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TREND-FOURIER TIME SERIES REGRESSION MODEL FOR SECULAR-CYCLICAL DATASETS

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ABSTRACT: *The study proposed a Trend-Fourier Regression (TFR) model to handle time series datasets with simultaneous trend and cyclical variations. The model steps involve identification, estimation, diagnosis and forecasting. The Nigerian monthly Crude Oil Price (NMCOP) was used to implement the model and NMCOP was identified as trend-cyclical. The model estimation using Ordinary Least Squares method indicates that an increase in time will result in changes in NMCOP. Durbin-Watson statistics, histogram and autocorrelation function of residual plots were used to diagnose and specify the model to be stable. The coefficient of determination* (R^2) *indicates that over 80% of dependent variable variations were explained, with an adjusted* (R^2) *indicating a predictive ability exceeding 80%. The model efficiency was confirmed through out-sample and forecast evaluations, revealing superiority due to its smaller MAE, RMSE, and MAPE values, indicating minimal error. Conclusively, the TFR model is suitable for datasets that exhibit trend-cyclical variations simultaneously.*

KEYWORDS: Trend-cyclical variation, Diagnostic checking, Trend-Fourier regression, Nigerian monthly crude oil price, Forecasting, Estimation.

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INTRODUCTION

The swift alterations seen over the years in the time series dataset can be attributed to social challenges, economic recession, psychological issues, COVID-19 pandemic, global crude oil crash, climate change and political instability (Ibn-Mohammed *et al*. [7], Duarte *et al.* [5], Maqbool *et al*. [11]). These changes happen over time, and this depends on structural changes that are subject to large historical changes (Constantine [4]). These historical changes have affected the time series dataset across all spheres and in terms of the global economy which include the Great Contraction of the 1930s and the post-depression reforms (Zarnowitz [29]), the partial recession in 2015 (Kose *et al.* [9]) and COVID-19 pandemic, which has hurt the world's finances (Adegboye *et al.* [1]). Crude oil prices, which have decreased by roughly one-third from their peak but are still very volatile, are another example of variation. This has affected the revenue generation of many countries that rely heavily on crude oil exportation (Nagle & Temaj [14]). Price fluctuations for crude oil are a worldwide phenomenon that is felt by all nations. Since emerging economies lack financial stability and are vulnerable to outside shocks, the impact of oil prices on these economies is particularly significant (Lescaroux & Mignon [10]). Price level fluctuations may also result in other economic developments that impact the state of the economy. This is why the inflation rate is regarded as the primary economic indicator that best represents the state and performance of the economy (Seka *et al.* [24]).

In the Nigerian context, it is anticipated that the drop in oil prices has caused the worst economic downturn since the 1980s (World Bank [28]). This is the case since oil accounts for over 80% of Nigeria's exports, 30% of the country's bank sector credit, and 50% of total government income. Government revenues declined from a projected 8% of GDP in 2019 to a projected 5% in 2020 because of the decline in oil prices. This coincides with the urgent need for financial resources to curb the COVID-19 pandemic and boost the economy (OECD [15]). The changes seen in the crude oil price time series dataset are important as this is considered very important and must receive serious attention because the effect is devastating to human daily life and peaceful existence (Ighosewe *et al.* [8]; Taiwo *et al.* [25], Ogundunmade *et al.* [16]). To have an in-depth insight into this critical factor that determines the state of the Nigerian economy, several researchers have conducted numerous works by modelling and forecasting Nigerian crude oil prices. These include the works of Oriavwote and Ojie [20], Umaru and Awoyemi [27], Oyeniran *et al.* [21], Afolabi *et al.* [3], Afia and Effiong [2], Ighosewe *et al.* [8], Olujobi *et al.* [19], Sami and Taiwo [23], Ekara and Usoro [6], and Mujtaba *et al.* [12]. These previous works have yielded good outcomes but a critical look at the variation in Nigerian crude oil prices signified a combination of secular and cyclical variations. This is not often considered and ascertained before modelling, and this may affect the robustness of the results. Therefore, to cater for the secular and cyclical variations simultaneously seen in Nigerian crude oil prices, this study is used to propose a combination of time series regression and Fourier regression models where the former will be used to handle linearity and the latter for cyclical components. Since the Fourier regression model can capture the cyclical changes from period to period (Olatayo & Taiwo [18], Taiwo *et al.* [26], Ozyilmaz *et al.* [22]). The efficiency of the proposed model will be compared with linear trend, Fourier regression and ARIMA models based on some validation and forecast evaluation metrics.

