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CONSTRAINED MAXIMIZATION OF THE ECONOMIC PRODUCTION MODEL OF LITHIUM ORE EXPLORATION IN NASARAWA

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ABSTRACT: *Over 20,000 years ago, mining was discovered as one of the oldest production industries generating over US$700 billions' of revenue in the world by a few mining companies. For some time, mining work has resulted in a very demanding affair as a result of greater depth, low-grade, limited resources, and complex geo-mining conditions. Therefore, optimization of the mining system plays a vital role in profit maximization with the satisfaction of many constraints. However, today's mining industry uses complex and sophisticated systems whose reliability has become a critical issue. This work adopted the financial market theory of development to propose a maximized constrained optimization economic production model for lithium ore exploration in Nasarawa, Nigeria using three methods such as: the break-even principle method of cut-off grade between revenue earned and cost incurred; the mortimer's method principle to determine ore based in two cut-off criteria (original and average good) and the lane method for net profit value thereby maximizing processing capacity. Data obtained from the field and the Ministry of Mines and Solid Minerals were analyzed using energy dispersive x-ray fluorescence (EDXRF) showing the presence of lithium. The froth flotation technique used showed the beneficiation thereby achieving improved lithium concentrates and the inductively coupled atomic emission revealed a high presence of lithium of over 1859 parts per million (ppm) and other minerals.*

KEYWORDS: Brine, Granite, Kunzite, Lithium, Pegmatites.

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INTRODUCTION

The world economic growth rapidly has resulted in a vast increase in the demand for exploitation of other natural resources instead of crude oil in order to meet metal and energy demands for several industrial uses. The United States Geological Survey (2015) opined that the need and demand for Lithium is increasing geometrically with a projection of about 10% annual consumption growth. The advancement in the technological driven devices and new energy industry has enhanced the consumption, need and high demand for lithium. Pratima et al. (2014) estimated lithium to be the 25th most abundant element in the earth's crust, implying that lithium covers one-quarter of the earths' crust. The British Geological Survey risk level report of 2013 as contained in the British Geological Survey (2016) also revealed that the crustal abundance was 16 parts per million (ppm), thus it is rated as a medium-risk on a global scale. Bradely et al. (2017) identified two primary sources of lithium as Brines and Pegmatites of lithium-bearing minerals. The work further revealed that the most important rock deposits of lithium are granitic pegmatites and greisens. Bradely et al. (2017) stated that there exists an average grade of 16 ppm of lithium in the earth's crust, 4.3ppm of it in the oceanic crust and a 20 ppm of lithium in the upper continental crust. Rudnick et al. (2014) asserted that some Lithium occurs in magmatic crystalline rocks with the possession of industrial and commercial values. These include lepidolite, spodumene, petalite, amblygonite-montebrasite, and zinnwaldite (Rudnick et al., 2014). Furthermore, these minerals are the major constituents of lithium with the following percentages: lepidolite (20% to 30%), spodumene (50% to 60%), petalite (15% to 20%), amblygonite-montebrasite (5% to 10%), Kunzite (60% to 70%) and zinnwaldite (10% to 30%) (Rudnick et al., 2014).

Nigeria, the largest nation in the African continent, consists of about 35 industrial minerals in abundance that could improve the national income generation thereby bettering the economy when exploited appropriately. One of these 35 mineral resources is the Lithium ore deposited in many Lithium-bearing pegmatites. The constituents of the Lithium-bearing mineral deposits in Nigeria include Kunzite (most suitable for extraction) followed by Spodumene, Lepidolite, Petalite, Amblygonite and many more yet to be discovered (Kathryn et al., 2021). The characterization and beneficiation was used to identify the real deposits of Lithium ore in Nigeria and the recovery of the lithium became expedient as it adds value and also increases the revenue generation and economic potential from selling the concentrate to industries for its uses (Abdulfattah et al., 2023). The scope of this research work is centered on optimizing the different impacts of lithium ore exploration in the Nigerian economy with special consideration on Nasarawa Local Government Area in Nasarawa State of North Central, Nigeria. The work is limited to only sixteen (16) mining sites in Nasarawa Local Government Area of Nasarawa State.

