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INVERSE SHANKER DISTRIBUTION: ITS PROPERTIES AND APPLICATION

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ABSTRACT: *In this paper, new lifetime distribution has been proposed called the Inverse Shanker distribution. Its statistical properties including stochastic ordering, survival function, hazard rate function, Renyi entropy and Stress-strength reliability measure have been discussed. Maximum likelihood estimation method was used to estimate the parameter of the distribution. We compared the applicability of Inverse Shanker distribution with one parameter Inverse distributions, Inverse Lindley distribution (ILD), and Inverse Rayleigh distribution (IRD), based on two real data sets. Finally, the proposed distribution has been shown to have superiority over other lifetime distributions.*

KEYWORDS: Moments, inverse shanker distribution, Stress-strength reliability, Maximum likelihood estimator and Renyi entropy.

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INTRODUCTION

Predicting differences in life occurrences, situations, areas with high level of precision has become unpredictable and challenging. Due to these differences statisticians has continue to derive, and modify models for use to solve diverse problems, these has led to the emergence of several probability distributions which can be used to model varying situations in diverse field of human endeavors. Consequently, the construction of new distributions or modification of existing ones has continued to gain serious attention in the area of statistics. The essence of developing new or modifying existing distributions is to obtain probability distributions that are more reliable, flexible, versatile, robust, unbiased, efficient and sufficient to capture the situation under study. One of the proofs, is the emergence of the Shanker distribution proposed for modelling real lifetime data-sets from various fields of knowledge as claimed by the author. One major defect of the shanker distribution is its inability to capture or model datasets with decreasing function in their hazard rate. Shanker (2015) proposed a one parameter lifetime distribution with the following probability density function (PDF) and cumulative distribution function (CDF) respectively;

$$f(x;\theta) = \frac{\theta^2}{\theta^2+1}(\theta + x)e^{-\theta x}; \quad x > 0, \theta > 0 \tag{1.1}$$

$$F(x;\theta) = 1 - \frac{(\theta^2+1)+\theta x}{\theta^2+1}e^{-\theta x}; \quad x > 0, \theta > 0 \tag{1.2}$$

This distribution is known as the Shanker distribution. The mathematical and statistical properties including its parameter estimation can been shown. It's important applications in biological and engineering data have been described in his paper, and its superiority over other one parameter life time classical distributions such as Lindley and Exponential distribution has been demonstrated. The Lindley distribution was proposed by Ghitany *et al.* (2008) and its PDF and CDF are defined respectively by

$$f(x;\theta) = \frac{\theta^2}{\theta+1}(1 + x)e^{-\theta x}; \quad x > 0, \theta > 0 \tag{1.3}$$

$$F(x;\theta) = 1 - \frac{(\theta+1)+\theta x}{\theta+1}e^{-\theta x}; \quad x > 0, \theta > 0 \tag{1.4}$$

He also derived its statistical properties like the hazard rate etc and demonstrated its application from lifetime data. The Inverse Lindley distribution has been proposed by Sharma *et al.* (2015) which is an Inverse of Lindley distribution (ILD). Its statistical properties, stress and strength reliability measure including its stress and strength parameter estimation method on real lifetime data were all derived in their paper. Its PDF and CDF are given respectively by

$$f(x;\theta) = \frac{\theta^2}{(\theta+1)x^3}(1 + x)e^{-\theta/x}; \quad x > 0, \theta > 0 \tag{1.5}$$

$$F(x;\theta) = 1 + \frac{\theta}{(\theta+1)x}e^{-\theta/x}; \quad x > 0, \theta > 0 \tag{1.6}$$

In another study, Trayer *et al.* (1964) recommended the Inverse Rayleigh distribution (IRD) and since then it has been studied and discussed as a lifetime model with vast applicability in survival analysis. Voda (1972) studied various properties of the ILD. The PDF and CDF are given respectively by

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$$f(x;\theta)=\left(\frac{2\theta}{x^3}\right)e^{(-\theta/x^2)};\quad x>0,\theta>0 \tag{1.7}$$

$$F(x;\theta)=e^{\theta/x^2};\quad x>0,\theta>0 \tag{1.8}$$

A comparative study about its mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany *et al*. (2008). Over the years, the Lindley distribution has been generalized, extended and modified by different researchers including Nadarajah *et al*. (2011), Deniz and Ojeda (2011), Shanker and Mishra (2013 a, 2013 b), Shanker *et al*. (2013), Elbatal *et al.* (2013), Ghitany *et al* (2013), Liyanage and Pararai (2014), Ashour and Eltehiwy (2014), Oluyede and Yang (2014), Singh *et al.* (2014), Sharma *et al*. (2015), Alkarni (2015) are some among others. Notably, Shanker *et al.* (2015) who conducted a study of the comparative study of exponential, and Lindley distributions for modeling various real lifetime data-sets and found a contrasting result based on data type, that in some data-sets the Lindley distribution is better than the exponential while in other data sets exponential distribution is better than Lindley.

