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A COMPARISON OF PARAMETERS OF WEIBULL DISTRIBUTION ON NIGERIA STOCK PRICES

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ABSTRACT: *This paper compared 2-parameter and 3-parameter Weibull Distribution using monthly average stock price returns of five different bottling companies enlisted in the Nigerian Stock market comprising Nigerian Breweries PLC, Coca-Cola Bottling Company, Guinness Nigeria PLC, Seven Up Bottling Company and International Breweries from 2012 to 2016 obtained from www.market.ft.com/data/equities/tearsheet/historical. The parameters were obtained using the Method of Moments (MOM) and were compared by the means of minimum Mean Squared Error; the result shows that Coca-Cola bottling company stock price return performed better under 2-parameter Weibull estimate while International Breweries was a choice under 3-parameter Weibull estimate. Also, the goodness-of-fit test was conducted using Kolmogorov-Smirnov Test and the result reviewed that the price of the stock returns does not follow a Weibull Distribution (P-value<0.05).*

KEYWORDS: Two-Parameter Weibull Distribution, Three-Parameter Weibull Distribution, Method of Moments, Stock Prices.

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INTRODUCTION

The Weibull distribution is a continuous probability distribution named after Swedish Mathematician Wallodi Weibull. The distribution was originally proposed by Weibull as a model for material breaking strength, but recognized the potential of the distribution in a paper in 1951. The wide applicability of Weibull distribution today has cut across all fields, including assessing product reliability, analysis of life data and model failure times. The Weibull can also fit a wide range of data from many other fields including biology, economics, engineering science and hydrology, for example, Mann and Fertig (1973), Stone and Heesswijk (1977), Tikku and Signh (1981), Rinne (2008), Johnson and Balakrishan (1995), and Scholz (1999). Weibull distribution has two common probability density functions, namely the two and three parameters Weibull distribution. Murthy et al. (2003) presented a monograph that contains nearly every fact relating to the Weibull distribution. Statistically, modeling of real life scenarios helps us to better understand and explain random events when they occur, thereby enabling us to reproduce such a scenario either on a large and/or on a simplified scale aimed at describing only critical parts of the phenomenon. These real life phenomena are captured by means of statistical distribution models, which are extracted or learned directly from data gathered about them. Every distribution model has a set of parameters that needs to be estimated. These parameters specify the model and provide a mechanism for efficient and accurate use of data, for example, Wong (1993), Johnson et al. (1994), Mudholkar and Kolia (1996), Wu (2002), Tang (2004), and Nwobi and Ugomma (2014).

Generally, several authors have applied the Weibull distribution in different areas of life and study, for example, Lawless (1982), Al-Fawazan (2000), and Nelson (1982). Hallinan (1993) provided an excellent review of the Weibull distribution by presenting historical facts, and the many different forms of this distribution as used by practitioners, and possible confusions and errors that arise due to this non-uniqueness. The Weibull distribution is undeniably the distribution that has received the maximum attention due to its contribution so far. The evidence against the assumption that monthly stock returns are normally distributed has been mounting for over thirty years. Most of the empirical evidence analyzed United States data, although some recent studies have considered European markets, stock price being a determinant on the price fluctuations which can be an important indicator of a supply of a particular product/stock that influences the potential output of an economy. Hence, it is observed that a rise in the price of a product shows a decrease in demand and supply as well as its scarcity, and a fall in price may indicate an increase in demand of a product which leads to an increase in sales of the product as part of the basic input of production, and as well impacting positively on the economic situation of the country. Based on this, it has been a great concern for statisticians or financial practitioners to provide a model that always accommodates stock prices in order to ascertain the best fit.

Therefore, this study is to determine whether Weibull distribution is the best distribution that can fit the stock prices in some of our industries, with emphasis on five selected industries enlisted in Nigerian Stock Market, which include Guinness Nigeria PLC, Seven up Bottling Company, Coca-Cola Bottling Company, Nigerian breweries PLC and International Breweries PLC, from 2012–2016, for better economic planning and future decision making.