Statement of Problem

The global world is undergoing green energy transitions with diversities of challenges with respect to each country's economic and social existence. To provide this change in technology, many new raw materials are required for the manufacturing of fossil free energy sources and storage such as solar panels, wind turbines, geothermal energy, batteries, and hydrogen technology. Failure of the source of the mainstay of the Nigerian economy has contributed to high rates of poverty, famine and a drop in the Gross Domestic Product (GDP). These challenges had failed in transforming raw materials of minerals from fossil fuel dependent to a zero-emission economy. This results in multi-fold mineral requirements for technologies such as wind turbines, solar panels, and electric vehicles with exorbitant amounts that cannot be

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fulfilled by just recycling alone. Different conflicts in the continents have also accelerated the teething problems of developing economies and this transition requires many raw materials in form of minerals to be mined for these important technologies. Mining occurs not until mineral deposits have been discovered and explored. However, the success rate of exploration in Nigeria is seen to be less than 1 mine from 1000 exploration projects with projects taking about one to three decades to convert from discovery to producing mine. Lithium ore in Nigeria is exported as mined without any value addition to it, which reduces the revenue generated by the country from the mining and mineral industry. Historically, mining has contributed to many of the challenges that the SDGs are addressing; however, in recent decades, the industry has made significant advances on minimizing and managing impacts by improving how companies manage both their social and environmental impacts. In the light of the above, this work seeks to identify both lower and higher-graded pegmatites of lithium ore deposits, consider its constituents as well as optimize the impacts of the lithium ore values to the development of a dwindling economy of Nigeria seen as the giant of Africa.

Lithium Ore Deposit and Mineralization in Nigeria

A great number of African countries have low concentrations of lithium ore deposits in igneous rocks which could be real sources for lithium concentrate if well exploited (MMSD, 2022). Lithium ore in Nigeria can be found in the Northern and Southern parts of Nigeria like Kogi, Nasarawa, Kwara, Oyo, Plateau, Bauchi, Gombe, and Adamawa. This lithium ore exists in Nigeria in the following order of basic constituents with high ranking percentages starting with kunzite, followed by spodumene, lepidolite, petalite and amblygonite. However, lithium ore in Nigeria is exported as mined without any value addition to it, thereby reducing the revenue generated by the country from the mining and mineral industry (Kathryn et al., 2021).

Sustainable Development Goals and Mining Impacts

The 2030 Agenda for Sustainable Development Goals (SDGs) was in 2015 adopted by the United Nation Member States including seventeen (17) SDGs to achieve peace coexistence and prosperity for people and the planet for now and years to come (United Nations, 2015). These SDGs consist of topical social and environmental issues regarding ending poverty and other inequalities, strategies for improved health, education, economic development, combating climate change and protecting life on land and underwater (United Nations, 2015). Many nations like Nigeria have after the official commissioning of the SDGs incorporated them into their national policies. This incorporation in 2017 has resulted in over 62% of global firms engaging in SDGs in their global reporting (Conway, 2018).

From the works and reports of UNDP in 2016, the mining industry contributed positively to all 17 SDGs. Historically, mining has contributed to many of the challenges that the SDGs are addressing; however, in recent decades, the industry has made significant advances on minimizing and managing impacts by improving how companies manage both their social and environmental impacts (UNDP, 2016).

Business Model Innovation

Innovation is defined as a new idea or method (Cambridge University Press, 2022); hence innovative models related to business referred to as “Business Models” means to change or alter the business model. Volans (2016) stated that the Business and Sustainable Development Commission warned that population growths are creating new economic orders that are rendering many existing business models outdated. That, mining organizations ought to forgo

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the "archaic or old ways" of exploring or mining minerals rather than think on the new perspectives that will enhance the wider economic, social, and environmental systems (Volans, 2016). Business models have gained influence and researchers' intentions recently as the society recognizes the desire to drift from the traditional linear, profit-focused way of doing business and have been identified as a key bottleneck in the transition to sustainability (Bidmon & Knab, 2018). The works of Treece (2010) revealed that business models are key features of market economies with consumers' choices, transaction costs, and diverse views amongst consumers and producers, and competition. Kavadias, Ladas and Loch (2016) believed that some experts assert that business models are essential to a company's success compared to technological innovation. According to Chesbrough (2010), "a mediocre technology pursued within a great business model may be more valuable than a great technology exploited via a mediocre business model".

THEORETICAL FRAMEWORK

The theoretical framework of this research is centered on the Financial Market Theory of Development. The Financial Market Theory of development is an economic theory that uses private flows of capital in new stock markets to encourage domestic development in developing countries. This theory was put forward by the World Bank's World Development Report for 2000. The theory states that foreign investors should have access to "well-regulated" financial markets which would provide the "surest path" to economic development. Businesses in low-income countries would gain direct access to the private capital from industrialized countries.

