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MODELLING OF LIFETIME INVERTER BATTERIES AND ENERGY STORAGE SYSTEMS

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ABSTRACT: *Lifetime batteries are essential component of energy storage systems. These batteries are designed to store electrical energy and provide power during periods of power outages. Energy storage systems are broader solutions for storing electrical energy which often include inverter batteries (Lithium ion) as a component. Energy storage systems are becoming increasingly important for integrating renewable energy sources like solar ensuring stability and reliability. To maximize battery performance and extend lifespan, precise assessment of the batteries' state of charge, battery impedance, and remaining capacity is necessary. In this paper, Gamma Distribution (G (α, β)) and Weibull Distribution (W (α, β)) were used to model the lifetime batteries and energy storage systems in other to determine the maximum lifetime of an inverter batteries and secondly to compare the results obtained from the two distributions. The two probability distributions (The Gamma and The Weibull) were employed in the analysis of our simulated data because of their flexibility in modeling data longevity. From the results obtained, it was observed that the average lifespan of inverter batteries is seven years (7 years). Weibull distribution demonstrated superior fit when compared with Gamma distribution in handling the modeling of lifetime inverter batteries.*

KEYWORDS: Batteries, Gamma, Inverters.

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INTRODUCTION

The modelling of lifetime inverter batteries and energy storage systems has become a crucial frontier in the energy sector amid the global shift to greener and more sustainable energy sources. The effective management and storage of energy become critical as the globe works to lessen its dependency on fossil fuels and include renewable energy sources like solar and wind into the electrical grid. The high energy density, extended cycle life, and lightweight nature of lithium-ion batteries make them indispensable in the field of energy storage. Lithium-ion batteries are crucial for energy storage because they allow for the stable supply of power and the storing of electricity produced by renewable energy sources like solar and wind. Given that inverter batteries are essential to the general functionality and dependability of energy storage systems; it is critical to comprehend the variables affecting their longevity. This paper investigates the modelling of lifespan inverter batteries and its implications for energy storage, filling a large knowledge gap. This study intends to stimulate battery longevity and investigates useful applications in energy storage system optimization. By doing this, it hopes to lower costs, increase the overall efficiency of energy storage systems and promote the development of sustainable energy solutions. According to research done expressly by Kafetsiz et. al, (2020) on inverters, load conditions have the biggest impact on inverter efficiency also based on their case study's ideal power output, the highest inverter efficiency was over 90 percent.

Jiajun, (2013) stated that material discovery is the foundation of lithium ion battery development and commercialization. The study covers new developments in polyoxyanion cathodes for lithium-ion batteries and anode materials based on silicon and tin. Creating materials with a high energy density for use in alternate modes of transportation is the aim of his research. According to Salcedo-Sanz et. al, (2009) The data that the inverter measures, whose nature may influence the reference level of efficiency as of right now. As a result, performance diagnosis technology is required in order to precisely determine system performance from solar energy input to system output, as well as any process loss or malfunction. Peng Wei et. al, (2023) stated that Energy storage system research has been done in response to the growing importance of energy storage technologies and the requirement for the integration of renewable energy sources. Their study focuses on a number of topics including technology related to battery storage, electricity to gas conversion and ideal configuration. In the study carried out by Surender et. al, (2021) emphasises how storage systems have fast response time, high ramp rates, and the capacity to function as both generation and load which make them viable choices for lowering peak demand and promoting the integration of renewable energy sources. From Ritchie (2004), the two main changes are that polymer electrolyte batteries are now being produced and, as anticipated, lithium cobalt oxide cathode material is replacing lithium cobalt/nickel oxide. New materials for the cathode and electrolyte to lower costs and increase safety are probably in the works. Abdul et. al, (2021), The most recent development in energy storage applications are reviewed critically, along with their potential and drawbacks. Additionally, it examines ways to lower expenses and address obstacles in order to enhance performance and promote the use of these technologies.

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METHODOLOGY

A Monte Carlo simulation research is conducted to model the inverter battery's lifetime. A random sample of 100 for different power range of 450Watts, 450Watts - 1500Watts and above 1500Watts. The shape parameters are chosen as $\propto$ = 2.0, 1.5 and 3.0 respectively and scale parameters are chosen as β = 365, 730, 1095, 1460, 1825, 2190, and 2555 which represent the number of days in a year for Gamma Distribution and Weibull Distribution.

Weibull Distribution

Nawal et. al, (2022), The probability density function and the cumulative distribution function of a two parameter Weibull distribution with scale parameter, $\alpha > 0$ and shape parameter, $\beta > 0$, are given by,

$$f\left(x_{i;}\,\alpha,\beta\right) = \left(\frac{\beta}{\alpha}\right)\left(\frac{x_i}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{x_i}{\alpha}\right)\right]^{\beta} \quad (1)$$

The cumulative distribution function is,

$$F\left(x_{i;}\,\alpha,\beta\right) = 1 - exp\left[-\left(\frac{x_i}{\alpha}\right)\right]^{\beta} \quad (2)$$

where X is the random variable.

