considering the left-hand side of the equation, we employ the method of partial fractions

$$\frac{1}{y\left(1-\frac{y}{K}\right)} = \frac{A}{y} + \frac{B}{1-\frac{y}{K}}$$
(17)

Clearing the denominators, we obtain the equation

$$1 = A\left(1 - \frac{y}{K}\right) + By = A - \frac{A}{K}y + By$$
(18)

Simplifying this gives

$$A = 1, B = \frac{1}{K}$$

Hence, the equation becomes

$$\frac{1}{y\left(1-\frac{y}{K}\right)} = \frac{1}{y} + \frac{1/K}{1-\frac{y}{K}} = \frac{1}{y} + \frac{1}{K-y}$$
(19)

Therefore,

$$\int \frac{dy}{y\left(1-\frac{y}{K}\right)} = \int \frac{dy}{y} + \int \frac{dy}{K-y}$$

$$\int \frac{dy}{y\left(1-\frac{y}{K}\right)} = \ln \ln y - \ln \ln |K-y|$$
(20)

$$\int \frac{dy}{y\left(1-\frac{y}{K}\right)} = \ln \ln \left|\frac{y}{y-K}\right|$$
(21)

Hence, the complete equation becomes

$$\ln \ln \left|\frac{y}{y-K}\right| = rt + C$$
(22)

Simplifying gives

$$y = \frac{K}{1+Ae^{-rt}}$$
(23)

The figure below shows the shape of the logistic curve with parameters A = K = r = 1
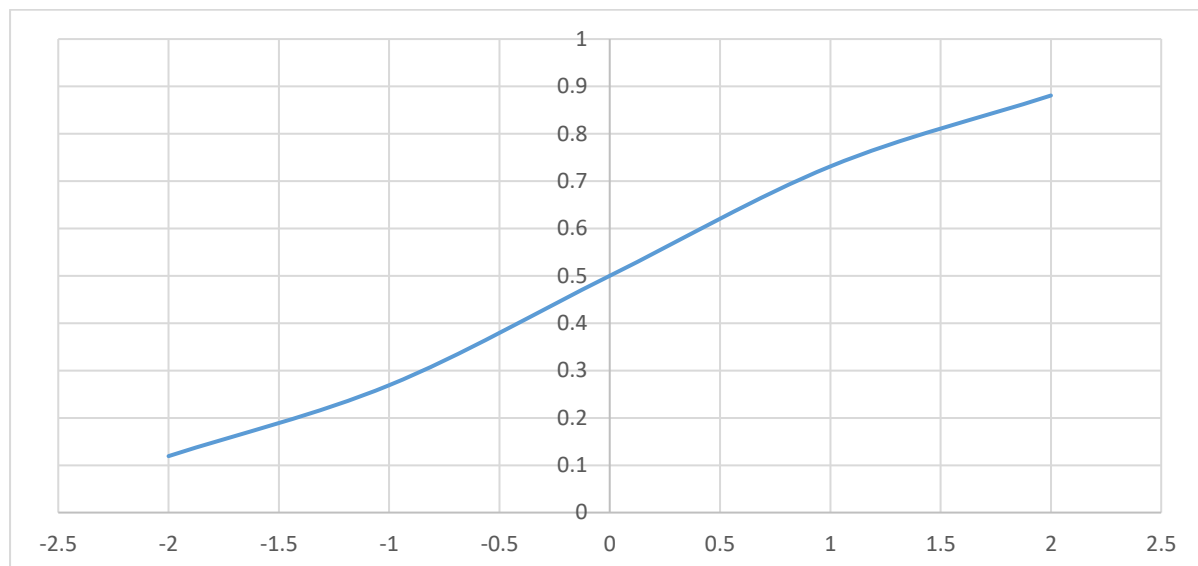

Figure 2: Graph of the logistic model

Which is the logistic model, where A is a constant. The graph of the function is asymptotic to the y-axis on the left, asymptotic to y = K on the right, and symmetric with respect to the point where y = K/2.

Model Selection

This study employed two model selection criteria, namely, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC)

The Akaike Information Criterion (AIC): The Akaike Information Criterion (AIC), named after the Japanese statistician Hirotugu Akaike, who formulated it. It is a widely used model selection criterion in statistical inference. It is a means of statistical model selection which estimates prediction error in a statistical model. It presents a quantity that serves as a measure of the relative quality of statistical models for a given set of data. Given a collection of models for a given set of data, AIC estimates the quality of each model relative to each of the other models. Thus, AIC provides a means for model selection. AIC is founded on information theory. It is noted that there is always a loss of a certain amount of information whenever a statistical model is used to represent the process that generated the data. This is because the representation will almost never be exact. AIC estimates the relative amount of information lost by a given model; hence, the less the value of AIC for a model, the less the information lost using the model, and of course, the higher the quality of that model.

The Akaike Information Criterion is given as

$$AIC = 2k - 2ln\,(\hat{L}) \tag{24}$$

$where\ k = number\ of\ estimated\ parameters\ in\ the\ model$ $\hat{L} =$ the maximum value of the likelihood function of the model.

