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FORMULATION OF GAME MODEL AS A LINEAR PROGRAMMING PROBLEM USING VARIOUS MODELS

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ABSTRACT: *Game theory is the examination of strategic interactions between two or more individuals known as players who act based on their individual self-interest within a framework known as the game. Every player possesses a set of possible actions referred to as strategies, from which they make selections. In a two-person zero sum game, each of the two players has at least two strategies. In such a game problem where both players have no inferior strategies, we can determine the optimal mixed strategies of the game problem by converting it to a linear programming problem and solving it using the simplex method or variations of it. In this paper, consideration of some existing models along with our proposed model on the conversion of game problem to LPP was made. We compared the results across the various models considered. The results obtained revealed that our proposed model on the conversion of game problem to LPP produced a higher value of the game compared to the others considered; and thus, produced better performance.*

KEYWORDS: Game Theory, Linear Programming Formulation, linear programming problem, simplex method.

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INTRODUCTION

The search for optimal solutions has always fascinated humanity, leading to the development of various operations and strategies across different fields. Mathematics has a long history of optimization, dating back to Euclid's work. However, it was the contributions of renowned mathematicians such as Newton, Lagrange, and Cauchy to the development of Differential Calculus that paved the way for optimization techniques in certain problems. Since then, the introduction of high-speed digital computers in the 20th century, saw significant progress in development of new optimization methods and vast implementation of these new methods, as well as existing ones. A wide range of optimization problems is addressed by techniques under Mathematical programming.

Mathematical programming, particularly linear programming, is a numerical technique that seeks optimal solutions through iterative processes consisting of a finite number of steps starting from an initial solution. Dantzig's formulation of linear forms with constraints gave birth to the simplex method, beneficial for solving allocation problems involving limited resources and effectiveness. Certain optimization problems, especially in economic situations can be treated as games, where competitors vie for conflicting interests based on controlled and uncontrolled variables. These optimization problems, known as game problems, are closely tied to linear programming. Game theory, founded by John von Neuman, plays a vital role in understanding such scenarios. Game theory is a mathematical discipline and as such has its own scientific interest, independent of any applications. Its character has been explored rather deeply during the last two decades.

Game theory examines decision-making in conflict situations, addressing problems where the decision-maker lacks complete control over the factors that influence outcome. In a game problem, people with different goals are connected, making it more challenging than simple maximization. In game theory, individuals need to figure out how to achieve the best results, considering others with different goals whose actions affect everyone.

Decision-makers in games face a tricky maximization problem, needing to plan for optimal outcomes while keeping an eye on what their opponents might do. A game is defined by its players/decision makers, the rules of the game, the resulting payoffs, the values assigned to these payoffs, and the variables controlled by each player. In a game, a player makes decisions independently. A player is not necessarily one person; it may be a group of individuals acting in an organization, a firm, or an army. The key characteristic of a player is their specific objective within the game, and that they act autonomously to pursue that objective.

The game's outcome depends on the strategies used by each player. The set of possible strategies for the $j\ th$ player, denoted as Q, includes all potential actions, considering their resources and accounting for possible moves by opponents. The two-person zero-sum game is characterized by the principle that one player's loss equals the other player's gain. The key features of this game can be illustrated using a payoff matrix. The graphical method and the method of Linear Programming exist for solving games of mixed strategy. The graphical method can be used only for game problems whose payoff matrices are of order 2×2, $2 \times n$ or $m \times 2$. (Ekoko, 2016). When both players have more than two operational strategies with no inferior strategies then we can determine the optimal mixed strategies of the game problem by converting it to a linear programming problem and solving the LP problem by the appropriate method.

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It is needful to add here that just like game problem has been formulated as an LPP, there are other operations research techniques that have been formulated as an LPP. These include formulation of transportation problem and dynamic programming problem as LP problem. In the literature there abound many research works on the conversion of game problem into linear programming problem. Notably among these are Hillier and Lieberman (2020), Taha (2017) and Ekoko (2016). The reason for the formulation of game problem, dynamic programming problem, transportation problem, etc. to LP problem is not unconnected with the fact that linear programming is one of the most applicable areas of operations research. More so, there are various computer programs that are available to solve LP problems using the simplex method or variations of it.