This finding was the motivation of introducing the Shanker distribution which was shown to give a better fit than both exponential and Lindley distributions for modeling different real lifetime data-sets from various fields of knowledge. The motivation of this paper therefore, is to introduce a new life time distribution based on the following observations:

(i) The Shanker distribution is more flexible distribution than Lindley and exponential distribution especially in modelling biological and engineering data.

(ii) The Inverse of Shanker distribution may also be more flexible and gives good fit over Inverse Lindley distribution as well as Inverse Rayleigh distribution.

Since then several authors have modified the shanker distribution either by increasing the flexibility of the distribution without adding another parameter or vice-versa. Notably are (X shanker distribution, Harrison *et al*., 2023; Transmuted shanker distribution, Onyekwere *et al*.,2022; Power Shanker, Shanker and Shukla 2017; weighted power Shanker; length biased weight shanker distribution, Modified inverse shanker distribution, Nwadiogbu *et al*., 2022. This study has joined this trend by introducing a new one parameter distribution called the inverse shanker distribution.

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MATRIALS AND METHOD

Inverse Shanker Distribution

If a random variable Y has a Shanker distribution SD (θ), then the random variable X = ($^1/_Y$) is said to be follow the Inverse Shanker distribution (ISD) having a scale parameter θ with its probability density function (PDF) defined by

$$f(x;\theta) = \frac{\theta^2}{(\theta^2+1)x^3}(\theta x + 1)e^{-\theta/x}; \quad x > 0, \theta > 0 \tag{2.1}$$

It is denoted by ISD (θ). The cumulative distribution function (CDF) of Inverse Shanker distribution is given by

$$F(x;\theta) = \left[1 + \frac{\theta}{(\theta^2+1)x}\right] e^{-\theta/x}; \quad x > 0, \theta > 0 \tag{2.2}$$

Since this distribution has a good closed form expression for the CDF, hazard function as well as stress-strength reliability, its relevance in the study of survival analysis can never be undermined. Similar PDF and CDF of the inverse shanker distribution has previously been derived by Nwadiogbu *et al.*, (2022) to enable them derive the properties of the modified inverse shanker distribution but they did not derive other properties of the inverse shanker distribution neither do they use it in the comparison for the goodness of fit.

Shape Characteristics of the Density

The first derivation of (2.1) is given by

$$\frac{d}{dx}f(x) = -\left(\frac{\theta^2}{\theta^2+1}\right)\frac{e^{-\frac{\theta}{x}}}{x^5}(2\theta x^2 - (\theta^2 - 3)x - \theta) \tag{2.3}$$

And $\left.\frac{d}{dx}f(x)\right|_{x=M_0} = M_0$ is the mode of an Inverse Shanker random variable and it is given by

$$M_0 = \frac{(\theta^2-3)+\sqrt{(\theta^2-3)^2+8\theta^2}}{4\theta} \tag{2.4}$$

Thus, the graphs of the PDF and CDF are given below.

