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THE FRECHET RELIABILITY FOR (2+2) CASCADE MODEL

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ABSTRACT: *In this paper, a reliability function for one of the cascade models was found; the model consists of two basic components* $(B_1 \text{ and } B_2)$ *and two spare components* $(B_3 \text{ and } B_4)$*, and it is sufficient for the model to be in a working state with the presence of two active components and, in case of failure of one of the two basic components, it is compensated with one of the two standby components to keep the model running. It was assumed that the strength-stress factors traced the Frechet, the parameters of the Frechet were estimated by three different estimation methods (Moments, Least Squares and Weighted Least Square,), after which the reliability of the model was estimated. Monte Carlo simulations were also conducted to compare the results and find out which estimation methods are the best to estimate the reliability of the model using two statistical criteria: MSE and MAPE, where it was presented that least square estimation is the preferred for estimating the function of reliability.*

KEYWORDS: Frechet Distribution, Cascade Model, Component, Least Squares Estimation, Simulation.

INTRODUCTION

Due to the great interest in increasing the reliability of industrial machinery systems, additional spare components can be added to replace components that fail during the operation of the systems $R = (X \geq Y)$; the cascade model is considered one of the most important reliability models, as it contributes to increased reliability. The cascade model is a hierarchical formation of basic components and standby components, in which the basic components are replaced, when they fail, by standby components sequentially and in a hierarchical order.
Several important studies exist in this area: Karam and Khalil (2019) studied the (2+1) sequential model for inverse generalized Rayleigh distributions. Khalil and Karam (2019) found the reliability of the (2+1) Cascade model of the inverse Weibull distribution. Jabbour, Klaff, and Salman (2020) found a parameter estimation and reliability of a system where the model has numerous components. Hassan, Naji, Mohammed, and Saad (2020) found an estimate of the stress-strength model containing a multicomponent system. Kanaparthi, Balakurthy, and Narayana (2020) studied the estimation of the reliability of the new Rayleigh-Pareto model. Khaleel (2021) found the reliability of the cascade (2+2) model of the Weibull distribution.
This paper aims to find a reliability function of the cascade model (2+2), where we assume that the random variables of strength and stress follow the Fréchet distribution, as well as to estimate the model by three different estimation methods (Mo, LS, WLS) and make a simulation to compare the results by using the "mean square error" and "mean absolute percentage error."

MATHEMATICAL MODEL

In this model, which consists of four components, where two components $(B_1 \; and \; B_2)$ are basic for the functioning of the model and two components $(B_3 \; and \; B_4)$ are surplus with an active standby state, if case one of the two basic components fails, it is replaced by one of the two spare components, assuming that the random variable X represents durability, where $X_n \sim Fr(\varrho, \lambda_n)\,; n = 1,2,3,4$, and the random variable Y represents stress, where $Y_m \sim F(\varrho, \theta_m)\,; m = 1,2,3,4$. Also, X_n and Y_m are "independently" and "identically" distributed Frechet with known joint shape ϱ parameter and unknown scale parameters $\lambda_n\,; n = 1,2,3,4$ and $\theta_m\,; m = 1,2,3,4$.
The CDF and PDF of $Fr(\varrho, \lambda)$ are [1]:

$$F(x) = 1 - e^{-\lambda x^{-\varrho}} \qquad x > 0; \varrho, \lambda > 0 \qquad \ldots(1)$$

$$f(x) = \varrho \lambda x^{-(\varrho+1)}\, e^{-\lambda x^{-\varrho}} \qquad x > 0; \varrho, \lambda > 0 \qquad \ldots(2)$$

The CDF and PDF of $Fr(\varrho, \theta)$ are:

$$G(y) = 1 - e^{-\theta y^{-\varrho}} \qquad y > 0; \varrho, \theta > 0 \qquad \ldots(3)$$

$$g(y) = \varrho \theta y^{-(\varrho+1)}\, e^{-\theta y^{-\varrho}} \qquad y > 0; \varrho, \theta > 0 \qquad \ldots(4)$$

In this model, there are six cases to be found and the sum of these cases represents the reliability function of model and can be formulated as follows:

$$R = pr[X_1 \geq Y_1, X_2 \geq Y_2] + pr[X_1 < Y_1, X_2 \geq Y_2, X_3 \geq Y_3] + pr[X_1 < Y_1, X_2 \geq Y_2, X_3 < Y_3, X_4 \geq Y_4] + pr[X_1 \geq Y_1, X_2 < Y_2, X_3 \geq Y_3] + pr[X_1 \geq Y_1, X_2 < Y_2, X_3 < Y_3, X_4 \geq Y_4] + pr[X_1 < Y_1, X_2 < Y_2, X_3 \geq Y_3, X_4 \geq Y_4]$$

$$R = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 \qquad \ldots(5)$$

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First state of the model operation is when the two main components $(B_1\ and\ B_2)$ are working and the two spare components $(B_3\ and\ B_4)$ remain in an active standby state, and this state can be derived as follows:

$S_1 = pr[X_1 \geq Y_1, X_2 \geq Y_2]$

$S_1 = \int_0^\infty [\underline{F}_{x_1}(y_1)] g(y_1) dy_1 \int_0^\infty [\underline{F}_{x_2}(y_2)] g(y_2) dy_2$

$S_1 = \int_0^\infty \left[e^{-\lambda_1 y_1^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1 \, . \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$

