African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 7, Issue 1, 2024 (pp. 64-79)
www.abjournals.org

A PRODUCTION INVENTORY MODEL WITH LINEAR TIME DEPENDENT PRODUCTION RATE, LINEAR LEVEL DEPENDENT DEMAND AND DEMAND AND CONSTANT HOLDING COST

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Cite this article:

Atama A. M., Sani B. (2024), A Production Inventory Model with Linear Time Dependent Production Rate, Linear Level Dependent Demand and Demand and Constant Holding Cost. African Journal of Mathematics and Statistics Studies 7(1), 64-79. DOI: 10.52589/AJMSS-8IYDEQEU

Manuscript History

Received: 25 Nov 2023

Accepted: 12 Jan 2024

Published: 11 Feb 2024

ABSTRACT: *In this paper, a production inventory model with linear time dependent production rate is considered. The market demand is assumed to be linear level dependent while the holding cost is a constant. The model considered a small amount of decay without having any shortage. Production starts with a buffer stock reaching its maximum desired level and then the inventory begins to deplete due to demand and deterioration. The model is formulated using a system of differential equations and typical integral calculus was used to analyze the inventory problems. These differential equations were solved to give the best cycle length* $T_1^* = 0.8273(303\, days)$, *Optimal time for maximum inventory* t_1^*= *0.7015, Optimal order quantity* Q_1^*=*38.3404 and total average inventory cost per unit time* $TC(T_1)*$ =*170.5004. The cost function has been shown to be convex and a numerical example to show the application of the model has been given. A sensitivity analysis is then carried out to see the effects of parameter changes*

KEYWORDS: Production rate, Level dependent demand, Production inventory, Linear time dependent.

Article DOI: 10.52589/AJMSS-8IYDEQEU
DOI URL: https://doi.org/10.52589/AJMSS-8IYDEQEU

INTRODUCTION

Many scholars have worked in the fields of retail and production inventory to solve real life problems. They do this by building a suitable inventory model to take care of what is on the ground depending on various types of demands. Demand may be constant, linearly increasing or decreasing, increasing or decreasing with time, level or stock-dependent demand, time varying demand pattern, quadratic demand and so on. Based on the model developed and the nature of the demand, the firm decides how much to produce and when to produce it. The literature on production inventory management provides a solid foundation for understanding complexities of managing different items.

Hill [5] proposed a time dependent demand pattern by considering it as a combination of two different types of demand in two successive time periods over the entire time horizon i.e ramp-type time dependent pattern. This type of demand pattern usually appears in the case of a new brand of consumer goods coming to the market.

Manna and Chaudhuri [8] studied a production-inventory system for time-dependent deteriorating items. They assumed the demand rate to be a ramp type function of the item. The demand rate increases with time up to a certain point and then ultimately stabilizes becoming a constant. The system was first studied by allowing no shortages in the inventory and then it was extended to cover shortages. Chakraborti and Chaudhuri [4] proposed an EOQ model with linear trend in demand and having shortages for deteriorating products in all cycles. Roy [12] proposed an inventory model for deteriorating items with price dependent demand and time-varying holding cost. Mishra and Singh [9] studied an inventory model for deteriorating items with time dependent demand rate allowing partial backlogging. Sushil and Rajput [14] studied an inflationary inventory model with constant demand considering Weibull rate of deterioration and partial backlogging under permissible delay in payment. They assumed average carrying inventory to be approximately one half of the maximum inventory model and obtained approximate expressions for the optimal production lot size, the production cycle time and the inventory cycle time. Venkateswarlu and Mohan [16] studied an EOQ model for price dependent quadratic demand with time varying deterioration allowing salvage value for deteriorating items. Amutha and Chandrasekaran [2] studied an inventory model for deterioration items, quadratic demand and time dependent holding cost. They considered a constant partial backlogging rate during the shortage period. Ouyang and Cheng [11] studied an inventory model for deteriorating items with exponential declining demand and partial backlogging. The model considered a reduction on the selling price of the items after deterioration. The holding cost was considered to be constant and the optimal solution of the system was determined by the model. Shirajul and Sharifuddin [13] developed an inventory model with constant holding cost and production rate, linear level dependent demand with buffer stock. The demand during production is the same with the demand after production. Ali *et al.* [1] developed an inventory model for non-instantaneous deteriorating items, price and stock dependent demand, fully backlogged shortage and under inflation. The demand function was assumed to be generally dependent on price and stock and when there was a shortage then the demand would depend only on the price of the product. They considered the price of the product to be dependent on different kinds of fixed rates and the deterioration was assumed to be non–instantaneous. Shortages were allowed and fully backlogged. Bashair and Lakdere [3] proposed an EOQ inventory model with non-instantaneous deteriorating items and partial backlogging. They assumed that the time at which deterioration begins is greater than or equal to the time at which shortages begin.

