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ON THE SELECTION OF OPTIMAL BALANCED INCOMPLETE BLOCK DESIGN USING DIFFERENT TYPES OF DESIGNS

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ABSTRACT: *The choice of selecting the best among the types of balanced incomplete block designs (BIBDs) have brought about some controversies in combinatorial design. Prominent methods of constructing BIBD have been proposed by different authors, but none has been juxtaposed the designs with respect to type. In this paper, we proposed the adoption of the concept of optimality criteria such as: A-, D-, E- , and T – optimality criterion to select the best design based on their optimality values using a (15, 15, 7, 7, 3) simulated balanced incomplete block design. A dual design, residue design and derived design were selected as the major three types of BIBDs for investigation. The findings revealed that dual design is the best balanced incomplete block design based on its high optimal value. We discovered that residue and derived design obtained from the simulated balanced incomplete block design by method of block selection and intersection becomes a standard balanced incomplete block design.*

KEYWORDS: Optimal, Block design, Incomplete block design and Balanced design.

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INTRODUCTION

A block design is a non-empty domain $T = \{t_1, t_2, \ldots, t_n\}$ whose elements are varieties and a non-empty collection of subsets of T is blocks. This design has wide applications in many areas, including experimental design, finite geometry, physical chemistry, software testing, cryptography and algebraic units. When the copious numbers of similar experimental units are not accessible to contain all the treatments in a block, then the used of incomplete block design is applied [1]. A block design is incomplete if the number of varieties is greater than the block size that is $(k < t)$, then the design is incomplete block design. [2] Investigated the best balanced incomplete block design used to estimate the parameter rates for a smaller variance than estimates obtained using the similar design.

A balanced incomplete block design (BIBD) is an incomplete block design which consists of a set of t points that is divided into b subsets in such a way that each point in t is contained in r different subsets and any couple of points in t is contained in $\lambda < b$ subsets with $k < t$ points in each subset. A BIBD is known with the five parameters (t, b, r, k, λ) which satisfy the following axioms that $bk = tr$ and $\lambda(t-1) = r(k-1)$. A standard way of representing a BIBD is in terms of its incidence matrix $M \equiv \{m_{ij}\}_{v \times b}$, which is a $t \times b$ binary matrix with exactly r once per row, k once per column, a scalar product of λ between any pair of distinct rows, and where $m_{ij} \in \{0,1\}$ is equal to 1 if the i^{th} object is contained in the j^{th} block, and 0 otherwise. In this context, m_{ij} represents the incidence of object i in block j of M. There are various types of balanced incomplete block design named according to different authors by the method of their construction.

It has been seen in the literature that all symmetric balanced incomplete block design is a balanced incomplete block design. Different methods of balanced incomplete block designs have been given in the literature, like, [3] applied Galois field to construct balanced incomplete block design. [4] constructed of efficiency-balanced design, [5] constructed balanced incomplete block designs using method of lattice or orthogonal designs of series I and II due to Yates Algorithm and Khare and Federer, [6] developed balanced incomplete block design using Hadamardrhotrices, [7] constructed orthogonal balanced incomplete block design, [8] evaluation of some algebraic structures in BIBD, [9] showed construction of mutually orthogonal Latin square, [10] applied the methods of modern algebra to construct a BIBD, [11] construction of BIBD, [12] constructed balanced incomplete block design using finite Euclidean geometry approach and the construction of balanced incomplete block design using cyclic shift was done by [13].

However, this paper emphasized on three types of designs namely dual design, residue design and derived design for investigation. This singular problem of making a choice of best design has brought a lot of controversies to be handled in the field of combinatorial design. The outcome of the findings will serve as a significant evidence and also prove the best out of the three types of the balanced incomplete block designs.

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Definition 1: Dual design

The dual design is a design with the parameters $t' = b,\ b' = t,\ r' = k,\ k' = r$ obtained by interchanging the treatment and block symbols in the original design. The dual of a balanced incomplete block design is not always a balanced incomplete block design (BIBD). If the original design is a symmetric balanced incomplete block design (SBIBD), then its dual is also a balanced incomplete block design (BIBD) with the same parameter.

Definition 2: Residual design

A residue design is a design with the parameters $t^* = t-k,\ b^* = t-1,\ r^* = r,\ k^* = k-r,\ \lambda^* = \lambda$ a where the given block and treatment from the other blocks occurs in the block are omitted.