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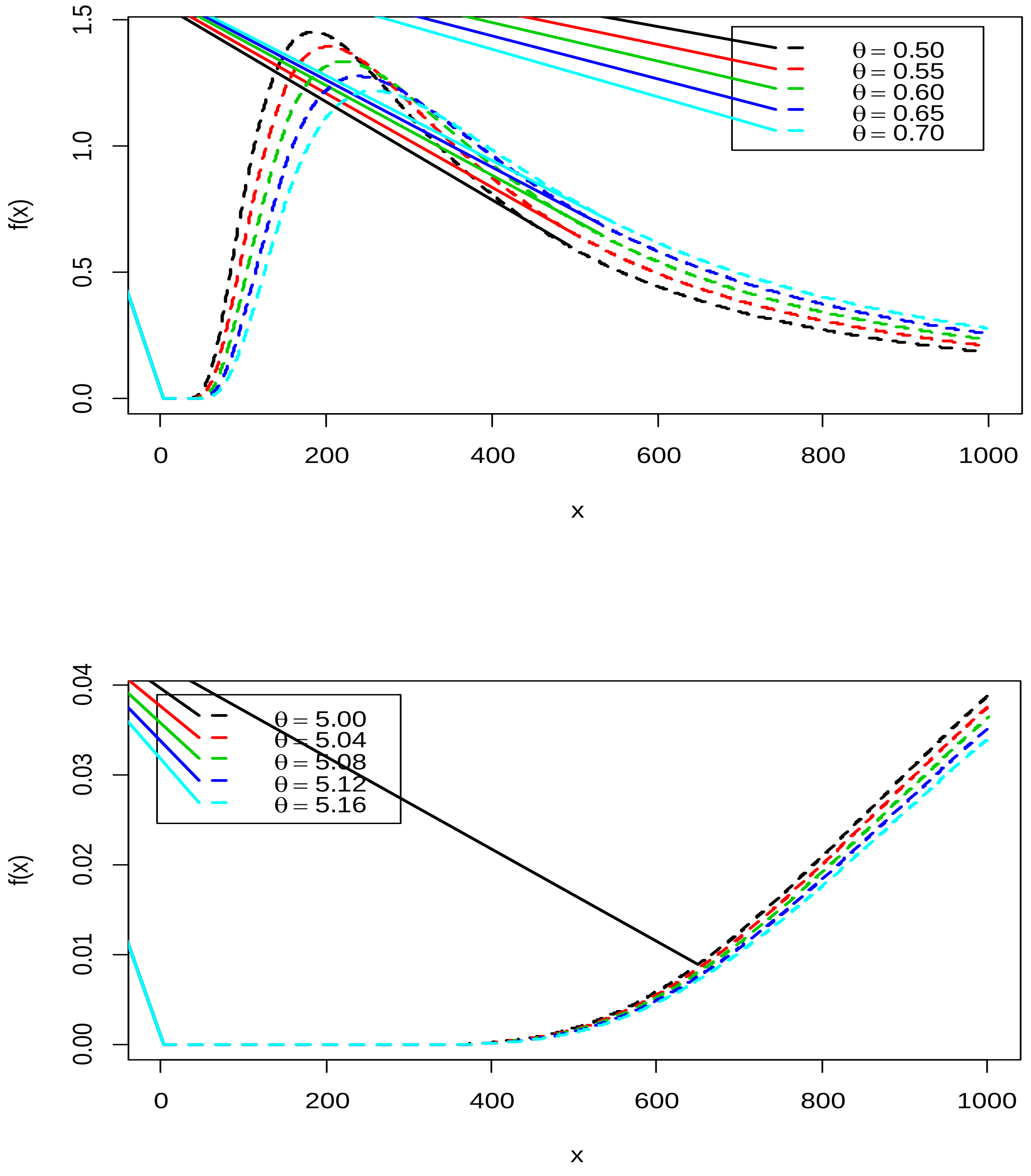

Figure 1. Plots of the PDF of Inverse Shanker distribution for varying values of the parameter θ.

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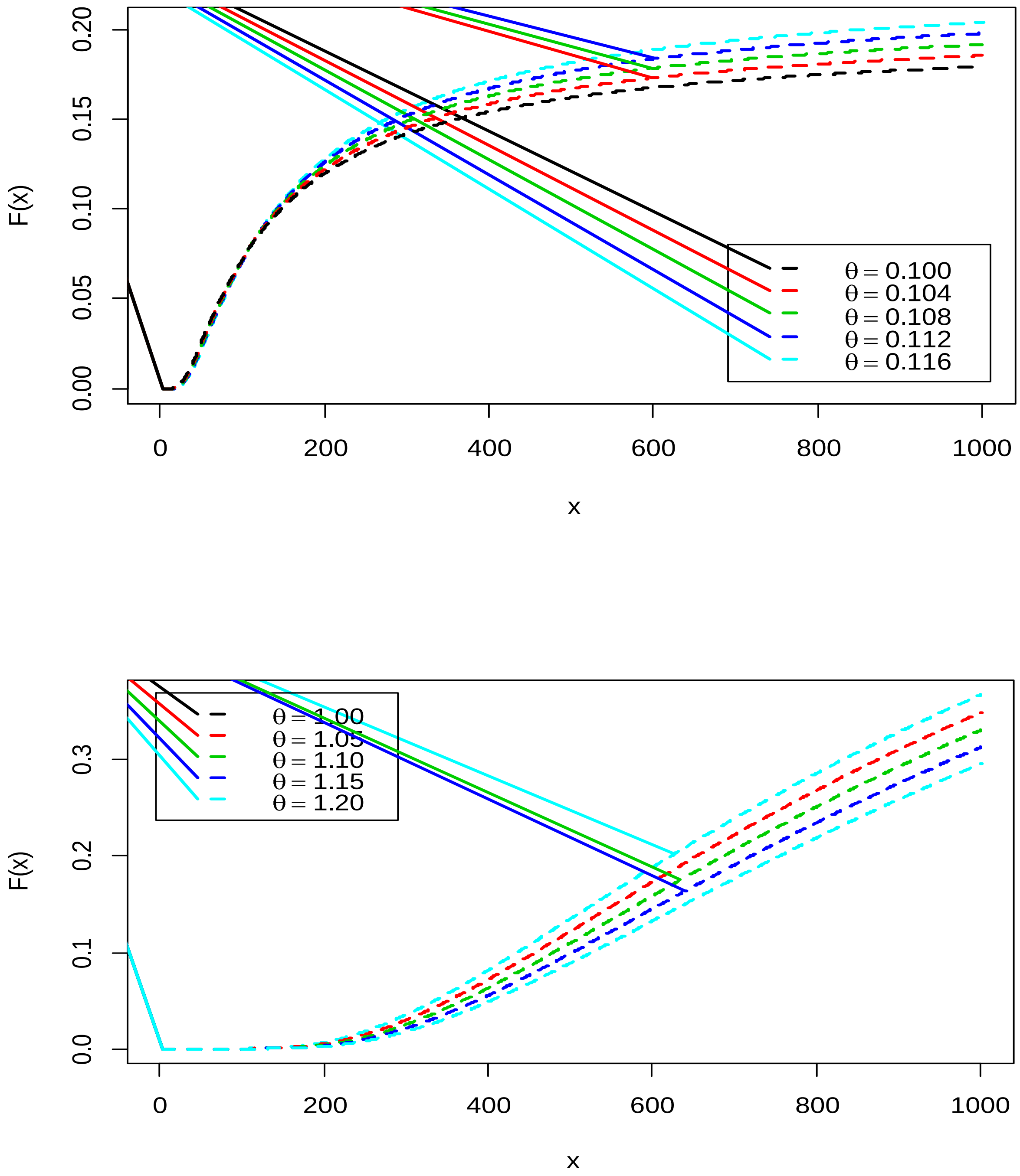

