African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 7, Issue 2, 2024 (pp. 26-34)
www.abjournals.org

ENTROPY GENERATION FOR NON-NEWTONIAN FLUID FLOW WITH CONSTANT VISCOSITY

Obi Boniface Inalu

Department of Mathematics,

Imo State University, Owerri, Nigeria

Email: appliedbon@yaho.com

Cite this article:

Obi Boniface Inalu (2024), Entropy Generation For Non-Newtonian Fluid With Constant Viscosity. African Journal of Mathematics and Statistics Studies 7(2), 26-34. DOI: 10.52589/AJMSS-3FOWTMHB

Manuscript History

Received: 28 Jan 2024

Accepted: 8 Mar 2024

Published: 3 Apr 2024

ABSTRACT: *In this research, entropy generation for non-Newtonian fluid flow with constant viscosity is considered. The governing nonlinear equations of motion are solved analytically with regular perturbation techniques. Third grade fluid is employed to account for the non-Newtonian influence. The influence of some physical parameters involved in the analysis is studied. Results show that the parameter has the tendency of increasing the velocity of the fluid flow as well as the temperature of the cylindrical pipe. It is observed that as the Brinkman parameter* B_r *increases, the cylindrical wall temperature is enhanced. Entropy generation number for both heat transfer and fluid friction for various values of* β *and* B_r *is examined. Results indicate that increase in these parameters increases the temperature of the cylinder, thereby increasing the entropy generation number.*

KEYWORDS: Entropy, Non-Newtonian, Fluid, Viscosity.

INTRODUCTION

The flow in pipes finds wide application in many industries such as the pulp industries and the petroleum industries. The non-Newtonian fluid of the Rivlin-Ericksen [11] has been applied in many flow problems such as the third grade fluid whose theory was developed by Fosdic and Rajagpal [3].

Many leading researchers in this field have carried out some considerable investigation into the non-Newtonian fluid flow. Amongst the early researchers is [9]. He dealt with the stability characteristics of third grade fluid. [11] investigated the flow between parallel plates. [13] applied perturbation technique for the analysis of third grade fluid. The flow was temperature-dependent and Reynold's model viscosity was applied to account for it. They found that the effects of the non-Newtonian parameter and viscosity index are more pronounced in the region of the plate surface. [1] examined the unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. They found that the analysis was time-dependent.

Entropy generation analysis gives the information for the quantification of thermodynamic irreversibility [12]. It was found that entropy minimization lowers friction and heat transfer losses in the system. They discovered that a decrease in the non-Newtonian parameter increases both the velocity and temperature profiles within the annulus. It was found that the entropy generation number attains high values in the vicinity of the wall of the inner cylinder. Further investigation shows that increase in the Brinkman number enhances the entropy generation number in the inner wall of the annular pipe.

Entropy generation and minimization in thermal systems was treated by [2] and found that entropy minimization can be applied in thermal system designing. [6] analyzed the flow of an incompressible MHD third grade fluid in an inclined rotating cylindrical pipe with isothermal wall and Joule heating. [8] examined entropy generation for pipe flow of a third grade fluid. They introduced Vogel's model viscosity to account for the temperature-dependent viscosity. It was discovered that increase in viscosity parameter A, reduces entropy generation number. They discovered that a decrease in the non-Newtonian parameter increases both the velocity and temperature profiles within the annulus. It was found that the entropy generation number attains high values in the vicinity of the wall of the inner cylinder. Further investigation shows that increase in the Brinkman number enhances the entropy generation number in the inner wall of the annular pipe.

[7] examined the flow of third grade fluid and Vogel's model viscosity in cylindrical pipe. He applied the regular perturbation technique for the solution of the nonlinear ordinary differential equations of motion. In the analysis, Vogel's model viscosity was inserted to account for the temperature-dependent viscosity. It was observed that as the viscosity index increases, the velocity of the fluid reduces indicating that the shear strain which reduces the velocity , increases the viscosity. [4] investigated the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in pipe. They numerically analyzed the velocity and temperature distribution in the pipe.