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TREND-FOURIER REGRESSION MODEL

The proposed model is a combination of trend linear and Fourier regression models. The model is given as

$$y_t = \beta_0 + \beta_1 t + \beta_2 cos\omega t + \beta_3 sin\omega t + \varepsilon_t \qquad (1)$$

where y_t is the dependent variable, β_0 is the constant, β_1 is the coefficient of the trend regression model, β_2 and β_3 are the coefficients of the Fourier regression, $sin\omega$ and $cos\omega$ are the trigonometric functions, $\omega = \frac{2\pi k}{n}$, y_t is the dependent variable, t is the indexes time and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ is the uncorrelated error term.

The model steps involve identification by using the time plot and estimation of the coefficients using the Ordinary Least Squares method by minimising the error term. In matrix form, the coefficient of the Trend-Fourier regression model is

$$(\beta_0\ \beta_1\ \beta_2\ \beta_3\ ) = (n\ \Sigma t\ \Sigma cos\omega t\ \Sigma sin\omega t\ \ \Sigma t\ \Sigma(t)^2\ \Sigma tcos\omega t\ \Sigma tsin\omega t\ \ \Sigma(cos\omega t)\ \Sigma sin\omega t\ \ \Sigma t(cos\omega t)\ \Sigma t(sin\omega t)\ \ \Sigma(cos\omega t)^2$$

The model diagnostic test for residual serial correlation will be achieved using Durbin-Watson Statistics and the test statistic is given as

$$d = \frac{\sum_{t=2}^{T} (\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T} \varepsilon_t^2} \qquad (3)$$

The autocorrelation function (ACF) as well will be used to validate the statistical significance of the residual term. The ACF is defined as

$$r_k = \frac{\sum_{t=k+1}^{T} \left(y_k - \underline{y}\right)\left(y_{t-k} - \underline{y}\right)}{\sum_{t-1}^{T} \left(y_t - \underline{y}\right)^2} \qquad (4)$$

Furthermore, the histogram of residuals will be used to show the residuals are normally distributed. The Coefficient of Determination (R^2) will be utilized to explain the level of variability in the independent variables collectively accounting for the dependent variable. This is defined in matrix form as

$$R^2 = 1 - \frac{(\beta_0\ \ \beta_1\ \beta_2\ \beta_3\ )(\ \Sigma y_t\ \Sigma(ty_t)\ \ \Sigma(cos\omega ty_t)\ \Sigma(sin\omega ty_t)\ \ ) - n\underline{y}^2}{y'y - (\beta_0\ \ \beta_1\ \beta_2\ \beta_3\ )(\ \Sigma y_t\ \Sigma(X_t y_t)\ \ \Sigma(cos\omega ty_t)\ \Sigma(sin\omega ty_t)\ )} \qquad (5)$$

The Adjusted Coefficient of Determination ($\underline{R}^2$) will be to ascertain the predictive ability of the proposed Fourier-Time series regression model. This is defined in matrix form as

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$$\underline{R}^2 = \frac{1}{n-k}\left[n\left(1 - \frac{(\beta_0 \ \ \beta_1\, \beta_2\, \beta_3 \ )(\ \Sigma y_t\, \Sigma(ty_t) \ \ \Sigma(cos\omega ty_t)\, \Sigma(sin\omega ty_t) \ ) - n\underline{y}^2}{y'y - (\beta_0 \ \ \beta_1\, \beta_2\, \beta_3 \ )(\ \Sigma y_t\, \Sigma(X_t y_t) \ \ \Sigma(cos\omega ty_t)\, \Sigma(sin\omega ty_t) \ )}\right) - k\right] \quad (6)$$

where k is the number of fitted parameters and n is the number of observations.