Figure 2. Plots of the CDF of Inverse Shanker distribution for varying values of the parameter θ.

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Survival Function and Hazard Rate Function

Survival function $S(x;\theta)$ of the Inverse Shanker distribution (ISD) can be defined as

$$S(x;\theta) = 1 - F(x;\theta) \tag{3.1}$$

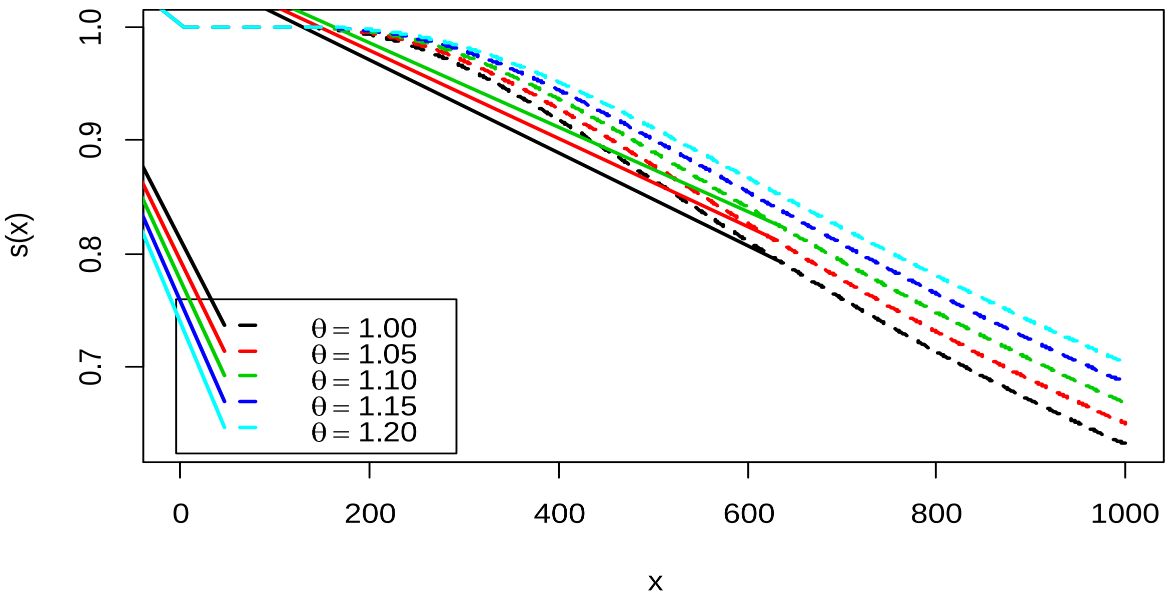
$$S(x;\theta) = 1 - \left[1 + \frac{\theta}{(\theta^2+1)x}\right] e^{-\theta/x} \tag{3.2}$$