METHODOLOGY

Optimization is a searching process of obtaining the most efficient or cost-effective solution of any system or subsystems within some specified constraints in a problem for either maximizing desired factors and minimizing undesired ones. In the optimization problems, we focus on a single objective function aimed at measuring the decision making quality. Example:

$$Minimize \ f(x) \qquad (1)$$

$$subjext \ to \ \ h_i(x)=0, \quad i=1,2,3,....,m$$

$$g_j(x)\le 0, \quad j=1,2,3,......,r$$

$$where \ \ f(x), \ h_i(x) \ \ and \ \ g_j(x) are \ \ linear \ constraints.$$

Optimization problems can be classified based on the nature of constraints, decision variables characteristics, type of equations used, and many objective functions. The classification can be:

1. Based on the existence of constraints:

• Constrained optimization problem: if it has one or more constraints.

• Unconstrained optimization problem: if it has no constraints. Examples of constrained and unconstrained optimization problems are given as equations (2) and (3) below respectively.

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$$Maximize\left(Minimize\right)\ z = \sum_{i} C_i\, x_i^{p_k},\ \ p_k \in \Re$$

$$sunject\ \ to: \quad \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ji} x_i^{p_k} \le b_i \qquad (2)$$

$$x_i \ge 0,\quad i = 1,2,3,\ldots\ldots\ldots, m;\qquad k = 1,2,3,\ldots, m$$

Where, z=objective function that needs to be optimized; x_i the decision variables subjected to some constraints and p_k are any real numbers.

$$Maximize\left(Minimize\right)\ z = \sum_{i} C_i\, x_i^{p_k},\ \ p_k \in \Re \qquad (3)$$

$$as\quad i = 1,2,3,\ldots\ldots\ldots, m;\qquad k = 1,2,3,\ldots, m$$

Where, z=objective function that needs to be optimized; x_i the decision variables that are not subjected to any constraints and p_k are any real numbers.

Three Methods Used for this Research

The Break-Even-Principle (BEP) is the Break-Even cutoff grade where the revenue earned is equivalent to the costs incurred to generate the revenue. That is, each grade is greater than the break-even calculation which in turn generates greater costs. The BEP is a one dimensional process that only considers financial parameters. Many mining operations and companies that use the BEP do not provide the expected rate of return (ERR).

$$Break - Even\ \ Grade = \frac{Cost\ \ (naira\ \ / production)}{\Pr oduct\ \ \Pr ice\ * \operatorname{Re} cov ery} \qquad (4)$$

The Mortimer’s Method Principle of determining ore based on two cut-off criteria called the Mortimer’s theory. These criteria are based on the original break-even cut-off grade and the average grade with the addition of the minimum expected profit margin per ton that the company expects.

The Lane Method uses the net profit value (NPV) concept to maximize the value of the mine by optimizing the cut-off grade value at each mining stage. The optimization is carried out by understanding the cut-off grade derivation economic concept (COGDEC) at each mining stage. Lane Method's most striking parameter is the use of three main constraints in the optimization algorithm. These limits or constraints include the Mining, Concentration and the Refining.

$$Break - Even\ \ Grade = \frac{Cost\ \ +\ profit\ \ m\arg in}{\Pr oduct\ \ \Pr ice\ * \operatorname{Re} cov ery} \qquad (5)$$

The Lane Method also identifies that Mining production has three (3) stages.

Stage 1: Mining, Treatment and Marketing.

Stage 2: Processing the mined material, Ore treatment and the waste.

Stage 3: All the smelting which results to the product

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$$G = (d * p_c) * q_r - m_c * q_m - CP * q_c - f * p \qquad (6)$$

Where "q_r" in the equation above is used to calculate the mining, processing and refining of Ore. That is $q_r = a*y*q_c$. Equation (3.3) can further be written as:

$$G = (d * p_c) * a * y * q_c - m_c * q_m - p_c q_c - f * p \qquad (7)$$

Where "G" is the profit, d is the commodity price value in naira per dollar, "p_c" is the production cost (smelting, refining and selling), "a" is the processed average grade of mined material, "y" metallurgical recovery from smelting, "q_r" is the amount of material mined, procured and refined, "m_c" is the mining cost in naira per ton, q_m" is the amount of material to be mined, "q_c" is the amount of Ore to be sent out for processing in ton, f is the fixed cost in every production period, p is the mine life time in years or the production time, CP is the processing cost and int=percentage interest rate.