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MATERIALS AND METHOD

Materials

The Probability Density Functions of Weibull Distribution

The 2-Parameter and 3-Parameter Weibull Probability density functions of are given by:

$$f(x) = \left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{\beta}{\alpha}\right)^{\beta}\right\}, x \geq 0 \tag{1}$$

and

$$f(x) = \left(\frac{\beta}{\alpha}\right)\left(\frac{x-\nu}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right\}, x \geq 0, \beta, \alpha \geq 0, \nu \geq 0 \tag{2}$$

where

β is the shape parameter (threshold parameter)

α is the scale parameter (characteristic life parameter)

ν is the location parameter (waiting time parameter or shift parameter).

The value of the shape parameter (β) determines the failure rates as:

i. If $\alpha < 1$, then the failure rate decreases with time (the process has a large number of infinite or early-life failures and fewer failures as time passes)

ii. If $\beta = 1$, the failure rate is constant, which means it is indicative of useful life or random failures.

iii. If $\beta > 1$, the rate of failure increases with time (i.e., the distribution models wear-out failures, which tend to happen after some time has passed).

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METHOD OF ANALYSIS

The Method of Moments (MOM)

The method of moments is another technique commonly used in the field of parameter estimation. Let $x_1, x_2 \ldots, x_n$ be a random sample and then unbiased estimator for is moment is given by:

$$\hat{m}_k = \sum_{i=1}^{n} x_i^k \tag{3}$$

where $\hat{m}_k$ stands for the estimate of the kth moment. In Weibull the kth moment follows from Equation (3) (Al-Fawazan 2000) as:

$$\hat{\mu}_k = \left(\frac{1}{\alpha^{\beta}}\right)^{-\frac{k}{\beta}} \Gamma\left(1+\frac{k}{\beta}\right) \tag{4}$$

where

Γ is a gamma function evaluated at the value of $\left(1+\frac{1}{\beta}\right)$ which provides the values $\Gamma\,(k)$ at any value of k. From (4), we can find the 1st and 2nd moments as follows:

$$\hat{m}_1 = \hat{\mu}_1 = \left(\frac{1}{\alpha^{\beta}}\right)^{\frac{1}{\beta}} \Gamma\left(1+\frac{1}{\beta}\right) \tag{5}$$

and

$$\hat{m}_2 = \hat{\mu} + \sigma^2 = \left(\frac{1}{\alpha}\right)^{\frac{2}{\beta}} \left[\Gamma\left(1+\frac{2}{\beta}\right) - \left(1+\frac{1}{\beta}\right)\right]^2 \tag{6}$$

m_1 is divided by the square of m_2, an expression which is a function of β only is obtained, as in Al-Fawazan (2000).

$$\frac{\mu^2}{\sigma^2+\mu^2} = \frac{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right)}{\Gamma\left(1+\frac{2}{\beta}\right)} \tag{7}$$

where

$\hat{\mu} = \sum_{i=1}^{n} \ln\left(\frac{S_t}{S_{t-1}}\right)$, $E\left(X_t\right) = \frac{1}{n}\sum_{t=1}^{n} X_t$, $\sigma^2 = E\left(X_t^2\right) - \left[E\left(X_t\right)\right]^2$ and $Z = \frac{1}{\beta}$

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Equation (7) is transformed in order to estimate β and α respectively, as in Nwobi and Ugomma (2014):

$$\frac{\mu^2}{\sigma^2+\mu^2}=\frac{\Gamma(1+Z)\Gamma(1+Z)}{\Gamma(1+Z)} \tag{8}$$

The value of the scale parameter $\hat{\alpha}_{mom}$ can be estimated thus:

$$\hat{\alpha}_{mom}=\frac{\hat{\mu}}{\Gamma\left(1+\frac{1}{\beta}\right)} \tag{9}$$

where $\hat{\mu}$ is the mean of the original data.

Goodness of Fit Tests for the Weibull Distribution

In this section, we also explore and compare the goodness of fit test for the Weibull distribution and select the best method of parameter estimation.

The goodness of fit test procedure for the Weibull can be generally described as:

H_0: The data does not follow a Weibull model

H_1: The data follows a Weibull model.

In the case of our study, we apply one sample test of the Kolmogorov-Smirnov test of goodness of fit. To perform the Kolomogorov-Smirnov test (see for example, Chakravarti, 1967; Lawless, 1982), the maximum distance between the cumulative frequency of the failure times and the theoretical cumulative frequency provided by the estimated model is required. If this distance is large enough, the hypothesis that the chosen model fits the failure times will be rejected. The distance between theoretical frequency and observed frequency are expressed as:

$$D_n^+=Max_{(1,\alpha)}\left(\frac{1}{n}-F(X_t)\right)\text{, for right tailed test} \tag{10}$$

$$D_n^-=Max_{(1,\alpha)}\left(F(X_t)-\frac{i-1}{n}\right)\text{, for left tailed test} \tag{11}$$

and

$$\hat{D}=Max_{(1,\alpha)}\left(D_n^+,D_n^-\right)\text{, for two tailed test} \tag{12}$$

where

$F(X_t)$ is the cumulative distribution function of t.

