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INVENTORY REPLENISHMENT FROM A SINGLE SUPPLIER TO MULTIPLE RETAILERS IN RURAL AREA IN SUPPLY CHAIN

L. T. Encho[1,2], O. C. Asogwa[2*], N. M. Eze[3] and C. C. Christopher[4]

[1]The University of Bamenda, North West Region, Cameroon.

[2]Department of Mathematics and Statistics, Alex Ekweme Federal University Ndufu-Alike, Nigeria.

[3]Department of Statistics, University of Nigeria, Nsukka.

[4]Department of Management and Information Systems, Cowan University, Perth, Western Australia.

[*]Corresponding Author's Email: qackasoo@yahoo.com

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ABSTRACT: *Supply chain management (SCM) is the management of operations that are involved in the procurement of raw materials, its processing into finished goods, and distribution to the end consumer. In order to maximize profits in the supply chain, more customers should be explored from the rural areas but there is difficulty in reaching the customers due to road network accessibility for heavy loaded trucks. Our objective seeks to find out the optimum quantity and optimal cost required by the supplier and customer to maximize the supply chain profit. We applied a quantity-based mathematical model with renewal theory and shipment consolidation to obtain the profit function in the supply chain system. Then, we presented a solution to the model to determine the optimal solution in the supply chain. Our results show that the demand rate and the supply chain's profit are higher, especially for the more retailers demand. Therefore, involving small vehicles for delivery of products to customers into areas where road network accessibility is difficult for heavy trucks is advantageous for the supply chain.*

KEYWORDS: Delivery, Inventory, Replenishment, Shipment consolidation, Supply chain.

INTRODUCTION

Supply chain management (SCM) is the management of operations that are involved in the procurement of raw materials, its processing into finished goods, and distribution to the end consumer. It also involves the flow of information and products between and among supply chain stages to maximize profitability. The major functions involved are the procurement of raw materials, product development, marketing, operations, distribution, finance, and customer services (O'Byrne, 2016). SCM also involves the active streamlining of the supply-side activities of a business to maximize customer value and gain an overall competitive advantage in the marketplace.

Supply chain processes employ a combination of people, systems and technology and are performed by the firm itself or in collaboration with external supply chain partners (Simon, 2012). The ultimate goal of supply chain is profit maximization, regardless of what service or product they offer or the nature of their supply chain practices (Rothschild, 2006). They consider how the products are delivered to customers faster and with greater accuracy than you could with a manual system and track shipments to ensure they reach their destinations safely and on time (Rob O'Byrne, 2016). For fast delivery to customers, Adyang (2012) argued that proper supply chain practices led to higher profitability. Muthoni (2010) suggested that enhancement of operational excellence in the retail service workshop processes increased service quality, customer satisfaction and service performance. Zohreh and Amir (2018) explained that the most important features that can be mentioned in order to manage supply chain orders for profitability are long-term orders earnings, increased customer loyalty, long-term cooperation with the company, minimizing the total costs, involves forward flows in order to reduce fixed and variable costs and increase customer responsiveness; and that applying dispatch volume limit, increases both the ordering cycle and the total annual costs.

In order to maximize profit, Judit et al. (2017) suggested that the manufacturing companies should keep their inventory value at the lowest possible rate to minimize costs. However, several processes such as Vendor Managed Inventory [VMI] were also encouraged by Judit et al. (2017). Here the supplier manages the inventory at the customer and secondly the consignment inventory processing in which suppliers store goods at the customer location.

This study investigated the level of profitability in the supply chain when the company management employs high service level drivers who are conversant with the road network and apply quantity-based shipment consolidation for the delivering of finished products to customers located in rural areas. Shipment consolidation allows small loading vehicles to enhance the service level since many of the customers have small scale businesses and are unable to buy a huge quantity and combination of two or more orders or multiple small batches into a single larger quantity to be dispatched or delivery is possible (Qishu, 2011). That is, instead of shipping individual loads whenever an order arrives, the transporters will hold the outbound quantities for a period of time, and then dispatch them on the same vehicle (Bookbinder et al., 2012). This strategy is ensuring continuous performance improvement of the company from huge losses to profitability and a higher level of quantity supply.