The Net Profit Value (NPV) is seen to be the same as the Maximizing Net Present Profit Value (MNPV). That is given as:

$$Net\ Profit\ Value \equiv Maximum\ Net\ Profit\ Value = \frac{G}{P}\left[\frac{(1+\text{int})^P \_1}{\text{int}(1+\text{int})^P}\right] \qquad (8)$$

The Maximum Processing capacity (Max Pro) in ton per year is given as

$$Max\ Pro\ \frac{Re}{a*y} \qquad (9)$$

The Maximum Mining Capacity in ton per year is given as the Max Pro $(1+D_m)$. D_m is the Dilution of mine with a 10% assumption and "Re" is the maximum amount refined.

$$Dilution = \frac{waste\ material}{Ore\ material + waste\ material} \times \frac{100}{1} \qquad (10)$$

$$Max\ Min = \frac{Re}{a*y}\left(1 + \frac{waste\ material}{Ore\ material + waste\ material} \times \frac{100}{1}\right) \qquad (11)$$

$$p = Max\left[\frac{q_m}{Max\ Min}, \frac{q_c}{Max\ Pro}, \frac{q_r}{Re}\right] \qquad (12)$$

The seven (7) Mining Cost Components are mine plan, drilling, blasting and smoke cleaning, mucking and loading, supporting, Ore and waste hauling and back filing. The ten (10) Processing Cost Components are crushing and separation, fine separation, milling, leaching and adsorption, elution, electro-winning, smelting, thickening, destruction and waste water treatment while the ten (10) Mining and Financial parameters include maximum capacity mined (Max min), maximum capacity of processing (Max Pro), maximum capacity refined (Re), cost of mining (m_c), cost of processing (CP), cost of refining (p_c), recovery (a), fixed cost (f), selling price (SP) and discount rate (r_d).

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Mineral Exploration Axioms

Axiom I:

A mineral occurrence and exploration will grow at a rate which is proportional to its size. A deterministic model that describes such a solid mineral growth in continuous time is the differential equation given as

$$\frac{dl}{dt} = al \qquad (13)$$

Where l(t) is the lithium mineral size at time "t" and "a" is the proportionality constant. Integrating the equation (3) above gives a solution as

$$l(t) = l(0)e^{at} \qquad (13.1)$$

Where l(0) is the mineral size at time zero. This implies that the lithium grows in size at an exponential rate. However, not all minerals grow exponentially fast. Minerals' growth is subjected to some constraints like land, soil type etcetera.

Axiom II:

The occurrence and exploration takes place continually.

$$d_{i+1} = bd_i \qquad (13.2)$$

Where d_i is the size of the i^{th} exploration or mining and "b" is the occurrence. Equation (3.2) has solution as

$$d_1 = b^i d_0 \qquad (13.3)$$

Where d_0 is the initial occurrence and exploration or mining of the mineral. But the solution of differential and difference equation can coincide at time t=i if

$$d_0 = \quad l(0) \qquad and \quad b = e^a \qquad (13.4)$$

Axiom III:

The occurrence and exploration (mining) behaves or exist according to a deterministic law. Large occurrences of minerals lead to randomness or variation limitations. However, if the occurrence is small, we intuitively expect stochastic variability to be important. As such we consider the occurrence of minerals separately from exploration (Mining of minerals). A model for occurrence and exploration of lithium between time "t" and (t+δt) is given in the table below.

Table 1: Model of Occurrence and Exploration of Lithium

Event	Effect on Mineral Lithium (l)	Probability of Event
Occurrence	l(t+δt)= l(t) +1	cl(t)δt
Exploration (Mining)	l(t+δt)= l(t) -1	fl(t)δt
No Change	l(t+δt)= l(t)	1- cl(t)δt- fl(t)δt

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The description of the model shows how to approximately simulate the stochastic model by choosing a sufficiently small time step δt (such that all the probabilities are less than 1); and then choosing one of the possible events with probability shown. The final stage is obtained by drawing a random number "y" uniformly on [0,1] with many packages having a random number generator. An occurrence "y" of any of the different sources of lithium could be

$$y \langle\ cl(t)\delta t, \tag{13.5}$$

And an exploration (mining) as

$$y \langle\ cl(t)\delta t + fl(t)\delta t \tag{13.6}$$

Otherwise, none of such events occur. This stochastic model has the same expected value as the differential equation model if (c-f) = a.

Generally, the degree of correspondence between deterministic and stochastic models can be obtained using the lithium size (l), rates of occurrence d (l) and rate of change dl(t) given as follows.

