KAMAL TRANSFORM OF UNIT STEP FUNCTIONS

Onuoha N. O.

Imo State University, Owerri, Imo State, Nigeria.

Cite this article:

Onuoha N. O. (2024), Kamal Transform of Unit Step Functions. African Journal of Mathematics and Statistics Studies 7(3), 1-8. DOI: 10.52589/AJMSS-A8VKKQOA

Manuscript History

Received: 13 Apr 2024

Accepted: 19 Jun 2024

Published: 12 Jul 2024

ABSTRACT: *This research presents the Kamal transform of unit step functions. Kamal transform is an integral transform that can be applied to solve mathematical problems. Kamal transform has some similarities with Laplace transform. Both transforms are half line and one fold integral transforms. Due to the applications of unit step functions in diverse areas, this paper showcases the Kamal transform of unit step functions. We applied Kamal transform to the following unit step functions: (a) Heaviside unit step function, (b) Shifted unit step function, and (c) Unit impulse function. The results obtained showed that the new integral transform (Kamal transform) can be applied to unit step functions.*

KEYWORDS: Kamal transform, Unit step function, Integral transform, Heaviside unit step function, Shifted unit step, Unit impulse function.

INTRODUCTION

Unit step function is commonly seen in mathematics of control theory and signal processing. It is mainly seen in functions that change values at specified time. It represents the unit output of a given system with possible time lag. In electrical engineering, it is used to calculate currents when an electricity circuit is switched on. The switching process (switch on/off) can be described mathematically by unit step function. Unit step function can equally be used to represent any real life signal that switches on at a specified time and stays switched on indefinitely. The unit step function enables one to represent piecewise continuous functions. Approximations to the unit step functions are of use in biochemistry and neuroscience. Unit step functions are used in structural mechanics to describe different types of structural loads. Logistic approximation of step functions may also be used to approximate binary cellular switches in response to chemical signals. Many integral transforms of unit step functions have been studied. Burrow and Colwell [6] discussed the Fourier transform of unit step functions. Sukhen and Puneet [22] studied the Laplace transform and its application to differential equations containing unit step functions while this research focuses on the Kamal transform of unit step functions. Asatur [4] proved in his paper that any positive real power of the Heaviside function is almost everywhere equal to the Heaviside function of power one using nonlinear Green's function. Mohamed [12] discussed the advantage of unit step function in solving initial value problems having discontinuous functions in the area of electrical circuit theory, biological modeling and atomic control. Zakharov et al. [24] applied Heaviside step function to investigate the influence of various climatic conditions and operating modes in the city on the formation of the battery charge level in the winter period.

Abdelilah and Zahra [2] in their research derived the Kamal transform of partial derivatives and applied it to solve different partial differential equations. Rachana Khandelwal et al. [18] in their paper solved a coupled system of nonlinear partial differential equations. Muhammad et al. [13] applied Kamal transform to temperature equations in mechanical, chemical and other areas of engineering to show the effectiveness of the integral transform—Kamal transform. Mechanics and electrical circuits problems were solved using Kamal transform by Huda et al. [8]. Abhale et al. [3] presented using various examples the application of Kamal transform of hyperbolic and algebraic functions with Sandip's method to cryptography. In solving ordinary differential equations with constant coefficients, Abdelilah [1] applied the new Kamal transform. Zainab and Nejmaddin [23] solved linear systems of Volterra integro-differential equations of the second kind by Kamal transform. Sudhanshu et al. [19] in their study applied Kamal transform to population growth and decay problems and concluded that Kamal transform is an effective mathematical tool to solve such problems. Katre N.T. and Katre R.T. [10] did a comparative study of Laplace and Kamal transform for solving linear difference equations. Kamal transform has been applied to the error function. Many engineering problems such as vibrating beam problems, heat and mass transfer problems contain error functions. Sudhanshu and Gyanvendra [21] showed the usefulness of Kamal transform of error functions. Owolabi and Oderinu [15] solved generalized Hirota-satsuma coupled KDV equations numerically using Kamal transform. Ayush [5] applied Kamal transform based cryptographic technique to network security involving ASCII value. Sudhanshu et al. [20] demonstrated with examples the effectiveness of the application of Kamal transform to linear Volterra integral equations. Onuoha [14] applied Kamal transform to coupled systems of linear ordinary differential equations. Kamal transform has been applied to fractional ordinary differential equations by Rachana et al. [17]. The Kamal decomposition method has been applied by

