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PORTFOLIO MANAGEMENT STRATEGIES USING KNAPSACK PROGRAMMING

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ABSTRACT: *One of the major tasks in portfolio management is to determine the number of stocks with relatively high net value on the stock market. This work presents a knapsack based portfolio selection model that considers the expected returns, prices and budget. It represents a typical resource allocation model in which limited resource is apportioned among a finite number of stocks. The objective is to maximize an associated return function. The work is implemented for some numerical data to illustrate the application of the model and demonstrate the effectiveness of the designed algorithm. Numerical results have shown that the optimization model yields promising results.*

KEYWORDS: Portfolio management, Knapsack programming, Integer linear programming, Backward recursive equation.

INTRODUCTION

The concept of portfolio refers to the combination of stocks, bonds and cash which can also be referred to as financial assets. The major concern of investors in the portfolio selection model is to allocate capital among different assets in a way that the risk and return are optimized simultaneously in the portfolio. A mean-variance (MV) model for the selection of an optimal asset portfolio was introduced by Markowitz [1, 2]. In reality, some of the assumptions in the Markowitz model are rarely feasible. The model did not consider some real financial market constraints like transaction costs [3, 4] cardinality constraint [5, 6], and multi-period cardinality constraint mean-variance (CCMV) [7, 8]. Since then, there have been efforts by many researchers to provide methods for stocks analysis in financial markets [9]. Complexity and uncertainty in decision-making processes is another problem in the financial market. Based on the uncertain conditions, many approaches have been developed to look into the real condition of financial markets in portfolio selection models. They include: approach in a robust-based [10]; approach in a scenario-based [11, 12] and fuzzy methods [13, 14]. Zadeh [15] introduced fuzzy sets while Turksen [16] presented interval-valued fuzzy sets. Liu [17] proposed two approaches on linear programming for obtaining the upper and lower bounds of the interval number of returns in the Markowitz model. In extending the work, Liu et al. [18, 19] proposed a multi-period MV model with risk, turnover rates of risky assets and interval returns. Mercangoz [20] moved into Markowitz mean-variance model which is a bedrock of modern portfolio theory. The work focused on Particle Swarm Optimization (PSO). Bitar et al. [21] combined covariance matrix estimators with different portfolio allocation techniques to meet two types of client's requirements. Zhou and Xu [22] proposed some qualitative portfolio models under the fuzzy environment when all or part of the qualitative data is not reachable. Furthermore, variance which is known as a symmetric risk measurement factor is considered in the Markowitz model as a risk measurement factor. This has been criticized by many researchers. A number of studies have used other forms of risk measurement in the portfolio optimization models. These studies used interval uncertainty to examine the model. Examples include mean-Value-at-Risk (VAR) portfolio optimization model [23] and Mean-Variance Skewness (MVS) portfolio selection model [24]. A portfolio composition using the knapsack problem was introduced by Bevilaqua da Silva, & De Mattos [25]. They compared its performance to an investment website's share portfolio.

This work presents a knapsack based portfolio selection model that considers the expected returns, prices and budget. It represents a typical resource allocation model in which limited resource is apportioned among a finite number of stocks. The objective is to maximize an associated return function. The work is implemented for some numerical data to illustrate the application of the model and demonstrate the effectiveness of the designed algorithm.

METHODOLOGY

Integer Linear Programming

Integer Linear Programs (ILPs) are linear programs in which some or all the variables are restricted to integer (or discrete) values. Its algorithms are based on exploiting the tremendous computational success of Linear Program (LP). The strategy of these algorithms involves three steps.

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Step I: Relax the solution space of the ILP by deleting the integer restriction on all integer variables and replacing any binary variable, say y with the continuous range $0 \leq y \leq 1$. The result of the relaxation is a regular LP.

Step II: Solve the LP, and identify its continuous optimum.

Step III: Starting from the continuous optimum point add special constraints that iteratively modify the LP solution space in a manner that will eventually render an optimum extreme point satisfying the integer requirements.

Backward Recursive Method

The basic model is based on the knapsack programming, which is as follows. The backward recursive equation is developed for the general problem of an $n - stock, B$ revenue portfolio. This is represented by the following Integer Linear Programming Problem (ILPP).

$$Maximize\ P = r_1m_1 + r_2m_2 + \cdots + r_nm_n$$

$$Subject\ to: \quad b_1m_1 + b_2m_2 + \cdots + b_nm_n \leq B \qquad (1)$$

$$m_1, m_2, \cdots, m_n \geq 0\ and\ integer,$$

where P is the total profit, r_i is the return of stock i, m_i is the desired number of stock i which can be chosen in the portfolio, b_i is the price of stock i and B is the total available budget.