Estimation of Parameters

The method of MLE is a common procedure to estimate parameters of a model's distribution which are assumed to be independent and identically distributed (i.i.d). The parameters are estimated by maximizing the likelihood function. Let $x_1, x_2, \dots, x_n$ be a sample of size n obtained from a probability density function $f\left(x,\hat{\theta}\right)$ where $\hat{\theta}$is an unknown parameter. The likelihood function is given as,

$$L = \prod_{i=1}^{n} f\left(x,\hat{\theta}\right) \quad (3)$$

The MLE of $\hat{\theta}$ is the value of $\hat{\theta}$ that maximizes the likelihood function or the log-likelihood function where

$$\frac{d\log L}{d\hat{\theta}} = 0 \quad (4)$$

By applying Eqn. (3) to the Weibull probability density function in Eqn. (2), the likelihood function will be,

$$L(x_i,\alpha,\beta) = \prod_{i=1}^{n}\left(\frac{\beta}{\alpha}\right)\left(\frac{x_i}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{x_i}{\alpha}\right)\right]^{\beta} \quad (5)$$

Taking the logarithms of Eqn. (5), differentiating with respect to α and β and equating to zero, the equations become,

$$In\ L(\alpha,\beta) = n\ In\ \beta - n\beta\ In\ \alpha - \frac{1}{\alpha^{\beta}}\sum_{i=1}^{n} x^{\beta}{}_{i} + (\beta - 1)\sum_{i=1}^{n} In\ x_i \quad (6)$$

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$$\frac{\partial\, In\, L\,(\alpha,\beta)}{\partial\alpha} = -\frac{n\beta}{\alpha} + \frac{\beta}{\alpha^{\beta+1}}\sum_{i=1}^{n} x^{\beta}{}_{i} \tag{7}$$

$$\frac{\partial\, In\, L\,(\alpha,\beta)}{\partial\beta} = \frac{n}{\beta} - n\, In\, \alpha - \frac{\sum_{i=1}^{n} x^{\beta}{}_{i} - In\,\alpha\,\sum_{i=1}^{n} x^{\beta}{}_{i}}{\alpha^{\beta}} + \sum_{i=1}^{n} In\, x_i = 0 \tag{8}$$

By eliminating α from both Eqns. (7) and (8) and simplifying the equations,

$$\hat{\alpha} = \left(\frac{1}{n}\sum_{i=1}^{n} x^{\beta}{}_{i}\right)^{\frac{1}{\beta}} \tag{9}$$

$$\frac{1}{\beta} - \frac{\sum_{i=1}^{n} x^{\beta}{}_{i}\, In\, x_i}{\sum_{i=1}^{n} x^{\beta}{}_{i}} + \frac{1}{n}\sum_{i=1}^{n} In\, x_i = 0 \tag{10}$$

The estimate $\hat{\alpha}$ can be obtained using Eqn. (9). However, the estimate $\hat{\beta}$ must be solved numerically as the Eqn. (10) has not produced the analytical solution. It can be accomplished by applying the optimization method. One of the most used methods for optimization is the Newton-Raphson method. The Newton-Raphson method requires finding the inverse of the Hessian, H_f at each iteration. Newton- Raphson method will be used to get the iteration value until a convergent estimator is achieved. It can be written as,

$$\beta_{i+1} = \beta_i - \frac{f(\beta_i)}{f'(\beta_i)} \tag{11}$$

where i is the iteration. Eqn. (10) is used as the initial point, β_0. Next, Eqn. (9) is substituted into the log-likelihood function in Eqn. (6) to obtain,

$$l_{\beta}(\beta) = l(\hat{\alpha}\,, \beta) \tag{12}$$

$$l_{\beta}(\beta) = n\, In\,(\beta) - n\beta\, In\left(\left[\frac{1}{n}\sum_{i=1}^{n} x^{\beta}{}_{i}\right]^{\frac{1}{\beta}}\right) + (\beta-1)\sum_{i=1}^{n} In\,(x_i) - \frac{\sum_{i=1}^{n} x^{\beta}{}_{i}}{\frac{1}{n}\sum_{i=1}^{n} x^{\beta}{}_{i}} \tag{13}$$

$$l_{\beta}(\beta) = n\, In\,(\beta) - n\, In\left(\frac{1}{n}\sum_{i=1}^{n} x^{\beta}{}_{i}\right) + (\beta-1)\sum_{i=1}^{n} In\,(x_i) - n \tag{14}$$

The partial maximized log-likelihood function $l_{\beta}(\beta)$ is called the profile log-likelihood. Then, Eqn. (14) is differentiated twice with respect to β to form,

$$f(\beta_i) = \frac{n}{\beta} - n\frac{\sum_{i=1}^{n} x^{\beta}{}_{i}\, In\,(x_i)}{\sum_{i=1}^{n} x^{\beta}{}_{i}} + \sum_{i=1}^{n} In\,(x_i) \tag{15}$$

And,

$$f'(\beta_i) = -\frac{n}{\beta^2} - n\frac{\sum_{i=1}^{n} x^{\beta}{}_{i}\, In\,(x_i)^2 \sum_{i=1}^{n} x^{\beta}{}_{i} - \left(\sum_{i=1}^{n} x^{\beta}{}_{i}\, In\,(x_i)\right)^2}{\left(\sum_{i=1}^{n} x^{\beta}{}_{i}\right)^2} \tag{16}$$

The convergence criterion is given as,

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$$|\beta_i - \beta_{i-1}| < 0.000001 \quad (17)$$

Gamma Distribution

Nwankwo (2017), The Gamma distribution is characterized by two parameters scale (α) and shape (β). The probability density function of the Gamma distribution is given by

$$f(x_{i;}\, \alpha, \beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} X^{\alpha-1} e^{-x_i/\beta} \quad (18)$$

Estimation of Parameters

The likelihood function is given by

$$L = \prod_{i=1}^{n} f(x, \hat{\theta})$$

The MLE of $\hat{\theta}$ is the value of $\hat{\theta}$ that maximizes the likelihood function or the log-likelihood function where

$$\frac{d \log L}{d\hat{\theta}} = 0$$

By applying Eqn. (3) to the Gamma probability density function in Eqn. (18), the likelihood function will be,

$$L(x_i, \alpha, \beta) = \prod_{i=1}^{n} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} X^{\alpha-1} e^{-x_i/\beta} \quad (19)$$