And the hazard rate function $h(x;\theta)$ of the Inverse Shanker distribution (ISD) can be defined as

$$h(x;\theta) = \frac{f(x;\theta)}{1-F(x;\theta)} = \frac{f(x;\theta)}{S(x;\theta)} \tag{3.3}$$

$$h(x;\theta) = \frac{\theta^2(\theta x+1)}{x^2\left[x(\theta^2+1)\left(e^{\theta/x}-1\right)-\theta\right]} \tag{3.4}$$

Thus, the graph of the survival function and hazard rate function are given below



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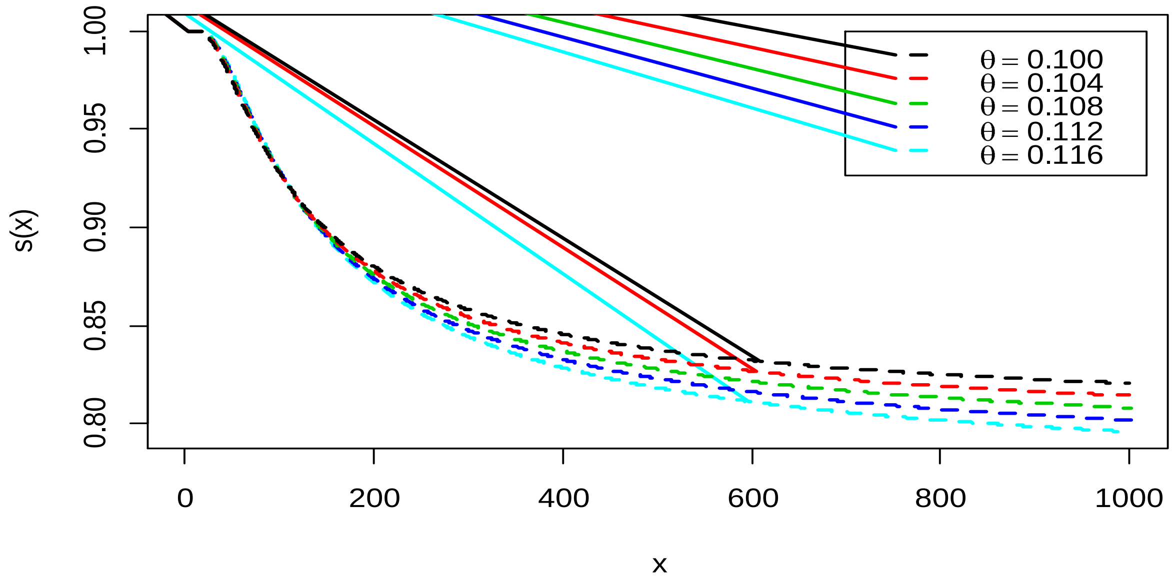
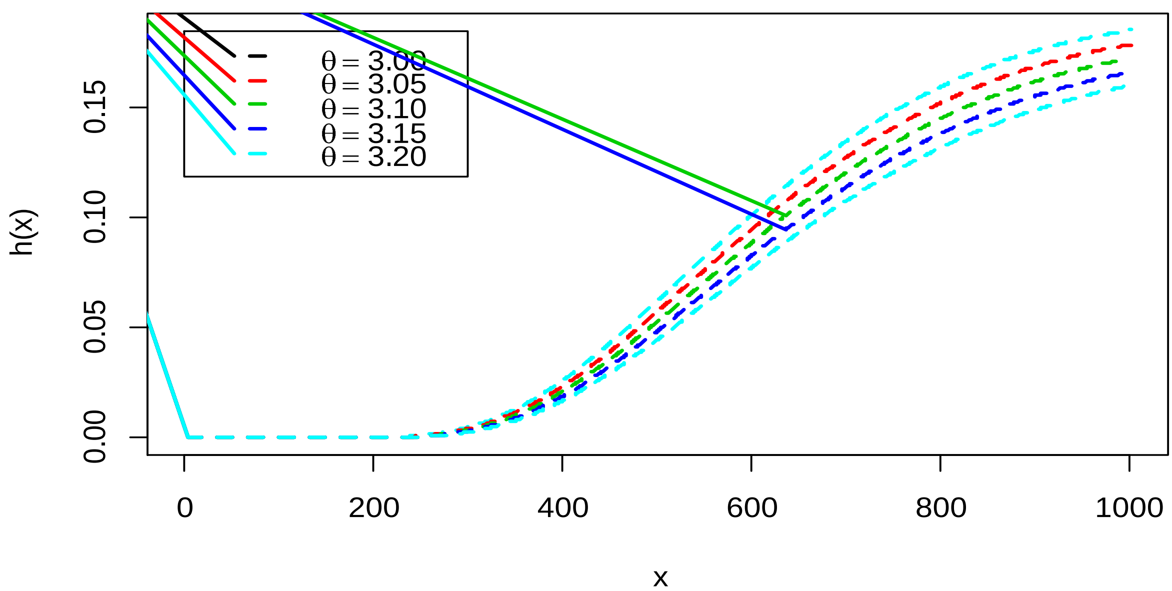


Figure 3. Plots of the Survival function of Inverse Shanker distribution for varying values of the parameter θ.