In estimating the amount of information lost by a model, AIC deals with the trade-off between the goodness of fit of the model and the simplicity of the model. In other words, AIC deals with both the risk of overfitting and the risk of underfitting. Thus, AIC rewards goodness of fit (as assessed by the likelihood function) but also incorporates a penalty which is an increasing function of the number of estimated parameters. This penalty discourages overfitting, which is necessary because increasing the number of parameters in the model almost always improves the goodness of the fit.

The Akaike Information Criterion is based on information theory. Assuming a given set of data is generated by some process f, and we consider two candidate models g_1 and g_2 to represent f: If f is known, then we could find the information lost from using any of the candidate models by calculating the Kullback–Leibler divergence, $D_{KL}(f \parallel g_1)$ and $D_{KL}(f \parallel g_2)$. But, in practice, f is unknown; hence Akaike (1974) showed, however, that we can estimate via AIC how much more (or less) information is lost by g_1 than by g_2. This estimate, though, is only valid asymptotically; if the number of data points is small, then some correction is often necessary. The corrected AIC is given in equation (25) as

$$AIC_c = 2k - ln\, ln\, (\hat{L})\; + \frac{2k(k+1)}{n-k-1}$$
(25)

where $k = number\ of\ estimated\ parameters\ in\ the\ model$ $\hat{L} =$ the maximum value of the likelihood function.

It is worth noting, however, that AIC tells nothing about the absolute quality of a fitted model, only the quality relative to other models. Thus, if all the candidate models fit poorly, AIC will not give any indication of that. Hence, after selecting a model via AIC, it is usually advisable to validate the absolute quality of the model by checking the model's residuals (to determine whether the residuals seem random) or testing the model's prediction power

Bayesian Information Criterion (BIC): The Bayesian Information Criterion (BIC) is a model selection criterion used to determine the best among a finite set of models. It was developed in 1978 by Gideon E. Schwarz. The BIC is closely related to the Akaike Information Criterion (AIC), and it is based in part on the likelihood function. When computed for different candidate models, the model having the least BIC is deemed best. Generally, it is customary to increase the likelihood of a fitted model by adding more parameters to the model. This sometimes leads to overfitting. Both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) attempt to resolve this problem by introducing a penalty term for the number of parameters included in the model. This penalty term is larger in BIC than in AIC (Stoica and Selen, 2004).

The Bayesian Information Criterion (BIC) is given as

$$BIC = kln(n) - 2ln\,(\hat{L})$$
(26)

where $k = number\ of\ estimated\ parameters\ in\ the\ model$

$\hat{L} = the\ maximum\ value\ of\ the\ likelihood\ function$

$n\ is\ the\ sample\ size$

The BIC is an increasing function of the error variance and an increasing function of *k*.

It is advisable to note that the BIC can be used to compare estimated models only when the numerical values of the dependent variable are identical for all models being compared. That is, when all estimated models are done using the same data set.

The BIC can also measure the efficiency of the parameterised model in terms of predicting the data, but cannot handle complex collections of models as in the variable selection problem in high dimension (Giraud, 2015).

RESULTS AND DISCUSSION

In this section, we present the result of our findings. The data collected were modelled using the Gompertz population growth model in equation (13). The aim is to estimate the model parameters in order to obtain some important features of the pandemic. The model was fit and parameters were estimated using the non-linear least square method. This was implemented using the nls tool in the minpack package of R statistical software.

Modeling the cumulative confirmed cases of Covid-19

First, we visualise the data collected; the daily cumulative cases of covid-19 pandemic in Nigeria were plotted as shown in Figure 3 below.
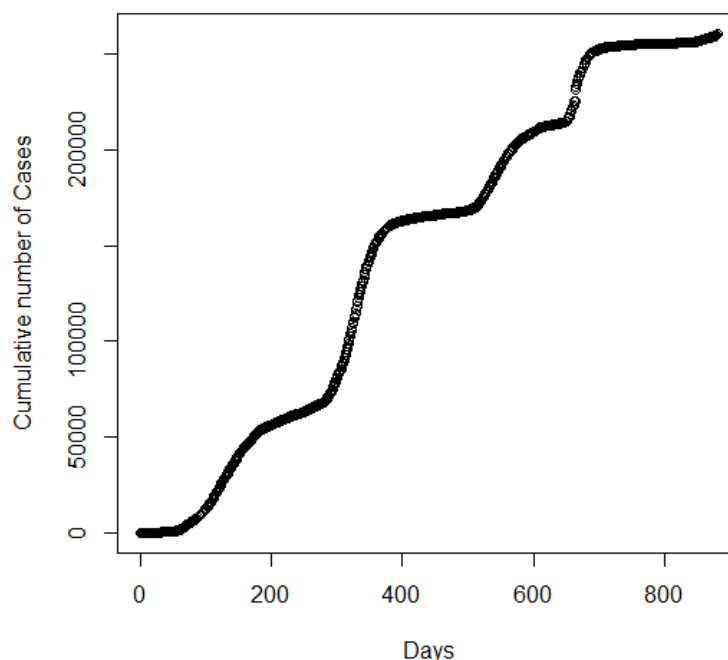

Figure 3: Plot of Cumulative total confirmed cases of Covid-19

Figure 3 above is a plot of cumulative daily confirmed cases of the Covid-19 pandemic in Nigeria from February 2020 to July 2022. The figure shows an increasing pattern from day 1 to around day 290, then a sharp increase from day 290 to around day 390. There was a steady rise again from day 390 to around day 620 when the growth rate declined, and cumulative confirmed cases approaches an asymptote. A cursory look at Figure 3 also reveals the possibility of the data being modelled by a Gompertz population growth model, the logistic growth model or any other sigmoid-type model. In practical terms, the data has the possibility of being modelled by a number of s-shaped models. This study, however, explored the Gompertz and the logistic growth models to reveal the various properties of the dataset and also made comparisons to identify the best model for the cumulative covid-19 confirmed cases in Nigeria from February 2020 to July 2022.