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The Null hypothesis for Weibull distribution will be rejected if the maximum distance in Equation (8) is greater than or equal to critical value.

Comparison of the Performance of Stock Prices under the Estimation of 2-Parameter and 3-Parameter Weibull Distributions

To compare the performance of the each of the stock prices under the estimation of 2-Parameter and 3-Parameter Weibull Distributions, the Minimum Mean Squared Error (MSE) test criterion will be used, and it is given by:

$$MSE_{(\min)} = \frac{\sum_{i=1}^{n}\left(\hat{F}(X_t) - F(X\ )\right)^2}{N} \tag{13}$$

where

$\hat{F}(X_t)$ is the cumulative distribution function obtained as:

$$\hat{F}(X_t) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\beta}} \tag{14}$$

and

$$F(X_t) = \sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + 59$$

such that the company with the smallest MSE outperforms the other companies both in the estimation of 2 and 3 parameter Weibull distributions.

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RESULT AND DISCUSSION

Graphical Analysis of Data

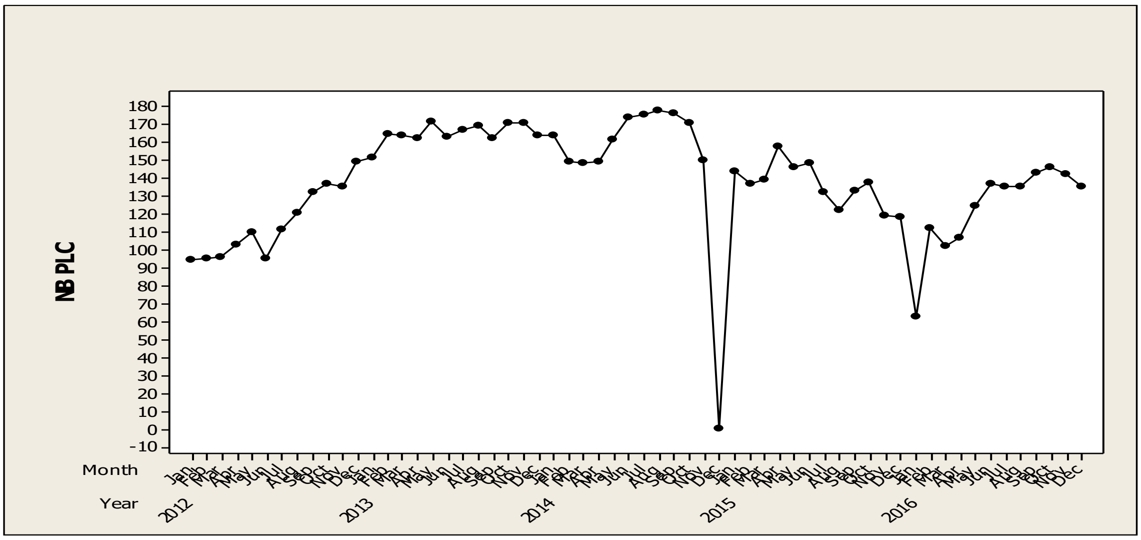

Fig 1: Time Series Plot of Average Monthly Stock Prices of Nigerian Breweries PLC

Fig 1 shows that the monthly average stock price of Nigerian Breweries almost maintained the same level of prices at a higher rate from January 2012 to October 2014 and dropped significantly at a lower price in December 2014. It also showed that the prices were not steady between January 2015 and December 2016. From the analysis, we also observed that the monthly prices of stock in Nigerian Breweries PLC also fluctuated during the period of study.