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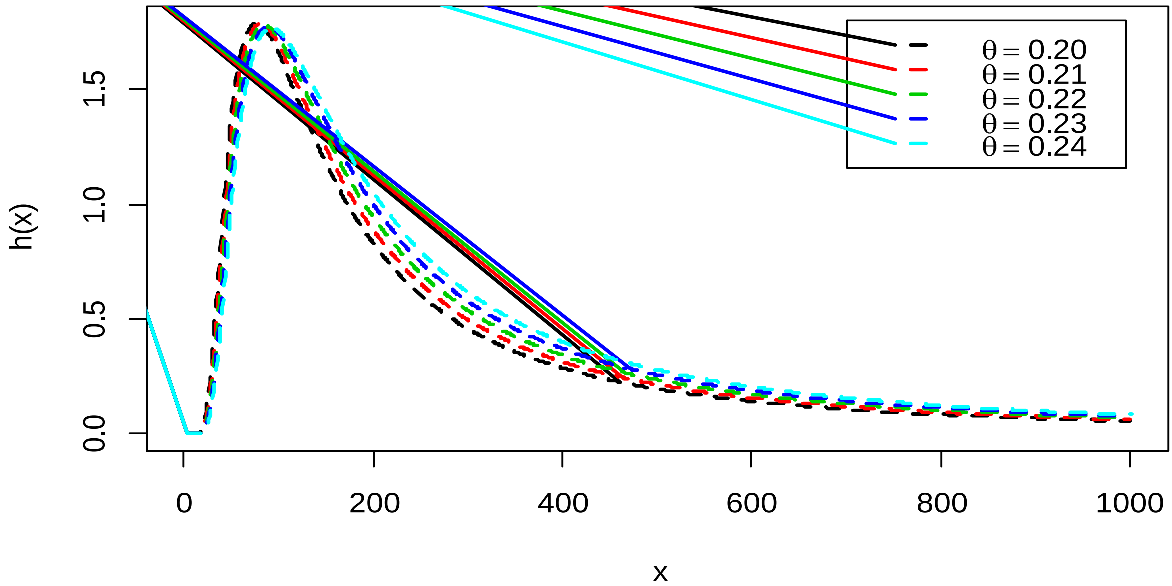

Figure 4. Plots of the Hazard Rate function of Inverse Shanker distribution for varying values of the parameter θ.

Stochastic Ordering

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. According to Shanker (2015), a random variable X is said to be smaller than a random variable Y in the;

i. Stochastic order $(X \leq_{st} Y)$ if $F_X(x) \geq F_Y(x)$ for all x.

ii. Hazard rate order $(X \leq_{hr} Y)$ if $h_X(x) \geq h_Y(x)$ for all x.

iii. Mean residual life order $(X \leq_{mrl} Y)$ if $m_X(x) \leq m_Y(x)$ for all x.

iv. Likelihood ratio order $(X \leq_{lr} Y)$ if $\left(\frac{f_X(x)}{f_Y(x)}\right)$ decreases in x.

The following results due to Shaked *et al*. (1994) are well known for establishing stochastic ordering of distributions

$$\begin{array}{c}(X \leq_{lr} Y) \Rightarrow (X \leq_{hr} Y) \Rightarrow (X \leq_{mrl} Y) \\ \Downarrow \\ (X \leq_{st} Y)\end{array} \tag{4.1}$$

The Inverse Shanker distributions are ordered with respect to the strongest likelihood ratio ordering as shown in the following theorem.

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Theorem: Let $X \sim ISD(\theta_1)$ and $Y \sim ISD(\theta_2)$.