Article DOI: 10.52589/AJMSS-A8VKKQOA
DOI URL: https://doi.org/10.52589/AJMSS-A8VKKQOA

Johnson et al. [9] to solve two dimensional unsteady flow. Padama and Yogesh [16] applied the Kamal transform of the derived function to the Heaviside function. Kumar and Vasuki [11] applied Kamal transform in cryptography. Ghanwat and Gaikwad [7] in their research applied Kamal transform for solving linear integral equations of second kind.

Definition of Unit Step Function

A unit step function $f(t)$ can be considered as a function that maintains a zero (o) value for all values of t up to a specified point and a unit (1) value for all values of t greater than or equal to the specified point.

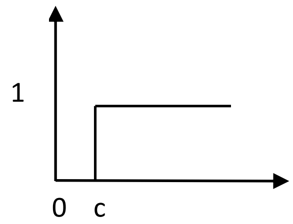

Fig. 1: Unit step function

Here, $f(t)=0$ for $t<c$, $f(t)=1$ for $t\geq c$ where c is the specified point.

Kamal Transform of a Function

Abdelilah [1] defined the Kamal transform of a function $f(t)$ as follows:

$$K\{f(t)\}=\int_0^\infty e^{-\frac{1}{v}t} f(t)\,dt \qquad v>0 \tag{1}$$

We shall apply Equation (1) to different unit step functions.

Kamal Transform of a Heaviside's Unit Step Function

A unit step function denoted by $u(t)$ and defined as

$$u(t)=\begin{cases}0\,, & t<0\\ 1\,, & t\geq 0\end{cases} \tag{2}$$

is called a Heaviside's unit step function. In this case, $u(t)$ is a function of time (t) and it has only two values: zero and one.

Article DOI: 10.52589/AJMSS-A8VKKQOA
DOI URL: https://doi.org/10.52589/AJMSS-A8VKKQOA

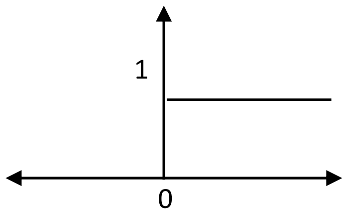

Fig. II: Heaviside unit step function

The Kamal transform of a Heaviside's unit step function, $u(t)$ from Equation (1) is defined as:

$$K\left[u(t)\right]=\int_0^\infty e^{-\frac{1}{v}t}u(t)\,dt \tag{3a}$$

$$K\left[u(t)\right]=\int_{-\infty}^0 e^{-\frac{1}{v}t}u(t)\,dt+\int_0^\infty e^{-\frac{1}{v}t}u(t)\,dt \tag{3b}$$

Simplifying further Equation (3b), we get:

$$K\left[u(t)\right]=ve^{-\frac{1}{v}} \tag{4}$$

Equation (4) is the Kamal transform of the Heaviside unit step function, $u(t)$.

Kamal Transform of a Shifted Unit Step Function

Shifted unit step functions are observed mainly in circuits where waveforms are applied at specified intervals than at $t=0$. This type of unit step function is one that has value "0" up to a certain time $t=c$ and thereafter the value changes to one.

$$u(t-c)=\begin{cases}0\,, & t<c\\ 1\,, & t\geq c\end{cases} \tag{5}$$

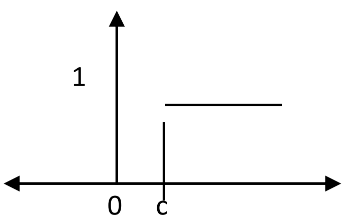

Fig. III: Shifted unit step function

Article DOI: 10.52589/AJMSS-A8VKKQOA
DOI URL: https://doi.org/10.52589/AJMSS-A8VKKQOA