Stage i is represented by item i where $i = 1,2,3, \cdots, n$. The alternatives at stage i are represented by m_i. The associated return is r_im_i. Let $\left[\frac{B}{b_i}\right]$ be the largest integer less than or equal to $\frac{B}{b_i}$. hence, $m_i = 0,1,2, \cdots, \left[\frac{B}{b_i}\right]$. The state at stage i is represented by x_i, which is the total revenue assigned to items $i, i+1, i+2, \cdots, n$. The above definition reflects the fact that the revenue constraint is the only restriction that links all n stages together. Let $f_i(x_i)$ be the maximum return for stages $i, i+1$ and n given state x_i. The recursive equation can then be determined by a two-step procedure. Firstly, express $f_i(x_i)$ as a function of $f_i(x_{i+1})$ as follows:

$$f_i(x_i) = \{r_im_i + f_{i+1}(x_{i+1})\}, i = 1,2, \cdots, n \qquad (2)$$

$$f_{n+1}(x_{n+1}) \equiv 0. \qquad (3)$$

Secondly, express x_{i+1} as a function of x_i to ensure that the left-hand side, $f_i(x_i)$ is a function of x_i only. By definition, $x_i - x_{i+1} = b_im_i$ represents the revenue used at stage, i. Thus,

$x_{i+1} = x_i - b_im_i$, and the proper recursive equation is given as

$$f_i(x_i) = \{r_im_i + f_{i+1}(x_i - b_im_i)\}, i = 1,2, \cdots, n . \qquad (4)$$

The Model

An investor with a maximum budget of B wants to build his portfolio with one or more of n stocks. Using the backward recursive equation, we start from stage n and move backward till we get to stage 1.

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Stage n: The exact revenue to be allocated to stage n is not known in advance, but must assume one of the values $0,1,2,3,\cdots,B$. The states $x_n = 0 \; and \; x_n = B$, respectively represent the extreme cases of not including stock n at all in the portfolio and allocating the entire budget to it. The remaining values of $x_n = 1,2,3,\cdots,n$ imply a partial allocation of the portfolio capacity to stock n. In effect, the given range of values for x_n covers all possible allocations of the portfolio capacity to stock n.

The maximum number of units of stock n that can be allocated is $\frac{B}{b_n}$, which means that the possible values of m_n are $0,1,2,3,\cdots,n$. An alternative m_n is feasible only if $b_n m_n \leq x_n$. Thus, all the infeasible alternatives (those for which $b_n m_n > x_n$) are excluded. The following equation is the basis for comparing the alternatives of stage n:

$$f_n(x_n) = \{r_n m_n\}, \{m_n\} = \left[\frac{B}{b_n}\right]. \tag{5}$$

The following table compares the feasible alternatives for each value of x_n.

Table I: Comparing the alternatives of stage n using $f_n(x_n) = \{r_n m_n\}, \{m_n\} = \left[\frac{B}{b_n}\right]$.

$r_n m_n$						Optimum Solution
x_n	$m_n = 0$	$m_n = 1$	$m_n = 2$	$\cdots$	$m_n = \left[\frac{B}{b_n}\right]$	$f_n(x_n)$
0	0	$r_n : b_n \leq 0$	$2r_n : 2b_n \leq 0$	$\cdots$	$\left[\frac{B}{b_n}\right] r_n : \left[\frac{B}{b_n}\right] b_n \leq 0$	$\{r_n m_n\}$
1	0	$r_n : b_n \leq 1$	$2r_n : 2b_n \leq 1$	$\cdots$	$\left[\frac{B}{b_n}\right] r_n : \left[\frac{B}{b_n}\right] b_n \leq 1$	
2	0	$r_n : b_n \leq 2$	$2r_n : 2b_n \leq 2$	$\cdots$	$\left[\frac{B}{b_n}\right] r_n : \left[\frac{B}{b_n}\right] b_n \leq 2$	
3	0	$r_n : b_n \leq 3$	$2r_n : 2b_n \leq 3$	$\cdots$	$\left[\frac{B}{b_n}\right] r_n : \left[\frac{B}{b_n}\right] b_n \leq 3$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
B	0	$r_n : b_n \leq B$	$2r_n : 2b_n \leq B$	$\cdots$	$\left[\frac{B}{b_n}\right] r_n : \left[\frac{B}{b_n}\right] b_n \leq B$	

Stage $n-1$: Similarly, the exact revenue to be allocated to stage $n-1$ is not known in advance, but must assume one of the values $0,1,2,3,\cdots,B$. The states $x_{n-1} = 0 \; and \; x_{n-1} = B$, respectively represent the extreme cases of not including stock $n-1$ at all in the portfolio and allocating the entire budget to it. The remaining values of $x_{n-1} = 1,2,3,\cdots,n-1$ imply a partial allocation of the portfolio capacity to stock $n-1$. Therefore, the given range of values for x_{n-1} covers all possible allocations of the portfolio capacity to stock $n-1$.