Definition 3: Derived design

A design with the parameters $t^{**} = k,\ b^{**} = b-1,\ r^{**} = r-1,\ k^{**} = \lambda,\ \lambda^{**} = \lambda - 1$ is a derived design when blocks are omitted but keep in the others blocks those λ treatment occurs in the given block.

CONCEPT OF OPTIMALITY CRITERIA

Design optimality is a variance-type criterion that involves optimizing various individual properties of the information matrix $(N'N)$. Optimal designs are experimental designs that are generated based on a particular optimality criterion and are generally optimal only for a specific design. An optimality criterion is a criterion which summarizes how good a design is, and it is maximized or minimized by an optimal design. Design optimality is often called the alphabetical optimality criteria because they are named by some of the letters of the alphabet.

(a) A-optimality criterion

This criterion seeks to minimize the trace of the inverse of the information matrix ($N'N$).

$$A^* = \arg\min \mathit{trace}\,[A^{-1}(\psi)]$$

$$= \arg\min \mathit{trace}\,[(N'N)^{-1}] \qquad (1)$$

Where $A(\psi) = N'N$

(b) D-optimality criterion

D-optimality seeks to maximize the determinant of the information matrix $N'N$ or equivalently seeks to minimize the inverse of the information matrix. Symbolically

$$D_{Opt} = \max|N'N| \text{ or } \min(N'N)^{-1} \qquad (2)$$

Where $N'N$ the information matrix, N represents the incidence matrix associated with the design and N' represents its transpose.

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(c) E – optimality criterion

This criterion minimizes the maximum eigenvalues of the dispersion matrix $A(\psi)^{-1}$. In other word, a design $A(\psi)$ is said to be E – optimality if it gives $\min\{\max(\lambda^{-1})\}$, where λ is the largest eigenvalue of the information matrix $A(\psi)$. Symbolically, we have;

$$E_{Opt} = \min\{\max(\lambda^{-1})\} \tag{3}$$

(d) T – optimality criterion

This criterion seeks to maximize the trace of the information matrix ($N'N$). $A^* = \arg\max trace\,[A(\psi)]$

$$= \arg\max trace\,[A(N'N)] \tag{4}$$

Where $\psi = N'N$

Incidence matrices

Incidence matrix is a useful tool and a convenient way of expressing balanced incomplete block designs in a matrix form. By definition, we have;

Let (X,ψ) be a design where $T = (t_1, t_2, ..., t_n)$ and $\psi = \{\psi_1, \psi_2, ..., \psi_b\}$. The incidence matrix of (T,ψ) is the $t \times b_{0-1}$ matrix $M = (m_{ij})$ defined as:

$$m_{ij} = \begin{cases} 1 \text{ if } t_i \in \psi_j \\ 0 \text{ if } t_i \notin \psi_j \end{cases} \tag{5}$$

IMPLEMENTATION/RESULTS

Consider the simulated balanced incomplete block design (SBIBD) with parameters $(t = b = 15,\ r = k = 7,\ \lambda = 3)$ where the treatment (T) and nonempty collection subset of blocks (B) are given as;

$T = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15$

$$B = \begin{Bmatrix} (1,2,3,4,5,6,7), (1,2,3,8,9,10,11), (1,4,5,8,9,12,13), (2,4,6,8,10,12,14), (1,2,3,12,13,14,15), \\ (1,4,5,10,11,14,15), (2,4,6,9,11,13,15), (1,8,9,6,7,14,15), (2,8,10,5,7,13,15), (4,8,12,3,7,11,15), \\ (3,5,6,8,11,13,14), (3,9,10,4,7,13,14), (5,9,12,2,7,11,14), (6,10,12,1,7,11,13), (3,5,6,9,10,12,15) \end{Bmatrix}$$

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Residue design analysis

From the $(t = b = 15, \; r = k = 7, \; \lambda = 3)$ BIB design, a residue balanced incomplete block design (BIBD) with parameters $t = 8, \; b = 14, \; r = 7, \; k = 4, \; \lambda = 3$ is obtained as;

$T = 0,1,3,4,7,8,11,12$

$$B^* = \begin{Bmatrix} (3,4,7,8),(3,4,11,12),(3,7,11,0),(3,4,0,1),(11,12,1,0), \\ (3,7,12,1),(7,8,0,1),(3,11,8,1),(4,8,12,1),(4,7,12,0) \\ (4,11,8,0),(3,8,12,0),(7,11,8,12),(4,7,11,1) \end{Bmatrix}$$