For deterministic case, the rate of change of lithium size with respect to time is

$$\frac{dl(t)}{dt} = b(l) - d(l) \tag{13.7}$$

Where b(l) is the occurrence rate and d(l) is the exploration rate.

For the stochastic case, the definition for a small δt of the probabilities of occurrence and exploration of events is given as follows:

Table 2: Model of Occurrence and Exploration of Lithium and Probabilities

Event	Effect on Mineral Lithium (l)	Probability of Event
Occurrence	l(t+δt)= l(t) +1	b(l)δt
Exploration (Mining)	l(t+δt)= l(t) -1	d(l)δt
No Change	l(t+δt)= l(t)	1- b(l)δt- d(l)δt

Axiom IV:

To formulate the model, we assume that our lithium exploration/mining model is to be uniformly distributed on earth, and has no contact with other solid minerals. Another idea of consideration is the spatially explicit model with rates describing the movement of the different sources of lithium from one geographical area (land) to another. More so, the only relevant information about the lithium mineral is its weight value (a stable variable), the only source variable of energy or weight value is its natural occurrence and that the weight or value lost is as a result of improper exploration or premature exploration or due to converting or processing it from its five basic natural sources.

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Data Exploration and Fitting the Model

We restrict comments to occurrence and exploration or mining of lithium processes described by differential equations. Data /observations given as l_i at times t_i, i >1, estimating the derivative between time t_i and t_{i+1} by d_{i+1} equals

$$(y_{i+1} - y_i) \div (t_{i+1} - t_i) \qquad (13.8)$$

Plotting d_{i+1} against $(y_{i+1} + y_i)/2$, an estimate of average value of "y" yields a relationship between dy/dt and y. Exponentially, the graph becomes a straight line. Limited occurrence leads to departure from linearity leading to a quadratic curve.

$$\frac{dy}{dt} = ry(a-y) \qquad (13.9)$$

The left hand side of the equation below is the occurrence growth level of the mineral which is solved as a logistic equation.

$$\frac{1dy}{y\,dt} = ry(a-y) \qquad (13.10)$$

Plotting the proportional occurrence growth against "y" yields a straight line

$$\frac{1dy}{y\,dt} = \frac{d(\log y)}{dt} \qquad (13.11)$$

Then

$$d_i = \frac{(\log y_{i+1} - \log y_i)}{(t_{i+1} - t_i)} \qquad (13.12)$$

Analytically,

$$\frac{dL}{dt} = au + bv + cw + dx + ey + fz \qquad (13.13)$$

Where a, b, c, d, e and f are all constants representing percentage composition of each mineral and u, v, w, x, y and z are all solid minerals.

Model Formulation

Assumptions for the Model Formulation:

1. The degree of access to the lithium pre is in working order and well mature.

2. Production processes or activities such as drilling, blasting, loading, unloading of Lithium is carried out as when due.

3. The method for lithium extraction used is the open stoping mining method.

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4. The stoping blocks sizes are deterministic using the thickness and depth of the lithium ore.

5. The crown pillar size is fixed and deterministic using the stress analysis of the different openings and extractions in different manners as decided by the geotechnical parameters.

Model Notations:

The notations used for the model formulation are stated below.

V(i,j,k) = Value of the grades of the (i, j, k)th blocks in percentages.

SP = Selling price of the mineral ore (dollar per ton)

CP = Processing cost for the ore (dollar per ton)

CM = Mining cost for the ore (dollar per ton)

MB = Single block tonnage

N_x = Number of blocks in the x-direction

N_y = Number of blocks in the y-direction

N_z = Number of blocks in the z-direction

m_x = Minimum number of blocks in the x-direction in one stoping block

m_y = Minimum number of blocks in the y-direction in one stoping block

m_z = Minimum number of blocks in the z-direction in one stoping block

GC = Grade cutoff marks for a particular type of ore

Min_m = Minimum tonnage needed to be mined in one period

Max_m = Maximum tonnage needed to be mined in one period

Min_p = Minimum tonnage needed to be processed in one period

Max_p = Maximum tonnage needed to be processed in one period

Decision Variable:

$$y_{i,j,k} = \begin{cases} 1 & if \ (i,j,k)^{th} \ block \ is \ in \ stope \\ 0, & otherwise \end{cases} \qquad (14)$$

Objective Function:

