



SIMPLE REGRESSION MODELS: A COMPARISON USING CRITERIA MEASURES

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ABSTRACT: *The study is on simple regression models: a comparison using criteria measures. The source of the dataset used for this study was extracted from records of the Federal Medical Centre, Owerri, Imo State, on weight of babies and hemoglobin level of mothers. The response variable is weight of babies while the explanatory variable is hemoglobin level of mothers. Eleven simple regression models—Linear, Growth, Quadratic, Polynomial, Logarithmic, Hyperbolic, Power, Exponential Growth, Square Root, Sinusoidal and Arctangent—were stated and employed for the study. For ease of data analysis, E-views package was implemented. Three model selection criteria measures for comparison, known as Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC), were employed. The result of the study showed that, when it comes to analyzing the association between baby weight and mothers' hemoglobin levels, the exponential growth regression model performs better than the other ten models that were examined. Therefore, researchers should investigate other models that were not included in this analysis and compare the findings using goodness of fit metrics other than the criteria measures used in this work.*

KEYWORDS: Simple Nonlinear Regression, Simple Linear Regression, AIC, SIC, HQIC, Model Comparison.



INTRODUCTION

While fitting a simple linear model to data is rare because most data follow nonlinear models, simple regression model fitting is typically used in many scientific domains, including pharmaceutical and biochemical test quantification (Duong & Lim, 2023). There are nonlinear models and choosing the best model for the data requires a combination of expertise, understanding of the underlying mechanism, and statistical analysis of the fitting result (Esemokumo et al., 2020). Quantifying the validity of a fit using a metric that distinguishes between "good" and "bad" fits is crucial. When performing calibration experiments for samples to be measured, many researchers typically use a common measure known as the coefficient of determination (R^2) employed in linear regression (Montgomery et al., 2006).

Because values between 0 and 1 make it simple to grasp how much of the variation in the data is explained by the fit, this measure is therefore particularly intuitive from a linear perspective (Chicco et al., 2021). Many scientists and academics continue to utilize R^2 in studies pertaining to nonlinear data processing, despite the fact that it has been proven for some time to be an inappropriate metric for nonlinear regression (Berk, 2020). This problem had been highlighted by a number of earlier descriptions of R^2 being useless in nonlinear fitting, but they have presumably now been forgotten (Bartlett et al., 2020). This observation may be the result of the disparities in mathematical training between researchers and trained statisticians, who frequently use statistical techniques but lack in-depth statistical understanding (Spiess & Neumeier, 2010).

R^2 is not the best option in a nonlinear regime because, unlike in linear regression, the total sum-of-squares (TSS) is not equal to the regression sum-of-squares (REGSS) plus the residual sum-of-squares (RSS), and as a result, it lacks the appropriate interpretation. It has been stated that researchers arbitrarily use R^2 to evaluate the validity of a specific model when dealing with nonlinear data fit. One possible explanation for the prevalence of relying just on R^2 values to assess the validity of nonlinear models is that researchers may not be aware of this common misunderstanding.

This study only employed three criteria models known as the Akaike Information Criterion, Schwarz Information Criterion, and Hannan-Quinn Information Criterion for model selection, correct interpretation, and conclusion because using R^2 alone to assess the performance of nonlinear data analysis has been discouraged.

In terms of medicine, it has been demonstrated that a patient's weight and pulse rate have a linear relationship. But many researchers, particularly those in other fields where they most likely lack enough statistical skills, typically used the linear regression technique to find a relationship between these two variables without considering the nonlinear models. Because of this, the goal of this study is to compare several non-linear models with linear models in order to determine which model best fits the patient's weight and pulse rate based on the data collected for this investigation.



METHODOLOGY

Regression Models

Eleven Regression models were considered in this study, which are Linear, Growth, Quadratic, Polynomial, Logarithmic, Hyperbolic, Power, Exponential Growth, Square Root, Sinusoidal and Arctangent Regression models as written in Equations (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) and (11) respectively:

$$Y = \lambda_0 + \lambda_1 Z + \varepsilon \quad (1)$$

$$Y = \frac{\lambda_0 Z}{\lambda_1 + Z} + \varepsilon \quad (2)$$

$$Y = \lambda_0 + \lambda_1 Z + \lambda_2 Z^2 + \varepsilon \quad (3)$$

$$Y = \lambda_0 + \lambda_1 Z + \lambda_2 Z^2 + \lambda_3 Z^3 + \varepsilon \quad (4)$$