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Pankey and Pijus [10] formulated an imperfect production model to rework the imperfect products. The model assumed the demand to be time dependent with price and advertisement for delay decaying items. The model aims at maximizing the non-linear profit function of the system. Madaki and Sani [7] developed a production inventory model with constant production rate, linear level dependent demand and linear holding cost. It is an extension of the model developed by Shirajul and Sharufudding [13] whereby the demand during production was considered to be different from the demand after production. The model determines the total average optimal inventory cost per unit time and optimal cycle time. Swagatika *et al.* [15] contributed to the literature of instantaneous deterioration. They developed inventory models for both crisp and fuzzy single commodities with three rates of production where the demand rate was a function of both advertising and selling price. Jamil *et al.* [16] proposed a model of an inventory with stock dependent demand allowing few defective items constituting little amount of decay. The production rate was constant and the aim was to find out the total optimum inventory cost, optimal cycle time and the Economic Production Quantity (EPQ).

In this paper, a production inventory model has been proposed considering time linear production rate and a constant holding cost. Shortages are not allowed and the model determines the total average optimal inventory cost per unit time and the optimal cycle time. The cost function has been shown to be convex. The difference between this paper and that of Shirajul and Sharifuddin [13] is in the fact that in Shirajul Islam Sharifuddin [13], the production rate is constant whereas in this paper the production rate is a function of time. A sensitivity analysis is carried out at the end, to see the effect of parameter changes.

Assumptions

The production rate $\lambda+\beta t$ is a linear function of time and always greater than the demand rate. The rate of decay μ is constant and small. Since the decay is small it is assumed that there is no deterioration cost as in Shirajul Islam and Sharifuddin (2016). The demand rate during production at any instant t is given by $a+bI(t)$ where a and b are constants and satisfying the condition that $\lambda+\beta t>a+bI(t)$. The demand rate after production is $c+fI(t)$ and assumed to be greater than the demand during production at any instant t where c and f are constants. Production starts with a buffer stock and shortages are not allowed. Inventory is highest at the end of production and after this time the inventory depletes due to demand and deterioration.

Notations

$I(t)$ = Inventory level at any instant t

I_1 = Total inventory for the period from $t=0$ to $t=t_1$

I_2 = Total inventory for the period from $t=t_1$ to $t=T_1$

D_1 = Deteriorated inventory for the period from $t=0$ to $t=t_1$

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D_2 = Deteriorated inventory for the period from $t = t_1$ *to* $t = T_1$

dt = Very small portion of instant t

K_0 = Set up cost

h = Constant holding cost

Q and Q_1 are the inventory levels at time $t=0$ and $t=t_1$ respectively. Here, Q is the buffer stock.

$TC(T_1)$ = Total average inventory cost in a unit time

t_1 = Time when the inventory is at the maximum level

T_1 = Time for a complete cycle

Q^* = Optimal order quantity

t_1^* = Optimal time for maximum inventory

$TC(T_1)^*$ = Optimal average inventory cost per unit time

Model Formation

The production starts with a buffer stock from the beginning of the cycle at $t=0$ where the production rate $\lambda + \beta t$ is a linear function of time. The inventory changes (increases) at the rate of $\lambda + \beta t - a - bI(t) - \mu I(t)$ between $t=0$ to $t=t_1$. The market demand is $a + bI(t)$ and $\mu I(t)$ is the decay of $I(t)$ inventory at any instant t. By considering the above facts, we can formulate the differential equation as below:

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Volume 7, Issue 1, 2024 (pp. 64-79) www.abjournals.org