Taking the logarithms of Eqn. (19), differentiating with respect to α and β and equating to zero, the equations become,

$$In\ L(\alpha, \beta) = n\left(-\alpha \log \beta - \log \Gamma(\alpha) + (\alpha - 1) \sum_{i=1}^{n} \log x_i - \frac{1}{\beta} \sum_{i=1}^{n} x_i\right) \quad (20)$$

$$\frac{\partial\ In\ L\ (\alpha,\beta)}{\partial \alpha} = n(-\log \beta - \psi(\alpha)) + \sum_{i=1}^{n} \log x_i \quad (21)$$

$$\frac{\partial\ In\ L\ (\alpha,\beta)}{\partial \beta} = n\left(\frac{-\alpha}{\beta}\right) + \frac{1}{\beta^2} \sum_{i=1}^{n} x_i \quad (22)$$

Equating equation (21) and (22) to zero gives;

$$n(-\log \beta - \psi(\alpha)) + \sum_{i=1}^{n} \log x_i = 0 \quad (23)$$

$$\frac{\partial L}{\partial \beta} = n\left(\frac{-\alpha}{\beta}\right) + \frac{1}{\beta^2} \sum_{i=1}^{n} x_i = 0 \quad (24)$$

From equation (24)

$$n\left(\frac{-\alpha}{\beta}\right) = -\frac{1}{\beta^2} \sum_{i=1}^{n} x_i$$

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$$\frac{-n\alpha}{\beta} = -\frac{1}{\beta^2}\sum_{i=1}^{n} x_i$$

$$\frac{n\alpha}{\beta} = \frac{1}{\beta^2}\sum_{i=1}^{n} x_i$$

Multiply through by β;

$$n\alpha = \frac{1}{\beta}\sum_{i=1}^{n} x_i$$

$$\beta n\alpha = \sum_{i=1}^{n} x_i$$

$$\beta = \frac{\sum_{i=1}^{n} x_i}{n\alpha}$$

$$\widehat{\beta} = \frac{\bar{x}}{\alpha}$$

From equation (23)

$$n(-\log\beta - \psi(\alpha)) = -\sum_{i=1}^{n} \log x_i$$

Diving through by n gives;

$$-\log\beta - \psi(\alpha) = -\frac{1}{n}\sum_{i=1}^{n} \log x_i$$

Multiply through by "-"

$$\log\beta + \psi(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

Recall that $\widehat{\beta} = \frac{\bar{x}}{\alpha}$

$$\log\left(\frac{\bar{x}}{\alpha}\right) + \psi(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

$$\log\bar{x} - \log\alpha + \psi(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

$$\log\alpha - \psi(\alpha) = \log\bar{x} - \frac{1}{n}\sum_{i=1}^{n} \log x_i \tag{25}$$

Where $\psi(\alpha) = \frac{\partial}{\partial\alpha}(\log\Gamma(\alpha))$ is the Gamma function

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Solving equation (25) does not yield an explicit expression for the scale parameter(α). Therefore, an iterative procedure (like Newton Raphson Method) will be used with the help of statistical software (R).

RESULTS

The power range of the inverter batteries chosen were 450Watts, 450Watts – 1500Watts and above 1500Watts.The shape (α) and scale (β) parameters of Weibull distribution and Gamma distribution were estimated. The shape (α) parameter represents the failure/degradation process and its values for the three power ranges of the inverter batteries for 450Watts, 450Watts – 1500Watts and Above 1500Watts are 2, 1.5 and 3 respectively for both Weibull distribution and Gamma distribution. The scale (β) parameter represents the average lifetime of batteries/average time until failure or degradation (days) and its values for Weibull distribution and Gamma distribution are 730 days, 1095 days, 1460 days, 1825 days, 2190 days and 2555 days. The tabular results and the graphical results from the both distributions are presented below.

Tabular Results and Graphical Results for Weibull Distribution

Table 3.1: result of the three power range of inverter batteries of lifetime of 730 days

Power-Rating	Lifetime (730 days)
Length: 300	Min: 55.34
Class: Character	1st Quarter: 406.94
Mode: Character	Median: 625.79
	Mean: 640.22
	3rd Quarter: 847.33
	Max: 1982.88

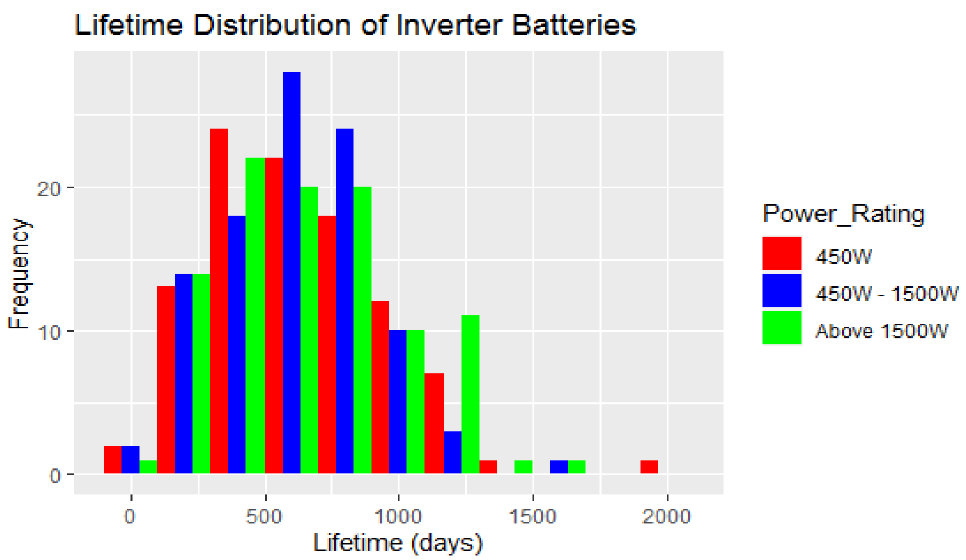

Figure 3.1: The Histogram of the three power range of inverter batteries of lifetime of 730 days