Our objective seeks to find out the optimum quantity and optimal cost the customer's require for the supply chain to maximize profit. The questions we answered are; what is the optimal values of the supplier quantity ($\boldsymbol{S_Q}$) and retailer ($\boldsymbol{R_Q}$) that minimized the expected long-run average cost in supply chain management? What is the variation of the optimality of

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replenishment quantity of the supplier and retailer and total relevant cost? To answer these questions, we considered that shipment is made only when a certain quantity of outstanding demand is accumulated. A consolidation cycle begins immediately after the previous dispatch, and ends upon arrival of the order which causes the accumulative weight to reach the supplier or the retailer quantity. The cycle length is random, depending on the inter-arrival times between orders. The load dispatched to replenish the supplier's inventory from the manufacturer is often greater than that delivered to the retailers from the supplier.

MATERIAL AND METHODOLOGY

We used quantity-based policy with renewal theory to obtain the long-run average cost function in a coordinated supply chain system. This is to help determine the order-up-to levels of the retailers from the supplier. We considered replenishments to represent the events that the retailer receives products from the supplier and deliveries represent the events that the retailer delivers the products to the consumers. The batch size is not the same but equal to the order quantity to be delivered and may vary due to stochastic demand from customers (that is the end consumers). Demands from supplier to the retailer and from the retailer to consumers follow a compound Poisson distribution.

The replenishment (delivery) cycle denotes the time interval between two consecutive replenishments (deliveries). Also the replenishment and delivering cost of the supplier and retailers is composed of a fixed cost which is incurred when there is a positive replenishment quantity and a linear variable cost which is linearly proportional to the quantity. This variable cost includes the cost for loading products on vehicles at the company, transporting them to the supplier, and unloading them from the vehicles at the retailer. Figure 1 shows the inventory levels at the supplier and the retailers. The reorder points of the supplier and the retailers can be easily determined to be zero.

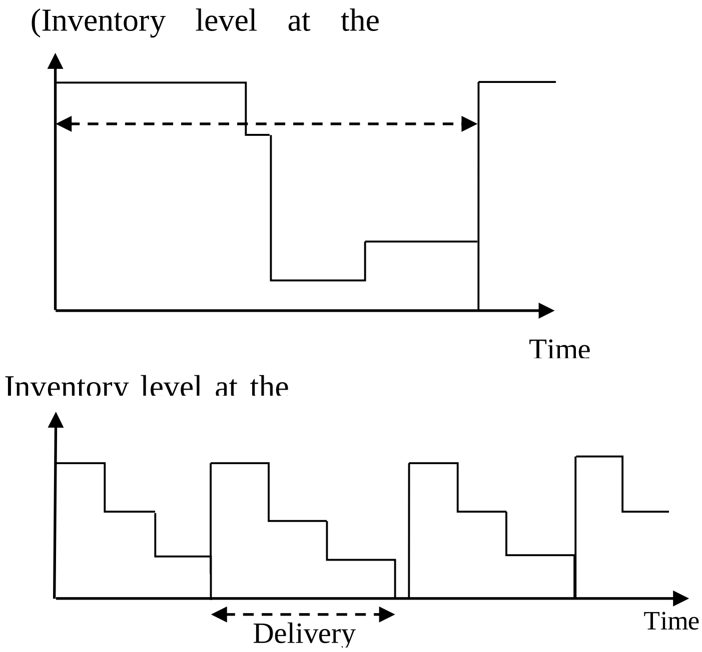

Figure 1: Inventory levels of the supplier and retaile

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Mathematical Model

We develop a mathematical model for the quantity in a coordinated supply chain system in which the retailers initiate the ordered quantity from the supplier who has been replenished by a supplier using small vehicles. In order to keep the model mathematically tractable, we consider a simplified framework based on Jac-Hun (2010) who considered Q for the size of a replenishment quantity and r the number of dispatches in a cycle. In our model, we considered the replenishment quantity of the supplier (S_Q) and the delivery quantity of the retailers (R_Q).