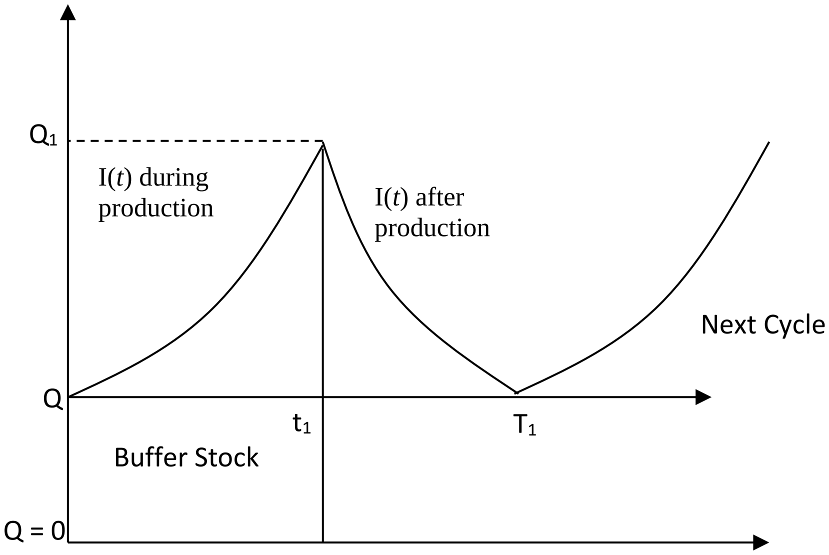

Figure 1: Inventory situation before and after production

$$I(t+dt) = I(t) + \{\lambda + \beta t - a - bI(t) - \mu I(t)\} dt$$

$$\lim_{dt \to 0} \frac{I(t+dt) - I(t)}{dt} = \lambda + \beta t - a - bI(t) - \mu I(t)$$

$$I(t) = \frac{\lambda + \beta t - a}{\mu + b} - \frac{\beta}{(\mu + b)^2} + Ae^{-(\mu+b)t} \tag{1}$$

which is the general solution of the differential equation. Applying the initial condition,

at $t = 0$, $I(t) = Q$ we get

$$\therefore A = Q - \frac{\lambda - a}{\mu + b} + \frac{\beta}{(\mu + b)^2} \tag{2}$$

We now substitute for A into equation (1) to get

$$I(t) = \frac{\lambda + \beta t - a}{\mu + b} - \frac{\beta}{(\mu + b)^2} + \left(Q - \frac{\lambda - a}{\mu + b} + \frac{\beta}{(\mu + b)^2} \right) e^{-(\mu+b)t}$$

(3)

From the boundary/matching condition at $t = t_1$, $I(t) = Q_1$

(4)

The exponential value of μ decreases as its exponential power increases. This is because μ is very small and so its higher exponential powers will give insignificant values. Thus we use

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Taylor series to expand $e^{-(\mu+b)t_1}$ and take up to the first degree of μ to be a good approximation.

$$\Rightarrow Q_1 = Q + \left(-Q(\mu+b) + \lambda - a\right)t_1 \tag{5}$$

From equation (3) the total inventory from the period $t=0$ to $t=t_1$, is

$$I_1 = \int_0^{t_1}\left[\frac{\lambda-a}{\mu+b} + \frac{\beta t}{\mu+b} - \frac{\beta}{(\mu+b)^2} + \left(Q - \frac{\lambda-a}{\mu+b} + \frac{\beta}{(\mu+b)^2}\right)e^{-(\mu+b)t}\right]dt$$

Again we approximate $e^{-(\mu+b)t_1}$ using Taylor's series by considering only the first three terms so that

$$I_1 = \frac{(\lambda-a)t_1}{\mu+b} + \frac{\beta t_1^2}{2(\mu+b)} - \frac{\beta t_1}{(\mu+b)^2} - \left(Q - \frac{\lambda-a}{\mu+b} + \frac{\beta}{(\mu+b)^2}\right)\left(\frac{1}{\mu+b}\right)\left(1-(\mu+b)t_1 + \frac{(\mu+b)^2 t_1^2}{2} - 1\right)$$

$$= Qt_1 - \frac{Q(\mu+b)t_1^2}{2} + \frac{(\lambda-a)t_1^2}{2} \tag{6}$$

Again using equation (3), we calculate the deteriorated items from the period $t=0$ to $t=t_1$ getting

$$D_1 = \int_0^{t_1}\mu I(t)\,dt = \mu\int_0^{t_1}\left[\frac{\lambda+\beta t-a}{\mu+b} - \frac{\beta}{(\mu+b)^2} + \left(Q - \frac{\lambda-a}{\mu+b} + \frac{\beta}{(\mu+b)^2}\right)e^{-(\mu+b)t}\right]dt$$