If $\theta_1 \geq \theta_2$, then $(X \leq_{lr} Y)$ and hence $(X \leq_{hr} Y), (X \leq_{mrl} Y)$ and $(X \leq_{st} Y)$

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^2(\theta_2^2+1)}{\theta_2^2(\theta_1^2+1)}\left(\frac{\theta_1 x+1}{\theta_2 x+1}\right)e^{-(\theta_1-\theta_2)/x}; \quad x > 0$$

Now

$$log\frac{f_X(x)}{f_Y(x)} = log\left[\frac{\theta_1^2(\theta_2^2+1)}{\theta_2^2(\theta_1^2+1)}\right] + log\left(\frac{\theta_1 x+1}{\theta_2 x+1}\right) - (\theta_1-\theta_2)/x$$

This gives

$$\frac{d}{dx}log\frac{f_X(x)}{f_Y(x)} = \frac{(\theta_1-\theta_2)}{(\theta_1 x+1)(\theta_2 x+1)} + (\theta_1-\theta_2)/x^2$$

Thus, for $\theta_1 \geq \theta_2$, $\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} > 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Entropy Measure

Entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is the Renyi entropy (1961). If X is a continuous random variable having probability density function$f(.)$, then Renyi entropy is defined as

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\{\int f^{\gamma}(x)\,dx\} \tag{5.1}$$

Where $\gamma > 0$ *and* $\gamma \neq 1$. For the inverse Shanker distribution, the Renyi entropy measure is defined by:

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\int_0^\infty \frac{\theta^{2\gamma}}{(\theta^2+1)^\gamma}\left[\frac{(1+\theta x)^\gamma}{x^{3\gamma}}\right]e^{-\frac{\theta\gamma}{x}}dx$$

We know that $(1+z)^j = \sum_{j=0}^{\infty}\binom{\gamma}{j}z^j$ and $\int_0^\infty e^{-\frac{b}{x}}x^{-a-1}dx = \frac{\Gamma(a)}{b^a}$

$$= \frac{1}{1-\gamma}\log\left[\frac{\theta^{2\gamma+j}}{(\theta^2+1)^\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\int_0^\infty \frac{e^{-\frac{\theta\gamma}{x}}}{x^{3\gamma-j}}\mathrm{dx}\right]$$

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$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left[\frac{\theta^{2\gamma+j}}{(\theta^2+1)^\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\Gamma(3\gamma-j-1)}{(\theta\gamma)^{3\gamma-j-1}}\right] \quad (5.2)$$

Stress-Strength Reliability

The stress- strength reliability describes the life of a component which has random strength X that is subjected to a random stress Y. When the stress applied to it exceeds the strength, the component fails instantly and the component will function satisfactorily till X >Y. Therefore, $R = P(Y < X)$ is a measure of component reliability and in statistical literature it is known as stress-strength parameter. Let Y and X be independent stress and strength random variables that follow Inverse Shanker distribution with parameter θ_1 *and* θ_2 respectively. Then, the stress-strength reliability R is defined as

$$R = P[Y<X] = \int_0^\infty P[Y < X \mid X = x]\, f_x(x)dx$$

$$= \int_0^\infty f(x,\theta_1)\ F(x,\theta_2)dx$$

$$= \int_0^\infty \frac{\theta_1^2}{\theta_1^2+1}\left(\frac{\theta_1 x+1}{x^3}\right)e^{\theta_1/x} \times \left[1+\frac{\theta_2}{(\theta_2^2+1)x}\right]e^{\theta_2/x}dx$$

Using inverse gamma function, it can be written as

$$R = \theta_1^2\left[\frac{\theta_1(\theta_2^2+1)(\theta_1+\theta_2)^2+(\theta_2^2+1)(\theta_1+\theta_2)+\theta_1\theta_2(\theta_1+\theta_2)+2\theta_2}{(\theta_1^2+1)(\theta_2^2+1)(\theta_1+\theta_2)^3}\right]$$

Maximum Likelihood Estimation Method

Let $L(x_1, x_2, \dots, x_n;\ \theta)$ be a random sample from the Inverse Shanker distribution (2.1). The likelihood function, L of (2.1) is given by:

$$L(x_1, x_2, \dots, x_n; \theta) = \prod_{i=1}^{n}[f(x_i;\theta)]$$

$$L = \left(\frac{\theta^2}{\theta^2+1}\right)^n \prod_{i=1}^{n}\frac{(\theta x_i+1)}{x_i^3}e^{\sum\theta/x_i}$$

The log likelihood function is thus obtained as:

$$logL = nlog\left(\frac{\theta^2}{\theta^2+1}\right)+\sum_{i=1}^{n}log(\theta x_i+1)-\sum_{i=1}^{n}log(x_i^3)-\theta\sum_{i=1}^{n}(1/x_i)$$

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the maximum likelihood estimates $\hat{\theta}$ of parameter θ is the solution of the log-likelihood equation $\frac{\partial logL}{\partial \theta} = 0$. It is obvious that $\frac{\partial logL}{\partial \theta} = 0$ will not be in closed form and hence some numerical optimization technique can be used in the equation for θ . In this paper the nonlinear method available in R software has been used to find the MLE of the parameter θ .