Objectively, mining is to maximize the net profit cash flow from single stope with blocks of mineral Ore and minimize the cost of production. Modifying the economic function model of Gangawat (2014) that optimized and also obtained the profit for a single block, it is derived as follows:

$$e_i = a_i * u_i * (f_i * r * s - c) * \rho_i \qquad (15)$$

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Where i = blocks, a_i = blocks density, u_i = volume of blocks, f_i = average grades of blocks, r = level of recovery, s = selling price per unit, c = mining and processing costs per unit and ρ is the stope stability determinant strength factor.

$$\rho_i = \frac{Rock \quad mass \quad strength}{Induced \quad factor} \qquad (16)$$

The present value of the cost gotten at each period is denoted as W_m

$$W_m = \frac{M}{(1+i)^p} \qquad (17)$$

Where M = the amount of money received, i = rate of interest, p = index time period. From the above explanations and equations, the objective function aimed at maximizing the cash flow single stope is given as:

$$\max h(x) \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{z=1}^{N_z} \{[(W_m * \frac{V_{(i,j,k)}}{100} * SP - CP) - CM] * y_{(i,j,k)} * MB\} \qquad (18)$$

Where $y_{(i,j,k)}$ is the decision variable.

Constraint Related to the Angle of Stope Extraction

The predecessor's blocks for a particular block (i, j, k) is given below. If the blocks reside to the left hand side then j-1 does not exist similarly if the blocks reside to the right hand side, then j+1 does not exist.

Mathematically, the model for the constraints will be represented using 9 different predecessors' blocks with their positions represented in the table below.

Table 3: Predecessors Blocks Arrangements

i+1, j-1, k-1	i+1, j k-1	i+1, j+1, k-1
i j-1 k-1	i j k-1	i j+1 k-1
i-1 j-1 k-1	i-1 j k-1	i-1 j+1 k-1

The model for the constraints using the 9 different predecessor's blocks is represented below with "T" as the set containing {-1, 0, 1} as:

$$9 * y_{(i,j,k)} - \sum_{\alpha \in T} \sum_{\beta \in T} y[i+\alpha,\ j+\beta, k-1]: \quad \forall i, j, k \in \Re \qquad (19)$$

Constraints Related to the Minimum Stope Height

Extraction of each and every block in the underground mining is based on the technical and economic perceptions. A block is mined if and only if the other blocks adjacent to it have the potential to be extracted. The constraint ensures that a given stope contains at least the minimum number of blocks represented as equation (20) below.

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$$\sum_{w=0}^{m_x}\{ y(i,\ j, k+w) - y(i,\ j, k+m_x-1) - y\} \leq\ N_x - 2 \qquad (20)$$

Constraints Related to the Maximum Stope Height

This constraint restricts them to avoid roof failure and sagging problems. Therefore, the mathematical logic for this constraint is given as:

$$\sum_{w=0}^{m_x}\{y(i,\ j,\ k+w)\} \leq m_z,\quad \forall\ i,\ j,\ k = \{1,2,3,....,m_z\} \qquad (21)$$

Details of Lithium Mining Activities at Major Mining Sites in Nasarawa Local Government Area of Nasarawa State

(1) At Angwan Mada, Onda village of Nasarawa Local Government Area, at least 40 tons of low grade spodumene of 2% and above lithium concentration is mined from the site daily. Although mining activities have stopped due to legal issues or disagreement between the community and the company.

(2) At Opa, in Nasarawa Local Government Area, a high grade of kunzite of 5% - 8% of lithium concentration is mined everyday. At least 10 to 15 tons of this high grade kunzite is mined everyday at Opa. At the same time, 30 to 40 tons of high grade petalite is mined from Opa every day. In addition to kunzite and petalite at Opa in Nasarawa Local Government Area of Nasarawa State, 2 – 4% concentration of amblygonite, and 1 to 2 tons of this mineral is mined at Opa every day.

(3) At Endo, which happens to be the largest lithium site in Nigeria, low grade spodumene of 2.5% above in concentration and between 50 to 100 truck loads containing between 30 to 45 tons each of these minerals is mined per day. This is the busiest lithium mining site producing the highest quantity of the mineral in Nasarawa Local Government Area and Nigeria at large. However, mining has also been stopped due to legal issues between the companies, communities and the government of the day.

(4) At Gidan Kwanu in Nasarawa Local Government, two to three trucks of high grade kunzite of lithium concentration of 4% and above are mined per week which account for 8.6 tons per day. And at the same site, a high grade petalite of 8% - 10% is mined from this site with one truck in quantity per week which accounts for 4.2 tons of the high grade minerals mined at the site.