$S_1 = \int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-(\lambda_1+\theta_1) y_1^{-\varrho}} dy_1 \, . \int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-(\lambda_2+\theta_2) y_2^{-\varrho}} dy_2$

$$S_1 = \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] \qquad \ldots(6)$$

Second state is when a main component (B_1) fails and is replaced by the active backup component (B_3) and the main component (B_2) remains working while the backup component (B_4) is in an active standby state, and this state can be derived as follows:

$S_2 = pr[X_1 < Y_1, X_2 \geq Y_2, X_3 \geq Y_3] = pr[X_1 < Y_1, MX_1 \geq KY_1] pr[X_2 \geq Y_2]$

where "M" and "K" strength–stress attenuation factors [ ]:

$X_3 = MX_1$ and $Y_3 = KY_1$

$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty [F_{x_1}(y_1)] \left[\underline{F}_{x_1}\left(\frac{K}{M} y_1\right)\right] g(y_1) dy_1$

$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \left[e^{-\lambda_1 y_1^{-\varrho}}\right]\left[1 - e^{-\lambda_1 (\frac{K}{M})^{-\varrho} y_1^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$

$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \left[e^{-\lambda_1 (\frac{K}{M})^{-\varrho} y_1^{-\varrho}} - e^{-\lambda_1 \left(1+(\frac{K}{M})^{-\varrho}\right) y_1^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\left(\lambda_1 (\frac{K}{M})^{-\varrho} + \theta_1\right) y_1^{-\varrho}} dy_1 - \int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\left(\lambda_1 (1+\frac{K}{M})^{-\varrho} + \theta_1\right) y_1^{-\varrho}} dy_1$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \left[\frac{\lambda_1 \left(\frac{K}{M}\right)^{-\varrho} \theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]$$

As equation (6) can get $pr[X_2 \geq Y_2]$ as :

$pr[X_2 \geq Y_2] = \left[\frac{\lambda_2}{\lambda_2+\theta_2}\right]$

$$S_2 = \left[\frac{\lambda_1 \left(\frac{K}{M}\right)^{-\varrho} \theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]\left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] \qquad \ldots(7)$$

Third state of the functioning of the model is when the main component fails (B_1) as well as the backup component (B_3) and is replaced by the active backup component (B_4) and the main component (B_2) remains working, and this case can be derived as follows:

$S_3 = pr[X_1 < Y_1, X_2 \geq Y_2, X_3 < Y_3, X_4 \geq Y_4]$

$S_3 = pr[X_1 < Y_1, MX_1 < KY_1, MX_3 \geq KY_3] pr[X_2 \geq Y_2]$

$S_3 = pr[X_1 < Y_1, MX_1 < KY_1, M^2X_1 \geq K^2Y_1] pr[X_2 \geq Y_2]$

where $X_4 = MX_3 = M(MX_1) = M^2X_1$

and $Y_4 = KY_3 = K(KY_1) = K^2Y_1$, then :

$pr[X_1 < Y_1, MX_1 < KY_1, M^2X_1 \geq K^2Y_1] =$

$\int_0^\infty [F_{x_1}(y_1)] \left[F_{x_1}\left(\frac{K}{M} y_1\right)\right] \left[\underline{F}_{x_1}\left(\frac{K^2}{M^2} y_1\right)\right] . g(y_1) dy_1$

$= \int_0^\infty \left[e^{-\lambda_1 y_1^{-\varrho}}\right]\left[e^{-\lambda_1 (\frac{K}{M})^{-\varrho} y_1^{-\varrho}}\right] . \left[1 - e^{-\lambda_1 (\frac{K}{M})^{2\varrho} y_1^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$

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$$= \int_0^\infty \left[e^{-\lambda_1\left(1+(\frac{K}{M})^{-\varrho}\right)y_1^{-\varrho}} - e^{-\lambda_1\left(1+(\frac{K}{M})^{-\varrho}+(\frac{K}{M})^{-2\varrho}\right)y_1^{-\varrho}} \right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$$

$$= \left[\int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\left(\lambda_1\left(1+(\frac{K}{M})^{-\varrho}\right)+\theta_1\right)y_1^{-\varrho}} dy_1 \right] - \left[\int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\left(\lambda_1\left(1+(\frac{K}{M})^{-\varrho}+(\frac{K}{M})^{-2\varrho}\right)+\theta_1\right)y_1^{-\varrho}} dy_1 \right]$$

$$pr[X_1 < Y_1, MX_1 < KY_1, M^2X_1 \geq K^2Y_1] = \left[\frac{\lambda_1\left(\frac{k}{m}\right)^{-2\varrho}\theta_1}{\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_1\right)\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_1\right)} \right]$$

and $pr[X_2 \geq Y_2] = \left[\frac{\lambda_2}{\lambda_2+\theta_2}\right]$

$$S_3 = \left[\frac{\lambda_1\left(\frac{k}{m}\right)^{-2\varrho}\theta_1}{\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_1\right)\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_1\right)} \right] \left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] \quad \text{...(8)}$$

Fourth state is when the main component (B_1) remains working but the main component (B_2) fails, so it is replaced by the active standby component (B_3) and the standby component (B_4) remains in an active standby state, and this state can be derived as follows:
$S_4 = pr[X_1 \geq Y_1, X_2 < Y_2, X_3 \geq Y_3] = pr[X_1 \geq Y_1]pr[X_2 < Y_2, MX_2 \geq KY_2]$
where $X_3 = MX_2$ and $Y_3 = KY_2$.