The aim of this research is to examine the conversion of the game problem to a linear programming problem, study and implement various models of this conversion so as to ascertain the model with best performance. The objectives of the study therefore are;

i. To convert a game problem to a linear programming problem of various forms

ii. To solve games of mixed strategy by the LPP method

iii. To compare the results obtained by solving the various LP models from game problem in (i) above

LITERATURE REVIEW

The applications of game theory span beyond economics, influencing fields such as political science, psychology, biology, and computer science. Game theory has been used by authors in solving or addressing real life problem.

Wolford (2019) used game theory and international relations to analyze the complex historical conflict of World War I. He presented thirteen historical puzzles related to various aspects of the war, such as its outbreak, attrition, unrestricted submarine warfare, and the entry of the United States into the conflict. Through guided exercises, he showed how game theoretical models can provide insights into these strategic puzzles, offering a deeper understanding of the role of individual leaders, coalition cooperation, the impact of international law, conflict resolution, and the challenges of achieving peace. Spaniel (2014) also explored the application of game theory in solving simple models of war.

Ahrabi (2022) examined Russia's 2022 invasion of Ukraine using game theory, analyzing various war scenarios based on realistic strategies available to both Russia and the West, resulting in a simultaneous-move 3x3 non-cooperative game. The players are categorized as approachable or aggressive for the West and militant or insecure for Russia, affecting their preferences over available strategies. Four different non-cooperative games are modeled and solved to find their Nash equilibria. Furthermore, in the study Ukraine is introduced as a third player with the potential to influence the war's outcome. The objective is to predict possible future outcomes of the ongoing conflict and gain insights into the world's future economic and political landscape

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Brams (2011) demonstrates how game theory can shed light on the rational choices made by characters in literary texts and provide strategic insights in law, history, and philosophy. He introduced the theory of moves (TOM), which is rooted in game theory, he applied it to help analyze the dynamics of player choices, which includes their misperceptions, and the various types of power they employ. Through the lens of TOM, Brams (2011) examines several intriguing topics. For example, he explores the payoff matrix of Pascal's wager on the existence of God, the strategic games played by presidents and Supreme Court justices, and the gradual revelation of information in the game played by Hamlet and Claudius.

Carraro and Fragnelli (2004) offered a comprehensive overview of the application of game theory to address environmental issues. They delve into the application of game theory to address challenges related to climate negotiations and policy, the equitable sharing of environmental costs, and environmental management and pollution control. By applying game theory, they propose innovative tools and strategies to support decision-making in environmental policy. Additionally, they offer insights into effective environmental management and pollution control, emphasizing the role of game theory in optimizing resource allocation and fostering cooperation. Corchón (2013) sought to investigate the theory of imperfectly competitive markets, focusing on the strategic interactions among firms, utilizing equilibrium concepts from Game Theory to analyze this issue.

Ekoko (2016) posited that a significant portion of game theory research has focused on two-person zero-sum games. These games involve only two adversaries or players. The term "zero-sum" indicates that one player's gains are achieved at the cost of the other player, resulting in a net sum of zero. Game problems can be categorized based on the methods used to find the best strategies for players. There are two main classifications: games of pure strategies and games of mixed strategies. In games of pure strategies, each player consistently employs a specific pure strategy based on a given payoff table. On the other hand, in games of mixed strategies, players use their pure strategies in varying proportions. He went further to discuss the methods of dominated strategy and maximin and minimax criteria as methods for solving games of pure strategies; then the methods of graph, algebraic evaluation and linear programming as the method for solving games of mixed strategies.

METHODS

When both m and n are greater than two (i.e., $m, n > 2$), the graphical method breaks down and cannot be used. In such a case, an alternative procedure when both players have more than two operational strategies with no inferior strategies is to convert the problem of finding an optimum mixed strategy into a LP problem. The primary benefit of employing linear programming techniques is their capability to address mixed-strategy games with larger dimension payoff matrices.

To illustrate the conversion of a game problem into a linear programming problem, consider a payoff matrix of size $m \times n$, as shown below (Table 1)

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Table 1:

Player A's strategies		Player B's strategies: B_1	B_2	...	B_n
	A_1	a_{11}	a_{12}	...	a_{1n}
	A_2	a_{21}	a_{22}	...	a_{2n}
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	A_m	a_{m1}	a_{m2}	...	a_{mn}

Let a_{ij} be the element in the ith row and jth column of game payoff matrix. Also, let x_i, be the probability of A using his ith strategy $(i = 1,2,\dots,m)$. Player A's expected gains when player B uses his jth strategy is

$$E_j = \sum_{i=1}^{m} x_i a_{ij}\,, j = 1,2,\dots,n \tag{1}$$

The value of the game, V, represents the expected payoff at the conclusion of the game, assuming each player employs their optimal strategy. Player A's objective is to choose a set of strategies with probabilities x_i, $(i = 1,2,\dots,m)$ in each play of the game, aiming to maximize his minimum expected gains.