$$Y = \lambda_0 + \lambda_1 \ln(Z) + \varepsilon \quad (5)$$

$$Y = \lambda_0 + \lambda_1 (1/Z) + \varepsilon \quad (6)$$

$$Y = \lambda_0 Z^{\lambda_1} + \varepsilon \quad (7)$$

$$Y = \lambda_0 + \exp(\lambda_1 Z) + \varepsilon \quad (8)$$

$$Y = \lambda_0 + \lambda_1 \sqrt{z} + \varepsilon \quad (9)$$

$$Y = \lambda_0 + \lambda_1 \sin(Z) + \varepsilon \quad (10)$$

$$Y = \lambda_0 + \lambda_1 \arctan(\lambda_2 Z + \lambda_3) + \varepsilon \quad (11)$$

Simple Linear Regression

This is a regression line involving only two variables as it is applicable in this study. A widely used procedure for obtaining the regression line of Y and Z is the least square method.

The linear regression of Y on Z is stated in Equation (1)

If there are n pairs of sample observations $(Z_1, Y_1), (Z_2, Y_2), \dots, (Z_n, Y_n)$, then we get

$$Y_i = \lambda_0 + \lambda_1 Z_i + \varepsilon_i, \quad i = 1, 2, \dots, n \quad \dots \quad (12)$$

Then seeking for the estimators $\hat{\lambda}_0$ and $\hat{\lambda}_1$ of λ_0 and λ_1 respectively in such a way that P is minimized.

$$\text{Let } P = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (Y_i - \lambda_0 - \lambda_1 Z_i)^2 \quad \dots \quad (13)$$

Differentiate (13) partially w.r.t. λ_0 and λ_1 , to get Equations (14) and (15) respectively



$$\sum_{i=1}^n Y_i - n\lambda_0 - \lambda_1 \sum_{i=1}^n Z_i = 0 \quad \dots \quad (14)$$

$$\sum Z_i Y_i - \lambda_0 \sum Z_i - \lambda_1 \sum Z_i^2 = 0 \quad \dots \quad (15)$$

Solving Equations (14) and (15) simultaneously, we get

$$\hat{\lambda}_1 = \frac{n\sum Z_i Y_i - \sum Z_i Y_i}{n\sum Z_i^2 - (\sum Z_i)^2} \quad \dots \quad (16)$$

$$\hat{\lambda}_0 = \frac{\sum Z_i^2 \sum Z_i - \sum Z_i Y_i}{n\sum Z_i^2 - (\sum Z_i)^2} \quad \dots \quad (17)$$

The calculation is usually set out in ANOVA form as shown (see Table 1).

Table 1: Regression ANOVA Table

Variance	Degree of freedom	Sum of square	Mean square
Regression	1	$RSS = \lambda_1 \sum zy$	$RMS = \frac{RSS}{1}$
Error	$n - 2$	$ESS = TSS - RSS$	$EMS = \frac{ESS}{n - 2}$
Total	$n - 1$	$TSS = \sum y^2$	

In the same procedure, the parameters of other nonlinear models can be obtained.

Akaike Information Criterion (AIC)

The degree of goodness of fit for an assessed measurable equation is known as AIC (Maguilla et al., 2021) and it can be employed for model choice. It is scientifically characterized as:

$$AIC = \exp^n \frac{\sum \hat{e}_i^2}{n} = \exp^n \frac{SS_R}{n} \quad (18)$$

where p is the number of parameters with the inclusion of the intercept. Equation (18) is stated mathematically for convenience sake as:

$$\ln(AIC) = \left(\frac{2p}{n}\right) + \ln\left(\frac{SS_R}{n}\right) \quad (19)$$

Schwarz Information Criterion (SIC)

The degree of goodness of fit for an evaluated measurable equation is known as SIC (Obaji & Nwagor, 2021) and it can be employed for model choice. It is mathematically characterized as:

$$SIC = n^n \frac{\sum \hat{e}_i^2}{n} = n^n \frac{SS_R}{n} \quad (20)$$



The log of (20) gives (21):

$$\log_e(SIC) = \frac{p}{n} \log_e(n) + \log_e\left(\frac{SS_R}{n}\right) \quad (21)$$

Hannan-Quinn Information Criterion (HQIC)

The degree of goodness of fit for an evaluated measurable equation is known as HQIC (Obaji & Nwagor, 2021) and it can be utilized for model choice. It is mathematically characterized as:

$$HQIC = n \ln \frac{SS_E}{n} + 2p \ln(\ln n) \quad (22)$$

The equation with least AIC, SIC or HQIC value is chosen as the best model.