As Equation (6) can get $pr[X_1 \geq Y_1]$ as:
$pr[X_1 \geq Y_1] = \left[\frac{\theta_1}{\lambda_1+\delta_1}\right]$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty [F_{x_2}(y_2)]\left[\underline{F}_{x_2}\left(\frac{K}{M}y_2\right)\right] g(y_2)dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}}\right]\left[1 - e^{-\lambda_2\left(\frac{K}{M}\right)^{-\varrho} y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}} - e^{-\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-(\lambda_2+\theta_2)y_2^{-\varrho}} dy_2 - \int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right]$$

$$S_4 = \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right] \quad \text{...(9)}$$

Fifth state of the functioning of the model is when the main component (B_1) remains working but the main component (B_2) fails, as well as the replacement component (B_3) fails, so it is replaced by the active backup component (B_4), and this case can be derived as follows:
$S_5 = pr[X_1 \geq Y_1, X_2 < Y_2, X_3 < Y_3, X_4 \geq Y_4]$
$= pr[X_1 \geq Y_1]pr[X_2 < Y_2, MX_2 < KY_2, MX_3 \geq KY_3]$
$= pr[X_1 \geq Y_1]pr[X_2 < Y_2, MX_2 < KY_2, M^2X_2 \geq K^2Y_2]$
and $Y_4 = KY_3 = K(KY_2) = K^2Y_2$, then :

$pr[X_1 \geq Y_1] = \left[\frac{\theta_1}{\lambda_1+\theta_1}\right]$
and

$$pr[X_2 < Y_2, MX_2 < KY_2, M^2X_2 \geq K^2Y_2] = \int_0^\infty [F_{x_2}(y_2)]\left[F_{x_2}\left(\frac{K}{M}y_2\right)\right]\left[\underline{F}_{x_2}\left(\frac{K^2}{M^2}y_2\right)\right]$$

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$$.g(y_1)dy_1$$

$$= \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}}\right]\left[e^{-\lambda_2\left(\frac{K}{M}\right)^{-\varrho} y_2^{-\varrho}}\right].\left[1 - e^{-\lambda_2(\frac{K}{M})^{-2\varrho} y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$= \int_0^\infty \left[e^{-\lambda_2\left(1+(\frac{K}{M})^{-\varrho}\right)y_2^{-\varrho}} - e^{-\lambda_2\left(1+(\frac{K}{M})^{-\varrho}+(\frac{K}{M})^{-2\varrho}\right)y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 < KY_2, M^2X_2 \geq K^2Y_2] = \left[\int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\left(\lambda_2\left(1+(\frac{K}{M})^{-\varrho}\right)+\theta_2\right)y_2^{-\varrho}} dy_2\right]$$

$$-\left[\int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\left(\lambda_2\left(1+(\frac{K}{M})^{-\varrho}+(\frac{K}{M})^{-2\varrho}\right)+\theta_2\right)y_2^{-\varrho}} dy_2\right]$$

$$= \left[\frac{\lambda_2\left(\frac{k}{m}\right)^{-2\varrho}\theta_2}{\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_2\right)\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_2\right)}\right]$$

$$S_5 = \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2\left(\frac{k}{m}\right)^{-2\varrho}\theta_2}{\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_2\right)\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_2\right)}\right] \quad ...(10)$$

Sixth state is when the two main components (B_1 and B_2) fail and are replaced by the two standby active components (B_3 and B_4) and this case can be derived as follows:

$$S_6 = pr[X_1 < Y_1, X_2 < Y_2, X_3 \geq Y_3, X_4 \geq Y_4]$$

$$= pr[X_1 < Y_1, MX_1 \geq KY_1]pr[X_2 < Y_2, mX_2 \geq kY_2]$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty [F_{x_1}(y_1)]\left[\underline{F}_{x_1}\left(\frac{K}{M}y_1\right)\right] g(y_1)dy_1$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \left[e^{-\lambda_1 y_1^{-\varrho}}\right]\left[1 - e^{-\lambda_1(\frac{K}{M}y_1)^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \left[e^{-\lambda_1 y_1^{-\varrho}} - e^{-\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)y_1^{-\varrho}}\right] \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\theta_1 y_1^{-\varrho}} dy_1$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-(\lambda_1+\theta_1)y_1^{-\varrho}} dy_1$$

$$-\int_0^\infty \varrho\theta_1 y_1^{-(\varrho+1)} e^{-\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)y_1^{-\varrho}} dy_1$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \left[\left(\frac{\theta_1}{\lambda_1+\theta_1}\right) - \left(\frac{\theta_1}{\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1}\right)\right]$$

$$pr[X_1 < Y_1, MX_1 \geq KY_1] = \left[\frac{\lambda_1\left(\frac{K}{M}\right)^{-\varrho}\theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]$$

$$pr[X_2 < Y_2, mX_2 \geq kY_2] = \int_0^\infty [F_{x_2}(y_2)]\left[\underline{F}_{x_2}\left(\frac{K}{M}y_2\right)\right] g(y_2)dy_2$$

$$pr[X_2 < Y_2, mX_2 \geq kY_2] = \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}}\right]\left[1 - e^{-\lambda_2(\frac{K}{M}y_2)^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty \left[e^{-\lambda_2 y_2^{-\varrho}} - e^{-\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)y_2^{-\varrho}}\right] \varrho\theta_2 y_2^{-(\varrho+1)} e^{-\theta_2 y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, MX_2 \geq KY_2] = \int_0^\infty \varrho\theta_2 y_2^{-(\varrho+1)} e^{-(\lambda_2+\theta_2)y_2^{-\varrho}} dy_2$$

$$.\int_0^\infty o\theta_2 y_2^{-(\varrho+1)} e^{-\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{o}\right)+\theta_2\right)y_2^{-\varrho}} dy_2$$

$$pr[X_2 < Y_2, mX_2 \geq kY_2] = \left[\left(\frac{\theta_2}{\lambda_2+\theta_2}\right) - \left(\frac{\theta_2}{\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2}\right)\right]$$