This is because the rate of decay is μ from equation (6)

$$\Rightarrow D_1 = Q\mu t_1 - \frac{Q\mu(\mu+b)t_1^2}{2} + \frac{\mu(\lambda-a)t_1^2}{2} \tag{7}$$

On the other hand, the inventory decreases at the rate of $c + fI(t)$ during $t=0$ to $t=T_1$ as there is no production after time t_1. During this period, the inventory depletes due to market demand and deterioration of the items. We use similar approach to formulate the differential equation as follows:

$$I(t+dt) = I(t) + \left\{-c - fI(t) - \mu I(t)\right\}dt$$ or

$$\lim_{dt\to 0}\frac{I(t+dt)-I(t)}{dt} = -c - fI(t) - \mu I(t)$$ or

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$$\Rightarrow \frac{dI(t)}{dt} + (\mu + f)I(t) = -c$$

$$I(t) \quad = \frac{-c}{\mu + f} + Be^{-(\mu+f)t} \tag{8}$$

which is the general solution of the differential equation. Applying the boundary condition at $t = T_1$, $I(t) = Q$, we obtain.

$$\therefore B = \left(Q + \frac{c}{\mu + f}\right)e^{(\mu+f)T_1}$$

Substituting the value of B into equation (8) we get

$$I(t) = \frac{-c}{\mu + f} + \left(Q + \frac{c}{\mu + f}\right)e^{(\mu+f)T_1}e^{-(\mu+f)t}$$
(9)

Now putting the other boundary/matching condition at $t = t_1$, $I(t) = Q_1$ we obtain

$$Q_1 = \frac{-c}{\mu + f} + \left(Q + \frac{c}{\mu + f}\right)\left\{e^{(\mu+f)(T_1-t_1)}\right\}$$
(10)

Again using Taylor Series to expand $e^{(\mu+f)(T_1-t_1)}$ to the first degree of μ, we obtain

$$\therefore Q_1 = Q + Q\left((\mu + f) + c\right)(T_1 - t_1)$$
(11)

Now from equation (9) the total inventory from $t = t_1$ to $t = T_1$ is as follows:

$$I_2 = \int_{t_1}^{T_1} I(t)\,dt = \int_{t_1}^{T_1}\left[\frac{-c}{\mu + f} + \left(Q + \frac{c}{\mu + f}\right)\left\{e^{(\mu+f)(T_1-t)}\right\}\right]dt$$

$$= \left(\frac{-c}{\mu + f}\right)(T_1 - t_1) + \left(Q + \frac{c}{\mu + f}\right)\left\{\frac{e^{(\mu+f)(T_1-T_1)} - e^{(\mu+f)(T_1-t_1)}}{-(\mu + f)}\right\}$$

Using Taylor Series to expand $e^{(\mu+f)(T_1-t_1)}$ to the first degree of μ the reason as stated before gives

$$\therefore I_2 = Q(T_1 - t_1) \tag{12}$$

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ISSN: 2689-5323
Volume 7, Issue 1, 2024 (pp. 64-79) www.abjournals.org

Considering the decay of the items, we calculate the total deteriorated inventory items during the same period with the help of equation (9). That is

$$D_2 = \int_{t_1}^{T_1} \mu I(t)\,dt = \mu \int_{t_1}^{T_1} \left[\frac{-c}{\mu+f} + \left(Q + \frac{c}{\mu+f} \right) \left\{ e^{(\mu+f)(T_1-t)} \right\} \right] dt$$

$\Rightarrow D_2 = Q\mu(T_1 - t_1)$ from equation(12). (13)

Because of continuity at t_1 we equate equation (5) and (11)

$$Q + \left(-Q(\mu+b) + \lambda - a\right)t_1 = Q + Q\left((\mu+f)+c\right)(T_1 - t_1)$$

or $t_1 = \dfrac{\left(Q(\mu+f)+c\right)T_1}{-Qb+\lambda-a+Qf+c}$ so that if (14)

$$V = \frac{Q(\mu+f)+c}{Q(-b+f)+c+\lambda-a}$$ then (15)

$$t_1 = VT_1$$ (16)