In this present research, entropy generation for non-Newtonian fluid flow with constant viscosity is considered. The governing nonlinear equations of motion are solved analytically with regular perturbation techniques. Third grade fluid is employed to account for the non-Newtonian influence.

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

Mathematical Formulation

Considering the non-Newtonian fluid flow of the differential type, the non-dimensional form of equations f motion between parallel plates was derived by [12] as:

$$\frac{d}{dr}\left(\mu\frac{du}{dr}\right)+6\Lambda\left(\frac{du}{dr}\right)^2\frac{d^2u}{dr^2}=C \tag{1}$$

$$\frac{d^2\theta}{dr^2}+\Gamma\left(\mu\frac{du}{dr}\right)^2+2\Lambda\Gamma\left(\frac{du}{dr}\right)^4=0 \tag{2}$$

$$u(0)=\theta(0)=0,u(0)=0,\theta(1)=1 \tag{3}$$

where u is the fluid velocity, θ is the temperature, μ is the fluid viscosity. The terms are related to the non-dimensional parameters

$$r=\frac{\bar{r}}{a},\theta=\frac{\bar{\theta}-\theta_1}{\theta_2-\theta_1},u=\frac{\bar{u}}{u_0},\mu=\frac{\bar{\mu}}{\mu_0}, \tag{4}$$

where a is the distance between parallel plates, θ_1 and θ_2 are the plate temperatures in upper and lower plates respectively, u_0 is the reference velocity, μ_0 is the reference viscosity. The dimensionless parameters are

$$C=\frac{C_1a^2}{\mu_0u_0},C_1=\frac{dp}{dx},\Gamma=\frac{\mu_0u_0^2}{k(\theta_2-\theta_1)},\Lambda=\frac{\beta u_0^2}{\mu_0a^2} \tag{5}$$

where

$\Gamma=B_r$ is the Brinkman number

$\Lambda=\beta$ is the material constant related to the third grade parameter

$C=1$ is the constant pressure drop in the axial direction.

Equations (1-3) with eqn (4), transforms to

$$\frac{d}{dr}\left(\frac{du}{dr}\right)+6\beta\left(\frac{du}{dr}\right)^2\frac{d^2u}{dr^2}=-1 \tag{6}$$

$$\frac{d^2\theta}{dr^2}+B_r\left(\frac{du}{dr}\right)^2+2\beta B_r\left(\frac{du}{dr}\right)^4=0 \tag{7}$$

$$u(0)=u(1)=0,\theta(0)=0,\theta(1)=1 \tag{8}$$

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 7, Issue 2, 2024 (pp. 26-34)
www.abjournals.org

METHOD OF SOLUTION

The approximate analytical solutions for the velocity and temperature profiles can be of the form

$$u(r) = u_0(r) + \beta u_1(r) + O(\beta^2), \theta(r) = \theta_0(r) + \beta\theta_1(r) + O(\beta^2) \tag{9}$$

Substituting eqn (9) into eqns (7) and (8),and separating each order of β, yields

$$\beta^0 : \frac{d}{dr}\left(\frac{du_0}{dr}\right) = -1 \tag{10}$$

$$\beta : \frac{d}{dr}\left(\frac{du_1}{dr}\right) + 6\left(\frac{du_0}{dr}\right)^4 \frac{d^2u_0}{dr^2} = 0 \tag{11}$$

$$\beta^0 : \frac{d^2\theta_0}{dr^2} + B_r\left(\frac{du_0}{dr}\right)^2 = 0 \tag{12}$$

$$\beta : \frac{d^2\theta_1}{dr^2} + 2B_r\frac{du_0}{dr}\frac{du_1}{dr} + 2B_r\left(\frac{du_0}{dr}\right)^4 = 0 \tag{13}$$