Let first define the following notations:

T_i: Inter-arrival time between the arrivals of the $(n-1)^{st}$ and the n^{th} retailers

A_t: Arrival time of the n^{th} retailer $(A_t = \sum_{i=1}^{n} T_i)$

λ: Arrival rates of the customers

$\frac{1}{\lambda}$: The mean of the inter-arrival time of customers

$N(t)$: Number of orders that have arrived by time t, $(N(t) = max\{n/A \leq t_t\})$;

it is assumed that this follows the Poisson distribution with mean λt

d_n: Demand quantity (or weight) of the n^{th} retailer

μ: Mean of demand quantities

σ^2: Variance of the demand quantities

D_n: Cumulative demand quantity of the first n retailers $(D_n = \sum_{i=1}^{n} d_i)$

$N_2(x)$: Minimum number of retailers whose cumulative demand quantity

exceeds, i.e., $N_2(x) = min\{n/D_n > x\}$

$L^j(x)$: Minimum number of retailers whose cumulative demand quantity exceeds

x in the j^{th} delivery cycle.

S_Q: The order-up-to level of the supplier

R_Q: The order-up-to level of the retailer

h_S: The inventory holding cost per unit per unit time at the supplier

h_R: The inventory holding cost per unit per unit time at the retailer.

$I_S(t)$: Inventory level of the supplier at time t

$I_R(t)$: Inventory level of the retailer at time t

C_R: The cost replenishing one unit at supplier

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A_R:The fixed cost of replenishing the inventory at the retailer from the supplier

C_D: The cost of delivering one unit from the supplier to the retailer;

A_D: The fixed cost of delivering of a shipment from the supplier to the retailer;

K: Number of delivery cycles within replenishment cycles (a random variable)

$F(x)$: Distribution of $D_{N_2(S_R)}$, the sum of demand quantities of the customers that arrive during a delivery cycle, i.e., $F(x) = P\{ D_{N_2(S_R)} \leq x \}$

$F^{(k)}(x)$: k-fold convolution of $F(x)$

$C(S_Q, R_Q)$: The expected long-run average cost incurred when the order-up-to-levels of the supplier and the retailer are S_Q and R_Q respectively.

Assumptions of the Model

To enable us to achieve the quantity-based dispatching model for the coordinated supply chain, the following are the assumptions of the model.

(a) The inventory level is under continuous review.

(b) The load is dispatched whenever the size of demands is accumulated.

(c) The mean and variance of the quantities is known to each supplier

(d) Inter-arrival times of the order quantities are mutually independent.

(e) Shortages are not allowed.

(f) Lead times for inventory replenishments are fixed and negligibly short.

(g) There are an integer number of delivery cycles in each replenishment cycle.

(h) The distances between the supplier and retailers are not very large.

Since we assume that dispatching decisions are made on a recurrent basis, one can make use of the renewal theory (Çetinkaya & Lee, 2000) to obtain an optimal solution for our problem. Here let T_i $(i = 1, 2, \cdots, K)$ be the instants that the demands have accumulated to a level of S_Q and R_Q and a dispatch takes place. At a time instant T_K, an inventory replenishment takes place and the replenishment arrives at once (as we assume zero lead time).

The objective here is to obtain the optimal values of S_Q and R_Q so that the average long-run cost of the system is minimized. The average long-run cost of the system is given by

$$TC(S_Q, R_Q) = \frac{E[Replenishment\ cycle\ cost]}{E[Replenishment\ cycle\ length]} \quad (1)$$

by using the renewal reward theorem. The cost of a replenishment/delivery cycle consists of the following parameters or variables; expected delivery cycle length; expected delivery quantity to the retailer in a delivery cycle; expected number of delivery cycles within a

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replenishment cycle; expected replenishment cycle length; expected replenishment quantity to the supplier in the replenishment cycle; expected inventory holding cost at the retailer in a delivery cycle; and expected inventory holding cost at the supplier in a replenishment cycle as components in the objective function (the expected long-run average cost).

Note that the inter-arrival times of demands $\{T_n: n \geq 1\}$ are exponentially distributed with parameter, λ. $d_n, n = 1, 2, 3, \cdots$ are random variables representing the demand quantity of the n^{th} customer, and d_n's are assumed to be identically and independently distributed and are independent of $N_1(t)$ as well.