With such an objective, the expected payoff to player A for each strategy selected by player B must be at least V. Thus, the LPP representing the player A problem can be stated as:

$$\left.\begin{array}{l}
\text{Maximize } V \\
\text{subject to} \\
a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \geq V \\
a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \geq V \\
\vdots \\
a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \geq V \\
\text{where } x_1 + x_2 + \cdots + x_m = 1 \\
\text{and } x_i \geq 0\ \forall\, i
\end{array}\right\} \quad (2)$$

Since $V > 0$, we can divide both sides of the n inequalities and equation by V. We also let $\frac{x_i}{V} = s_i \geq 0\ \forall\, i$; and the objective of player A which is to maximize the value of the game, V be

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given as minimize $\frac{1}{V}$ (they are both equivalent). This is done since all the linear inequalities are of the type "≥".

Since, negative a_{ij} could result in V being negative also; one sure way to make V positive is to add a positive constant to every element a_{ij} of the original payoff matrix. This constant should be sufficiently large to render the new payoff matrix with non-negative a_{ij}, resulting in a non-negative V. Such a change to all elements of the payoff matrix will just change V by the added amount. The optimal x_is will remain the same.

Momentarily we assume that all a_{ij}s we deal with do not return a negative V.

The resulting linear programming can be stated as:

$$\left.\begin{aligned}
&\text{Minimize } z = \frac{1}{V} = s_1 + s_2 + \cdots + s_m \\
&\text{subject to} \\
&a_{11}s_1 + a_{21}s_2 + \cdots + a_{m1}s_m \geq 1 \\
&a_{12}s_1 + a_{22}s_2 + \cdots + a_{m2}s_m \geq 1 \\
&\vdots \\
&a_{1n}s_1 + a_{2n}s_2 + \cdots + a_{mn}s_m \geq 1 \\
&\text{and } s_i = \frac{x_i}{V} \geq 0 \ \forall \ i
\end{aligned}\right\} \quad (3)$$

Note that the last linear constraint, $\frac{x_1}{V} + \frac{x_2}{V} + \cdots + \frac{x_m}{V} = \frac{1}{V}$ (also written as $s_1 + s_2 + \cdots + s_m = \frac{1}{V}$) in system (3) was used to modify the objective function of the LPP (4) above, and thereafter excluded from the other linear constraints.

In this paper, the preceding result (given ultimately in (4)) obtained from the above conversion procedure of a game problem to an LP problem is our proposed model.

However, Dorfman et al. (1987) presented the following two other forms of the conversion of game problem to LP problem.

$$\left.\begin{aligned}
&\text{Minimize } z = s_1 + s_2 + \cdots + s_m \\
&\text{subject to} \\
&a_{11}s_1 + a_{21}s_2 + \cdots + a_{m1}s_m \geq 1 \\
&a_{12}s_1 + a_{22}s_2 + \cdots + a_{m2}s_m \geq 1 \\
&\vdots \\
&a_{1n}s_1 + a_{2n}s_2 + \cdots + a_{mn}s_m \geq 1
\end{aligned}\right\} \quad (4)$$

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$s_1 + s_2 + \cdots + s_m = 1$

$s_1, s_2, \dots, s_m \geq 0$

$$\left.\begin{array}{l} \text{Minimize } z = s_1 + s_2 + \cdots + s_m \\ \text{subject to} \\ a_{11}s_1 + a_{21}s_2 + \cdots + a_{m1}s_m \geq 1 \\ a_{12}s_1 + a_{22}s_2 + \cdots + a_{m2}s_m \geq 1 \\ \vdots \\ a_{1n}s_1 + a_{2n}s_2 + \cdots + a_{mn}s_m \geq 1 \\ s_1 + s_2 + \cdots + s_m \geq 1 \\ s_1, s_2, \dots, s_m \geq 0 \end{array}\right\} \quad (5)$$

The LPP (4) is the same as (5) except that it still retains the last linear constraint. But for the LPP (6), the last constraint was altered; the "=" was replaced with "≥". This is also congruent with the general linear programming problem of the minimization type. It is permissible to make such a change as long as V is positive. There will always be an optimal solution, V and optimal values for x_i for which the equality part of "≥" is fulfilled (Dorfman et al., 1987).