Solving eqns (10-13) with the condition (8), yields

$$u(r) = \frac{1}{2}r - \frac{1}{2}r^2 + \beta\left(\frac{1}{2}r^4 - r^3 + \frac{3}{4}r^2 - \frac{1}{4}r\right) \tag{14}$$

$$\theta(r) = -B_r\left(\frac{1}{8}r^2 - \frac{1}{6}r^3 + \frac{1}{12}r^4\right) + r\left(1 + \frac{1}{24}B_r\right) + \beta\left(-\frac{1}{8}r^2 + \frac{1}{3}r^3 - \frac{3}{4}r^4 + \frac{2}{5}r^5 - \frac{2}{15}r^6\right)$$
$$-B_r\left(\frac{1}{16}r^2 - \frac{1}{6}r^3 + \frac{1}{4}r^4 - \frac{1}{5}r^5\right) + r\left(1 - \frac{79}{240}B_r\right) \tag{15}$$

Entropy Generation

The dimensionless viscous dissipation term (ϕ) is obtained from the equation of motion as

$$\phi = B_r\left(\frac{du}{dr}\right)^2 + 2\beta B_r\left(\frac{du}{dr}\right)^4$$

or

$$\phi = B_r\left(\frac{du}{dr}\right)^2\left[1 + 2\beta\left(\frac{du}{dr}\right)^2\right] \tag{16}$$

The dimensionless volumetric entropy generation is obtained as in Bejan (1995)

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

$$S_{\text{gen}}^{III} = \frac{k}{\theta_0^2}\left(\frac{d\theta}{dr}\right)^2 + \frac{\phi}{\theta_0} \qquad (17)$$

where θ_0 is the reference temperature.

The first term in eqn (16) is the volumetric entropy generation due to heat transfer and the second term is the entropy generation due to fluid friction.

The solution for velocity and temperature profiles obtained using perturbation techniques have been presented for the above equations (6-8) in equations (14) and (15).

Substituting eqn (16) in eqn (17), we obtain

$$NS = \left(\frac{d\theta}{dr}\right)^2 + \theta_0 B\left(\frac{du}{dr}\right)^2\left[1 + 2\beta\left(\frac{du}{dr}\right)^2\right] \qquad (18)$$

Where NS is the entropy generation number defined as the ratio of the dimensionless volumetric entropy generation to a reference volumetric entropy generation given as

$$\text{NS}=\frac{S_{\text{gen}}^{III}}{S_G^{III}}, S_G^{III} = \frac{k\left(\theta_m - \theta w\right)}{\theta_0^2}, \theta_0 = \frac{\bar{\theta}_0}{\theta_m - \theta_w} \qquad (19)$$

We assigned the first term in eqn (18) as NS_a and the second term as NS_b . NS is the irreversibility due to heat transfer and entropy generation due to viscous dissipation and friction.

The ratio

$$\upsilon = \frac{NS_a}{NS_b} \qquad (20)$$

Is the irreversibility distribution that expresses the heat transfer dominants when

$0 < \upsilon < 1$ and the fluid friction dominates when $\upsilon > 1$. The Bejan number (Be) is defined as

$$Be = \frac{NS_a}{NS} = \frac{1}{1+\upsilon}, 0 \le Be \le 1 \qquad (21)$$

Consequently, the values of u and θ can be substituted from equations (14) and (15) for final evaluation.

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

ISSN: 2689-5323
Volume 7, Issue 2, 2024 (pp. 26-34)

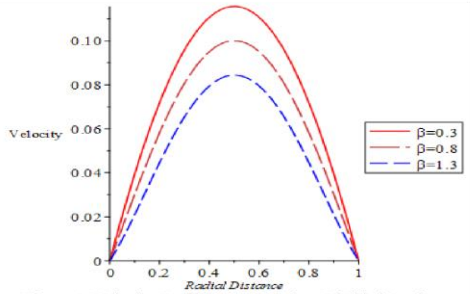

Figure 1: Velocity Profiles For Variation Of Third Grade Parameter (β)