The last step is forecasting, and the out-sample forecast will be obtained for the proposed model using

$$\hat{y}_{t+h} = \sum_{i=0}^{m} \hat{\beta}_i X_{t+i} + \hat{\beta}_{i+j}\, cos\, cos\, (\omega X_{t+i}) + \hat{\beta}_{i+j}\, sin\, sin\, (\omega X_{t+i}),\ \ i = 1,2,3, \dots\ j = 1,2,3, \dots\ h = 1,2,3 \dots \quad (7)$$

where the out point values are $\hat{y}_{t+h}$, $\hat{\beta}_0, \hat{\beta}_i$ and $\hat{\beta}_j$ which are fitted parameters; the single out point prediction is h and Fourier predictors are $cos\, cos\, \left(\omega t_{j+h}\right)$ and $sin\, sin\, \left(\omega t_{j+h}\right)$.

The forecast evaluation metrics used are Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error MAPE given as

$$MAE = \frac{1}{h+1}\sum_{t=s}^{h+s} (\hat{y}_t - y_t)^2 \quad (8)$$

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{h+s} (\hat{y}_t - y_t)^2} \quad (9)$$

$$MAPE = \frac{100}{h+s}\sum_{t=s}^{h+s} \left|\frac{\hat{y}_t - y_t}{\hat{y}_t}\right| \quad (10)$$

where $t = s, 1+s, \dots, h+s$. For corresponding t values, the values $\hat{y}_t\ and\ y_t$ represent actual and predicted values respectively (Olatayo & Taiwo, 2015 [17]).

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RESULTS AND DISCUSSION

The efficiency and performance of the proposed Trend-Fourier regression model was ascertained using Nigerian monthly crude oil prices from October 1973 to January 2023. The dataset was obtained from the Central Bank of Nigeria (CBN) website. A visual inspection of the Nigerian monthly crude oil price time plot in Figure 1 from October 1973 to January 2023 signified a non-stationary series. This series exhibits secular and cyclical variations, and this was indicated by continuous rise and fall. Cycles were completed every year and there was a significant continuous rise in crude oil prices from 2006 to 2008, a sharp fall in 2009, a crude oil price boom in 2010 to 2014, and another sharp fall from 2015 with a slow fluctuating price till 2021 and a continuous fall thereafter. The lowest price of Nigerian crude oil was in October 1973 with a price of 7.63 dollars per barrel and the highest price at 138 dollars per barrel in June 2008. All these indicated that the Nigerian crude oil price exhibits secular-cyclical variation. Therefore, the proposed model in this study is considered appropriate for modelling and forecasting Nigerian monthly crude oil prices in Figure (1).

Figure 1: Time plot of Nigerian monthly crude oil price from October 1973 to January 2023

The Trend-Fourier regression model estimated using OLS for Nigerian crude oil price is given as

$$y_t = 48.28 - 14.14t - 479.5cos\frac{2\pi}{12}t + 163.68sin\frac{2\pi}{12}t \quad (11)$$

with $R^2 = 0.867312, Adjusted\ R^2 = 0.883104\ and\ Durbin\ Watson = 1.85501$

The values of the Durbin-Watson statistics stated in Equation (12) showed that when using the OLS estimation technique, the error term is not serially correlated. The Trend-Fourier regression analysis equation for OLS showed that the coefficients indicated that for every increase or decrease in time, there will be a unit decrease or rise in crude oil prices every month.

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The coefficient of determination (R^2) in Equation (12) revealed that time explained the variations in Nigerian crude oil price by as much as 87%. Additionally, the model $\underline{R}^2$ value showed a goodness of fit with an 88% level of predictive power.

The histogram of the residual for the fitted Trend-Fourier regression model given in Figure 2 indicated that the residuals are normally distributed, while ACF of residuals is given in Figure 3 where the residuals do not have any usual structure and are significant at $\alpha = 0.05$. Therefore, with regards to Durbin Watson statistics value, Figures 2 and 3, the fitted Trend-Fourier regression model is considered adequate for forecasting Nigerian monthly crude oil prices.