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Table 3.2: Result of the three power range of inverter batteries of lifetime of 1095 days

Power-Rating	**Lifetime (1095 days)**
Length: 300	Min: 83.01
Class: Character	1st Quarter: 610.41
Mode: Character	Median: 938.69
	Mean: 960.33
	3rd Quarter: 1271.00
	Max: 2974.32

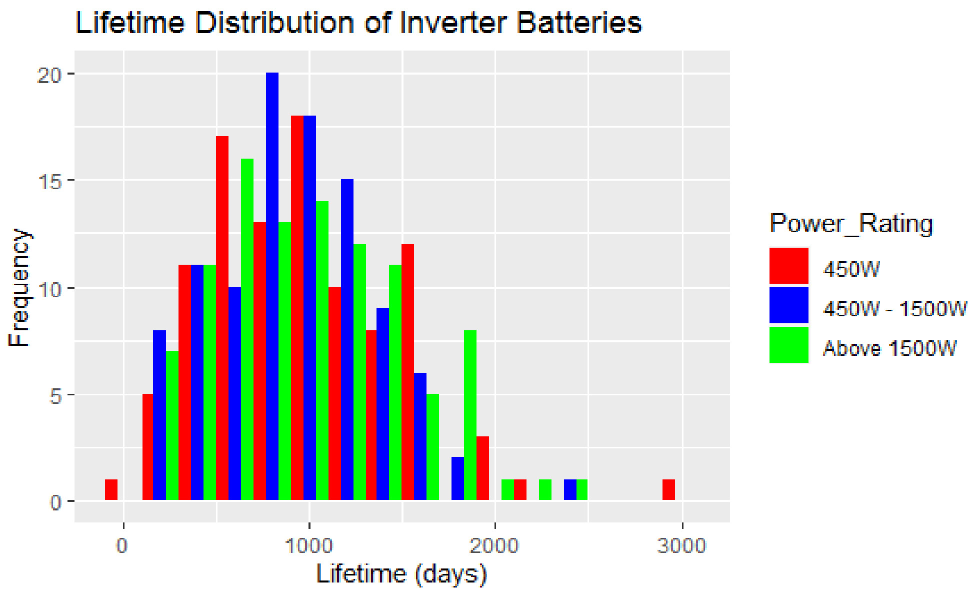

Figure 3.2: The Histogram of the three power range of inverter batteries of lifetime of 1095 days

Table 3.3: Result of the three power range of inverter batteries of lifetime of 1460 days

Power-Rating	**Lifetime (1460 days)**
Length: 300	Min: 110.7
Class: Character	1st Quarter: 813.9
Mode: Character	Median: 1251.6
	Mean: 1280.4
	3rd Quarter: 1694.7
	Max: 3965.8

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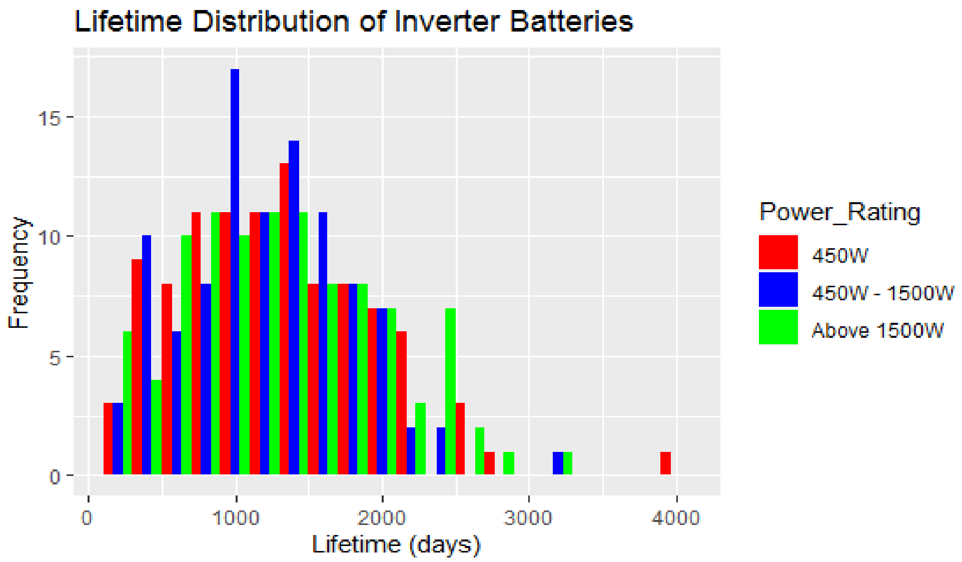

Figure 3.3: The Histogram of the three power range of inverter batteries of lifetime of 1460 days

Table 3.4: Result of the three power range of inverter batteries of lifetime of 1825 days

Power-Rating	**Lifetime (1825 days)**
Length: 300	Min: 138.3
Class: Character	1st Quarter: 1017.3
Mode: Character	Median: 1564.5
	Mean: 1600.6
	3rd Quarter: 2118.3
	Max: 4957.2