The Gompertz population growth model for cumulative confirmed cases of Covid-19.

The Gompertz population growth model was fitted to the data using the simple nonlinear least square algorithm. The estimates of the parameters of the fitted model are shown in the table below.

Table 1: Estimated Parameters of the Gompertz model of the cumulative covid-19 cases

Parameters	Estimate	Std. Error	t-value	p-value
k	2.824e+05	1.846e+03	152.92	<2e-16
b	4.326e+00	7.583e-02	57.06	<2e-16
c	4.649e-03	7.017e-05	66.26	<2e-16

The table above shows the estimated parameters of the Gompertz population growth model using the nonlinear least square algorithm. The model reveals that the cumulative daily confirmed cases of covid-19 pandemic in Nigeria would grow to 282400 with a standard error of 1846. The estimated growth rate was estimated to be 0.004649 with a standard error of 7.017e-05. Testing for the significance of the estimated parameters of the model showed that the three estimated parameters of the model are significant at a 5% level of significance (p-values less than 0.05).

The estimated Gompertz model is therefore given is given in equation (27) below.

$$y = 282400e^{-4.326e^{-0.004649t}}$$ (27)

Using equation (27) above, we predicted the cumulative COVID-19 confirmed cases and figure 4 below shows the plot of the actual data and the predictions made using the fitted model.
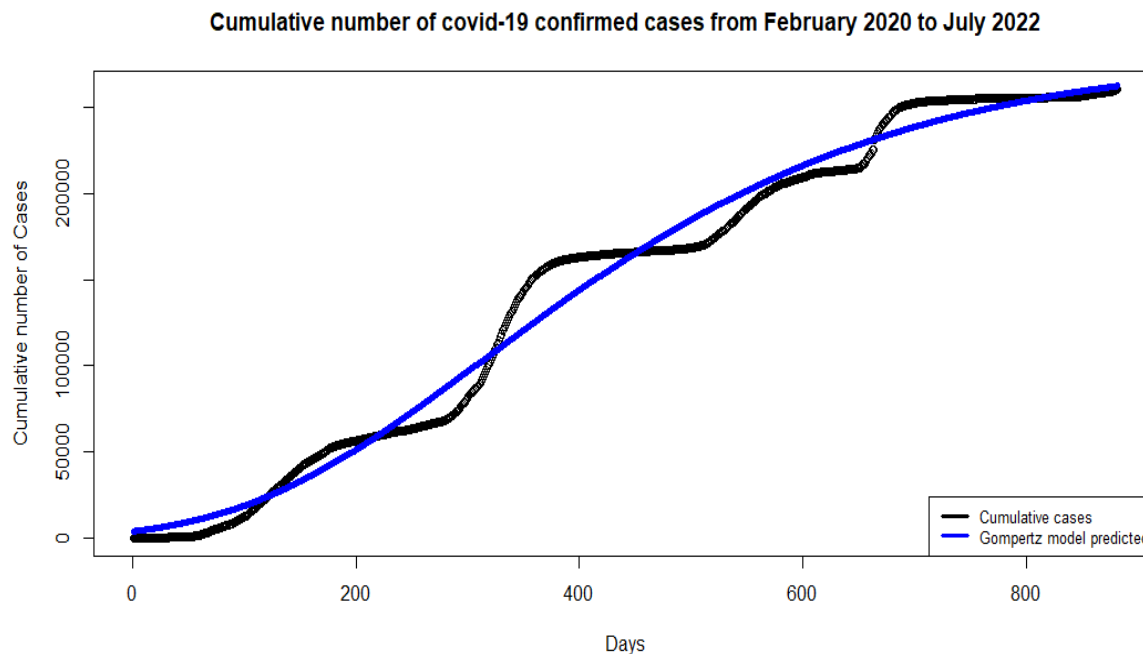
Figure 4: Plot of Cumulative total confirmed cases of Covid-19 (black) with fitted Gompertz growth model (blue)

The Logistic population growth model for cumulative confirmed cases of Covid-19.

The logistic growth model in equation (23) was also used to model the cumulative covid-19 confirmed cases, and the estimated parameters of the model are given in table 2 below.