Hitherto, we have been concerned with obtaining the optimal value for the probability, x_i of player A using his ith strategy. On the flip side, the LPP representing the player B problem can be stated as:

$$\left.\begin{array}{l} \text{Maximize } z^* = \frac{1}{V} = t_1 + t_2 + \cdots + t_n \\ \text{subject to} \\ a_{11}t_1 + a_{12}t_2 + \cdots + a_{1n}t_n \leq 1 \\ a_{21}t_1 + a_{22}t_2 + \cdots + a_{2n}t_n \leq 1 \\ \vdots \\ a_{m1}t_1 + a_{m2}t_2 + \cdots + a_{mn}t_n \leq 1 \\ \text{where } t_1, t_2, \dots, t_n \geq 0 \end{array}\right\} \quad (6)$$

Note that the LP problem for player B in system (6) is the dual of LP problem for player A in system (5) and vice versa. Therefore, we can obtain the solution of one from the other. By the Duality Theorem, $z = z^*$ when both players use their optimal strategies.

Just like the player A problem was expressed in Dorfman et al. (1987) game to LPP conversion forms, so can player B's problem be expressed correspondingly.

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ILLUSTRATION

As an illustration, let us solve the following game problem whose payoff matrix is given as

$$\mathbf{A}\overset{\mathbf{B}}{\begin{bmatrix} 3 & 7 & 4 \\ 1 & 6 & 8 \\ 5 & 2 & 3 \end{bmatrix}} \qquad (7)$$

Solution

Upon examination, the payoff matrix has no inferior strategy for either player. Also, Player A's maximin payoff is 3 and player B's minimax payoff is 5, i.e., the game has no equilibrium point (maximin ≠ minimax); therefore, the game is a mixed strategy game.

Following from the above discussion, the conversion of the game problem of player **A** to an LPP is given as;

$$\left.\begin{aligned} &\text{Minimize } z = s_1 + s_2 + s_3 \\ &\text{subject to} \\ &3s_1 + s_2 + 5s_3 \geq 1 \\ &7s_1 + 6s_2 + 2s_3 \geq 1 \\ &4s_1 + 8s_2 + 3s_3 \geq 1 \\ &s_1, s_2, s_3 \geq 0 \end{aligned}\right\} \qquad (8)$$

The above LPP can be solved using the Two-phase simplex method. The optimal solution to the game problem of player A is given as follows,

The probability of player A using his ith strategy $(i = 1,2,\dots,m)$

$$x_i = Vs_i \qquad (9)$$

and the value of the game, V is given as $V = \frac{1}{z}$ or $\frac{1}{s_1+s_2+s_3}$ (since $s_i = \frac{x_i}{V}$); $i = 1, 2, 3$ (10)

On the other hand, the player **B**'s problem is obtained by stating the dual of the LPP in system (8):

$$\left.\begin{aligned} &\text{Maximize } z^* = t_1 + t_2 + t_3 \\ &\text{subject to} \\ &3t_1 + 7t_2 + 4t_3 \leq 1 \\ &t_1 + 6t_2 + 8t_3 \leq 1 \\ &5t_1 + 2t_2 + 3t_3 \leq 1 \\ &t_1, t_2, t_3 \geq 0 \end{aligned}\right\} \qquad (11)$$

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Where $t_j = \frac{y_j}{V}$ or $y_j = Vt_j, \forall\, j$

The above LPP can be solved using the simplex method.

- **Solution using the Simplex Method (obtained using TORA software)**

Tableau 1

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	R.H.S.
t_4	3	7	4	1	0	0	1
t_5	1	6	8	0	1	0	1
t_6	5	2	3	0	0	1	1
z^*	-1	-1	-1	0	0	0	0

Tableau 2

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	R.H.S.
t_4	0	5.8	2.2	1	0	-0.6	0.4
t_5	0	5.6	7.4	0	1	-0.2	0.8
t_1	1	0.4	0.6	0	0	0.2	0.2
z^*	0	-0.6	-0.4	0	0	0.2	0.2