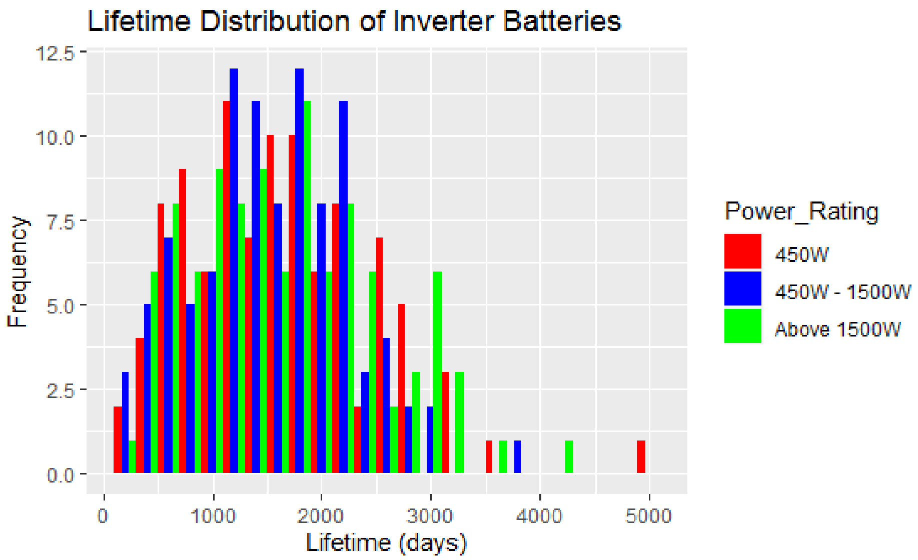

Figure 3.4: The Histogram of the three power range of inverter batteries of lifetime of 1825 days

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Table 3.5: Result of the three power range of inverter batteries of lifetime of 2190 days

Power-Rating	Lifetime (2190 days)
Length: 300	Min: 166
Class: Character	1st Quarter: 1221
Mode: Character	Median: 1877
	Mean: 1921
	3rd Quarter: 2542
	Max: 5949

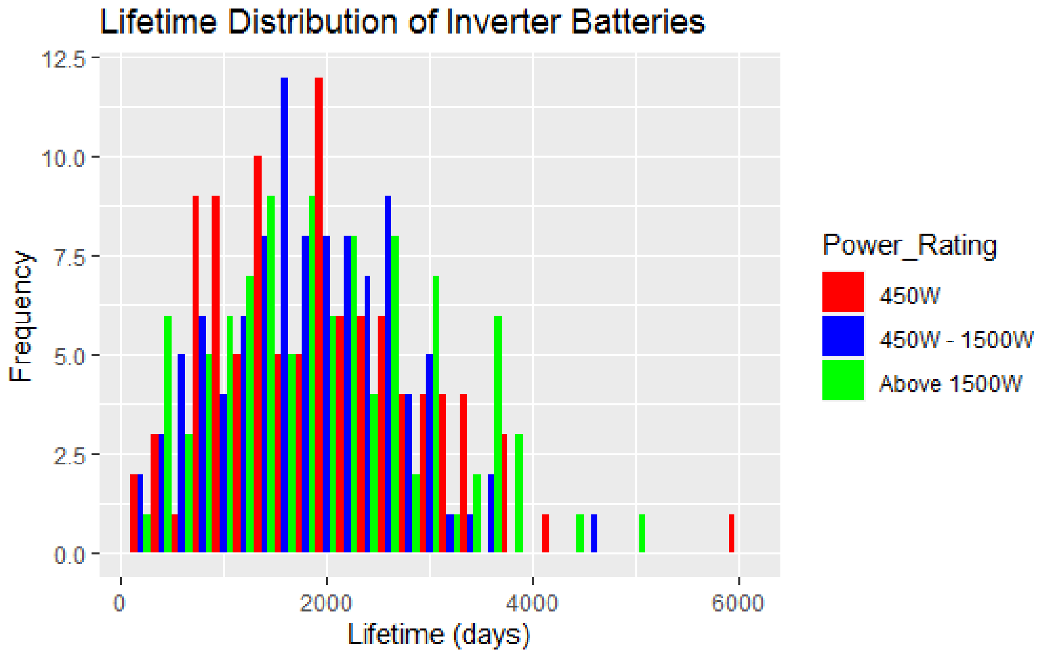

Figure 3.5: The Histogram of the three power range of inverter batteries of lifetime of 2190 days

Table 3.6: result of the three power range of inverter batteries of lifetime of 2555 days

Power-Rating	Lifetime (2555 days)
Length: 300	Min: 193.7
Class: Character	1st Quarter: 1424.3
Mode: Character	Median: 2190.3
	Mean: 2240.8
	3rd Quarter: 2965.7
	Max: 6940.1

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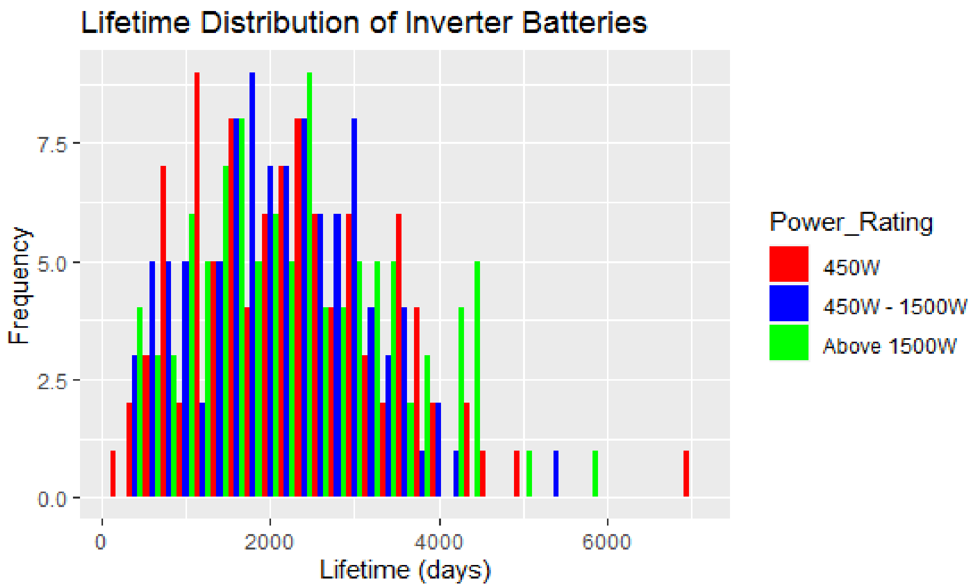