The maximum number of units of stock $n-1$ that can be allocated is $\frac{B}{b_{n-1}}$, which means that the possible values of m_{n-1} are $0,1,2,3,\cdots,n-1$. An alternative m_{n-1} is feasible only if

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$b_{n-1}m_{n-1} \le x_{n-1}$. Thus, all the infeasible alternatives (those for which $b_{n-1}m_{n-1} > x_{n-1}$) are excluded. The following equation is the basis for comparing the alternatives of stage $n - 1$:

$$f_{n-1}(x_{n-1}) = \{r_{n-1}m_{n-1} + f_n(x_{n-1} - b_{n-1}m_{n-1})\},\ \{m_{n-1}\} = \left[\frac{B}{b_{n-1}}\right]. \qquad (6)$$

The table below compares the feasible alternatives for each value of x_{n-1}.

Table II: Comparing the alternatives of stage $n - 1$ using

$$f_{n-1}(x_{n-1}) = \{r_{n-1}m_{n-1} + f_n(x_{n-1} - b_{n-1}m_{n-1})\},\ \{m_{n-1}\} = \left[\frac{B}{b_{n-1}}\right].$$

$r_{n-1}m_{n-1} + f_n(x_{n-1} - b_{n-1}m_{n-1})$					Optimum Solution
x_{n-1}	$m_{n-1} = 0$	$m_{n-1} = 1$	$\cdots$	$m_{n-1} = \left[\frac{B}{b_{n-1}}\right]$	$f_{n-1}(x_{n-1})$
0	$\{r_n m_n\}$	$r_{n-1} + \{r_n m_n\}$ $: b_{n-1} \le 0$	$\cdots$	$r_{n-1}\left[\frac{B}{b_{n-1}}\right] + \{r_n m_n\}$ $: b_{n-1}\left[\frac{B}{b_{n-1}}\right] \le 0$	$\{r_{n-1}m_{n-1} + f_n(x_{n-1} - b_{n-1}m_{n-1})\}, \{m_{n-1}\} = \left[\frac{B}{b_{n-1}}\right]$
1		$r_{n-1} + \{r_n m_n\}$ $: b_{n-1} \le 1$	$\cdots$	$r_{n-1}\left[\frac{B}{b_{n-1}}\right] + \{r_n m_n\}$ $: b_{n-1}\left[\frac{B}{b_{n-1}}\right] \le 1$	
2		$r_{n-1} + \{r_n m_n\}$ $: b_{n-1} \le 2$	$\cdots$	$r_{n-1}\left[\frac{B}{b_{n-1}}\right] + \{r_n m_n\}$ $: b_{n-1}\left[\frac{B}{b_{n-1}}\right] \le 2$	
3		$r_{n-1} + \{r_n m_n\}$ $: b_{n-1} \le 3$	$\cdots$	$r_{n-1}\left[\frac{B}{b_{n-1}}\right] + \{r_n m_n\}$ $: b_{n-1}\left[\frac{B}{b_{n-1}}\right] \le 3$	
$\vdots$		$\vdots$	$\vdots$	$\vdots$	
B		$r_{n-1} + \{r_n m_n\}$ $: b_{n-1} \le B$	$\cdots$	$r_{n-1}\left[\frac{B}{b_{n-1}}\right] + \{r_n m_n\}$ $: b_{n-1}\left[\frac{B}{b_{n-1}}\right] \le B$	

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The process of the ongoing allocation continues in the same manner to stage 1, which is the last stage.

Stage 1: Lastly, the exact revenue to be allocated to stage 1 is not known in advance, but must assume one of the values $0,1,2,3,\cdots,B$. The states $x_1 = 0$ and $x_1 = B$, respectively represent the extreme cases of not including stock 1 at all in the portfolio and allocating the entire budget to it. The remaining values imply a partial allocation of the portfolio capacity to stock 1. Therefore, the given range of values for x_1 covers all possible allocations of the portfolio capacity to stock 1.