The total cost per unit time is given as

$$TC(T_1) = \frac{K_o + h(I_1 + I_2)}{T_1} \tag{17}$$

Now we substitute equations (6) and (12) into equation (17) to obtain

$$TC(T_1) = \frac{1}{T_1}\left\{ K_o + h\left(\begin{array}{l} Qt_1 - \dfrac{Q(\mu+b)t_1^2}{2} + \dfrac{(\lambda-a)t_1^2}{2} \\ +Q(T_1 - t_1) \end{array} \right) \right\}$$

Using equation (16) to substitute for $t_1 = VT_1$

$$TC(T_1) = \frac{1}{T_1}\left\{ \left(\begin{array}{l} K_o + hQVT_1 - \dfrac{hQ(\mu+b)V^2T_1^2}{2} + \dfrac{h(\lambda-a)V^2T_1^2}{2} \\ +hQ(T_1 - VT_1) \end{array} \right) \right\}$$

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$$= \frac{K_o}{T_1} + hQV - \frac{hQ(\mu+b)V^2T_1}{2} + \frac{h(\lambda-a)V^2T_1}{2}$$
$$+hQ - hQV \tag{18}$$

The objective is to find the value of T_1 which gives the minimum variable cost per unit time. The necessary and sufficient conditions to minimize $TC(T_1)$ are as follows:

(i) $\dfrac{dTC(T_1)}{dT_1} = 0$ and (ii) $\dfrac{d^2TC(T_1)}{dT_1^2} > 0$

To satisfy the necessary condition we have to differentiate equation (18) with respect to T_1 as follows:

$$\frac{dTC(T_1)}{dT_1} = -\frac{K_o}{T_1^2} - \frac{hQ(\mu+b)V^2}{2} + \frac{h(\lambda-a)V^2}{2} \tag{19A}$$

Equating this value to zero, we obtain

$$\therefore \frac{K_o}{T_1^2} = -\frac{hQ(\mu+b)V^2}{2} + \frac{h(\lambda-a)V^2}{2} \tag{19B}$$

Lemma 1: The value of $T_1 = \sqrt{\dfrac{2K_o\{Q(-b+f)+c+\lambda-a\}^2}{h\left[(-Q(\mu+b)+(\lambda-a))\{Q(\mu+f)+c\}^2\right]}}$

Proof: From equations (15) and (19B) we get

$$\frac{K_o}{T_1^2} = -\frac{hQ(\mu+b)}{2}\left\{\frac{Q(\mu+f)+c}{Q(-b+f)+c+\lambda-a}\right\}^2 + \frac{h(\lambda-a)}{2}\left\{\frac{Q(\mu+f)+c}{Q(-b+f)+c+\lambda-a}\right\}^2$$

or $$T_1^2 = \frac{2K_o\{Q(-b+f)+c+\lambda-a\}^2}{h\left[(-Q(\mu+b)+(\lambda-a))\{Q(\mu+f)+c\}^2\right]} \tag{20}$$

Theorem 1: The value of $t_1 = \sqrt{\dfrac{2K_o}{h(-Q(\mu+b)+(\lambda-a))}}$

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ISSN: 2689-5323
Volume 7, Issue 1, 2024 (pp. 64-79)
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Proof: From equations (16) and (20) we get

$$t_1 = \frac{Q(\mu+f)+c}{Q(-b+f)+c+\lambda-a} \cdot \frac{(2K_o)^{\frac{1}{2}}\left[\left\{Q(-b+f)+c+\lambda-a\right\}^2\right]^{\frac{1}{2}}}{\left[\left(-hQ(\mu+b)+h(\lambda-a)\right)\left\{Q(\mu+f)+c\right\}^2\right]^{\frac{1}{2}}}$$

$$\Rightarrow t_1 = \sqrt{\frac{2K_o}{h\left(-Q(\mu+b)+(\lambda-a)\right)}} \qquad (21)$$

Theorem 2: The cost function $TC(T_1)$ is convex.

Proof: From equation (19A) we note that

$$\frac{dTC(T_1)}{dT_1} = -\frac{K_o}{T_1^2} - \frac{hQ(\mu+b)V^2}{2} + \frac{h(\lambda-a)V^2}{2}$$

$$\frac{d^2TC(T_1)}{dT_1^2} = \frac{2K_o}{T_1^3} > 0$$ Since K_0 and T_1 are both positive.