Table 2: Estimated Parameters of the Logistic model of the cumulative covid-19 cases

Parameters	Estimate	Std. Error	t-value	p-value
k	2.609e+05	1.372e+03	190.16	<2e-16
a	1.904e+01	6.879e-01	27.68	<2e-16
r	7.756e-03	1.144e-04	67.83	<2e-16

Table 2 above shows the estimated parameters of the logistic population growth model using the nonlinear least square algorithm. The model reveals that the cumulative daily confirmed cases of covid-19 pandemic in Nigeria would grow to 260900 with a standard error of 1372. The average growth rate was estimated to be 0.007756 with a standard error of 0.0001144. Testing for the significance of the estimated parameters of the model showed that the three estimated parameters of the model are significant at a 5% level of significance (p-value less than 0.05).

The fitted logistic model is therefore given in equation (28) below

$$y = \frac{260900}{1+19.04e^{-0.007756t}} \qquad (28)$$

Using equation (28) above, we predicted the cumulative COVID-19 confirmed cases and figure 5 below shows the plot of the actual data and the predictions made using the fitted model (red line).
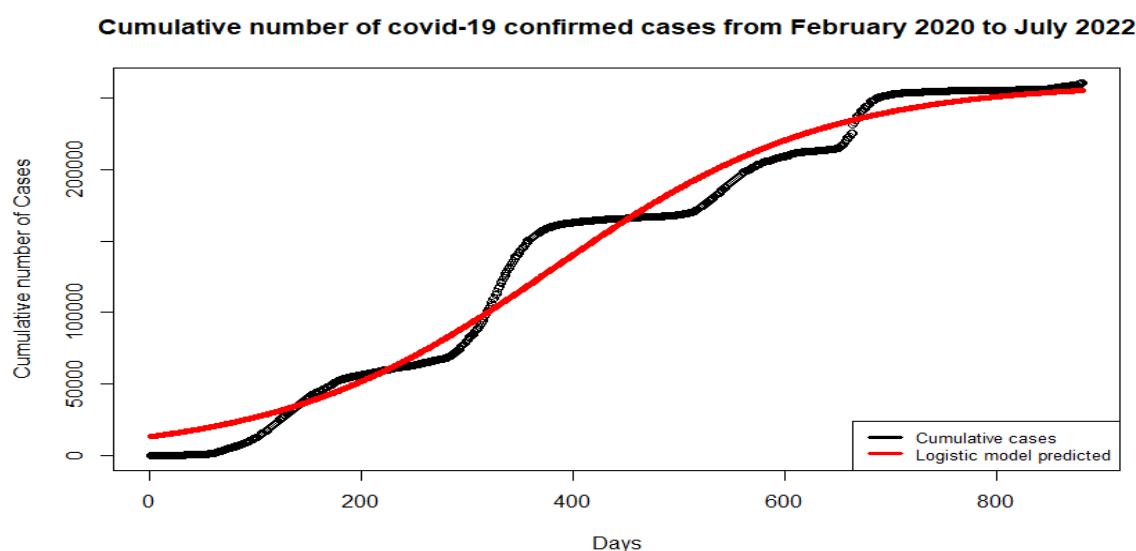

Figure 5: Plot of Cumulative total confirmed cases of Covid-19 with fitted logistic curve

Performance Comparison of the models

The performance of the models was evaluated using the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC), and a comparison was made to obtain the model that performs better in the modelling of the cumulative number of covid-19 laboratory-confirmed cases in Nigeria. Table 3 shows the values of the AIC and BIC for each of the models studied

Table 3: Performance Comparison of the Gompertz and Logistic Models on the cumulative covid-19 data in Nigeria.

Model	Residual Standard Error	No. of Iterations to Convergence	Achieved Convergence tolerance	AIC	BIC
Gompertz	11130	8	1.105e-06	18922.04	18941.17
Logistic	13370	12	1.878e-06	19245.1	19264.22

The table above shows the residual standard errors, number of iterations to convergence, achieved convergence tolerance as well as the AIC and BIC values of the fitted Gompertz and Logistic models. From the table, the Gompertz model gave a residual standard error of 11130 which is less than that of the logistic model of 13370. It took eight iterations for the Gompertz