The maximum number of units of stock 1 that can be allocated is $\frac{B}{b_1}$. An alternative m_1 is feasible only if $b_1 m_1 \leq x_1$. Thus, all the infeasible alternatives (those for which $b_1 m_1 > x_1$) are excluded. The following equation is the basis for comparing the alternatives of stage 1:

$$f_1(x_1) = \{r_1 m_1 + f_2(x_1 - b_1 m_1)\}, \{m_1\} = \left[\frac{B}{b_1}\right]. \qquad (7)$$

The table below compares the feasible alternatives for each value of x_1.

Table III: Comparing the alternatives of stage 1 using

$$f_1(x_1) = \{r_1 m_1 + f_2(x_1 - b_1 m_1)\}, \{m_1\} = \left[\frac{B}{b_1}\right].$$

$r_1 m_1 + f_2(x_1 - b_1 m_1)$					Optimum Solution
x_1	$m_1 = 0$	$m_1 = 1$	$\cdots$	$m_1 = \left[\frac{B}{b_1}\right]$	$f_1(x_1)$
0	$\{r_2 m_2 + f_3(x_2 - b_2 m_2)\}$	$r_1 + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \leq 0$	$\cdots$	$r_1 \left[\frac{B}{b_1}\right] + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \left[\frac{B}{b_1}\right] \leq 0$	$\{r_1 m_1 + f_2(x_1 - b_1 m_1)\}$: $\{m_1\} = \left[\frac{B}{b_1}\right]$
1		$r_1 + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \leq 1$	$\cdots$	$r_1 \left[\frac{B}{b_1}\right] + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \left[\frac{B}{b_1}\right] \leq 1$	
2		$r_1 + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \leq 2$	$\cdots$	$r_1 \left[\frac{B}{b_1}\right] + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\}$	

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				$: b_1 \left[\frac{B}{b_1}\right] \le 2$	
3		$r_1 + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \le 3$	$\cdots$	$r_1 \left[\frac{B}{b_1}\right] + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \left[\frac{B}{b_1}\right] \le 3$	
$\vdots$		$\vdots$	$\vdots$	$\vdots$	
B		$r_1 + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \le B$	$\cdots$	$r_1 \left[\frac{B}{b_1}\right] + \{r_2 m_2 + f_3(x_2 - b_2 m_2)\} : b_1 \left[\frac{B}{b_1}\right] \le B$	

The optimum solution is now determined in the following manner. Given the maximum budget, B, from stage 1, choose the optimum alternative, that is, $f_1(x_1)$. Determine the remaining budget, which is the total budget minus the amount allocated in stage, 1. Next, move up to stage, 2, choose $f_2(x_2)$ based on the remaining budget. The allocation continues in the same manner to stage, n, which is the final stage.

Numerical Application

An investor with a budget of 6 million USD is to build his portfolio with one or more of three stocks. The table (Table IV) below gives the unit price, b_i, in millions of USD and the unit return, r_i in thousands of USD for stock i. Advice the investor on the best way to allocate the USD 6 million to maximize his total return.

Table IV: Available stocks for the investor.

Stock i	b_i	r_i
1	4	70
2	1	20
3	2	40

Using the backward recursive method with $B = USD\ 6\ million$, we have the following tables.

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Stage III: $f_3(x_3) = \{40m_3\}, \{m_3\} = \left[\frac{6}{2}\right] = 3.$

Table V: Possible allocations of $stock$ 3.

$40m_3$					Optimum solution	
x_3	$m_3 = 0$	$m_3 = 1$	$m_3 = 2$	$m_3 = 3$	$f_3(x_3)$	m_3^*
0	0	-	-	-	0	0
1	0	-	-	-	0	0
2	0	40	-	-	40	1
3	0	40	-	-	40	1
4	0	40	80	-	80	2
5	0	40	80	-	80	2
6	0	40	80	120	120	3

Stage II: $f_2(x_2) = \{20m_2 + f_3(x_2 - m_2)\}, \{m_2\} = \left[\frac{6}{1}\right] = 6.$

Table VI: Possible allocations of $stock$ 2.