Therefore, the convex property (ii) above is satisfied. Hence, there is an optimal solution in T_1.

Numerical Example

We provide a numerical example to illustrate the developed model. The values of various parameters are as follows;

K_o= 50, λ= 50, Q=10, h= 5, f=20, μ= 0.01, a= 5.5, c= 25 and b= 0.4.

Substituting and simplifying the above parameters in equations (5), (18), (20) and (21) gives Q_1^*= 38.34043, $TC(T_1)^*$ = 170.5004, T_1^* = 0.8273497 (303 days) and t_1^*= 0.70146.

Sensitivity Analysis

We study the effects of the above parameters K_o, λ, Q, h, c, a, μ,b and f, on the optimal decision variables. We perform the sensitivity analysis by changing each of the parameters by 50%, 25%10%, 5%, -5%, -10%, -25% and -50% taking one parameter at a time while keeping the other parameters unchanged. The detail is shown in Table 1 below.

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Table1: Sensitivity analysis on the numerical example to see the changes in the values of T_1^*,t_1^*,Q_1^* and TC(T₁)*

parameter	% Change in Parameter	T_1*	t_1*	Q_1*	TC(T_1)*
K_0	50%	1.013699 (371 days)	0.859448	44.72171	197.5821
	25%	0.926027 (339 days)	0.785118	41.71876	184.7232
	10%	0.868493 (318 days)	0.736338	39.74807	176.3817
	5%	0.849315 (311 days)	0.720078	39.09117	173.4757
	0%	**0.827397 (303 days)**	**0.701496**	**38.34043**	**170.5004**
	-5%	0.810959 (297 days)	0.687559	37.77737	167.4492
	-10%	0.786301 (288 days)	0.666653	36.93279	164.3163
	-25%	0.717808 (263 days)	0.608582	34.58673	154.356
	-50%	0.586301 (215 days)	0.497086	30.08229	135.2063
Λ	50%	0.712329 (261 days)	0.551963	46.09837	190.1212
	25%	0.758904 (278 days)	0.514494	42.50673	181.6872
	10%	0.79726 (292 days)	0.66345	40.1206	175.378
	5%	0.810959 (297 days)	0.681145	39.22112	173.0143
	0%	**0.827397 (303 days)**	**0.701496**	**38.34043**	**170.5004**
	-5%	0.846575 (310 days)	0.724578	37.46152	167.8218
	-10%	0.868493 (318 days)	0.750471	36.56669	164.9622
	-25%	0.950685 (348 days)	0.845847	33.59912	155.0854
	-50%	1.216438 (445 days)	1.138546	27.53361	132.1308

Parameter	% Change in Parameter	T_1*	t_1*	Q_1*	TC(T_1)*
Q	50%	0.805479 (295 days)	0.7205	42.63116	198.8649
	25%	0.813699 (298 days)	0.711825	40.52809	185.2453
	10%	0.821918 (301 days)	0.70663	39.25814	176.5695
	5%	0.824658 (302 days)	0.704254	38.80749	173.5674
	0%	0.827397 (305 days)	0.701496	38.34043	170.5004
	-5%	0.827397 (305 days)	0.700616	37.94853	167.3602
	-10%	0.838356 (307 days)	0.699221	37.53521	164.1392
	-25%	0.857534 (314 days)	0.693454	36.22634	153.8813
	-50%	0.917808 (336 days)	0.685205	34.08697	133.7661
H	50%	0.676712 (248 days)	0.57374	33.17909	222.5817
	25%	0.739726 (271 days)	0.627165	35.33747	197.2238
	10%	0.789041 (289 days)	0.668976	37.02663	181.3818
	5%	0.808219 (296 days)	0.685236	37.68353	175.9759
	0%	0.827397 (303 days)	0.701496	38.34043	170.5004
	-5%	0.849315 (311 days)	0.720078	39.09117	164.9491
	-10%	0.873973 (320 days)	0.740984	39.93575	159.3163
	-25%	0.956164 (350 days)	0.810669	42.75103	141.8562
	-50%	1.172603 (429 days)	0.004173	50.16458	110.2063