model to converge to the estimate of the parameters, while the logistic model made 12 iterations before convergence. Furthermore, the Gompertz model gave an AIC value of 18922.04 and a BIC value of 18941.17, while the logistic model gave an AIC value of 19245.1 and a BIC value of 19264.22. These values show that the Gompertz model performed better on the cumulative covid-19 cases in Nigeria compared to the logistic model since it has the least values for both the AIC and BIC. Figure 6 below shows the original data and predictions made using both Gompertz and logistic growth models.
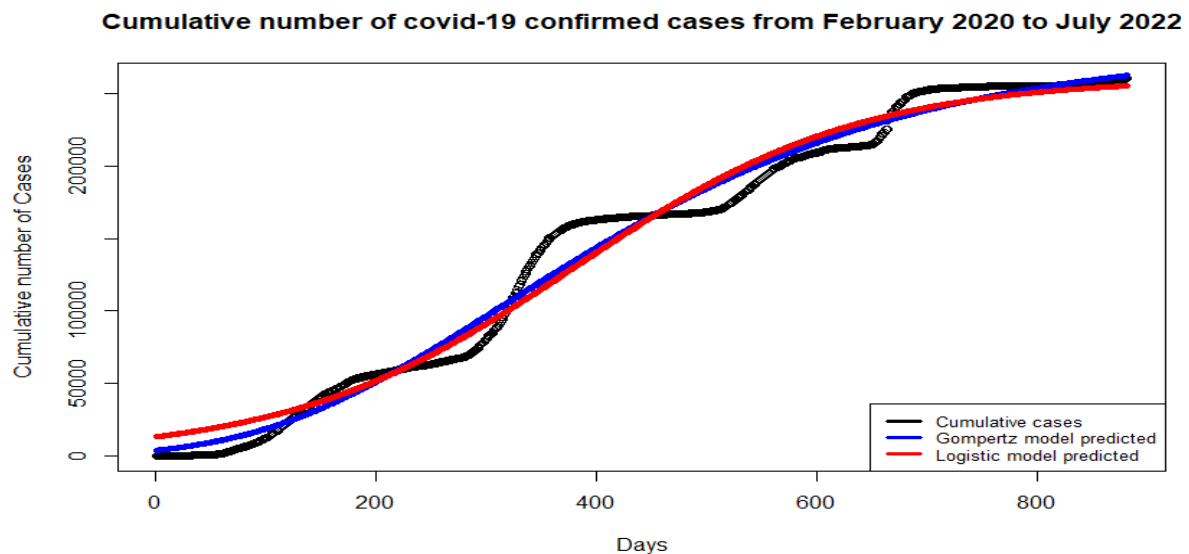

Figure 6: Plot of Cumulative total confirmed cases of Covid-19 with predictions from fitted Gompertz and logistic models

Modelling the fatality of the Covid-19 Pandemic

From the data collected, the daily cumulative deaths of the covid-19 pandemic in Nigeria was plotted as shown in the figure (7) below.
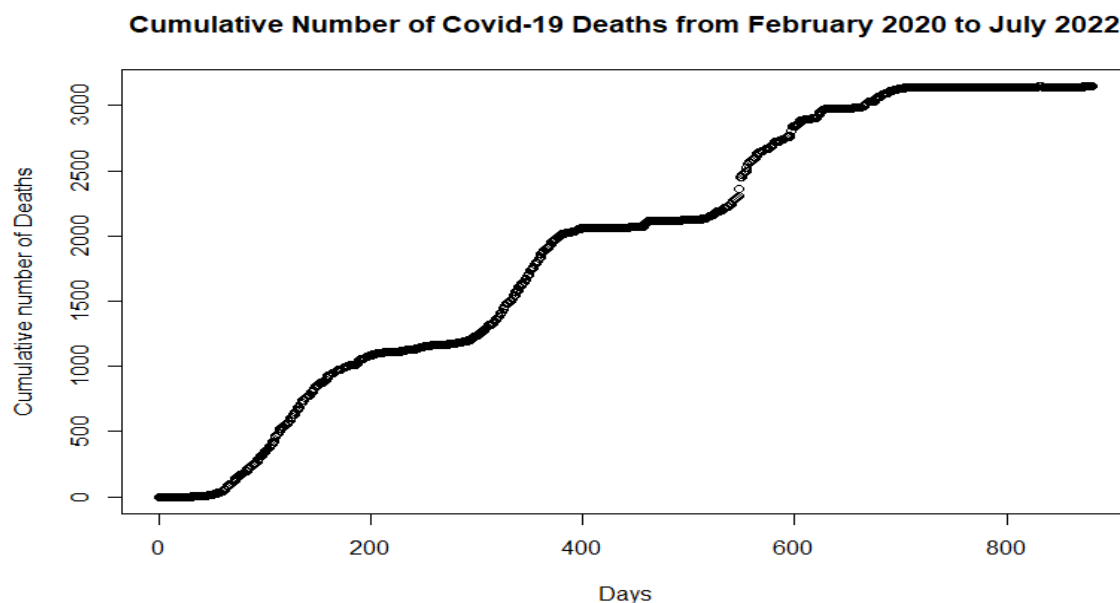
Figure 7: Plot of Cumulative Deaths Due to the Covid-19 Pandemic

Figure 7 above is a plot of cumulative Covid-19 death in Nigeria from February 2020 to July 2022. The figure shows an increasing pattern from day 1 to around day 290, then a sharp increase from day 290 to around day 350. There was a near zero growth from day 350 to around day 500, then a steady rise again from day 500 to around day 600. A close look at Figure 7 also reveals the possibility of the data being modelled by a Gompertz population growth model, the logistic growth model or any other sigmoid-type model. More generally, the data has the possibility of being modelled by a number of s-shaped models. This study, however, explored the Gompertz and the logistic growth models to describe the various properties of the dataset and also made comparisons to identify the best model for the cumulative covid-19 deaths in Nigeria from February 2020 to July 2022.

The Gompertz population growth model for the cumulative covid-19 deaths.

The Gompertz population growth model was fitted to the data using the simple nonlinear least square algorithm. The estimates of the parameters of the fitted model are shown in the table below.