$20m_2 + f_3(x_2 - m_2)$								Optimum solution	
x_2	$m_2 = 0$	$m_2 = 1$	$m_2 = 2$	$m_2 = 3$	$m_2 = 4$	$m_2 = 5$	$m_2 = 6$	$f_2(x_2)$	m_2^*
0	$0 + 0 = 0$	-	-	-	-	-	-	0	0
1	$0 + 0 = 0$	$20 + 0 = 20$	-	-	-	-	-	20	1
2	$0 + 40 = 40$	$20 + 0 = 20$	$40 + 0 = 40$	-	-	-	-	40	0,2
3	$0 + 40 = 40$	$20 + 40 = 60$	$40 + 0 = 40$	$60 + 0 = 60$	-	-	-	60	1,3
4	$0 + 80 = 80$	$20 + 40 = 60$	$40 + 40 = 80$	$60 + 0 = 60$	$80 + 0 = 80$	-	-	80	0,2,4
5	$0 + 80 = 80$	$20 + 80 = 100$	$40 + 40 = 40$	$60 + 40 = 100$	$80 + 0 = 80$	$100 + 0 = 100$	-	100	1,3,5
6	$0 + 120 = 120$	$20 + 80 = 100$	$40 + 80 = 120$	$60 + 40 = 100$	$80 + 40 = 120$	$100 + 0 = 100$	$120 + 0 = 120$	120	0,2,4,6

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Stage I: $f_1(x_1) = \{70m_1 + f_2(x_1 - 4m_1)\}, \{m_1\} = \left[\frac{6}{4}\right] = 1.$

Table VII: Possible allocations of $stock$ 1.

$70m_1 + f_2(x_1 - 4m_1)$			**Optimum solution**	
x_1	$m_1 = 0$	$m_1 = 1$	$f_1(x_1)$	m_1^*
0	$0 + 0 = 0$	-	0	0
1	$0 + 20 = 20$	-	20	0
2	$0 + 40 = 40$	-	40	0
3	$0 + 60 = 60$	-	60	0
4	$0 + 80 = 80$	$70 + 0 = 70$	80	0
5	$0 + 100 = 100$	$70 + 20 = 90$	100	0
6	$0 + 120 = 120$	$70 + 40 = 110$	120	0

ANALYSIS OF THE POSSIBLE OPTIMAL ALLOCATIONS

From stage I (Table VII), $x_1 = 6$ gives the optimum alternative with $m_1^* = 0$. Therefore, 0 unit of stock 1 will be added in the portfolio. This leaves,

$$x_2 = x_1 - 4m_1^* = 6. \tag{8}$$

From stage II (Table VI), $x_2 = 6$ gives $m_2^* = 0,2,4,6$. Hence,

$$x_3 = x_2 - m_2^*. \tag{9}$$

Considering the various values of m_2^* and using equation (9) gives the following.

$$m_2^* = 0: x_3 = x_2 - 0 = 6, \tag{10}$$

$$m_2^* = 2: x_3 = x_2 - 2 = 4, \tag{11}$$

$$m_2^* = 4: x_3 = x_2 - 4 = 2, \tag{12}$$

$$m_2^* = 6: x_3 = x_2 - 6 = 0. \tag{13}$$

From stage III (Table V),

$$x_3 = 6 \; implies \; m_3^* = 3, \tag{14}$$

$$x_3 = 4 \; implies \; m_3^* = 2, \tag{15}$$

$$x_3 = 2 \; implies \; m_3^* = 1, \tag{16}$$

$$x_3 = 0 \; implies \; m_3^* = 0. \tag{17}$$

Therefore, the optimum solution is

$$(m_1^*, m_2^*, m_3^*) = (0,0,3) \; or \; (0,2,2) \; or \; (0,4,1) \; or \; (0,6,0). \tag{18}$$

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From equation (18), $(m_1^*, m_2^*, m_3^*) = (0,0,3)$ means that the investor should add 0 unit of stock 1, 0 unit of stock 2 and 3 units of stock 3 to his portfolio. While $(m_1^*, m_2^*, m_3^*) = (0,2,2)$ means that the investor should add 0 unit of stock 1, 2 units of stock 2 and 2 units of stock 3 to his portfolio. Similarly, $(m_1^*, m_2^*, m_3^*) = (0,4,1)$ means that the investor should add 0 unit of stock 1, 4 units of stock 2 and 1 unit of stock 3 to his portfolio. Finally, $(m_1^*, m_2^*, m_3^*) = (0,6,0)$ means that the investor should add 0 unit of stock 1, 6 units of stock 2 and 0 unit of stock 3 to his portfolio.

CONCLUSION

This work represents a typical resource allocation model in which limited resource is apportioned among a finite number of stocks. The objective is to maximize an associated return function. The definition of the state at each stage is similar to the definition given for the model. That is, the state at stage i is the total resource amount allocated to stages $i, i+1, i+2, \cdots, n$. The optimization model has maximized the returns of the investment by taking into account the expected returns, prices and budget in the objective functions and constraints, simultaneously. The model has the ability to determine the optimal portfolio of assets. A numerical example has been given to illustrate the application of the model and demonstrate the effectiveness of the designed algorithm for solving the model. Numerical results have shown that the optimization model yields promising results.

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