Article DOI: 10.52589/AJMSS-8IYDEQEU
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Parameter	% Change in Parameter	T_1*	t_1*	Q_1*	$TC(T_1)$*
F	50%	0.789041 (289 days)	0.701826	38.35375	176.4173
	25%	0.805479 (295 days)	0.702337	38.37442	173.92975
	10%	0.819178 (300 days)	0.703259	38.41168	172.0149
	5%	0.821918 (301 days)	0.70139	38.33615	171.2855
	0%	0.827397 (303 days)	0.701496	38.34043	170.5004
	-5%	0.835616 (306 days)	0.703488	38.4209	169.6534
	-10%	0.841096 (308 days)	0.702683	38.38841	168.7381
	-25%	0.865753 (317 days)	0.70345	38.41937	165.4821
	-50%	0.928767 (340 days)	0.702047	38.3627	157.4326
μ	50%	0.830137 (304 days)	0.703975	38.40539	170.4521
	25%	0.827397 (303 days)	0.701574	38.32604	170.4766
	10%	0.827397 (303 days)	0.701527	38.33467	170.4909
	5%	0.827397 (303 days)	0.701511	38.33755	170.4957
	0%	0.827397 (303 days)	0.701496	38.34043	170.5004
	-5%	0.827397 (303 days)	0.70148	38.34331	170.5052
	-10%	0.827397 (303 days)	0.701465	38.34618	170.51
	-25%	0.827397 (303 days)	0.701418	38.35482	170.5243
	-50%	0.827397 (303 days)	0.70134	38.3692	170.5481

Parameter	% Change in Parameter	T_1*	t_1*	Q_1*	$TC(T_1)$*
A	50%	0.849315 (311 days)	0.727615	37.3947	167.5441
	25%	0.838356 (307 days)	0.714487	37.88287	169.0483
	10%	0.83287 (305 days)	0.707607	38.19815	169.9254
	5%	0.830137 (304 days)	0.704548	38.27	170.2138
	0%	**0.827397 (303 days)**	**0.701496**	**38.34043**	**170.5004**
	-5%	0.827397 (303 days)	0.70077	38.50382	170.7842
	-10%	0.824658 (302 days)	0.697728	38.57194	171.0667
	-25%	0.819178 (300 days)	0.690949	38.86439	171.9021
	-50%	0.810959 (297 days)	0.68051	39.36401	173.2569
C	50%	0.821918 (301 days)	0.702474	38.37993	171.4725
	25%	0.824658 (302 days)	0.702059	38.36318	170.9976
	10%	0.827397 (303 days)	0.70267	38.38788	170.7017
	5%	0.827397 (303 days)	0.702086	38.36426	170.6015
	0%	0.827397 (303 days)	0.701496	38.34043	170.5004
	-5%	0.830137 (304 days)	0.703221	38.41013	170.3976
	-10%	0.830137 (304 days)	0.702618	38.38576	170.2944
	-25%	0.832877 (305 days)	0.703086	38.40468	169.9785
	-50%	0.835616 (306 days)	0.702182	38.36815	169.4316

Article DOI: 10.52589/AJMSS-8IYDEQEU
DOI URL: https://doi.org/10.52589/AJMSS-8IYDEQEU

Parameter	% Change in Parameter	T_1*	t_1*	Q_1*	TC(T_1)*
B	50%	0.843836 (309 days)	0.720863	37.68114	168.3711
	25%	0.835618 (306 days)	0.711143	38.01902	169.4493
	10%	0.830137 (304 days)	0.704881	38.19522	170.0834
	5%	0.830137 (304 days)	0.704349	38.31484	170.292
	0%	0.827397 (303 days)	0.701496	38.34043	170.5004
	-5%	0.827397 (303 days)	0.700968	38.45929	170.707
	-10%	0.824658 (302 days)	0.698121	38.48334	170.9133
	-25%	0.821918 (301 days)	0.694235	38.74134	171.5245
	-50%	0.813699 (298 days)	0.684724	39.03228	172.5241

DISCUSSION OF RESULTS

Observing Table 1 carefully, we make the following deductions.

I. With increase in the values of the parameter K_o (set up cost), the values of T_1^*, t_1^*, Q_1^* and TC(T_1)*all increase. This means that increase in set up cost will result in the increase of the optimal cycle time T_1^*, optimal time for maximum inventory t_1^*, optimal production quantity $Q_1^*$and optimal total average inventory cost per unit time TC(T_1)*. This is clearly expected since excess stocking is encouraged as a result of higher set up cost. The total average inventory cost per unit time is therefore expected to increase due to increase in stock holding cost. The values of T_1^*, t_1^*, and Q_1^* all increase due to increase in inventory as a result of excess stocking.