Goodness of Fit

The Inverse Shanker distribution (ISD) was applied to two real life data sets (Data 1 and Data 2) in order to assess its statistical superiority over the Inverse Lindley distribution, and the Inverse Rayleigh distribution, to demonstrate that, the theoretical results in the previous sections can be used in practice. The first data set represents the 72 exceedances for the years 1958–1984 (rounded to one decimal place) of flood peaks (in *m*3/s) of the Wheaton River near Car cross in Yukon Territory, Canada while the second data set represent the lifetime of 50 devices. The data sets are as follows;

Data 1:

1.7, 2.2, 14.4, 1.1, 0.4, 20.6, 5.3, 0.7, 1.9, 13.0, 12.0, 9.3, 1.4, 18.7, 8.5, 25.5, 11.6, 14.1, 22.1,1.1, 2.5, 14.4, 1.7, 37.6, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0, 7.3, 22.9, 1.7, 0.1, 1.1, 0.6, 9.0, 1.7, 7.0, 20.1, 0.4, 2.8, 14.1, 9.9, 10.4, 10.7, 30.0, 3.6, 5.6, 30.8, 13.3, 4.2, 25.5, 3.4, 11.9, 21.5, 27.6, 36.4, 2.7, 64.0, 1.5, 2.5, 27.4, 1.0, 27.1, 20.2, 16.8, 5.3, 9.7, 27.5, 2.5, 7.0.

Data 2:

0.1, 0.2,1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32,36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67,72, 75,79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, 86.

First, we checked the validity of the Inverse Shanker distribution for the given data sets by using Akaike information criterion (AIC), Bayesian information criterion (BIC), Negative Log-Likelihood Function (-L), Consistent Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Standard Error Estimate of the Parameter (SE), and The Estimate of the Parameter. We compared the applicability of Inverse Shanker distribution with one parameter Inverse distributions, Inverse Lindley distribution (ILD), and Inverse Rayleigh distribution (IRD), based on real data sets.

For Data 1: The performance of the Inverse Shanker Distribution (ISD) with respect to the Inverse Lindley Distribution (ILD), and Inverse Rayleigh Distribution (IRD) using the Data set 1 is shown in Table 1 below.

Table 1: **Performance Ratings of Inverse Shanker distribution Using Data 1.**

Model	**MLE**	**Estimates**	**S.E**	**HQIC**	**BIC**	**CAIC**	**AIC**
ISD	284.7481	$\hat{\theta} = 2.26775$	0.207507	572.4026	573.7729	571.5534	571.4962
ILD	288.1557	$\hat{\theta} = 2.44124$	0.235164	579.2177	580.5880	578.3685	578.3113
IRD	453.7997	$\hat{\theta} = 0.51791$	0.061037	910.5057	911.8760	909.6565	909.5994

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From the above table, the ISD has the lowest -L value of 284.7481, the lowest AIC value of 571.4962, the lowest BIC value of 573.7729, the lowest HQIC value of 572.4026, and the lowest CAIC value of 571.5534 therefore, the ISD provides a better fit than the ILD and IRD.

For Data 2: The performance of the ISD with respect to the ILD, and IRD using the observations in Data 2 is shown in Table 2 below.

Table 2: **Performance Ratings of Inverse Shanker distribution Using Data 2.**

Model	MLE	Estimates	S.E	HQIC	BIC	CAIC	AIC
ISD	319.4775	$\hat{\theta} = 2.65855$	0.300111	641.6831	642.8670	641.0383	640.9550
ILD	324.0412	$\hat{\theta} = 2.84648$	0.334030	650.8105	651.9944	650.1657	650.0824
IRD	525.1389	$\hat{\theta} = 0.38329$	0.054205	1053.006	1054.190	1052.361	1052.278

From Table 2, the ISD has the lowest -L value of 319.4775, the lowest AIC value of 640.9550, the lowest BIC value of 642.8670, the lowest HQIC value of 641.6831, and the lowest CAIC value of 641.0383 therefore, the ISD provides a better fit as compared to ILD and IRD.

CONCLUSION

A one parameter lifetime distribution named, "Inverse Shanker distribution" has been proposed. The Shanker, Lindley and the exponential distribution are the particular cases of it. Its statistical properties including shape characteristics of density, survival function, hazard rate function, stochastic ordering has been discussed. Further, expressions for entropy measure and, Stress-Strength Reliability of the proposed distribution have been derived. The method of maximum likelihood estimation has also been discussed for estimating its parameter. Finally, the goodness of fit test using –L, AIC, BIC, HQIC, and CAIC based on two real lifetime data sets to examine the applicability and superiority of the proposed model over Inverse Lindley and Inverse Rayleigh distributions in modeling certain lifetime data have been established.

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