Kamal transform of shifted unit step function from Equation (1) is defined thus:

$$K\left[u(t-c)\right]=\int_0^\infty e^{-\frac{1}{v}t}u(t-c)\,dt$$
$$=\int_0^c e^{-\frac{1}{v}t}u(t-c)\,dt+\int_c^\infty e^{-\frac{1}{v}t}u(t-c)\,dt \quad (6)$$

Applying Equation (5) on Equation (6), we get:

$$K\left[u(t-c)\right]=\int_c^\infty e^{-\frac{1}{v}t}\,dt \quad (7)$$

Integrating Equation (7), we get

$$K\left[u(t-c)\right]=ve^{-\frac{c}{v}} \quad (8)$$

Kamal Transform of a Unit Impulse Function

This kind of function is obtained when a voltage is to be applied at a particular time $t=a$ and removed at time $t=b$. Using step function, the unit impulse function can be defined by:

$$u(t)=\begin{cases}1, & a<t<b\\ 0, & otherwise\end{cases} \quad (9)$$

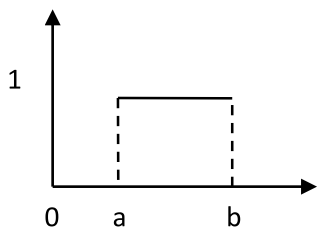

Fig. IV: Impulse unit step function

Kamal transform of unit impulse function from Equation (1) is defined thus:

$$K\left[u(t)\right]=\int_0^\infty e^{-\frac{1}{v}t}u(t)\,dt$$
$$=\int_0^a e^{-\frac{1}{v}t}u(t)\,dt+\int_a^b e^{-\frac{1}{v}t}u(t)\,dt+\int_b^\infty e^{-\frac{1}{v}t}u(t)\,dt \quad (10)$$

Applying Equation (9) on Equation (10), we get:

Article DOI: 10.52589/AJMSS-A8VKKQOA
DOI URL: https://doi.org/10.52589/AJMSS-A8VKKQOA

$$K\left[u(t)\right]=\int_a^b e^{-\frac{1}{v}t}dt \tag{11}$$

Integrating Equation (11), we get:

$$K\left[u(t)\right]=v\left[e^{-\frac{1}{v}b}-e^{-\frac{1}{v}a}\right] \tag{12}$$

CONCLUSION

In the study of unit step functions, other integral transforms have been applied to the functions. There are different equations that contain unit step functions, especially differential equations. Kamal transform has been applied to so many areas. This research shows that the Kamal transforms of unit step functions were derived. We conclude that Kamal transform can be applied to any differential equation containing any form of unit step function.

REFERENCES

[1]. Abdelilah Kamal. H. Sedeeg (2016): The new integral transform " kamal transform". Advances in Theoretical and Applied Mathematics. Vol. 11, No 4, 451- 458.

[2]. Abdelillah K. Hassan Sedeeg and Zahra I. Adam Mahamoud (2017): The use of Kamal transform for Solving partial differential equations. Advances in Theoretical and Applied Mathematics. Vol. 12 No. 1, 7-13.

[3]. Abhale N.S., Pawar S.S., Sonawane S.M., Kiwne S.B. and Sontakke B.R. (2022): Kamal transform in cryptography with sandip's method. International Journal of Current Engineering and Scientific Research. Vol. 9(6), 22-24.

[4]. Asatur Zh. Khurshudyan (2020): An identity for the Heaviside function and its application in representation of nonlinear Green's function. Computational and Applied Mathematics. Vol. 39(1), 1-14. DOI 10.1007/s40314-019-1011-5

[5]. Ayush Mittal (2019): Kamal transformation based cryptographic technique in network security involving ASCII value. International Journal of Advanced Scientific Research and Management. Issue 5, 16-19.

[6]. Burrows B.L. and Colwell D.J. (1990): The Fourier transform of the unit step function. International Journal of Mathematical Education in Science and Technology. Vol. 21(4), 629-635. DOI: 10.1080/0020739900210418.

[7]. Ghanwat A.J. and Gaikwad S.B. (2022): Application of Kamal transform for solving linear Volterra integral equations of second kind. Journal of Emerging Technologies and Innovative Research. Vol. 9(4), 7-14.