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$$pr[X_2 < Y_2, mX_2 \geq kY_2] = \left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right]$$

$$S_6 = \left[\frac{\lambda_1\left(\frac{K}{M}\right)^{-\varrho}\theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]\left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right] \quad ...(11)$$

Finally, adding the product of equations (6, 7, 8, 9, 10 and 11), obtain the function reliability of model:

$$R = \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] + \left[\frac{\lambda_1\left(\frac{K}{M}\right)^{-\varrho}\theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]\left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] + \left[\frac{\lambda_1\left(\frac{k}{m}\right)^{-2\varrho}\theta_1}{\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_1\right)\left(\lambda_1\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_1\right)}\right]$$
$$\cdot\left[\frac{\lambda_2}{\lambda_2+\theta_2}\right] + \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right] + \left[\frac{\lambda_1}{\lambda_1+\theta_1}\right]\left[\frac{\lambda_2\left(\frac{k}{m}\right)^{-2\varrho}\theta_2}{\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\theta_2\right)\left(\lambda_2\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\theta_2\right)}\right]$$
$$+\left[\frac{\lambda_1\left(\frac{K}{M}\right)^{-\varrho}\theta_1}{(\lambda_1+\theta_1)\left(\lambda_1\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_1\right)}\right]\left[\frac{\lambda_2\left(\frac{K}{M}\right)^{-\varrho}\theta_2}{(\lambda_2+\theta_2)\left(\lambda_2\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\theta_2\right)}\right] \quad ...(14)$$

MOMENT METHOD

To find the moment estimators, it needs mean population of $Fr(\varrho, \lambda)$, then begin by equation:

$$E(x) = \lambda^{\frac{1}{\varrho}}\Gamma\left(1-\frac{1}{\varrho}\right) \quad for\ \varrho > 1 \quad .....(15)$$

Then equating the mean population with corresponding the mean sample:

$$\frac{\sum_{i=1}^{n} x_i}{n} = \hat{\lambda}^{\frac{1}{\varrho}}\Gamma\left(1-\frac{1}{\varrho}\right) \quad .....(16)$$

Then:

$$\hat{\lambda}_{Mo} = \left[\frac{x}{\Gamma\left(1-\frac{1}{\varrho}\right)}\right]^{\varrho} \quad for\ \varrho > 1 \quad .....(17)$$

In the same way, we can obtain the moment estimator of θ say $\hat{\theta}_{Mo}$ is:

$$\hat{\theta}_{Mo} = \left[\frac{y}{\Gamma\left(1-\frac{1}{\varrho}\right)}\right]^{\varrho} \quad for\ \varrho > 1 \quad .....(18)$$

Now, the moment estimators to the unknown scale parameters for $\hat{R}_{Mo}$ then:

$$\hat{\lambda}_{\xi Mo} = \left[\frac{x_\xi}{\Gamma\left(1-\frac{1}{\varrho}\right)}\right]^{\varrho}, \xi = 1,2,3,4 \quad ..(19)$$

and

$$\hat{\theta}_{\xi Mo} = \left[\frac{y_\xi}{\Gamma\left(1-\frac{1}{\varrho}\right)}\right]^{\varrho}, \xi = 1,2,3,4 \quad ..(20)$$

By using (19) and (20) in (14) get as:

$$\hat{R}_{Mo} = \left[\frac{\hat{\lambda}_{1Mo}}{\hat{\lambda}_{1Mo}+\hat{\theta}_{1Mo}}\right]\left[\frac{\hat{\lambda}_{2Mo}}{\hat{\lambda}_{2Mo}+\hat{\theta}_{2Mo}}\right] + \left[\frac{\hat{\lambda}_{1Mo}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1Mo}}{(\hat{\lambda}_{1Mo}+\hat{\theta}_{1Mo})\left(\hat{\lambda}_{1Mo}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1Mo}\right)}\right]\left[\frac{\hat{\lambda}_{2Mo}}{\hat{\lambda}_{2Mo}+\hat{\theta}_{2Mo}}\right]$$
$$+\left[\frac{\hat{\lambda}_{1Mo}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{1Mo}}{\left(\hat{\lambda}_{1Mo}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{1Mo}\right)\left(\hat{\lambda}_{1Mo}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{1Mo}\right)}\right]\left[\frac{\hat{\lambda}_{2Mo}}{\hat{\lambda}_{2Mo}+\hat{\theta}_{2Mo}}\right]$$