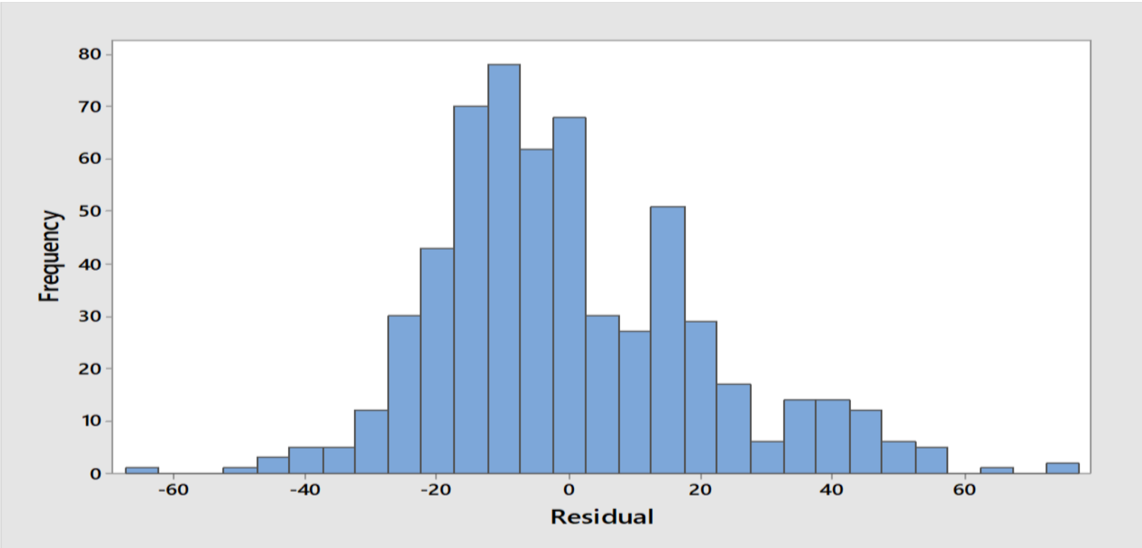

Figure 2: Histogram of the residual for Trend-Fourier regression model

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|*******</td><td>.|*******</td><td>1</td><td>0.974</td><td>0.974</td><td>563.92</td><td>0.000</td></tr>
<tr><td>.|*******</td><td>**|. |</td><td>2</td><td>0.933</td><td>-0.277</td><td>1083.1</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>3</td><td>0.890</td><td>-0.012</td><td>1556.0</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>4</td><td>0.849</td><td>0.028</td><td>1986.6</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>5</td><td>0.807</td><td>-0.054</td><td>2376.4</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|. |</td><td>6</td><td>0.766</td><td>0.013</td><td>2728.8</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>.|* |</td><td>7</td><td>0.735</td><td>0.152</td><td>3053.5</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>*|. |</td><td>8</td><td>0.706</td><td>-0.074</td><td>3353.3</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>9</td><td>0.676</td><td>-0.030</td><td>3629.0</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>10</td><td>0.648</td><td>0.046</td><td>3882.9</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>11</td><td>0.623</td><td>0.011</td><td>4118.2</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>12</td><td>0.599</td><td>-0.030</td><td>4336.0</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>13</td><td>0.572</td><td>-0.042</td><td>4535.0</td><td>0.000</td></tr>
</table>

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.|**** |	.|. |	14	0.546	0.029	4716.7	0.000
.|**** |	.|. |	15	0.522	-0.011	4882.7	0.000

Figure 3: Autocorrelation plot of the residual for Trend-Fourier regression model

The fitted Trend-Fourier regression model forecast for Nigerian crude oil prices from October 1973 to January 2025 is given in Figure 4. The forecast exhibits a replica of the actual data spanning October 1973 to January 2023. The out-sample forecast from February 2023 to January 2025 indicated that Nigeria's crude oil price may fall over the period forecasted.