(5) At Kama Otto, a high grade kunzite of 7% and above of lithium concentration is mined at the site every day with a quantity of at least 3 to 4 trucks load of the mineral which accounts for 12.8 tonnes of the mineral per day.

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DATA PRESENTATION AND ANALYSIS

Table 4: Showing the Quality and Quantity of the Various Minerals Containing Lithium at the Major Lithium Sites in Nasarawa LGA

Minerals	Onda	Opa	Endo	Gidan Kwanu	Kama Otto
Kunzite Quality Quantity	NIL	5% and above 10 – 15 tons per day	NIL	4% and above Approx. 8.6 tons per day	7% and above Approx.. 12.8 tons per day
Lepidolite Quality Quantity	NIL	0.45 at most Less than a ton per day	NIL	NIL	NIL
Spodumene Quality Quantity	2% and above At least 40 tons per day	NIL	2.5% and above At least 1,500 per day	NIL	NIL
Petalite Quality Quantity	NIL	6% and above At least 35 tons per day	NIL	8% to 10% At least 42. tons per day	NIL
Amblygonite Quality Quantity	NIL	2 – 4% At least 1 ton per day	NIL	NIL	NIL

Source: *Field Data*

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Table 5: Lithium Ore Deposits in Nigeria

Deposits	State	Associated Minerals
Panda	Nasarawa	Pegmatite
Wamba	Nasarawa	Quartzite
Kabba	Kogi	Quartzite
Kushaka, Birnin Gwari	Niger	Pegmatite/Petalite
Isanlu Egbe	Kogi	Pegmatite
Ilesha	Osun	Pegmatite
Ijero Aramoko	Ekiti	Pegmatite
ArikyaTsauni	Nasarawa	Pegmatite and Quartzite
Kafin Maiyarki	Nasarawa	Granite
Itakpe Area	Kogi	Quartzite and Pegmatite
Oke Ogun	Oyo	Quartzite
Ago Iwoye	Ogun	Pegmatite
Hong	Adamawa	Lepidolite/Kunzite
Zuru	Zamfara	Petalite
Kafanchan	Kaduna	Spodumene/Kunzite
Lere	Kaduna	Petalite
Jos- South	Plateau	Quartzite/Lepidolite
Ganjuwa	Bauchi	Lithium Oxide/Lithia
GidanBoda, Baruten	Kwara	Spodumene
Keffi	Nasarawa	Lepidolite

Source: *MMSD, 2022; Azomite ® Laboratory Report*

Table 6: Results of Energy Dispersive X-ray Fluorescence Spectrometer (ED - XRF) Chemical Analysis of Sourced Lithium Ore at Different Deposits within Nigeria

Sample Locations	SiO_2 wt. %	Al_2O_3 wt. %	Na_2O wt. %	K_2O wt. %	TiO_2 wt. %	P_2O_5 wt. %	MgO wt. %	Li_2O wt. %	MnO wt. %	CaO wt. %	Fe_2O_3 wt. %	L.O.I wt. %
Panda	73.8	3.21	2.14	0.12	2.14	1.04	0.26	9.53	0.06	2.36	4.31	0.01
Wamba	74.23	3.08	1.06	0.13	2.06	1.21	0.18	8.69	0.05	2.21	3.98	0.03
Kabba	82.86	2.14	1.05	2.10	0.06	0.03	0.50	8.69	0.01	0.16	2.07	0.01
Kushaka	94.08	0.06	0.03	0.12	0.18	0.002	0.05	3.51	0.01	0.14	2.04	0.01
Isanlu Egbe	78.4	2.63	2.01	0.25	3.14	0.001	0.05	9.336	0.37	1.05	1.29	0.02
Ilesha	85.51	1.01	2.01	3.62	0.21	0.002	0.43	4.379	0.089	0.96	1.93	0.01
Ijero Aramoko	87.21	1.86	1.05	0.34	0.069	0.006	0.061	6.102	0.42	1.71	1.16	0.03

Source: *MMSD (2022)*

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Table 7: Results of ED-XRF Chemical Analysis of Beneficiated Lithium Ores using Froth Flotation Processing Method