Analysis of Data

The dataset used for this study was extracted from the records of Federal Medical Centre, Owerri, Imo State, Nigeria and presented in Table 2.

Table 2: Weight of Babies and Hemoglobin Level of Mothers

S/ N	Weight of babies (Y)	Hemoglobin Level of Mothers (Z)	S/ N	Weight of babies (Y)	Hamoglobin Level of Mothers (Z)
1	3.6	14.7	41	2.8	7.7
2	3.1	13.6	42	3.3	7.9
3	3.7	12.2	43	3.1	8.9
4	3.8	14.8	44	3.2	9.4
5	3.0	11.7	45	3.2	5.7
6	3.2	12.1	46	3.4	14.7
7	2.9	7.5	47	3.0	10.1
8	3.1	12.5	48	2.5	8.9
9	2.5	11.2	49	3.6	9.7
10	2.6	12.7	50	2.9	7.4
11	3.7	12.9	51	3.2	9.4
12	2.4	10.8	52	2.6	8.4
13	2.6	11.1	53	2.3	5.7
14	2.7	11.6	54	2.3	14.7
15	3.7	12.1	55	3.0	13.0
16	3.1	5.5	56	2.9	10.1
17	2.8	10.5	57	2.9	7.3
18	3.2	10.9	58	4.0	6.3
19	3.0	10.1	59	3.4	9.5
20	2.5	8.9	60	3.3	12.3
21	3.6	9.7	61	3.3	10.9
22	2.8	7.4	62	2.8	9.9
23	3.2	9.4	63	3.3	10.8



24	2.6	8.4	64	3.4	11.5
25	2.3	5.7	65	3.2	10.3
26	2.8	11.7	66	2.7	8.9
27	3.2	13.4	67	2.9	9.9
28	2.9	10.1	68	3.0	10.7
29	2.7	7.3	69	2.8	7.7
30	4.2	12.3	70	3.3	10.9
31	3.4	9.5	71	3.1	8.9
32	3.3	8.3	72	3.0	8.3
33	2.9	10.9	73	2.5	8.1
34	2.5	9.9	74	3.6	9.7
35	3.3	10.8	75	2.9	7.4
36	3.4	13.5	76	3.2	9.5
37	3.2	13.3	77	2.6	8.4
38	2.7	7.9	78	2.3	5.7
39	2.9	9.9	79	3.8	14.7
40	3.0	10.7	80	3.1	13.0

Table 3: E-views Software Output for Linear Regression Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.375630	0.190979	12.43920	0.0000
Z	0.066374	0.018385	3.610266	0.0005
R-squared	0.143177	Mean dependent var	3.047500	
Adjusted R-squared	0.132193	S.D. dependent var	0.411842	
S.E. of regression	0.383656	Akaike info criterion	0.946543	
Sum squared resid	11.48099	Schwarz criterion	1.006094	
Log likelihood	-35.86174	Hannan-Quinn criter.	0.970419	
F-statistic	13.03402	Durbin-Watson stat	1.712895	
Prob(F-statistic)	0.000539			

**Table 4: E-views Software Output for Growth Regression Model**

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.802399	0.281716	13.49729	0.0000
C(2)	2.395918	0.894716	2.677855	0.0090
R-squared	0.130143	Mean dependent var	3.047500	
Adjusted R-squared	0.118991	S.D. dependent var	0.411842	
S.E. of regression	0.386564	Akaike info criterion	0.961641	
Sum squared resid	11.65565	Schwarz criterion	1.021192	
Log likelihood	-36.46565	Hannan-Quinn criter.	0.985517	
Durbin-Watson stat	1.743530			

Table 5: E-views Computer Software for Quadratic Regression Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.566495	0.674993	3.802253	0.0003
Z	0.026923	0.135020	0.199400	0.8425
Z ²	0.001932	0.006550	0.294964	0.7688
R-squared	0.144144	Mean dependent var	3.047500	
Adjusted R-squared	0.121914	S.D. dependent var	0.411842	
S.E. of regression	0.385922	Akaike info criterion	0.970414	
Sum squared resid	11.46804	Schwarz criterion	1.059740	
Log likelihood	-35.81656	Hannan-Quinn criter.	1.006227	
F-statistic	6.484229	Durbin-Watson stat	1.710524	
Prob(F-statistic)	0.002497			