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$$+\left[\frac{\hat{\lambda}_{1Mo}}{\hat{\lambda}_{1Mo}+\hat{\theta}_{1Mo}}\right]\left[\frac{\hat{\lambda}_{2Mo}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2Mo}}{\left(\hat{\lambda}_{2Mo}+\hat{\theta}_{2Mo}\right)\left(\hat{\lambda}_{2Mo}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2Mo}\right)}\right]$$
$$+\left[\frac{\hat{\lambda}_{1Mo}}{\hat{\lambda}_{1Mo}+\hat{\theta}_{1Mo}}\right]\left[\frac{\hat{\lambda}_{2Mo}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{2Mo}}{\left(\hat{\lambda}_{2Mo}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{2Mo}\right)\left(\hat{\lambda}_{2Mo}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{2Mo}\right)}\right]$$
$$+\left[\frac{\hat{\lambda}_{1Mo}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1Mo}}{\left(\hat{\lambda}_{1Mo}+\hat{\theta}_{1Mo}\right)\left(\hat{\lambda}_{1Mo}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1Mo}\right)}\right]\left[\frac{\hat{\lambda}_{2Mo}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2Mo}}{\left(\hat{\lambda}_{2Mo}+\hat{\theta}_{2Mo}\right)\left(\hat{\lambda}_{2Mo}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2Mo}\right)}\right] \quad \ldots(21)$$

LEAST SQUERS METHOD

Using the minimize equation the derivation starts in this method:

$$S(\varrho,\lambda)=\sum_{i=1}^{n}\left(\hat{F}(x_i)-F(x_i)\right)^2$$
$$=\sum_{i=1}^{n}\left(\hat{F}(x_i)-e^{-\lambda x_i^{-\varrho}}\right)^2 \quad ..(22)$$

Then

$$F(x_i)=e^{-\lambda x_i^{-\varrho}}$$
$$-ln\left(F(x_i)\right)=\lambda x_i^{-\varrho} \quad \ldots(23)$$

We minimize the squares of the difference between the linear form of $F(x_i)$ and the same transformation of $\hat{F}(x_i)$, we use $\hat{F}\left(x_{(i)}\right)$ as follows:

$\hat{F}\left(x_{(i)}\right)=\frac{i}{n+1}$; $i=1,2,\ldots.,n$, then:

$$S(\varrho,\lambda)=\sum_{i=1}^{n}\left(q_{(i)}-\lambda x_{(i)}^{-\varrho}\right)^2 \quad \ldots(24)$$

where $q_{(i)}=-ln\left(\hat{F}\left(x_{(i)}\right)\right)=-ln(P_i)$ and P_i is the plotting position.

Deriving (24) with λ, get as:

$$\frac{\partial S(\varrho,\lambda)}{\lambda}=\sum_{i=1}^{n}2\left(q_{(i)}-\lambda x_{(i)}^{-\varrho}\right)x_{(i)}^{-\varrho}$$
$$\sum_{i=1}^{n}q_{(i)}x_{(i)}^{-\varrho}-\lambda\sum_{i=1}^{n}x_{(i)}^{-2\varrho}=0$$

Then the LS estimator of λ; say $\hat{\lambda}_{(LS)}$:

$$\hat{\lambda}_{(LS)}=\frac{\sum_{i=1}^{n}q_{(i)}x_{(i)}^{-\varrho}}{\sum_{i=1}^{n}x_{(i)}^{-2\varrho}} \quad ...(25)$$

In the same way, we will LS estimator θ, say $\hat{\theta}_{LS}$:

$$\hat{\theta}_{(LS)}=\frac{\sum_{j=1}^{m}q_{(j)}y_{(j)}^{-\varrho}}{\sum_{j=1}^{m}y_{(j)}^{-2\varrho}} \quad ...(26)$$

where $\hat{G}\left(y_{(j)}\right)=\frac{j}{m+1}$; $j=1,2,\ldots.,m$ and $q_{(j)}=-ln\left(\hat{G}\left(y_{(j)}\right)\right)=-ln(P_j)$.

Now, the last squares estimators to the unknown scale parameters for $\hat{R}_{LS}$ then:

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$$\hat{\lambda}_{\xi(LS)} = \frac{\sum_{i_\xi=1}^{n_\xi} q_{\xi_{(i_\xi)}} x_{\xi_{(i_\xi)}}^{-\varrho}}{\sum_{i_\xi=1}^{n_\xi} x_{\xi_{(i_\xi)}}^{-2\varrho}} \qquad \ldots(27)$$

and

$$\hat{\theta}_{\xi(LS)} = \frac{\sum_{j_\xi=1}^{m_\xi} q_{\xi_{(j_\xi)}} y_{\xi_{(j_\xi)}}^{-\varrho}}{\sum_{j_\xi=1}^{m_\xi} y_{\xi_{(j_\xi)}}^{-2\varrho}} \qquad \ldots(28)$$