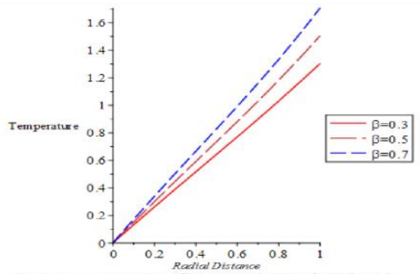

Figure 2: Temperature Profiles For Variation Of Third Grade Parameter (β)

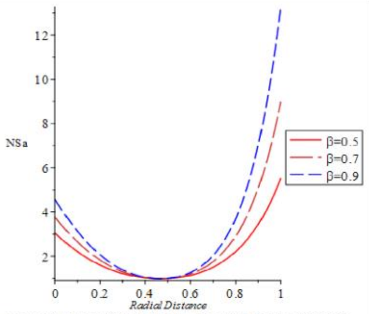

Figure 3: Entropy Generation Number Due To Heat Transfer With Different Values Of β

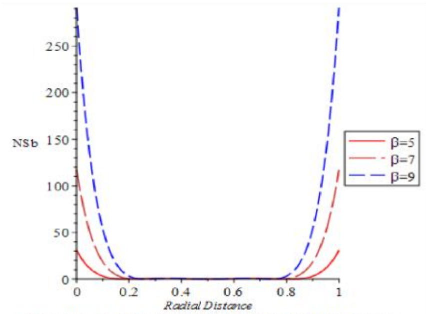

Figure 4: Entropy Generation Number Due To Fluid Friction With Different Values Of β

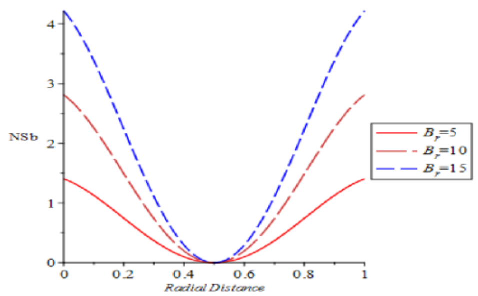

Figure 5: Entropy Generation Number Due To Fluid Friction With Different Values Of B_r

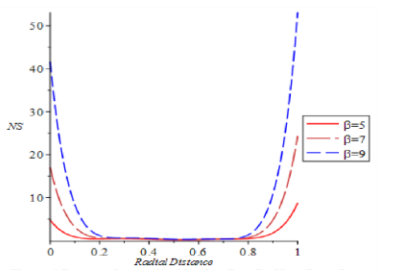

Figure 6: Entropy Generation Number Due To Heat Transfer and Fluid Friction With Different Values Of β

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

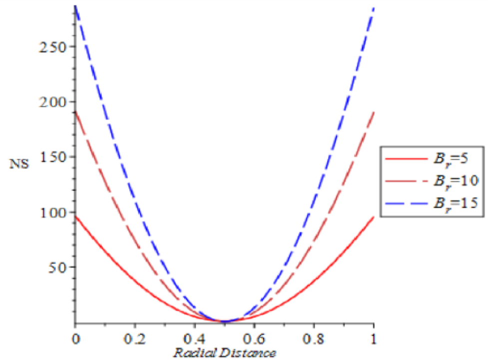

Figure 7:Entropy Generation Number Due To Heat Transfer and Fluid Friction With Different Values Of B_r

RESULTS AND DISCUSSION

In order to study the influence of some physical parameters involved in the analysis, graphs are presented in figures (1-7). Figures 1 and 2 show the velocity and temperature profiles for various values of the third grade parameter β. Results show that the parameter has the tendency of increasing the velocity of the fluid flow as well as the temperature of the cylindrical pipe. Figure 3 is the entropy generation number due to heat transfer. Results indicate that as β increases, the temperature increases which also increases the entropy generation number. In this case, it reduces the diffusion energy flow towards the cylindrical walls.