**Table 6: E-views Software Output for Polynomial Regression Model**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.583073	2.453272	1.052909	0.2957
Z	0.021548	0.776344	0.027755	0.9779
Z ²	0.002485	0.078881	0.031500	0.9750
Z ³	-1.81E-05	0.002578	-0.007033	0.9944
R-squared	0.144145	Mean dependent var	3.047500	
Adjusted R-squared	0.110361	S.D. dependent var	0.411842	
S.E. of regression	0.388452	Akaike info criterion	0.995413	
Sum squared resid	11.46803	Schwarz criterion	1.114515	
Log likelihood	-35.81654	Hannan-Quinn criter.	1.043165	
F-statistic	4.266698	Durbin-Watson stat	1.709922	
Prob(F-statistic)	0.007712			

Table 7: E-views Software Output for Logarithmic Regression Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.618701	0.408810	3.959541	0.0002
LOG(Z)	0.624862	0.177792	3.514564	0.0007
R-squared	0.136711	Mean dependent var	3.047500	
Adjusted R-squared	0.125643	S.D. dependent var	0.411842	
S.E. of regression	0.385101	Akaike info criterion	0.954062	
Sum squared resid	11.56764	Schwarz criterion	1.013612	
Log likelihood	-36.16247	Hannan-Quinn criter.	0.977937	
F-statistic	12.35216	Durbin-Watson stat	1.731528	
Prob(F-statistic)	0.000737			

**Table 8: E-views Software Output for Hyperbolic Regression Model**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.605930	0.173181	20.82175	0.0000
1/Z	-5.331010	1.600603	-3.330625	0.0013
R-squared	0.124511	Mean dependent var	3.047500	
Adjusted R-squared	0.113287	S.D. dependent var	0.411842	
S.E. of regression	0.387813	Akaike info criterion	0.968095	
Sum squared resid	11.73112	Schwarz criterion	1.027646	
Log likelihood	-36.72381	Hannan-Quinn criter.	0.991971	
F-statistic	11.09307	Durbin-Watson stat	1.761924	
Prob(F-statistic)	0.001326			

Table 9: E-views Software Output for Power Regression Model

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.881995	0.260943	7.212279	0.0000
C(2)	0.210221	0.059695	3.521559	0.0007
R-squared	0.138580	Mean dependent var	3.047500	
Adjusted R-squared	0.127536	S.D. dependent var	0.411842	
S.E. of regression	0.384684	Akaike info criterion	0.951895	
Sum squared resid	11.54260	Schwarz criterion	1.011446	
Log likelihood	-36.07581	Hannan-Quinn criter.	0.975771	
Durbin-Watson stat	1.725146			

**Table 10: E-views Software Output for Exponential Growth Regression Model**

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.497585	0.137485	10.89269	0.0000
C(2)	0.042796	0.008138	5.258567	0.0000
R-squared	0.144073	Mean dependent var	3.047500	
Adjusted R-squared	0.133100	S.D. dependent var	0.411842	
S.E. of regression	0.383456	Akaike info criterion	0.945497	
Sum squared resid	11.46899	Schwarz criterion	1.005048	
Log likelihood	-35.81990	Hannan-Quinn criter.	0.969373	
F-statistic	13.12928	Durbin-Watson stat	1.711333	
Prob(F-statistic)	0.000516			

Table 11: E-views Software Output for Square Root Regression Model

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.745097	0.366927	4.755986	0.0000
C(2)	0.412191	0.115328	3.574068	0.0006
R-squared	0.140723	Mean dependent var	3.047500	
Adjusted R-squared	0.129706	S.D. dependent var	0.411842	
S.E. of regression	0.384206	Akaike info criterion	0.949404	
Sum squared resid	11.51389	Schwarz criterion	1.008955	
Log likelihood	-35.97617	Hannan-Quinn criter.	0.973280	
F-statistic	12.77396	Durbin-Watson stat	1.720299	
Prob(F-statistic)	0.000607			

**Table 12: E-views Software Output for Sinusoidal Regression Model**

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.047498	0.046335	65.77086	0.0000
C(2)	-0.007996	0.064980	-0.123052	0.9024
R-squared	0.000194	Mean dependent var	3.047500	
Adjusted R-squared	-0.012624	S.D. dependent var	0.411842	
S.E. of regression	0.414433	Akaike info criterion	1.100874	
Sum squared resid	13.39690	Schwarz criterion	1.160424	
Log likelihood	-42.03495	Hannan-Quinn criter.	1.124749	
F-statistic	0.015142	Durbin-Watson stat	1.886517	
Prob(F-statistic)	0.902383			