Figure 3.6: The Histogram of the three power range of inverter batteries of lifetime of 2555 days

The Tabular Results and Graphical Results for Gamma Distribution

Table 3.7: Result of the three power range of inverter batteries of lifetime of 730 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	1257	1115	815	95.2	3861
450Watts – 1500Watts	1029	701	846	50.8	4045
Above 1500Watts	2028	2037	1059	160	5451

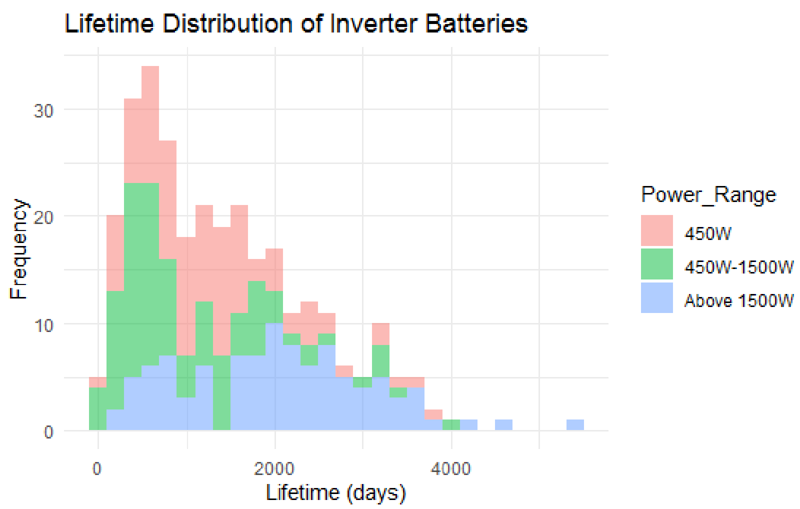

Figure 3.7: The Histogram of the three power range of inverter batteries of lifetime of 730 days

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Table 3.8: result of the three power range of inverter batteries of lifetime of 1095 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	1885	1672	1223	143	5791
450Watts – 1500Watts	1544	1051	1269	76.3	6068
Above 1500Watts	3042	3056	1588	240	8176

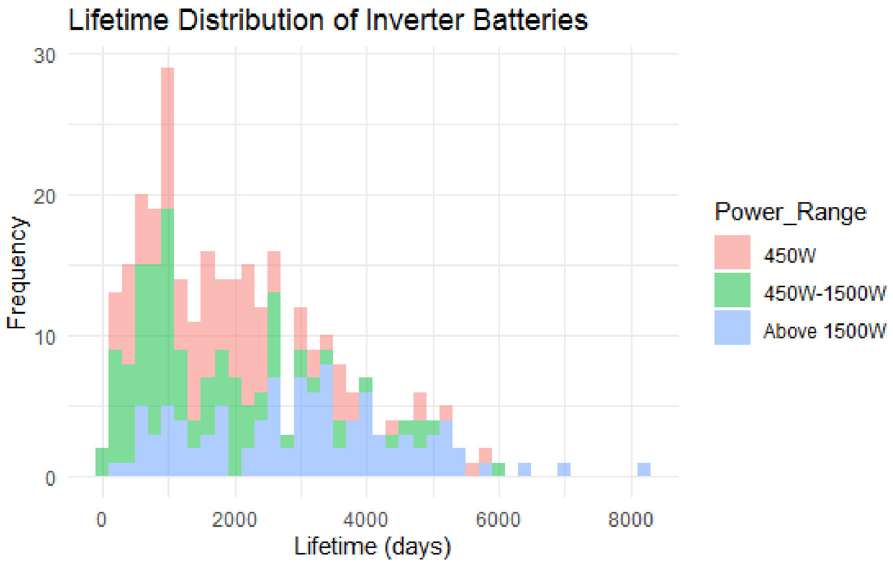

Figure 3.8: The Histogram of the three power range of inverter batteries of lifetime of 1095 days

Table 3.9: result of the three power range of inverter batteries of lifetime of 1460 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	2513	2229	1631	190	7721
450Watts – 1500Watts	2058	1402	1691	102	8091
Above 1500Watts	4056	4074	2117	321	10902

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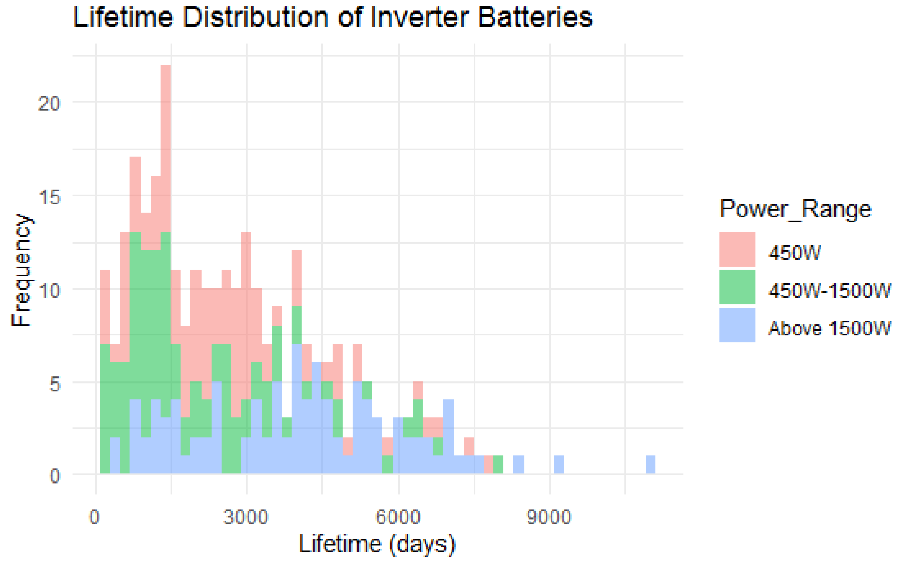