This design can be presented in an incidence matrix form as;

$$N = \begin{pmatrix} 0&0&1&1&1&1&0&0 \\ 0&0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1&0 \\ 1&1&1&1&0&0&0&0 \\ 1&1&0&0&0&0&1&1 \\ 0&1&1&0&1&0&0&1 \\ 1&1&0&0&1&1&0&0 \\ 0&1&1&0&0&1&1&0 \\ 0&1&0&1&0&1&0&1 \\ 1&0&0&1&1&0&0&1 \\ 1&0&0&1&0&1&1&0 \\ 1&0&1&0&0&1&0&1 \\ 0&0&0&0&1&1&1&1 \\ 0&1&0&1&1&0&1&0 \end{pmatrix}, \; N'N = \begin{pmatrix} 7&3&3&3&3&3&3&3 \\ 3&7&3&3&3&3&3&3 \\ 3&3&7&3&3&3&3&3 \\ 3&3&3&7&3&3&3&3 \\ 3&3&3&3&7&3&3&3 \\ 3&3&3&3&3&7&3&3 \\ 3&3&3&3&3&3&7&3 \\ 3&3&3&3&3&3&3&7 \end{pmatrix}$$

$Det(N'N) = 458752$

$$(N'N)^{-1} = \begin{pmatrix} 0.223&-0.027&-0.027&-0.027&-0.027&-0.027&-0.027&-0.027 \\ -0.027&0.223&-0.027&-0.027&-0.027&-0.027&-0.027&-0.027 \\ -0.027&-0.027&0.223&-0.027&-0.027&-0.027&-0.027&-0.027 \\ -0.027&-0.027&-0.027&0.223&-0.027&-0.027&-0.027&-0.027 \\ -0.027&-0.027&-0.027&-0.027&0.223&-0.027&-0.027&-0.027 \\ -0.027&-0.027&-0.027&-0.027&-0.027&0.223&-0.027&-0.027 \\ -0.027&-0.027&-0.027&-0.027&-0.027&-0.027&0.223&-0.027 \\ -0.027&-0.027&-0.027&-0.027&-0.027&-0.027&-0.027&0.223 \end{pmatrix}$$

Eigenv(λ^{-1}) = 0.875   0.875   0.875   0.875   0.875   0.875   0.875   0.000

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Optimality's result for residue design

1	A – Optimality	$= tra(N'N)^{-1} = 1.784$
2	D – Optimality	$= \max\{\det(N'N)\} = 458752$
3	E – Optimality	$= \max(\lambda^{-1}) = 0.875$
4	T – Optimality	$= tra(N'N) = 56$

Derived design analysis

From the $(t = b = 15,\ r = k = 7,\ \lambda = 3)$ BIB design, a derived balanced incomplete block design (BIBD) with parameters $t^{**} = 7,\ b^{**} = 14,\ k^{**} = 3,\ r^{**} = 6,\ \lambda^{**} = 2$ are obtained and is given as;

$T = 1, 4, 5, 10, 11, 14, 15$

$$B = \begin{Bmatrix} (1,4,5), (1,10,11), (5,4,1), (4,10,14), (1,14,15), (4,11,15), (15,14,1), \\ (10,5,15), (11,4,15), (14,5,11), (10,4,14), (5,11,14), (10,1,11), (5,10,15) \end{Bmatrix}$$

This design can be presented in an incidence matrix form as;

$$N = \begin{pmatrix} 1&1&1&0&0&0&0 \\ 1&0&0&1&1&0&0 \\ 1&1&1&0&0&0&0 \\ 0&1&0&1&0&1&0 \\ 1&0&0&0&0&1&1 \\ 0&1&0&0&1&0&1 \\ 1&0&0&0&0&1&1 \\ 0&0&1&1&0&0&1 \\ 0&1&0&0&1&0&1 \\ 0&0&1&0&1&1&0 \\ 0&1&0&1&0&1&0 \\ 0&0&1&0&1&1&0 \\ 1&0&0&1&1&0&0 \\ 0&0&1&1&0&0&1 \end{pmatrix}, N'N = \begin{pmatrix} 6&2&2&2&2&2&2 \\ 2&6&2&2&2&2&2 \\ 2&2&6&2&2&2&2 \\ 2&2&2&6&2&2&2 \\ 2&2&2&2&6&2&2 \\ 2&2&2&2&2&6&2 \\ 2&2&2&2&2&2&6 \end{pmatrix},$$