Figure 4 shows the entropy generation number due to fluid friction with variation in β. Result reveals that increase in β increases the wall temperature and in turn increases the entropy generation number. Figure 5 presents entropy generation number due to fluid friction for different values of the Brinkman parameter B_r. It is observed that as the Brinkman parameter B_r increases, the cylindrical wall temperature is enhanced. Figure 6 shows the entropy generation number for both heat transfer and fluid friction for various values of β and B_r. Results indicate that increase in these parameters increases the temperature of the cylinder, thereby increasing the entropy generation number.

CONCLUSIONS

In this research, entropy generation for non-Newtonian fluid flow with constant viscosity is considered. The governing nonlinear equations of motion are solved analytically with regular perturbation techniques. Third grade fluid is employed to account for the non-Newtonian influence. The influence of some physical parameters involved in the analysis is studied. Results show that the parameter has the tendency of increasing the velocity of the fluid flow as well as the temperature of the cylindrical pipe. It is observed that as the Brinkman parameter

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

B_r increases, the cylindrical wall temperature is enhanced. Entropy generation number for both heat transfer and fluid friction for various values of β and B_r is examined. Results indicate that increase in these parameters increases the temperature of the cylinder, thereby increasing the entropy generation number.

DECLARATIONS

1. Funding: Not applicable

2. Informed Consent Statement: Not applicable

3. Data Availability: Not applicable

4. Conflict of Interest Statement: No conflict of interest

REFERENCES

Aiyesimi, Y.M., Okedayo, G.T., and Lawal, O.W. Analysis of unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. International Journal of Applied and Computational Mathematics 2, 58-69(2014)

Bejan, A. Entropy generation minimization. CRC press: New York, (1995)

Fosdick R.L. and Rajagopal, K.R.: Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 339, 351-377, (1980).

Massoudi, M. and Christie, I.: Effects of variable viscosity and viscous dissipation on the flow of a third –grade fluid in a pipe. Int. J. of Nonlinear Mech.,30(5): 687-699,(1995).

Obi B.I.,Okedayo, G.T., Jiya, M. And Aiyesimi, Y.M.: Analysis of Flow of An Incompressible MHD third Grade Fluid In An Inclined Rotating Cylindrical Pipe With Isothermal Wall And Joule Heating. International Journal For Research In Mathematics And Statistics. Vol 7 Issue 6,(2021)

Obi B.I.: Perturbation Analysis Of Magnetohydrodynamic Flow of Third Grade Fluid In An Inclined Cylindrical Pipe: Journal Of Mathematical Science And Computational Mathematics (JMSCM) Vol 3, No. 3(2022)

Obi B.I.: Flow of third grade fluid and Vogel's model viscosity in cylindrical pipe. International Journal for Research in Applied Sciences and Engineering Technology (IJRASET) Vol.11, Issue 8 (2023).

Pakdemirli, M.and Yilbas, B.S.: Entropy generation for pipe flow of third grade fluid with Vogel model viscosity. International Journal of Nonlinear Mech. 41, 432-437(2006)

Rajagopal K.R.:On the stability of third grade fluid. Archive 32, 867-875(1980)

[10] Rivlin, R.S., Ericksen, J.I.: Stress deformation for isotropic materials. J. Ration. Mech. Anal. 4,323-329(1955).

Szeri, A.Z. and Rajagopal, K.R. Flow of a non-Newtonian fluid between heated parallel plates. Int. J. Nonlinear Mech.20(1985)91.

Yilbas, B.S., Yurusoy,M., and Pakdemirli, M, Entropy analysis for non-Newtonian fluid in annular pipe: Constant viscosity case. Entropy 6, 304-315(2004)

Article DOI: 10.52589/AJMSS-3FOWTMHB
DOI URL: https://doi.org/10.52589/AJMSS-3FOWTMHB

Yurusoy,M., Bayrakeeken,H., Kapuen, M. and Aksoy, F.: Entropy analysis for third grade fluid flow with Vogel model viscosity in an annular pipe. Int. J. of Linear Mech.43, 588-599(2008)