The Expectation Delivery Cycle Length

When the inventory at the retailer drops below a certain point, she replenished the items to bring the inventory back at a level R_Q. This implies that the inventory level of the retailer is a generative process. Since the number of customers that arrive at the retailer for a delivery cycle is $N_2(R_Q)$, from Wald's equation (Ross, 1996), the expected delivery cycle length is given as $E[T_i]E\left[N_2(R_Q)\right]$. But $E[T_i] = \frac{1}{\lambda}$ on the inter-arrival time of the customer, then the value of $E\left[N_2(R_Q)\right]$ can be estimated as $\frac{R_Q}{E[d_n]} + 1$ since $\frac{N_2(R_Q)-1}{R_Q} = \frac{1}{E[d_n]}$ as given in Ross (1996)

Thus, the expected delivery cycle length is

$$E[Delivery\ cycle\ length] = E\left[A_{N_2(R_Q)}\right] = E\left[\sum_{i=1}^{N_2(R_Q)} T_i\right] = E[T_i]E\left[N_2(R_Q)\right]$$

$$= E[T_n]\left(\frac{R_Q}{E[d_n]} + 1\right) = \frac{R_Q+\mu}{\lambda\mu}$$
(2)

Expected Delivery Quantity to the Retailer in a Delivery Cycle

The number of customers arriving at the retailer during a delivery cycle is $N_2(R_Q)$. The delivery quantity to the retailer is $D_{N_2(R_Q)}$.

Now, $E\left[D_{N_2(R_Q)} - R_Q\right] = \frac{E[d_n^2]}{2E[d_n]}$ by the inspection paradox (Ross, 1996), the expected delivery quantity in a delivery cycle is given as

$$E[Delivery\ quantity] = E\left[N_2\left(R_Q\right)\right] = R_Q + E\left[D_{N_2(R_Q)} - R_Q\right]$$

$$= R_Q + \frac{E[d_n^2]}{2E[d_n]} = R_Q + \frac{Var[d_n]+E[d_n]^2}{2e[d_n]} = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu} \quad (3)$$

Expected Number of Delivery Cycles within a Replenishment Cycle

To calculate the expected long-run average cost we most first of all know by obtaining the expected value and the variance of the number of delivery cycles within a replenishment cycle. From equation (2), we considered that $D_{N_2(R_Q)}$ follows the Poisson distribution with parameter $\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$

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i.e., $E\left[D_{N_{2(R_Q)}}\right] = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$ by allowing the value of $D_{N_{2(R_Q)}}$ to be less than or equal to R_Q. We know from the assumption that $F^{(k)}(S_Q)$ is the distribution function of the Poisson distribution with parameter $k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)$, since $F^{(k)}(S_Q)$ is the $k-fold$ convolution of the Poisson with $\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$

Since k can be expressed as $K = min\left\{k/\sum_{j=1}^{k} D_{L^j(R_Q)} > S_Q\right\}$, the event $\{K \geq k\}$ is equivalent to $\left\{\sum_{j=1}^{k-1} D_{L^j(R_Q)} \leq S_Q\right\}$ and

hence $P\{K \geq k\} = P\left\{\sum_{j=1}^{k-1} D_{L^j(R_Q)} \leq S_Q\right\} = F^{(k-1)}(S_Q)$.

Therefore, the distribution function of K is expressed as

$$P\{K \leq k\} = 1 - F^{(k)}(S_Q) = 1 - \sum_{i=0}^{S_Q} \frac{k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)^i esp\left(-k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)\right)}{i!} \quad (4)$$

This equation represents the distribution function of the (S_Q+1)-stage Erlang (Gamma) distribution with mean $\frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2}$ and variance

$\frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2}$. Therefore, we can approximate the expected value and the variance of the number of delivery cycles within a replenishment cycle as

$$E[K] = \frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2} \quad (5)$$

and $$Var[K] = \frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2} \quad (6)$$

Expected Replenishment Cycle Length

A replenishment cycle has k delivery cycles. From Wald's equation (1996), the expected replenishment cycle length is calculated by multiplying the expected delivery cycle length by the expected number of delivery cycles within a replenishment cycle. That is,