Table 13: E-views Software Output for Arctangent Regression Model

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.830565	28573.27	0.000134	0.9999
C(2)	37.15154	678694.2	5.47E-05	1.0000
C(3)	0.001787	32.61915	5.48E-05	1.0000
C(4)	-0.039174	76.22436	-0.000514	0.9996
R-squared	0.143180	Mean dependent var	3.047500	
Adjusted R-squared	0.109358	S.D. dependent var	0.411842	
S.E. of regression	0.388671	Akaike info criterion	0.996540	
Sum squared resid	11.48096	Schwarz criterion	1.115642	
Log likelihood	-35.86162	Hannan-Quinn criter.	1.044292	
F-statistic	4.233357	Durbin-Watson stat	1.712888	
Prob(F-statistic)	0.008026			

**Table 14: Summary Result of Different Regression Models**

Model	AIC	SIC	HQIC
Linear Regression	0.9465	1.0061	0.9704
Growth Regression	0.9616	1.0212	0.9855
Quadratic Regression	0.9704	1.0597	1.0062
Polynomial Regression	0.9954	1.1145	1.0431
Logarithmic Regression	0.9541	1.0136	0.9779
Hyperbolic Regression	0.9681	1.0276	0.9920
Power Regression	0.9519	1.0114	0.9758
Exponential Growth Regression	0.9455	1.0050	0.9694
Square Root Regression	0.9494	1.0090	0.9733
Sinusoidal Regression	1.1009	1.1604	1.1247
Arctangent Regression	0.9965	1.1156	1.0443

Source: *E-views Software*

Table 14 shows that the polynomial regression model had the lowest HQIC (0.9694), SIC (1.0050), and AIC (0.9455) criteria measures. This suggests that the exponential growth regression model is the most effective model using the dataset employed in this study. The linear regression model—whose criteria scores for AIC is 0.9465, BIC is 1.0061, and HQIC is 0.9704—is the second-best model. Once more, the least performed equation is the sinusoidal regression model, which has the highest HQIC (1.1247), SIC (1.1604), and AIC (1.1009).

CONCLUSION AND RECOMMENDATION

The result of the study showed that, when it comes to analyzing the association between baby weight and mothers' hemoglobin levels, the exponential growth regression model performs better than the other ten models that were examined. Therefore, researchers should investigate other models that were not included in this analysis and compare the findings using goodness of fit metrics other than the criteria measures used in this work.

REFERENCES

- Bartlett, P. L., Long, P. M., Lugosi, G. & Tsigler, A. (2020). Benign overfitting in linear regression. *Proceedings of the National Academy of Sciences of the USA* 117(48):30063–30070.
- Berk, R. A. (2020). Statistical learning as a regression problem. In: *statistical learning from a regression perspective*. Berlin: Springer International Publishing, 1–72.
- Chicco, D., Warrens, M. J. & Jurman, G. (2021). The coefficient of determination R-squared is more informative than SMAPE, MAE, MAPE, MSE and RMSE in regression analysis evaluation. *PeerJ Computer Science*, 7(2021), 1-24.
- Duong, C. M. & Lim, T.T. (2023). Use of regression models for development of a simple and effective biogas decision-support tool. *Scientific Report*, 13(2023), 1-11.
- Esemokumo, A. P., Bekesuoyeibo, M. & Nwobi, A. C. (2020). Model selection in bivariate regression models. *International Journal of Applied Science*, 3(4), 1-8.



- Esemokumo, P. E. (2023). Asymmetric distributions and nonlinear functions in a canonical correlation analysis using simulated and real-life medical data. An unpublished PhD Thesis submitted to the department of Mathematics and Statistics, Ignatius Ajuru University of Education Rivers State.
- Maguilla, E., Escudero, M., Jiménez-Lobato, V., Díaz-Lifante, Z., Andrés-Camacho, C. & Arroyo, J. (2021). Polyploidy expands the range of centaurium (Gentianaceae). *Frontiers in Plant Science*, 12(2021), 1-12.
- Montgomery, D. C., Peck, E. A. & Vining, G. G. (2006). Introduction to Linear Regression Analysis. Wiley & Sons, Hoboken.
- Obaji, I. & Nwagor, P. (2021). Multiple regression model selection via birth weight, mother age and gestation variables. *International Journal of Statistics and Applied Mathematics*, 6(6), 83-90.
- Spiess, A. & Neumeyer, N. (2010). An evaluation of R^2 as an inadequate measure for nonlinear models in pharmacological and biochemical research: a Monte Carlo approach. *BMC Pharmacol*, 10(2010), 34-45.