II. With increase in the value of the parameter ƛ (constant part of the production rate), there is a decrease in the values of t_1^* and T_1^* but increase in the values of $TC(\ T_1\ )*$ and Q_1^*. The value of t_1^* decreases due to an increase in the production rate which will in turn decrease the value of T_1^*. As a result of increase in the production rate, the value of the optimal order quantity Q_1^* will increase. TC(T_1)* will therefore increase due to higher value of Q_1^*.

III. With increase in the value of the parameter Q (buffer stock), the values of t_1^*,Q_1^* and TC(T_1)*increase while the value of T_1^* decreases. This is because if Q increases, the total average inventory cost increases as a result of increase in the value of the holding cost for buffer stock. The value of T_1^* decreases, since the buffer stock is much. Therefore the inventory will take a shorter time to finish. The values of t_1^* and Q_1^* increase, because Q increases.

IV. With increase in the value of the parameter h (holding cost), the values of T_1^*, t_1^*, and Q_1^*decrease while TC(T_1)*increases. This is because an increase in the holding cost of the items will also increase the total average inventory cost per unit time. Increase in stocking holding cost, encourages more setups. The value of Q_1* is expected to decrease due to increase in the number of setups. The values of both t_1^* and cycle time T_1^* decrease due to decrease in the value of Q_1^*, therefore the inventory will finish earlier.

V. With increase in the value of the parameter f (stock dependent part of the demand after production), the values of t_1^* and Q_1^* are unstable, while T_1^* decreases and TC(T_1)*increases.

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This is so because if the stock dependent part of the demand rate increases, the demand will increase. Due to high demand, stock will finish earlier and this lowers the value of T_1^*. The increase in the values of the total average inventory cost per unit time $TC(T_1)^*$ is probably due to instability of t_1^* and Q_1^*.

VI. With increase in the value of the parameter μ (deterioration rate), the values of T_1^* and t_1^* increase while the values of Q_1^* and $TC(T_1)^*$ decrease. Since there is increase in μ, *the* optimal order quantity Q_1^* decreases, which is supposed to reduce t_1^* *and* T_1^*. However, t_1^* and T_1^*, increase which is probably because the model was trying to reduce the cost.

VII. With increase in the value of the parameter a (constant part of the demand during production), the values of t_1^* and T_1^* increase while the values of $TC(T_1)*$ and Q_1^* decrease. Just as in case VI above, increase in the value of a, increases the demand and this will in turn reduce the optimal order quantity Q_1^* which is supposed to reduce T_1^* and t_1^*. However, T_1^* and t_1^* increase probably because the model was trying to reduce the cost.

VIII. With increase in the value of the parameter c (constant part of the demand after production), the values of t_1^* and Q_1^* are unstable, but the value of T_1^* decreases while the value of $TC(T_1)^*$ increases. This is because an increase in the value of the parameter c will result in higher demand. Due to high demand, stock will finish earlier and this lowers the value of T_1^*, increase in $TC(T_1)^*$ is probably due to instability of t_1^* and Q_1*.

IX. With increase in the value of the parameter b (stock dependent part of the demand before production), the values of t_1^* and T_1^* increase, while the values of $TC(T_1)$* and Q_1^* decrease. Since the value of b increases, the demand will be high and therefore the value of Q_1^* will decrease. This is supposed to reduce the values of T_1^* and t_1^*. However, t_1^* and T_1^* increase probably because the model was trying to reduce the cost.

CONCLUSION.

In this paper, we have attempted to develop a production inventory model for items with little decay starting from buffer stock. The demand is linear level dependent during after production. The paper considers linear time production rate along with constant holding cost. The cost function has been shown to be convex and a numerical example is given to show the applicability of the model. A sensitivity analysis is then carried out to see the effect of parameter changes in the model. The objective of the model is to get the optimum inventory cost, optimum maximum time and total average inventory cost per unit time.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

Article DOI: 10.52589/AJMSS-8IYDEQEU
DOI URL: https://doi.org/10.52589/AJMSS-8IYDEQEU

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