By using (27) and (28) in (14) get as:

$$\begin{aligned}\hat{R}_{LS} = &\left[\frac{\hat{\lambda}_{1LS}}{\hat{\lambda}_{1LS}+\hat{\theta}_{1LS}}\right]\left[\frac{\hat{\lambda}_{2LS}}{\hat{\lambda}_{2LS}+\hat{\theta}_{2LS}}\right] + \left[\frac{\hat{\lambda}_{1LS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1LS}}{(\hat{\lambda}_{1LS}+\hat{\theta}_{1LS})\left(\hat{\lambda}_{1LS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1LS}\right)}\right]\left[\frac{\hat{\lambda}_{2LS}}{\hat{\lambda}_{2LS}+\hat{\theta}_{2LS}}\right]\\
&+\left[\frac{\hat{\lambda}_{1LS}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{1LS}}{\left(\hat{\lambda}_{1LS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{1LS}\right)\left(\hat{\lambda}_{1LS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{1LS}\right)}\right]\left[\frac{\hat{\lambda}_{2LS}}{\hat{\lambda}_{2LS}+\hat{\theta}_{2LS}}\right]\\
&+\left[\frac{\hat{\lambda}_{1LS}}{\hat{\lambda}_{1LS}+\hat{\theta}_{1LS}}\right]\left[\frac{\hat{\lambda}_{2LS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2LS}}{(\hat{\lambda}_{2LS}+\hat{\theta}_{2LS})\left(\hat{\lambda}_{2LS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2LS}\right)}\right]\\
&+\left[\frac{\hat{\lambda}_{1LS}}{\hat{\lambda}_{1LS}+\hat{\theta}_{1LS}}\right]\left[\frac{\hat{\lambda}_{2LS}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{2LS}}{\left(\hat{\lambda}_{2LS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{2LS}\right)\left(\hat{\lambda}_{2LS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{2LS}\right)}\right]\\
&+\left[\frac{\hat{\lambda}_{1LS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1LS}}{(\hat{\lambda}_{1LS}+\hat{\theta}_{1LS})\left(\hat{\lambda}_{1LS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1LS}\right)}\right]\left[\frac{\hat{\lambda}_{2LS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2LS}}{(\hat{\lambda}_{2LS}+\hat{\theta}_{2LS})\left(\hat{\lambda}_{2LS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2LS}\right)}\right] \quad \ldots \quad (29)\end{aligned}$$

WEIGHTED LEAST SQUARES ESTIMATION METHOD

This method starts the derivation by using the minimize equation:

$$Q = \sum_{i=1}^{n} w_i \left(\hat{F}(x_i) - F(x_i)\right)^2 \qquad \ldots(30)$$

where $w_i = \frac{1}{Var[F(x_i)]} = \frac{(n+1)^2(n+2)}{i(n-i+1)}, i = 1,2,\ldots,n$

As steps in Equations (22) and (23), we get:

$$Q = \sum_{i=1}^{n} w_i\left(q_{(i)} - \lambda x_{(i)}^{-\varrho}\right)^2 \qquad ...(30)$$

Deriving Equation (30) and equating result to zero:

$$\frac{\partial Q}{\partial \lambda} = \sum_{i=1}^{n} 2w_i\left(-\lambda x_{(i)}^{-\varrho}\right)x_{(i)}^{-\varrho}$$

$$\sum_{i=1}^{n} w_i q_{(i)} x_{(i)}^{-\varrho} - \hat{\lambda}\sum_{i=1}^{n} w_i x_{(i)}^{-2\varrho} = 0$$

Then we get the estimator of λ; say $\hat{\lambda}_{WLS}$:

$$\hat{\lambda}_{WLS} = \frac{\sum_{i=1}^{n} w_i q_{(i)} x_{(i)}^{-\varrho}}{\sum_{i=1}^{n} w_i x_{(i)}^{-2\varrho}} \qquad ...(31)$$

We will estimate unknown parameter θ, $\hat{\theta}_{WLS}$, for the stress r.v. of $Fr(\varrho,\theta)$ distribution with the sample size m, and we will obtain:

$$\hat{\theta}_{WLS} = \frac{\sum_{j=1}^{m} w_j q_{(j)} y_{(j)}^{-\varrho}}{\sum_{j=1}^{m} w_j y_{(j)}^{-2\varrho}} \qquad ...(32)$$

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where $w_j = \frac{1}{Var[G(y_{(j)})]} = \frac{(m+1)^2(m+2)}{j(m-j+1)}, j = 1,2, \ldots, m$

Now, the WLS estimators to the unknown scale parameters for $\hat{R}_{WLS}$:

$$\hat{\lambda}_{\xi\, WLS} = \frac{\sum_{i_\xi=1}^{n_\xi} w_{i_\xi} q_{\xi_{(i_\xi)}} x_{\xi_{(i_\xi)}}^{-\varrho}}{\sum_{i_\xi=1}^{n_\xi} w_{i_\xi} x_{\xi_{(i_\xi)}}^{-2\varrho}}, \xi = 1,2,3,4 \qquad \ldots(33)$$

and

$$\hat{\theta}_{\xi\, WLS)} = \frac{\sum_{j_\xi=1}^{m_\xi} w_{j_\xi} q_{\xi_{(j_\xi)}} y_{\xi_{(j_\xi)}}^{-\varrho}}{\sum_{j_\xi=1}^{m_\xi} w_{j_\xi} y_{\xi_{(j_\xi)}}^{-2\varrho}}, \xi = 1,2,3,4 \qquad \ldots(34)$$