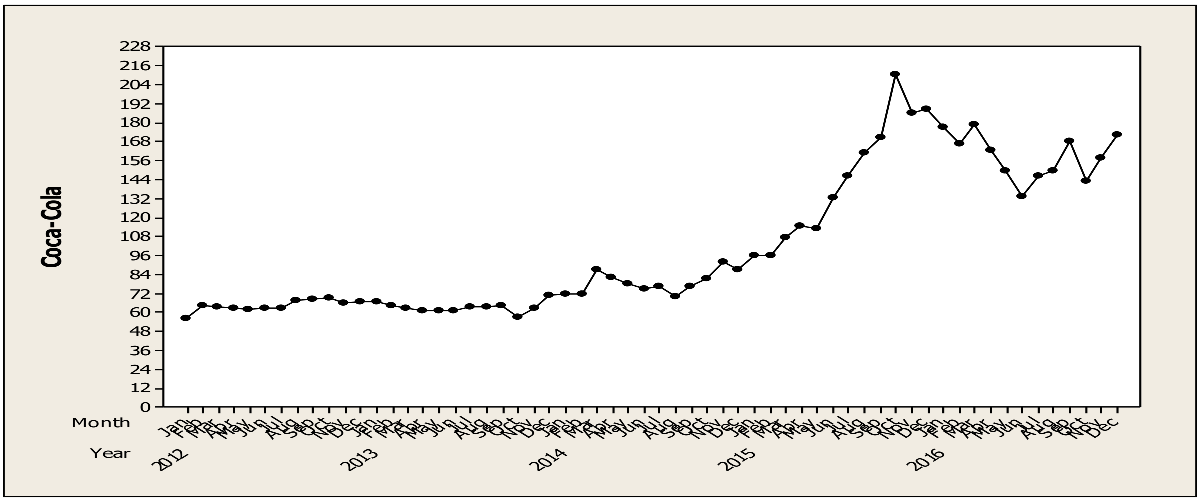

Fig 2: Time Series Plot of Average Monthly Stock Prices of Nigerian Bottling Company

Fig 2 depicts that the prices of Coca-Cola company maintained a steady monthly stock price between January 2012 and February 2014. It was also observed that the prices slightly increased from December 2015 and increased significantly in October 2015. The prices also fluctuated between January 2016 and December 2016. This indicates a random walk of the average monthly stock prices most especially from 2015 to 2016.

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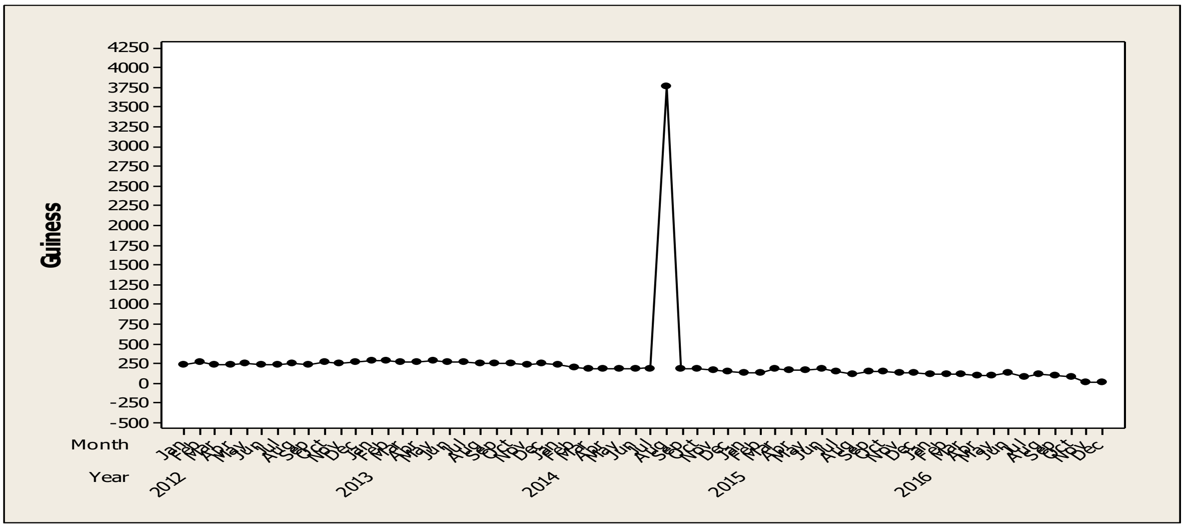

Fig 3: Time Series Plot of Average Monthly Stock Prices of Guinness Nigerian PLC

Fig 3 shows a steady price level in Guinness Nigeria PLC between January 2012 and August 2014 which increased at a higher rate of about 63% in September 2014 and also decreased at the same of 63% in October 2014. It was also observed that the prices maintained the same level from November 2014 to December 2016. This indicates that there were not much price changes in Guinness Nigeria PLC during the period of study.