Table 4: Gompertz model

Parameters	Estimate	Std. Error	t-value	p-value
k	3.562e+03	2.644e+01	134.70	2e-16
b	3.220e+00	4.628e-02	69.58	2e-16
c	4.139e-03	6.830e-05	60.60	2e-16

The table above shows the estimated parameters of the Gompertz population growth model using the nonlinear least square algorithm. The model reveals that the cumulative covid-19

related death in Nigeria would grow to 3562 with a standard error of 26.44. The average growth rate was estimated to be 0.004139 with a standard error of 0.00006830. Testing for the significance of the estimated parameters of the model showed that the three estimated parameters of the model are significant at a 5% level of significance.

The fitted Gompertz model is therefore given as

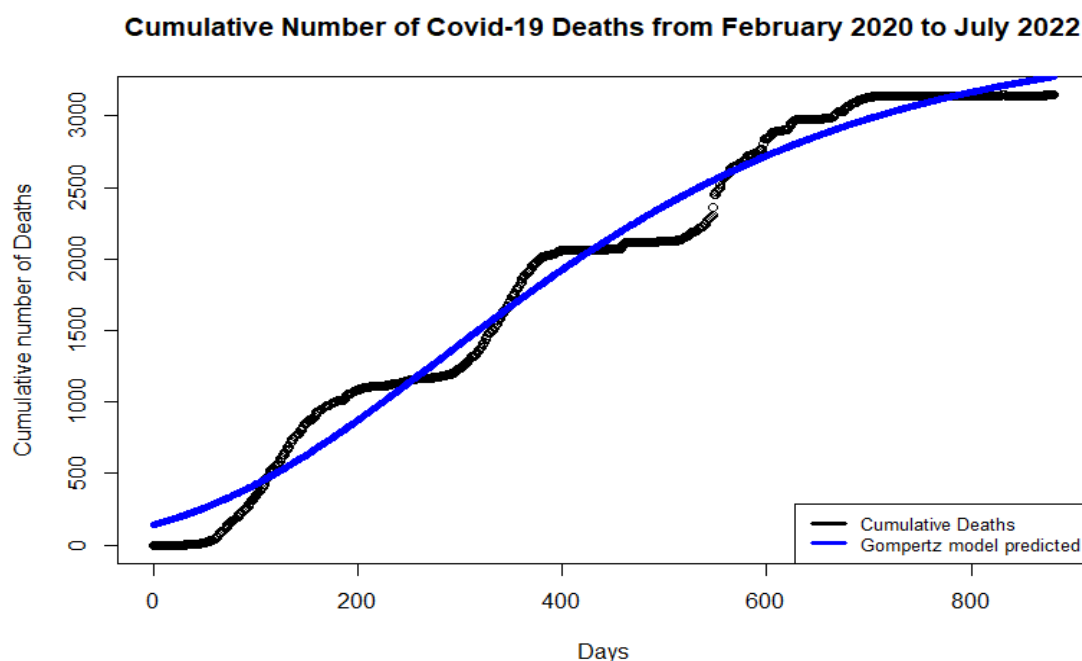$$y = 3562e^{-3.220e^{-0.004139t}} \qquad (29)$$

Figure 8: Plot of the Cumulative number of Covid-19 deaths (black) with fitted Gompertz growth model (blue)

The Logistic population growth model for cumulative deaths due to Covid-19.

The logistic growth model in equation (23) was also used to model the cumulative covid-19 deaths, and the estimated parameters of the model are given in table 5 below.

Table 5: Estimated Parameters of the Logistic model of the cumulative covid-19 deaths

Parameters	Estimate	Std. Error	t-value	p-value
K	3.304e+03	1.889e+01	174.86	<2e-16
A	1.123e+01	3.233e-01	34.75	<2e-16
R	6.767e-03	1.021e-04	66.26	<2e-16

Table 5 above shows the estimated parameters of the logistic population growth model using the nonlinear least square algorithm. The model reveals that the cumulative daily confirmed cases of covid-19 pandemic in Nigeria would grow to 3304 with a standard error of 18.89. The average growth rate was estimated to be 0.006767 with a standard error of 0.0001021. Testing for the significance of the estimated parameters of the model showed that the three estimated parameters of the model are significant at a 5% level of significance (p-value less than 0.05).