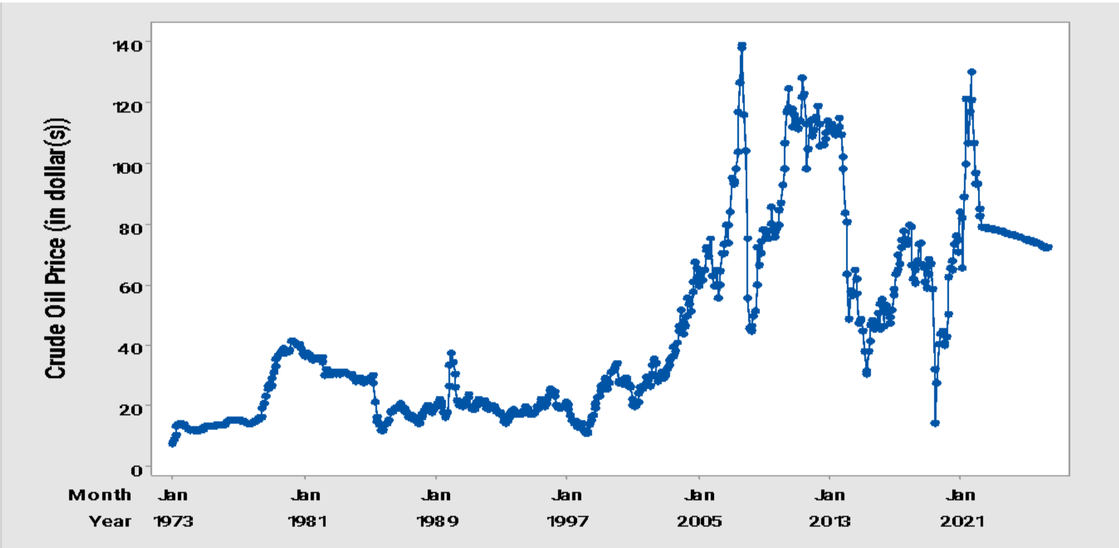

Figure 4: Time plot of Nigerian crude oil price from October 1973 to January 2025

The Trend-Fourier regression (TFR) model effectiveness is contrasted with the performance of linear trend time series regression, Fourier regression analysis and autoregressive integrated moving average models with regards to R^2, $\underline{R}^2$ and forecast evaluation metrics. Based on Table 1, the R^2 value for the TFR approach revealed that time explained the variations in Nigerian crude oil prices better than all other models considered. As well, based on Table 1, the value of $\underline{R}^2$ signified the TFR approach has better goodness of fit and predictive power.

The TFR and all other methods considered were utilized for forecasting Nigerian crude oil prices after the model is optimal, adequacy based on Durbin Watson statistics, Histogram of the residual and ACF plot of the residual. The proposed TFR, Fourier regression, linear trend and autoregressive integrated moving average (ARIMA) forecasts were compared based on their forecast evaluation, that is, MAE, RMSE and MAPE. The values of forecast evaluation metrics given in Table 2 indicated that the forecast evaluation of the trend-fourier regression has lower values. This makes the proposed model forecast to have better accuracy than all other models considered.

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Table 1: Values of Coefficient and Adjusted Coefficient of Determination for Trend-Fourier Regression Model (Crude Oil Price)

Model(s)	Coefficient of determination	Adj. coefficient of determination
Linear trend regression (LTR)	0.500591	0.499744
Fourier regression (FR)	0.528539	0.526939
Trend-Fourier regression (TFR)	0.867312	0.883104
ARIMA	0.096202	0.090033

Table 2: Forecast Evaluation for Trend-Fourier Regression Model

Model(s)	MAE	RMSE	MAPE
Linear trend regression (LTR)	17..66684	22.074050	53.97510
Fourier regression (FR)	16.34374	21.44749	45.00058
Trend-Fourier regression (TFR)	16.14114	20.546664	42.05903
ARIMA	32.17205	44.83127	60.44573

CONCLUSION

The variation seen in time series is changing and several statistical methods have been used inappropriately to analyse these movements. But when the series is trend-cyclical, this was analysed appropriately using mixed time series regression and Fourier regression approach termed Trend-Fourier regression model. The model building for the mixed time series regression and Fourier approach involves four stages and these are model identification, estimation, diagnostics, and forecasting. The model was constructed using the coupling of time series regression and Fourier regression models. This time plot is fundamentally used to establish the variation exhibited by the time series datasets. The OLS method was used to estimate the models. The Durbin-Watson statistics, histogram of residuals and ACF of residuals were used to diagnose the models. The models were validated using distributional properties obtained, that is, the coefficient of determination used to determine the level of variation explained and the adjusted coefficient of determination used to determine model goodness of fit and forecast ability. The forecast evaluation metrics were further used to ascertain the accuracy of the models using MAE, RMSE and MAPE. The efficiency of the Trend-Fourier regression model was ascertained by modelling and forecasting Nigerian crude oil price time series datasets. This outperformed linear trend, Fourier regression and ARIMA models based on the values of coefficient of determination, adjusted coefficient of determination and forecast evaluation metrics. In essence, the proposed Trend-Fourier regression model is suitable for modelling and forecasting Nigerian monthly crude oil price time series datasets that exhibit trend-cyclical variation. Conclusively, the proposed Trend-Fourier regression model is a better alternative for analysing trend-cyclical time series datasets.

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