Tableau 3

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	R.H.S.
t_2	0	1	0.3793	0.1724	0	-0.1034	0.069
t_5	0	0	5.2759	-0.9655	1	0.3793	0.4138
t_1	1	0	0.4483	-0.069	0	0.2414	0.1724
z^*	0	0	-0.1724	0.1034	0	0.1379	0.2414

Tableau 4

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	R.H.S.
t_2	0	1	0	0.2418	-0.0719	-0.1307	0.0392
t_3	0	0	1	-0.183	0.1895	0.0719	0.0784
t_1	1	0	0	0.0131	-0.085	0.2092	0.1373

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z^*	0	0	0	0.0719	0.0327	0.1503	0.2549

In Tableau 4, the optimality conditions are satisfied, since all values in the objective row are ≥ 0. Thus, the optimal solution to the LPP (11) is $t_1 = 0.1373$, $t_2 = 0.0392$, $t_3 = 0.0784$, $z^* = 0.2549$.

And by the duality theorem, the optimal objective value to the LPP (8) is $z = 0.2549$; and $s_1 = 0.0719$, $s_2 = 0.0327$, $s_3 = 0.1503$.

Therefore, the value of the game, $V = \frac{1}{z} = \frac{1}{0.2549} = 3.9231$

Now, the probability of player **A** using his 1st, 2nd and 3rd strategy is given as; $x_1 = Vs_1 = 0.2821$, $x_2 = Vs_2 = 0.1283$, and $x_3 = Vs_3 = 0.5896$ respectively.

On the other hand, the probability of player **B** using his 1st, 2nd and 3rd strategy is given as; $y_1 = Vt_1 = 0.5386$, $y_2 = Vt_2 = 0.1538$, and $y_3 = Vt_3 = 0.3076$ respectively.

Again, the result obtained above is for our research's proposed model.

As discussed earlier, there are two other forms of the conversion of game problem to LP problem:

i. Conversion form 1 (Dorfman et al., 1987)

$$\left.\begin{aligned}
&\text{Minimize } z = s_1 + s_2 + s_3\\
&\text{subject to}\\
&3s_1 + s_2 + 5s_3 \geq 1\\
&7s_1 + 6s_2 + 2s_3 \geq 1\\
&4s_1 + 8s_2 + 3s_3 \geq 1\\
&s_1 + s_2 + s_3 = 1\\
&s_1, s_2, s_3 \geq 0
\end{aligned}\right\} \quad (12)$$

PHASE I

Tableau 1

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
$\bar{s}_7$	3	1	5	-1	0	0	1	0	0	0	1
$\bar{s}_8$	7	6	2	0	-1	0	0	1	0	0	1
$\bar{s}_9$	4	8	3	0	0	-1	0	0	1	0	1

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$\bar{s}_{10}$	1	1	1	0	0	0	0	0	0	1	1
$-w$	-15	-16	-11	1	1	1	0	0	0	0	-4

Tableau 2

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
$\bar{s}_7$	2.5	0	4.625	-1	0	0.125	1	0	-0.125	0	0.875
$\bar{s}_8$	4	0	-0.25	0	-1	0.75	0	1	-0.75	0	0.25
s_2	0.5	1	0.375	0	0	-0.125	0	0	0.125	0	0.125
$\bar{s}_{10}$	0.5	0	0.625	0	0	0.125	0	0	-0.125	1	0.875
$-w$	-7	0	-5	1	1	-1	0	0	2	0	-2

Tableau 3

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
$\bar{s}_7$	0	0	4.7813	-1	0.625	-0.3438	1	-0.625	0.3438	0	0.7188
s_1	1	0	-0.0625	0	-0.25	0.1875	0	0.25	-0.1875	0	0.0625
s_2	0	1	0.4063	0	0.125	-0.2188	0	-0.125	0.2188	0	0.0938
$\bar{s}_{10}$	0	0	0.6563	0	0.125	0.0313	0	-0.125	-0.0313	1	0.8438
$-w$	0	0	-5.4375	1	-0.75	0.3125	0	0.125	0.6875	0	-1.5625

Tableau 4

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
s_3	0	0	1	-0.2092	0.1307	-0.0719	0.2092	-0.1307	0.0719	0	0.1503
s_1	1	0	0	-0.0131	-0.2418	0.183	0.0131	0.2418	-0.183	0	0.0719
s_2	0	1	0	0.085	0.0719	-0.1895	-0.085	-0.0719	0.1895	0	0.0327
$\bar{s}_{10}$	0	0	0	0.1373	0.0392	0.0784	-0.1373	-0.0392	-0.0784	1	0.7451
$-w$	0	0	0	-0.1373	-0.0392	-0.0784	1.1373	1.0392	1.0784	0	-0.7451