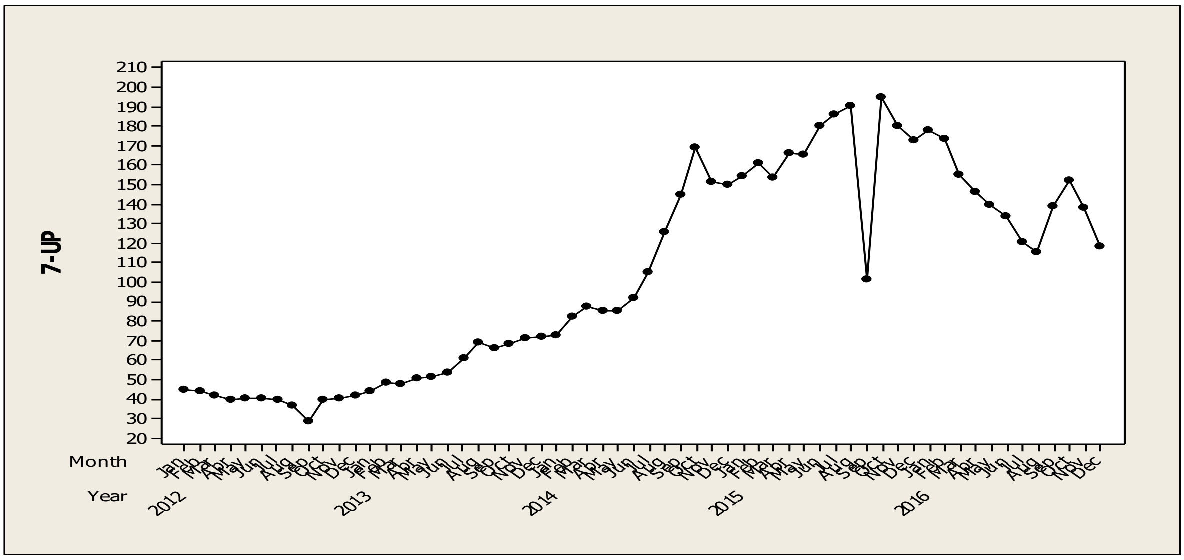

Fig 4: Time Series Plot of Average Monthly Stock Prices of 7 UP Bottling Company PLC

This company maintained a steady price level from January 2012 to September 2012 when the price decreased slightly. From November 2012, the prices of the stock increased slightly until October 2014 when the prices went higher and fluctuated between November 2014 and September 2015. From October 2015, there were random movements of the prices until December 2016. This result showed that there are price dynamics between 2012 and 2016, being the period of study.

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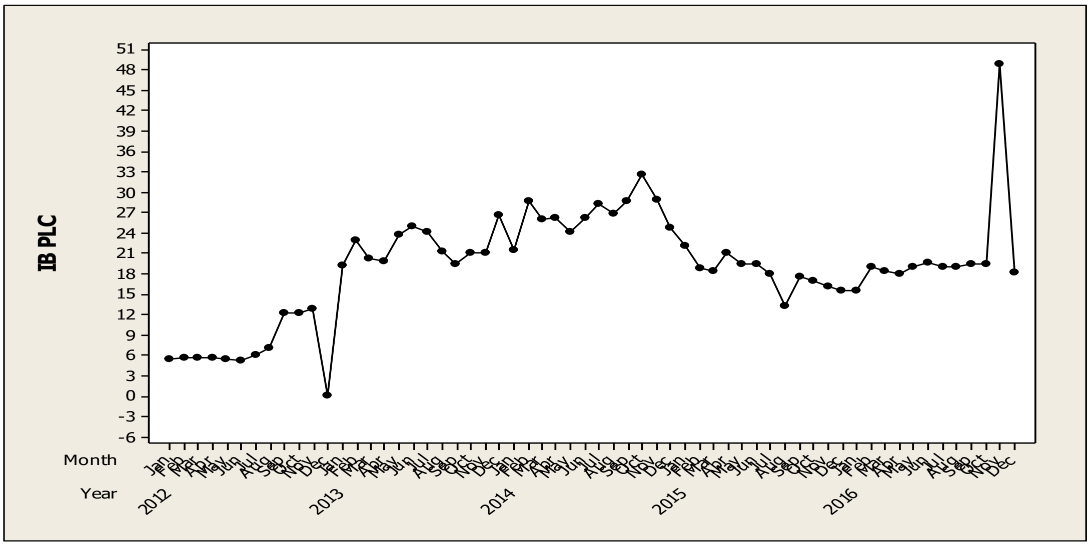