Figure 3.9: The Histogram of the three power range of inverter batteries of lifetime of 1460 days

Table 3.10: Result of the three power range of inverter batteries of lifetime of 1825 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	3142	2786	2038	238	9652
450Watts – 1500Watts	2573	1752	2114	127	10114
Above 1500Watts	5070	5093	2646	401	13627

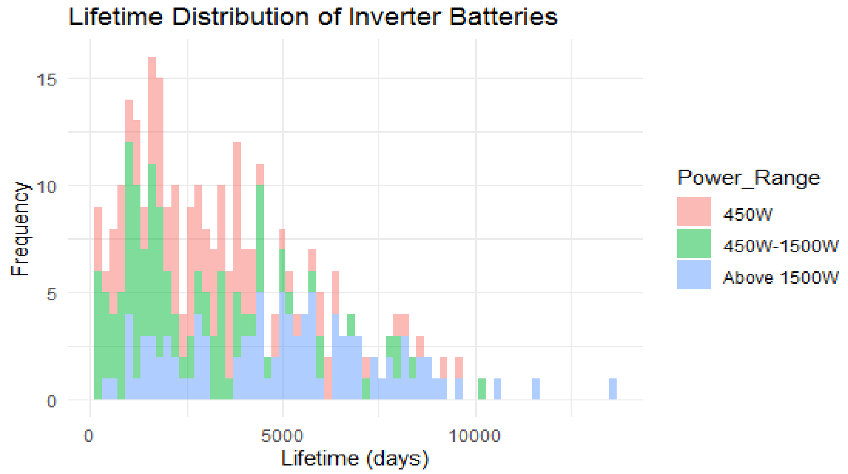

Figure 3.10: The Histogram of the three power range of inverter batteries of lifetime of 1825 days

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Table 3.11: result of the three power range of inverter batteries of lifetime of 2190 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	3770	3344	2446	286	11582
450Watts – 1500Watts	3087	2103	2537	153	12136
Above 1500Watts	6083	6111	3176	481	16352

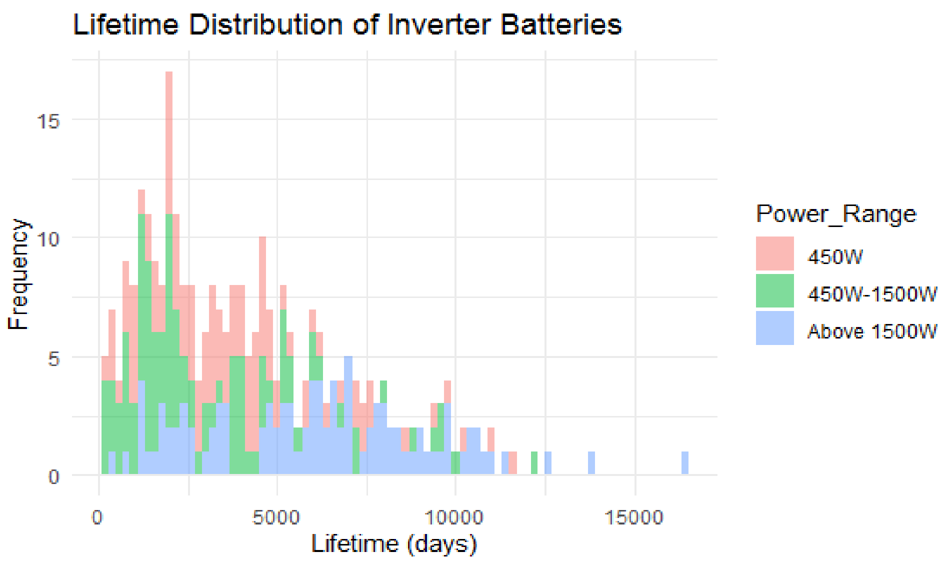

Figure 3.11: The Histogram of the three power range of inverter batteries of lifetime of 2190 days

Table 3.12: result of the three power range of inverter batteries of lifetime of 2555 days

Power range	Mean	Median	Standard Deviation	Minimum	Maximum
450Watts	4398	3091	2854	333	13512
450Watts – 1500Watts	3602	2453	2960	178	14159
Above 1500Watts	7097	7130	3705	561	19078