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Tableau 5

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
s_3	0	2.4615	1	0	0.3077	-0.5385	0	-0.3077	0.5385	0	0.2308
s_1	1	0.1538	0	0	-0.2308	0.1538	0	0.2308	-0.1538	0	0.0769
s_4	0	11.7692	0	1	0.8462	-2.2308	-1	-0.8462	2.2308	0	0.3846
$\bar{s}_{10}$	0	-1.6154	0	0	-0.0769	0.3846	0	0.0769	-0.3846	1	0.6923
$-w$	0	1.6154	0	0	0.0769	-0.3846	1	0.9231	1.3846	0	-0.6923

Tableau 6

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
s_3	3.5	3	1	0	0.3077	0	0	0.5	0	0	0.5
s_6	6.5	1	0	0	-0.2308	1	0	1.5	-1	0	0.5
s_4	14.5	14	0	1	0.8462	0	-1	2.5	0	0	1.5
$\bar{s}_{10}$	-2.5	-2	0	0	-0.0769	0	0	-0.5	0	1	0.5
$-w$	2.5	2	0	0	-0.5	0	1	1.5	1	0	-0.5

Tableau 7

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	$\bar{s}_7$	$\bar{s}_8$	$\bar{s}_9$	$\bar{s}_{10}$	R.H.S.
s_3	1	1	1	0	0	0	0	0	0	1	1
s_6	-1	-5	0	0	0	1	0	0	-1	3	2
s_4	2	4	0	1	0	0	-1	0	0	5	4
s_5	-5	-4	0	0	1	0	0	-1	0	2	1
$-w$	0	0	0	0	0	0	1	1	1	1	0

PHASE II

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Tableau 8

B.V.	s_1	s_2	s_3	s_4	s_5	s_6	R.H.S.
s_3	1	1	1	0	0	0	1
s_6	-1	-5	0	0	0	1	2
s_4	2	4	0	1	0	0	4
s_5	-5	-4	0	0	1	0	1
z	0	0	0	0	0	0	1

Therefore, the optimal solution to the LPP (12) is $s_3 = 1$, $s_2 = s_1 = 0$, $z = 1$; and the value of the game, $V = \frac{1}{z} = \frac{1}{1} = 1$. Now, the probability of player **A** using his 1st, 2nd and 3rd strategy is given as; $x_1 = Vs_1 = 0$, $x_2 = Vs_2 = 0$, and $x_3 = Vs_3 = 1$ respectively.
On the other hand, to obtain the solution player **B** problem, we considered the dual of (12):

$$\left.\begin{array}{l} \text{Maximize } z = t_1 + t_2 + t_3 + t_4 \\ \text{subject to} \\ 3t_1 + 7t_2 + 4t_3 + t_4 \leq 1 \\ t_1 + 6s_2 + 8t_3 + t_4 \leq 1 \\ 5t_1 + 2s_2 + 3t_3 + t_4 \leq 1 \\ t_1, t_2, t_3 \geq 0,\ t_4 \text{ is unrestricted in sign} \end{array}\right\} \quad (13)$$

Tableau 1

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_5	3	7	4	1	1	0	0	1
t_6	1	6	8	1	0	1	0	1
t_7	5	2	3	1	0	0	1	1
z^*	-1	-1	-1	-1	0	0	0	0

Tableau 2

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_5	0	5.8	2.2	0.4	1	0	-0.6	0.4
t_6	0	5.6	7.4	0.8	0	1	-0.2	0.8
t_1	1	0.4	0.6	0.2	0	0	0.2	0.2
z^*	0	-0.6	-0.4	-0.8	0	0	0.2	0.2

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Tableau 3

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_4	0	14.5	5.5	1	2.5	0	−1.5	1
t_6	0	−6	3	0	−2	1	1	0
t_1	1	−2.5	−0.5	0	−0.5	0	0.5	0
z^*	0	11	4	0	2	0	−1	1

Tableau 4

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_4	0	5.5	10	1	−0.5	1.5	0	1
t_7	0	−6	3	0	−2	1	1	0
t_1	1	0.5	−2	0	0.5	−0.5	0	0
z^*	0	5	7	0	0	1	0	1