[8]. Huda Ibrahim Osman Bashir, Mona Magzoub Mohammed Ahmed Hassan, Fatin Ali Mohamed Saeed, Nesreen Sirelkhtam Elmki Abdalla, Mubarak Dirar Abdallah and Sawsan Ahmed Elhouri Ahmed (2019): Applications of Kamal transform to mechanics & electrical circuits problems. Global Journal of Engineering Science and Researches. Vol. 6(10). DOI: 10.5281/zenodo.3475118.

[9]. Johnson Adekunle Owolabi, Olufemi Elijah Ige and Emmanuel Idowu Akinola (2019): Application of Kamal decomposition transform method in solving two dimensional unsteady flow. International Journal of Difference equations. Vol. 14 No. 2, 207-214.

[10]. Katre N.T. and Katre R.T. (2021): A comparative study of Laplace and Kamal transforms. Journal of Physics: Conference Series. DOI: 10.1088/1742-6596/1913/1/012115.

[11]. Kumar P. Senthil and Vasuki S. (2018): Application of Kamal transform in cryptography. International Journal of Interdisplinary Research and Innovations. Vol. 6(3), 182-186.

[12]. Mohamed M. Peer (2016): Applications of Laplace transform to unit step functions and Dirac delta functions. International Journal of Scientific and Research Publications. Vol. 6(8), 187-194.

[13]. Muhammad Waqas, Khurrem Shehzad, Ali Moazzam and Alizay Batool (2022): Applications of Kamal transformation in temperature problems. Scholars Journal of Engineering and Technology. Vol. 10(2), 5-8 Doi: 10.36347/sjet.2022.v10i02.001.

[14]. Onuoha N.O. (2023): Kamal transform technique to coupled systems of linear ordinary differential equations. IOSR Journal of Mathematics. Vol. 19(4), 24-29.

[15]. Owolabi J.A. and Oderinu R.A. (2021): Kamal transform based analytical solution of a generalized nonlinear Hirota-Satsuma coupled equations. Asian Journal of Pure and Applied Mathematics. Vol. 3(1), 69-78.

[16]. Padama Kumawat and Yogesh Khandelwal (2021): The Kamal transform of derived function demonstrated by Heaviside function. Indian Journal of Science and Research. Vol. 1(1), 17-19.

[17]. Rachana Khandelwal, Priyanka Choudhary and Yogesh Khandelwal (2018): Solution of fractional ordinary differential equation by Kamal transform. International Journal of Statistics and Applied Mathematics. Vol. 3(2), 279-284.

[18]. Rachana Khandewal, Padama Kumawat and Yogesh Khandelwal (2018): Kamal decomposition method and

its application in solving coupled system of nonlinear PDE's. Malaya Journal of Matematik. Vol. 6 No. 3, 619-625.

[19]. Sudhanshu Aggarwal, Anjana Rani Gupta, Nirbhaya Asthana and D.P. Singh (2018): Application of Kamal transform for solving population growth and decay problems. Global Journal of Engineering Science and Researches. Vol. 5(9), 254-260.

[20]. Sudhanshu Aggarwal, Raman Chauhan and Nidhi Sharma (2018): A new application of Kamal transform for solving linear Volterra integral equations. International Journal of Latest Technology in Engineering, Management & Applied Science. Vol. 7(4), 138-140.

[21]. Sudhanshu Aggarwal and Gyanvendra Pratap Singh (2019): Kamal transform of error function. Journal of Applied Science and Computations. Vol. 6(5), 2223-2235.

[22]. Sukhen Bhattacharyya and Puneet Kumar Agarwal (2019): A study of Laplace transform and applications to differential equations. Research Review Journals. Vol. 04(03), 2457-2462.

[23]. Zainab Rustam and Nejmaddin Sulaiman (2023): Kamal transform technique for solving system of linear Volterra integro-differential equations of the second kind. International Journal of Nonlinear Analysis and Applications. Vol. 14(1), 185-192.

[24]. Zakharov N.S., Sapozhenkov N.O., Tyan R.V., and Nazarow V.P. (2021): Applying the Heaviside step function to simulate the changes of temperature in automotive batteries. Journal of Physics: Conference series. DOI: 10. 1088/1742-6596/2016/1/012001.