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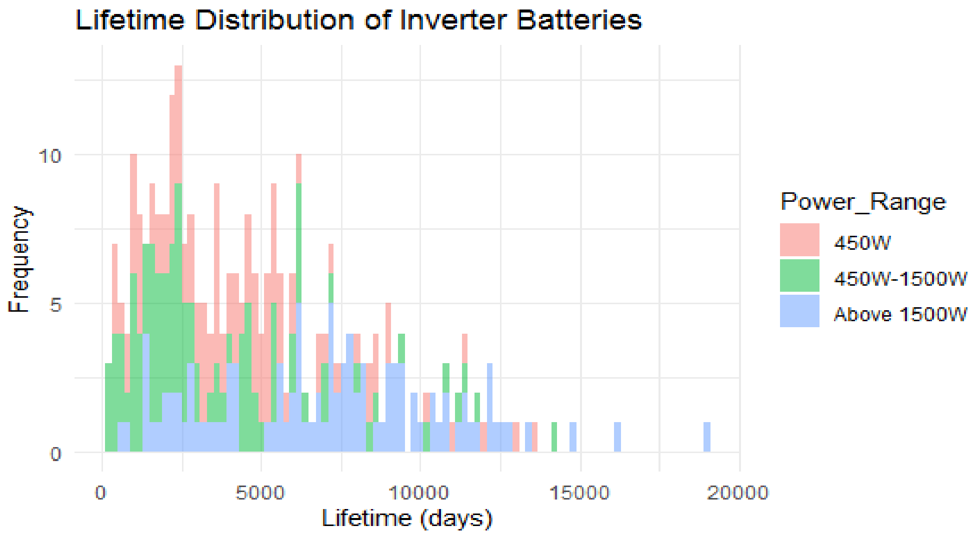

Figure 3.12: The Histogram of the three power range of inverter batteries of lifetime of 2555 days

CONCLUSION

In this study, Monte Carlo simulation study was carried out with sample size n = 100 for each of the power range of 450Watts, 450Watts – 1500Watts and above 1500Watts. The shape ($\propto$) and the scale (β) parameters for both Weibull distribution and Gamma distribution were estimated, where the shape ($\propto$) parameters represent the failure or degradation process and the scale (β) parameters represent the average lifespan of an inverter batteries before degradation. From the result of the simulated data, it was observed that the average lifespan on inverter batteries is 2555 days which is approximately seven (7) years. Weibull distribution also demonstrated superior fit when compared with Gamma distribution in handling the modelling of lifetime inverter batteries and energy storage systems.

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APPENDICE

```
# Load the necessary libraries

library(stats)

library(ggplot2)

library(dplyr)

# Set the random seed for reproducibility

set.seed(123)

# Number of data points to simulate for each power range

n_data_points <- 100

# Simulate data for inverter batteries with a power range of 450W

shape_450W <- 2  # Shape parameter (alpha) for 450W range

scale_450W <- 2555  # Scale parameter (beta) for 450W range

data_450W <- rgamma(n_data_points, shape = shape_450W, scale = scale_450W)

# Simulate data for inverter batteries with a power range of 450W - 1500W

shape_450W_1500W <- 1.5  # Shape parameter (alpha) for 450W - 1500W range

scale_450W_1500W <- 2555  # Scale parameter (beta) for 450W - 1500W range
```


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data_450W_1500W <- rgamma(n_data_points, shape = shape_450W_1500W, scale = scale_450W_1500W)

Simulate data for inverter batteries with a power range above 15000W

shape_above_15000W <- 3 # Shape parameter (alpha) for above 15000W range

scale_above_15000W <- 2555 # Scale parameter (beta) for above 15000W range

data_above_15000W <- rgamma(n_data_points, shape = shape_above_15000W, scale = scale_above_15000W)

Combine the data for all three power ranges

simulated_data <- c(data_450W, data_450W_1500W, data_above_1500W)

power_ranges <- c(rep("4500W", n_data_points), rep("4500W-15000W", n_data_points), rep("Above 15000W", n_data_points))

simulated_data_frame <- data.frame(Power_Range = power_ranges, Lifetime = simulated_data)

View the first few rows of the simulated data

cat("Simulated Data:\n")

print(head(simulated_data_frame))

Create histograms to visualize the data

ggplot(simulated_data_frame,aes(x=Lifetime,fill=Power_Range))+geom_histogram(binwidth = 200, alpha = 0.5)+labs(title = "Lifetime Distribution of Inverter Batteries", x = "Lifetime (days)", y = "Frequency") +

theme_minimal()

Summary statistics

summary_stats <- simulated_data_frame %>%

group_by(Power_Range) %>%

summarize(

Mean = mean(Lifetime),

Median = median(Lifetime),

SD = sd(Lifetime),

Min = min(Lifetime),

Max = max(Lifetime)

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```
  )
# View tabular summary
cat("\nSummary Statistics:\n")
print(summary_stats)
# Load required libraries
library(reliabilitydiag)
# Set a random seed for reproducibility
set.seed(123)
# Define the parameters for the Weibull distribution
shape <- 2
scale <- 2555  # You can adjust this parameter as needed
# Simulate lifetime data for three power ratings: 4500W, 4500W - 15000W, and above 15000W
sample_size <- 100  # Number of samples for each group
# Simulate data for 4500W batteries
data_4500W <- rweibull(sample_size, shape, scale)
# Simulate data for 4500W - 15000W batteries
data_4500W_15000W <- rweibull(sample_size, shape, scale)
# Simulate data for batteries above 15000W
data_above_15000W <- rweibull(sample_size, shape, scale)
# Create a data frame to store the simulated data
battery_data <- data.frame(
  Power_Rating = rep(c("4500W", "4500W – 15000W", "Above 15000W"), each = sample_size),
  Lifetime = c(data_4500W, data_4500W_15000W, data_above_15000W)
)
# Display the first few rows of the data frame
head(battery_data)
# Create a histogram to visualize the simulated data
```


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```
library(ggplot2)

ggplot(battery_data, aes(x = Lifetime, fill = Power_Rating)) +
  geom_histogram(binwidth = 200, position = "dodge") +
  labs(
    title = "Lifetime Distribution of Inverter Batteries",
    x = "Lifetime (days)",
    y = "Frequency"
  ) +
  scale_fill_manual(values = c("4500W" = "red", "4500W - 15000W" = "blue", "Above 15000W" = "green"))

# Perform a summary of the data

summary(battery_data)
```


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