Therefore, the optimal solution to the LPP (13) is $t_4 = 1$, $t_3 = t_2 = t_1 = 0$, $z = 1$; and the value of the game, $V = \frac{1}{z^*} = \frac{1}{1} = 1$. Now, the probability of player **B** using his 1st, 2nd, 3rd and 4th strategy is given as; $y_1 = Vt_1 = 0$, $y_2 = Vt_2 = 0$, $y_3 = Vt_3 = 0$ and $y_4 = Vt_4 = 1$ respectively.

ii. Conversion form 2 (Dorfman et al., 1987)

$$\left.\begin{aligned}
&\text{Minimize } z = s_1 + s_2 + s_3 \\
&\text{subject to} \\
&3s_1 + s_2 + 5s_3 \geq 1 \\
&7s_1 + 6s_2 + 2s_3 \geq 1 \\
&4s_1 + 8s_2 + 3s_3 \geq 1 \\
&s_1 + s_2 + s_3 \geq 1 \\
&s_1, s_2, s_3 \geq 0
\end{aligned}\right\} \quad (14)$$

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Solution

For computational efficiency, using the simplex algorithm we solved to optimality the dual of the above LPP, and the following result was obtained.

Tableau 1

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_5	3	7	4	1	1	0	0	1
t_6	1	6	8	1	0	1	0	1
t_7	5	2	3	1	0	0	1	1
z^*	-1	-1	-1	-1	0	0	0	0

Tableau 2

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_5	0	5.8	2.2	0.4	1	0	-0.6	0.4
t_6	0	5.6	7.4	0.8	0	1	-0.2	0.8
t_1	1	0.4	0.6	0.2	0	0	0.2	0.2
z^*	0	-0.6	-0.4	-0.8	0	0	0.2	0.2

Tableau 3

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_4	0	14.5	5.5	1	2.5	0	-1.5	1
t_6	0	-6	3	0	-2	1	1	0
t_1	1	-2.5	-0.5	0	-0.5	0	0.5	0
z^*	0	11	4	0	2	0	-1	1

Tableau 4

B.V.	t_1	t_2	t_3	t_4	t_5	t_6	t_7	R.H.S.
t_4	0	5.5	10	1	-0.5	1.5	0	1
t_7	0	-6	3	0	-2	1	1	0
t_1	1	0.5	-2	0	0.5	-0.5	0	0
z^*	0	5	7	0	0	1	0	1

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Therefore, the optimal solution to the LPP (4.10) is $s_2 = 1$, $s_1 = s_3 = 0$, $z = 1$; and the value of the game, $V = \frac{1}{z} = \frac{1}{1} = 1$. Now, the probability of player **A** using his 1st, 2nd and 3rd strategy is given as; $x_1 = Vs_1 = 0$, $x_2 = Vs_2 = 1$, and $x_3 = Vs_3 = 0$ respectively.

On the other hand, the probability of player **B** using his 1st, 2nd, 3rd and 4th strategy is given as; $y_1 = Vt_1 = 0$, $y_2 = Vt_2 = 0$, $y_3 = Vt_3 = 0$ and $y_4 = Vt_4 = 1$ respectively.

RESULTS

Table 2: Comparison of player A's solutions from the different models

Model	Optimal Solution	Value of Game
Our Model	$x_1 = 0.2821, x_2 = 0.1283, x_3 = 0.5896$	3.9231
Model 1 (Dorfman et al., 1987)	$x_1 = 0, x_2 = 0, x_3 = 1$	1
Model 2 (Dorfman et al., 1987)	$x_1 = 0, x_2 = 1, x_3 = 0$	1

Table 3: Comparison of player B's solutions from the different models

Model	Optimal Solution	Value of Game
Our Model	$y_1 = 0.5386, y_2 = 0.1538, y_3 = 0.3076$	3.9231
Model 1 (Dorfman et al., 1987)	$y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 1$	1
Model 2 (Dorfman et al., 1987)	$y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 1$	1

The value of the game is the objective function value which is a measure of model performance.

From the two tables, our model has the highest value of the game. This implies that our conversion model of game problem to LPP produces a better result. The conversion models of Dorfman et al. (1987) have one more linear constraint than our model; and by that, the quality of performance of the model based on the objective function is greatly sacrificed.

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