The fitted logistic model is therefore given in equation (30) below

$$y = \frac{3304}{1+18.89e^{-0.0067676t}} \tag{30}$$

Using equation (30) above, we predicted the cumulative COVID-19 confirmed cases and figure 6 below shows the plot of the actual data and the predictions made using the fitted model (red line).
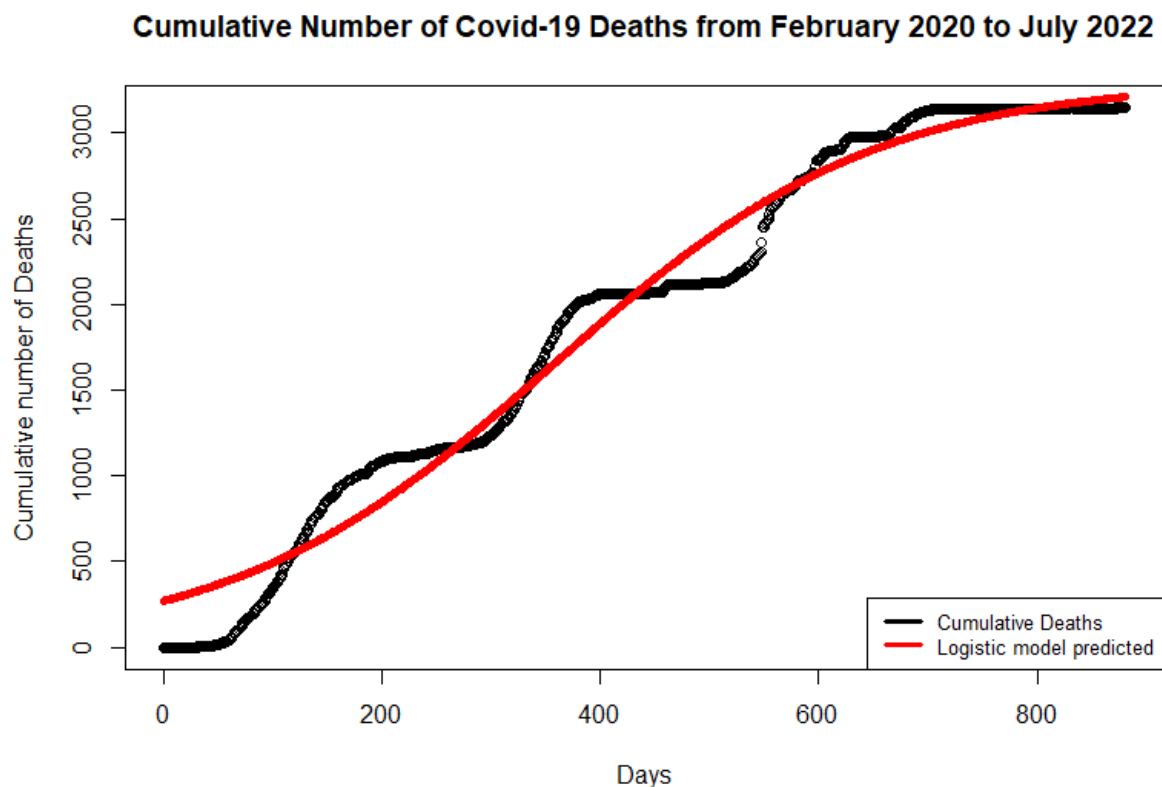

Figure 9: Plot of the Cumulative number of Covid-19 deaths with fitted logistic curve

Performance Comparison

The performance of the models was evaluated using the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) and a comparison was made to obtain the model that performs better in the modelling of the cumulative number of covid-19 related deaths in Nigeria.

Table 6: Performance Comparison of the Gompertz and Logistic Models on the cumulative covid-19 deaths in Nigeria.

Model	Residual Standard Error	No. of Iterations to Convergence	Achieved Convergence tolerance	AIC	BIC
Gompertz	147.2	10	3.423e-06	11300.99	11320.12
Logistic	166.2	12	2.973e-06	11514.80	11533.93

The table above shows the residual standard errors, number of iterations to convergence, achieved convergence tolerance as well as the AIC and BIC values of the fitted Gompertz and Logistic models. From the table, the Gompertz model gave a residual standard error of 147.2 which is less than that of the logistic model of 166.2. It took ten iterations for the Gompertz model to converge to the estimate of the parameters, while the logistic model made 12 iterations before convergence. Furthermore, the Gompertz model gave an AIC value of 11300.99 and a BIC value of 11320.12, while the logistic model gave an AIC value of 11514.80 and a BIC value of 11533.93. These values show that the Gompertz model provides a better fit on the cumulative covid-19 related deaths in Nigeria compared to the logistic model since it has the least values for both the AIC and BIC.

CONCLUSION

Modeling a deadly pandemic such as covid-19 provides useful insight into the behaviour of the disease among a population. Such information helps in understanding and tackling similar outbreaks in the future. The Gompertz and logistic growth models were originally developed to model the growth of diseases such as tumours. In the recent past, these models have been extended to model the growth of a population and other biological variables, such as the weight and sizes of poultry and aquatic life. The use of sigmoid curves such as the Gompertz and logistic growth curves have, however, not been fully explored and applied in modelling the spread of diseases among a population. This paper applied the Gompertz and logistic population growth models to the prevalence and fatality of the Covid-19 pandemic in Nigeria from February 2020 to July 2022. It is aimed to explore and compare the performance of the models to obtain the one with a better fit to the data. Comparing the performance of the two models, we employed some statistical outputs of the models, such as the residual standard error, the number of iterations before convergence, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The study revealed that the Gompertz model provides a better fit to the prevalence and fatality of covid-19 pandemic in Nigeria.

REFERENCES

Ahmad A. S., Aminu S., Usman A. A., Suleiman A. S. (2021). Estimation of the case fatality rate of COVID-19 epidemiological data in Nigeria using statistical regression analysis. Biosafety & Health. 3. Pp. 4 – 7.

Edike N., Braimah J. O., and Agbedeyi O. D. (2020). A Retrospective Analysis and Monitoring of the Spread of Covid-19 in Nigeria. Global Journal of Pure and Applied Sciences Vol. 26, pp. 157-163. doi: https://dx.doi.org/10.4314/gjpas.v26i2.7

Giraud, C. (2015). Introduction to High-Dimensional Statistics. Chapman & Hall/CRC. ISBN 9781482237948.