By using (33) and (34) in (14) get as:

$$\begin{aligned}
\hat{R}_{WLS} = {} & \left[\frac{\hat{\lambda}_{1WLS}}{\hat{\lambda}_{1WLS}+\hat{\theta}_{1WLS}}\right]\left[\frac{\hat{\lambda}_{2WLS}}{\hat{\lambda}_{2WLS}+\hat{\theta}_{2WLS}}\right] + \left[\frac{\hat{\lambda}_{1WLS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1WLS}}{(\hat{\lambda}_{1WLS}+\hat{\theta}_{1WLS})\left(\hat{\lambda}_{1WLS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1WLS}\right)}\right]\left[\frac{\hat{\lambda}_{2WLS}}{\hat{\lambda}_{2WLS}+\hat{\theta}_{2WLS}}\right] \\
& + \left[\frac{\hat{\lambda}_{1WLS}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{1WLS}}{\left(\hat{\lambda}_{1WLS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{1WLS}\right)\left(\hat{\lambda}_{1WLS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{1WLS}\right)}\right]\left[\frac{\hat{\lambda}_{2WLS}}{\hat{\lambda}_{2WLS}+\hat{\theta}_{2WLS}}\right] \\
& + \left[\frac{\hat{\lambda}_{1WLS}}{\hat{\lambda}_{1WLS}+\hat{\theta}_{1WLS}}\right]\left[\frac{\hat{\lambda}_{2WLS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2WLS}}{(\hat{\lambda}_{2WLS}+\hat{\theta}_{2WLS})\left(\hat{\lambda}_{2WLS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2WLS}\right)}\right] \\
& + \left[\frac{\hat{\lambda}_{1WLS}}{\hat{\lambda}_{1WLS}+\hat{\theta}_{1WLS}}\right]\left[\frac{\hat{\lambda}_{2WLS}\left(\frac{k}{m}\right)^{-2\varrho}\hat{\theta}_{2WLS}}{\left(\hat{\lambda}_{2WLS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}\right)+\hat{\theta}_{2WLS}\right)\left(\hat{\lambda}_{2WLS}\left(1+\left(\frac{k}{m}\right)^{-\varrho}+\left(\frac{k}{m}\right)^{-2\varrho}\right)+\hat{\theta}_{2WLS}\right)}\right] \\
& + \left[\frac{\hat{\lambda}_{1WLS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{1WLS}}{(\hat{\lambda}_{1WLS}+\hat{\theta}_{1WLS})\left(\hat{\lambda}_{1WLS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{1WLS}\right)}\right]\left[\frac{\hat{\lambda}_{2WLS}\left(\frac{K}{M}\right)^{-\varrho}\hat{\theta}_{2WLS}}{(\hat{\lambda}_{2WLS}+\hat{\theta}_{2WLS})\left(\hat{\lambda}_{2WLS}\left(1+\left(\frac{K}{M}\right)^{-\varrho}\right)+\hat{\theta}_{2WLS}\right)}\right] \quad ..(35)
\end{aligned}$$

SIMULATION AND RESULTS

The simulations are conducted to compare between the performance methods of estimation, especially for "mean square error" and" mean absolute percentage error," for different parameter values and different sample sizes. Ten experiments were conducted and the Fréchet parameters were estimated to determine which methods are the best for estimating reliability for the model shown in Equation (14). The results were noted for $(n_1, n_2, m_1, m_2) =$ (20,20,20,20) (small) , (50,50,50,50) (moderate) and (90,90,90,90) (large).

The following different values of parameters ($\varrho, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \theta_1, \theta_2, \theta_3, 4$) and factors of attenuation "k" and "m":

Table 1: The parameter values and reliability results

Experiment	ϱ	λ_1	λ_2	θ_1	θ_2	k	m	R
1	1.5	1.5	1.5	1.5	1.5	1.7	0.2	0.4489
2	1.2	1.5	1.5	1.5	1.5	1.7	0.2	0.4527
3	2.5	1.5	1.5	1.5	1.5	1.7	0.2	0.4450

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4	1.5	0.5	0.5	1.5	1.5	1.7	0.2	0.3922
5	1.5	2.5	2.5	1.5	1.5	1.7	0.2	0.5221
6	1.5	1.5	1.5	0.5	0.5	1.7	0.2	0.7850
7	1.5	1.5	1.5	2.5	2.5	1.7	0.2	0.2837
8	1.5	1.5	1.5	1.5	1.5	1.6	0.4	0.4576
9	1.5	1.5	1.5	1.5	1.5	1.4	0.6	0.4722
10	1.5	1.5	1.5	1.5	1.5	1.1	0.9	0.5065

The simulation results of the ten experiments are as follows:

Table 2: Simulation results of the Exp.1

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.5196	0.4752	0.4777
	MSE	0.0338	0.0047	0.0060
	MAPE	0.3412	0.1228	0.1381
(50,50,50,50)	Mean	0.4911	0.4650	0.4709
	MSE	0.0194	0.0019	0.0033
	MAPE	0.2533	0.0784	0.1017
(90,90,90,90)	Mean	0.4798	0.4595	0.4667
	MSE	0.0140	0.0010	0.0022
	MAPE	0.2113	0.0564	0.0830

Table 3: Simulation results of the Exp.2

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.7104	0.4790	0.4815
	MSE	0.1359	0.0050	0.0064
	MAPE	0.6983	0.1260	0.1413
(50,50,50,50)	Mean	0.6337	0.4692	0.4752
	MSE	0.0716	0.0020	0.0033
	MAPE	0.5146	0.0786	0.1018
(90,90,90,90)	Mean	0.5960	0.4649	0.4726
	MSE	0.0516	0.0011	0.0025
	MAPE	0.4382	0.0590	0.0866

Table 4: Simulation results of the Exp.3

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.4572	0.4708	0.4730
	MSE	0.0131	0.0045	0.0057
	MAPE	0.2063	0.1221	0.1364
(50,50,50,50)	Mean	0.4510	0.4609	0.4672
	MSE	0.0061	0.0019	0.0032
	MAPE	0.1388	0.0781	0.1018
(90,90,90,90)	Mean	0.4465	0.4564	0.4640
	MSE	0.0038	0.0010	0.0022
	MAPE	0.1073	0.0574	0.0840