Sample Locations	SiO_2 wt. %	Al_2O_3 wt. %	Na_2O wt. %	K_2O wt. %	TiO_2 wt. %	P_2O_5 wt. %	MgO wt. %	Li_2O wt. %	MnO wt. %	CaO wt. %	Fe_2O_3 wt. %	L.O.I wt. %
Panda	60.46	0.21	1.14	0.10	0.14	0.01	0.09	37.96	0.03	1.31	1.11	0.001
Wamba	58.21	3.08	0.68	0.13	0.96	0.11	0.08	36.86	0.05	0.21	1.78	0.001
Kabba	66.00	0.24	0.45	0.90	0.06	0.03	0.08	34.69	0.01	0.09	0.97	0.001
Kushaka	65.02	0.06	0.03	0.09	0.09	0.002	0.05	34.51	0.01	0.07	0.44	0.000
Isanlu Egbe	54.21	1.53	0.81	0.08	1.54	0.001	0.05	33.31	0.37	0.51	0.69	0.001
Ilesha	65.51	0.91	0.91	1.21	0.09	0.002	0.231	31.79	0.089	0.49	0.43	0.001
Ijero - Aramoko	63.72	0.76	0.85	0.14	0.05	0.006	0.061	32.11	0.42	0.64	0.66	0.001

Source: *Field Data*

DISCUSSION

From table 5, the elemental ED-XRF analysis shows the high abundance of lithium presence across the selected pegmatite ore deposits having a range of (3.52 – 9.53)% with Panda in Nasarawa State exhibiting the utmost presence of lithium oxide in the lithium-bearing pegmatite (Kathryn et al., 2021). The froth flotation technique was used to identify the beneficiation deposit thereby achieving an improved lithium concentrate (Kathryn et al., 2021). There exists a significant improvement in the lithium ranging from (31.79 – 37.9) % Li_2O and reduction in the silica as impurity from (54.21 – 66.00) % SiO_2 using ED-XRF as shown in table 6 above.

Table 6 also revealed a high presence of Li, Be, Cs and Rb contents with the average Li values of 1859 ppm, 1778 ppm, and 1656 ppm recorded for Oke-Asa, Oke-Igbo Aba, and Ijero Ekiti respectively using the physico-chemical characteristics of thirty lepidolite samples collected using the Inductively Coupled Plasma-Atomic Emission Spectrometry (ICP-AES) (Fosu et al., 2021). This buttresses the outcome of Agaku et al. (2020) who studied the qualitative analysis and hydrometallurgical extraction of lithium from polylithionite ore obtained from Keffi pegmatite field, Nasarawa state, North-Central, Nigeria. The study therefore, showed that, the hydrometallurgical extraction of Li using chloride roasting additives (NaCl and $CaCl_2$) is cost effective, less energy consuming and efficient, since the chloride additives rationed in fractions (1:0.6:0.4 ore, NaCl, $CaCl_2$) gave substantial optima lithium extraction efficiency of 89.90% at the optimized conditions (Akintola et al., 2014). The chemical characterization using both Energy Dispersive X-ray Fluorescence (EDXRF) and Flame Atomic Emission Spectroscopy (FAES) composition of the lithium rich rock was shown to be present in Li2O with Li content of 3.25 wt % (Agaku et al., 2020; Tassos et al., 2020).

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SUMMARY AND CONCLUSION

The research reveals that at Endo village, which happens to be the largest lithium site in Nigeria, low grade spodumene of 2.5% above in concentration and between 50 to 100 truck loads containing between 30 to 45 tons each of these minerals is mined per day. This is the busiest lithium mining site producing the highest quantity of the mineral in Nasarawa Local Government Area and Nigeria at large. Mining has, however, been stopped due to legal issues between the companies, communities and the government of the day. At Angwan Mada, Onda village, about 40 tons of low grade spodumene of 2% and above lithium concentration is mined from the site daily. Mining activities have stopped due to legal issues or disagreement between the community and the company. At Opa, a high grade of kunzite of 5% - 8% of lithium concentration is mined daily. At least 10 to 15 tons of this high grade kunzite is mined daily. At the same time, 30 to 40 tons of high grade petalite is mined from Opa daily. In addition to kunzite and petalite at Opa, 2 – 4% concentration of amblygonite, and 1 to 2 tons of this mineral is mined too. At Gidan Kwanu, two to three trucks of high grade kunzite of lithium concentration of 4% and above are mined per week which account for 8.6 tons per day. And at the same site, a high grade petalite of 8% - 10% is mined from this site with one truck in quantity per week which accounts for 4.2 tons of the high grade minerals mined at the site. At Kama Otto, a high grade kunzite of 7% and above of lithium concentration is mined at the site every day with a quantity of at least 3 to 4 trucks load of the mineral which accounts for 12.8 tonnes of the mineral per day.

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