Fig 5: Time Series Plot of Average Monthly Stock Prices of 7 UP Bottling Company PLC

Fig 5 shows that the average monthly stock price of International Breweries maintained a steady price from January 2012 to August 2012 and dropped significantly to zero by December 2012. The prices further increased in January 2013 and fluctuated between February 2013 and October 2016. It was also observed that the prices moved upwardly in November and dropped slightly in December 2016. Graphically, we observed that the average monthly stock price of International Breweries PLC was not steady throughout the period of study.

Descriptive Statistics for the Five Stock Prices

Table 1: The Output of Descriptive Statistics of Average Monthly Stock Prices of the Five Companies

Company	Mean	Std.Error	Std. Dev	Var	Sum	Skewness	kurtosis
NB PLC	138. 13	4.04	31.31	980.59	8287.56	-1.71	5.29
Coca-Cola	101.56	5.85	45.28	2050.38	6093.33	0.81	-0.87
Guinness	242.3	60.2	466.4	217507.4	14540.8	7.48	57.19
7-UP	104.09	6.85	53.06	2814.99	6245.51	0.14	-1.52
Int. Brew.	19.00	1.05	8.12	65.99	1140.11	0.30	2.36

From Table 1, we observe that Nigerian Breweries has mean and standard deviation values of 138.13 and 31.31 respectively, Coca-Cola has mean and standard deviation values of 101.56 and 45.28 respectively, Guinness has 242.3 and 466.4, Seven-Up company has 104.09 and 53.06 while International Breweries has values of 19.0 and of 8.12 respectively. Comparing the performance of the various five companies using their standard deviations, we noticed that International Breweries PLC has the minimum standard deviation among the considered companies; hence, it performed better in marinating its stock prices to be steadier than the others.

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Table 2: The Output of Descriptive Statistics of Average Monthly Return of Stock Prices of the Five Companies

Company	Sample	Mean	Var	Min	Max
NB PLC	59	-0.0068	0.017	-0.57	0.62
Coca-Cola	59	0.9841	0.0060	0.8125	1.1780
Guinness	59	0.0174	0.3327	-2.9978	3.0418
7-UP	59	-0.0164	0.0232	-0.6564	0.6340
Int. Brew.	59	-0.03197	0.0492	-0.9249	0.9931

COMPUTATION OF 2-PARAMETER WEIBULL DISTRIBUTION

Table 3: Summary of Computed Value of a 2-Parameter Weibull Distribution

Company	$\hat{\alpha}$	$\hat{\beta}$	$E(X)$	$Var(X)$
NB PLC	-0.0077	2.3814	6.8×10^{-3}	4.28×10^{-6}
Coca-Cola	1.0824	4.2191	9.841×10^{-1}	6.91×10^{-2}
Guinness	0.0182	9.5255	1.74×10^{-2}	1.23×10^{-6}
7-UP	-0.0173	9.5234	1.63×10^{-2}	4.17×10^{-6}
Int. Brew.	-0.0157	2.4690	1.39×10^{-2}	3.62×10^{-5}

Table 3 shows that Guinness Company has the minimum variance among the companies. This means that Guinness maintained a steady stock price over the years using a 2-parameter Weibull distribution.

Computation of a 3-Parameter Weibull Distribution

Table 4: Summary of computed Value of a 3-paramater Weibull Distribution

Company	$\hat{\alpha}$	$\hat{\beta}$	$E(X)$	$Var(X)$
NB PLC	0.5852	1.1085	0.5644	0.0021
Coca-Cola	0.0571	0.1658	0.1715	0.0053
Guinness	3.3898	2.7156	3.02	1.44
7-UP	0.7145	0.6176	0.64	0.0686
Int. Brew.	0.9753	0.8642	0.911	0.0957

Table 4 shows that Nigeria Breweries PLC has the least minimum variance of 0.0021. This means that Nigeria Breweries maintained a steady stock price over the years using a 3-parameter Weibull distribution.