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Table 5: Simulation results of the Exp.4

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.5428	0.4459	0.4509
	MSE	0.0693	0.0090	0.0115
	MAPE	0.5390	0.1886	0.2106
(50,50,50,50)	Mean	0.4814	0.4265	0.4372
	MSE	0.0333	0.0034	0.0060
	MAPE	0.3781	0.1160	0.1524
(90,90,90,90)	Mean	0.4616	0.4175	0.4311
	MSE	0.0223	0.0017	0.0042
	MAPE	0.3091	0.0844	0.1268

Table 6: Simulation results of the Exp.5

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.5515	0.5357	0.5368
	MSE	0.0265	0.0040	0.0050
	MAPE	0.2526	0.0967	0.1087
(50,50,50,50)	Mean	0.5388	0.5312	0.5346
	MSE	0.0173	0.0016	0.0027
	MAPE	0.1966	0.0617	0.0791
(90,90,90,90)	Mean	0.5340	0.5292	0.5335
	MSE	0.0123	0.0009	0.0019
	MAPE	0.1633	0.0467	0.0675

Table 7: Simulation results of the Exp.6

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.8586	0.8155	0.8191
	MSE	0.0331	0.0029	0.0038
	MAPE	0.1856	0.0556	0.0630
(50,50,50,50)	Mean	0.8248	0.8026	0.8096
	MSE	0.0177	0.0010	0.0019
	MAPE	0.1288	0.0336	0.0446
(90,90,90,90)	Mean	0.8150	0.7975	0.8062
	MSE	0.0125	0.0006	0.0013
	MAPE	0.1035	0.0243	0.0369

Table 8: Simulation results of the Exp.7

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.3465	0.3066	0.3089
	MSE	0.0250	0.0039	0.0050
	MAPE	0.4540	0.1734	0.1944
(50,50,50,50)	Mean	0.3239	0.2962	0.3016
	MSE	0.0145	0.0015	0.0026

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	MAPE	0.3416	0.1079	0.1416
(90,90,90,90)	Mean	0.3105	0.2923	0.2985
	MSE	0.0099	0.0008	0.0018
	MAPE	0.2792	0.0813	0.1183

Table 9: Simulation results of the Exp.8

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.5324	0.4859	0.4887
	MSE	0.0368	0.0054	0.0068
	MAPE	0.3483	0.1290	0.1449
(50,50,50,50)	Mean	0.5020	0.4751	0.4816
	MSE	0.0214	0.0021	0.0037
	MAPE	0.2622	0.0810	0.1051
(90,90,90,90)	Mean	0.4898	0.4694	0.4773
	MSE	0.0153	0.0012	0.0026
	MAPE	0.2169	0.0593	0.0874

Table 10: Simulation results of the Exp.9

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.5543	0.5035	0.5065
	MSE	0.0425	0.0064	0.0081
	MAPE	0.3622	0.1355	0.1520
(50,50,50,50)	Mean	0.5219	0.4910	0.4977
	MSE	0.0244	0.0025	0.0043
	MAPE	0.2709	0.0844	0.1097
(90,90,90,90)	Mean	0.5086	0.4851	0.4934
	MSE	0.0172	0.0013	0.0029
	MAPE	0.2244	0.0620	0.0904

Table 11: Simulation results of the Exp.10

S.S.	criteria	MoE	LSE	WLSE
(20,20,20,20)	Mean	0.6056	0.5415	0.5453
	MSE	0.0567	0.0084	0.0107
	MAPE	0.3894	0.1446	0.1628
(50,50,50,50)	Mean	0.5639	0.5286	0.5366
	MSE	0.0319	0.0034	0.0059
	MAPE	0.2907	0.0909	0.1186
(90,90,90,90)	Mean	0.5507	0.5221	0.5317
	MSE	0.0225	0.0018	0.0040
	MAPE	0.2406	0.0670	0.0980

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CONCLUSIONS

Table 1 shows the reliability values of the ten experiments that we have carried out, while Table 2 through Table 11 show the simulation values that we have carried out. Now, we clarify the conclusions we have reached:

A - From Table 1, the following was concluded:

1- When comparing Experiment 1 with Experiment 2 or 3, it turns out that the value of reliability model increases with the decrease in the value of parameter ϱ, and this seems obvious if we look at Tables 2, 3 and 4.
2- When comparing Experiment 1 with Experiments 4 and 5, it turns out that the reliability of the model increases in value with an increase in the values of parameters λ_1 and λ_2, and this seems obvious if we look at Tables 2, 5 and 6.
3- When comparing Experiment 1 with Experiments 6 and 7, it turns out that an increase in the values of parameters θ_1 and θ_1 causes a decrease in the reliability value of the model, and this seems obvious if we look at Tables 2, 7 and 8.
4- When comparing Experiment 1 with Experiments 8, 9 and 10, it turns out that the reliability of the model increases with a decrease in the value of the magnitude $\frac{k}{m}$, and this seems obvious if we look at Tables 2, 9, 10 and 11.

B - We conclude from the numbered tables (2, 3, 4, 5, 6, 7, 8, 9, 10 and 11) that the best estimator for the reliability of the model is the least squares estimator.

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