$Det(N'N) = 73728$

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$$(N'N)^{-1} = \begin{pmatrix} 0.222 & -0.028 & -0.028 & -0.028 & -0.028 & -0.028 & -0.028 \\ -0.028 & 0.222 & -0.028 & -0.028 & -0.028 & -0.028 & -0.028 \\ -0.028 & -0.028 & 0.222 & -0.028 & -0.028 & -0.028 & -0.028 \\ -0.028 & -0.028 & -0.028 & 0.222 & -0.028 & -0.028 & -0.028 \\ -0.028 & -0.028 & -0.028 & -0.028 & 0.222 & -0.028 & -0.028 \\ -0.028 & -0.028 & -0.028 & -0.028 & -0.028 & 0.222 & -0.028 \\ -0.028 & -0.028 & -0.028 & -0.028 & -0.028 & -0.028 & 0.222 \end{pmatrix}$$

Eigenv (λ^{-1}) = 0.857 0.857 0.857 0.857 0.857 0.857 0.000

Optimality's result for derived design

1 A – Optimality $= tra(N'N)^{-1} = 1.554$

2 D – Optimality $= \max\{\det(N'N)\} = 73728$

3 E – Optimality $= \max(\lambda^{-1}) = 0.857$

4 T – Optimality $= tra(N'N) = 42$

Dual design analysis

The SSBIBD with parameters $(t = b = 15,\ r = k = 7,\ \lambda = 3)$ is also a dual design balanced incomplete block design (BIBD) given as;

$T = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15$

$$B = \begin{Bmatrix} (1,2,3,4,5,6,7), (1,2,3,8,9,10,11), (1,4,5,8,9,12,13), (2,4,6,8,10,12,14), (1,2,3,12,13,14,15), \\ (1,4,5,10,11,14,15), (2,4,6,9,11,13,15), (1,8,9,6,7,14,15), (2,8,10,5,7,13,15), (4,8,12,3,7,11,15), \\ (3,5,6,8,11,13,14), (3,9,10,4,7,13,14), (5,9,12,2,7,11,14), (6,10,12,1,7,11,13), (3,5,6,9,10,12,15) \end{Bmatrix}$$

The incidence matrix for the design is;

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$$N = \begin{pmatrix} V/B & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 6 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 7 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 8 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 9 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 10 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 11 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 12 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 13 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 14 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 15 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$N'N = \begin{pmatrix} 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 7 \end{pmatrix}$$

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$$(N'N)^{-1} = \begin{pmatrix}
0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235 & -0.015 \\
-0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & -0.015 & 0.235
\end{pmatrix}$$

Eigenv (λ^{-1}) = 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.933, 0.000

Optimality's for dual design

1 A – Optimality = $tra(N'N)^{-1}$ = 3.525

2 D – Optimality = $\max\{\det(N'N)\}$ = 13153337344

3 E – Optimality = $\max(\lambda^{-1})$ = 0.933

4 T – Optimality = $tra(N'N)$ = 105

The optimality criteria for three types balanced incomplete block designs are summarized in Tables 1.

Table 1: BIBDs with optimality criteria

Designs	A – opt	D – opt	E – opt	T – opt
Residue	1.784	**458752**	0.875	56
Derived	1.554	**73728**	0.857	42
Dual	3.525	**1315334**	0.933	105

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DISCUSSION

A simulated balanced incomplete block design (SSBIBD) with the parameters $(t,\ v, r, k,\ \lambda) \rightarrow (15,\ 15,\ 7, 7,\ 3)$ produced a residue and derived balanced incomplete block design with the parameters $(t,\ b,\ r,\ k,\ \lambda) \rightarrow (8,\ 14,\ 7,\ 4,\ 3)$ and $(t,\ b,\ r,\ k,\ \lambda) \rightarrow (7,\ 14,\ 3,\ 6,\ 2)$ by methods of block selection and block intersection. Therefore, the two designs in turned become a standard balanced incomplete block design but the duality of the design is the same as the simulated design. From Table 1, D – optimality has the highest values compared with the three design followed by T-, A – and E – optimality. Again Table 1 shows that dual design has the highest values across all the optimality criteria followed by residue and derived design.

CONCLUSION

In the context of this paper, we discovered that D – optimality is the best optimality criterion followed by T - optimality, A – optimality and E – optimality. In relation of these four optimality criteria to the three designs, it is shown that dual design is the best symmetric balanced incomplete block design among others due to its high optimal values. The findings also revealed that all dual designs are symmetric balanced incomplete block design and balanced incomplete block design but the converse is not true especially for balanced incomplete block design. Therefore, it is recommended that dual design be used as a standard balanced incomplete block design.

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