Nigeria Centre for Disease Control NCDC, 2020. "COVID-19 outbreak in Nigeria: situation report." Available: http://covid19.ncdc.gov.ng www.healthknowledge.org

Pelinovsky, Kokoulina, Epifanova, Kurkin, Kurkina, Tang, Macau and Kirillin (2021). Gompertz model in COVID-19 spreading simulation. Chaos Solitons Fractals. 154:111699. doi:10.1016/j.chaos.2021.111699.

Rauf R. I. and Hannah O. O. (2020). Forecasting the spread of COVID-19 in Nigeria using Box-Jenkins Modeling Procedure. medRxiv preprint. doi: https://doi.org/10.1101/2020.05.05.20091686; made available under a CC-BY-NC-ND 4.0 International license

Samuel O. A., Muhammad A. B., Samuel O. O., and Haruna U. Y., and Rafiu O. A. (2020). Modelling COVID-19 Cases in Nigeria Using Some Selected Count Data Regression Models. International Journal of Healthcare and Medical Sciences. 6 (4) pp: 64-73.

Stoica, P. and Selen, Y. (2004). "Model-order selection: A review of Information Criterion rules", IEEE Signal Processing Magazine (July): 36 – 47.

World Health Organization. WHO Director-General's remarks at the media briefing on 2019-nCoV on 11 February 2020. [Accessed 11 February 2020] Available from: https://www.who.int/dg/speeches/detail/who-director-generals-remarks-at-the-media-briefing -on-2019-ncov-on-11-february-2020.

Stoica, P. and Selen, Y. (2004). "Model-order selection: A review of Information Criterion rules", IEEE Signal Processing Magazine (July): 36 – 47.

Schwarz, Gideion E. (1978). "Estimating the Dimension of a Model". Annals of Statistics, 6(2): 461 – 464.

Yusuf F. Z. (2020). Exposing Empirical Links between COVID-19 Situation Report and Available Data: The Case of Nigeria. MPDI: Diseases, 1-11.

APPENDIX I: R CODES

```
> gompertz<-function(t,a,b,c){

 y<-a*exp (-b*exp (-c*t))

 return(y)

 }
```

```
# c = 0.001

>

Gomp1<-nls(total_cases~gompertz(days,a,b,c),data=covid,start=list(a=261000,b=1,c=0.001))

> summary(Gomp1)
```

Formula: total_cases ~ gompertz(days, a, b, c)

Parameters:

   Estimate Std. Error t value Pr($>$|t|)

a 2.824e+05 1.846e+03 152.92   <2e-16 ***

b 4.326e+00 7.583e-02   57.06   <2e-16 ***

c 4.649e-03 7.017e-05   66.26   <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11130 on 878 degrees of freedom

Number of iterations to convergence: 8

Achieved convergence tolerance: 1.105e-06

```
> AIC(Gomp1)
```

[1] 18922.04

```
> BIC(Gomp1)
```

[1] 18941.17

#LOGISTIC GROWTH CURVE

c = 0.001

logis1<-nls(total_cases~logis(k,a,r,days),data=covid, start=list(k=261000, a=1, r=0.001))

> summary (logis1)

Formula: total_cases ~ logis(k, a, r, days)

Parameters:

Estimate Std. Error t value Pr(>|t|)

k 2.609e+05 1.372e+03 190.16 <2e-16 ***

a 1.904e+01 6.879e-01 27.68 <2e-16 ***

r 7.756e-03 1.144e-04 67.83 <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13370 on 878 degrees of freedom

Number of iterations to convergence: 12

Achieved convergence tolerance: 1.878e-06

> AIC(logis1)

[1] 19245.1

> BIC(logis1)

[1] 19264.22

FATALITY

 Gompf1<-nls(total_deaths~gompertz(days,a,b,c),data=covid,start=list(a=3000,b=1,c=0.001))

> summary(Gompf1)

Formula: total_deaths ~ gompertz(days, a, b, c)

Parameters:

Estimate Std. Error t value Pr(>|t|)

a 3.562e+03 2.644e+01 134.70 <2e-16 ***

b 3.220e+00 4.628e-02 69.58 <2e-16 ***

c 4.139e-03 6.830e-05   60.60   <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 147.2 on 878 degrees of freedom

Number of iterations to convergence: 10

Achieved convergence tolerance: 3.423e-06

AIC(Gompf1)

[1] 11300.99

> BIC(Gompf1)

[1] 11320.12

#LOGISTIC MODEL

logisf1<-nls(total_deaths~logis(k,a,r, days),data=covid, start=list(k=3000, a=1, r=0.001))

> summary(logisf1)

Formula: total_deaths ~ logis(k, a, r, days)

Parameters:

  Estimate Std. Error t value Pr(>|t|)

k 3.304e+03 1.889e+01 174.86   <2e-16 ***

a 1.123e+01 3.233e-01   34.75   <2e-16 ***

r 6.767e-03 1.021e-04   66.26   <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 166.2 on 878 degrees of freedom

Number of iterations to convergence: 12

Achieved convergence tolerance: 2.973e-06

AIC(logisf1)

[1] 11514.8

> BIC(logisf1)

[1] 11533.93