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Comparison of the Performance of Various Stock Prices under the Estimation of 2- and 3-Parameter Weibull Distributions

To compare the performance of the companies towards their stock prices stability, we use the Minimum Squared Error (MSE) given as:

$$MSE = \sum_{i=1}^{N} \frac{\left(\hat{F}(X_t) - F(X_t)\right)^2}{N}$$

where

$\hat{F}(X_t) = 1 - e^{-\left(\frac{x_t}{\alpha}\right)^{\beta}}$ is the cumulative distribution function (CDF) of each company

$$F(X_t) = \sum_{i=1}^{N} i = 1 + 2 + 3 + \ldots + 59$$

Table 5: Summary of Minimum Squared Errors of Estimated Weibull Parameters

Company	***MSE***	
	2-Parameter	**3-Parameter**
NB PLC	2.0734×10^{15}	5.13×10^{3}
Coca-Cola	4.90×10^{3}	4.89×10^{3}
Guinness	4.4528×10^{166}	3.24×10^{9}
7-UP	1.8963×10^{301}	4.99×10^{3}
Int. Brew.	8.6158×10^{133}	4.76×10^{3}
Minimum	4.90×10^{3}	4.76×10^{3}

Table 5 shows that Coca-Cola has the minimum squared error of 4.90×10^{3} under a 2-parameter Weibull distribution while the International Breweries PLC has the minimum squared error of 4.76×10^{3} using an estimate of a 3-parameter Weibull distribution. This analysis reviewed that Coca-Cola outperformed the other four companies under the estimation of a 2-parameter Weibull distribution; this means that Coca-Cola maintained stability of stock price returns among the other companies while the International Breweries maintained stock price stability among the other companies under the estimation of a 3-parameter Weibull distribution. Comparing both MSE of Coca-Cola and International Breweries PLC, we observed that stock prices returns of International Breweries were more stable than the other companies since it has the least Minimum Squared Error (MSE).

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Goodness of Fit Test of Weibull Distribution using Kolmogorov-Smirnov Test

Table 6: The Summary of 2-Parameter Case Using K-S Test

Company	D^{+}	D^{-}	$\hat{D}$	$WEI(\alpha,\beta)=\sqrt{n\hat{D}}$	**Critical value at 5%**	**Decision**
NB PLC	1.1060	0.9596	1.106	8.0780	0.1307	Reject
Coca-Cola	1.7008	0.1468	1.7008	10.0173	0.1307	Reject
Guinness	1.6209	0.9831	1.6209	9.7792	0.1307	Reject
7-UP	3.3449	0.9491	3.3449	14.0481	0.1307	Reject
Int. Brew.	7.1298	0.9958	7.1298	20.5100	0.1307	Reject

Table 6 shows that the computed test statistics for the various companies are greater than the critical value at 5% level of significance. We therefore have enough evidence to reject the null hypothesis and conclude that the data under 2-parameter Weibull do not follow the Weibull distribution.

Table 7: The Summary of 3-Parameter Case Using K-S Test

Company	D^{+}	D^{-}	$\hat{D}$	$WEI(\alpha,\beta)=\sqrt{n\hat{D}}$	**K-S Critical value at 5%**	**Decision**
NB PLC	5.0567	0.9480	5.0567	17.2727	0.1307	Reject
Coca-Cola	1.7008	0.0715	1.7008	10.0173	0.1307	Reject
Guinness	436819.55	0.4425	436819.55	5076.65	0.1307	Reject
7-UP	3.3449	0.9491	3.3449	14.0481	0.1307	Reject
Int. Brew.	0.00	1.000	1.000	7.6811	0.1307	Reject

Table 7 shows that the computed test statistics for the various companies using 3-parameter Weibull are greater than the critical value at 5% level of significance. We therefore have enough evidence to reject the null.

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CONCLUSION

From the analysis, based on comparison using Minimum Squared Error (MSE), it was observed that Coca-Cola Bottling Company outperformed other companies under the 3-parameter Weibull distribution and equally maintained a stable stock price return, while International Breweries maintained stock price stability among other companies under the 3-parameter estimation of Weibull distribution. Hence, the test using Kolmogorov-Smirnov (K-S), Weibull distribution showed the best fit for the distribution dataset at 5% level of significance.

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