$E\left[\sum_{j=1}^{K} X_i\right] = E[K]E[X]$

$where\ X_1, X_2, \ldots$ are independent and identically distributed random variables with finite expectations and K is a stopping time for $X_1, X_2, \ldots$ such that $E[K] < \infty$. The stopping time for $X_1, X_2, \ldots$ if the event $\{K = k\}$ is independent of $X_{k+1}, X_{k+2,} \ldots, k \geq 1$. From equations (2) and (4) the approximate expected replenishment cycles is

$E[Replenishment\ cycle\ length] = E[Delivery\ cycle\ length].E[K]$

$$= \frac{2(S_Q+1)(R_Q+\mu)}{\lambda(2\mu R_Q+\mu^2+\sigma^2)} \quad (7)$$

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Expected replenishment quantity to the supplier in a replenishment cycle

There are K delivery cycles in the replenishment cycle; we calculate the expected replenishment quantity in a replenishment cycle by multiplying the expected delivery quantity in a delivery cycle by the expected number of delivery cycles within a replenishment cycle. From (3) and (5), It can be given as

$$E[Replenishment\ quantity] = E[K]E\left[D_{2(R_Q)}\right] = S_Q + 1 \tag{8}$$

Expected Inventory Holding Cost at the Retailer in a Delivery Cycle

The inventory level of the retailer in a delivery cycle is expressed as

$$I_R(t) = \begin{cases} R_Q & if\ 0 \le t < F_1 \\ R_Q - D_1 & if\ F_1 \le t < F_2 \\ R_Q - D_2 & if F_2 \le t < F_3 \\ \vdots & \\ R_Q - D_{N_2(R_Q)-1} & if\ F_{N_2(R_Q)-1} \le t \le F_{N_2(R_Q)} \end{cases}$$

The expected inventory holding cost at the retailer in a delivery cycle is calculated as follows

$$h_R E\left[R_Q T_1 + (R_Q - D_1)T_2 + (R_Q - D_2)T_3 + \cdots + \left(R_Q - D_{N_2(R_Q)-1}\right)T_{N_2(R_Q)}\right]$$

$$= h_R E\left[R_Q \sum_{i=1}^{N_2(R_Q)} T_i - \sum_{i=1}^{N_2(R_Q)-1} D_i T_{i+1}\right]$$

$$= h_R R_Q E[T_i]E\left[N_2(R_Q)\right] - h_R E\left[E\left[\sum_{i=1}^{N_2(SR_Q)-1} D_i T_{i+1}\right]\right]$$

$$= h_R R_Q E[T_i]E\left[N_2(R_Q)\right] - h_R E\left[E\left[\sum_{i=1}^{N_2(R_Q)-1} D_i T_{i+1}/N_2(R_Q) = m + 1\right]\right]$$

$$= h_R R_Q E[T_i]E\left[N_2(R_Q)\right] - h_R E\left[E\left[\sum_{i=1}^{m} D_i T_{i+1}/N_2(R_Q) = m + 1\right]\right]$$

$$= h_R R_Q E[T_i]E\left[N_2(R_Q)\right] - h_R E[T_i]E\left[E\left[\sum_{i=1}^{m} D_i/N_2(R_Q) = m + 1\right]\right]$$

We assume that $d_i, i = 1, \cdots, N_2(R_Q) - 1$, follows an exponential distribution and also the cumulative demand quantities, $D_1, \cdots, D_m$, for $m = N_2(R_Q) - 1$, are mutually independent random variables following the uniform distribution with range $(0, R_Q)$ from the relationship between the arrival times of the Poisson arrival process and the uniform distribution (Ross, 1996).

Thus, $E[D_i]$ as $\frac{R_Q}{2}$, and

$$E\left[\sum_{i=1}^{m} m\, D_i T_{i+1}/N_2(R_Q) = m + 1\right] = mE[D_i T_{i+1}] = mE[D_i]E[T_{i+1}] = m\frac{1}{\lambda}\frac{R_Q}{2}.$$

Therefore,

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$$E\left[E\left[\sum_{i=1}^{m} m\, D_i T_{i+1} / N_2\left(R_Q\right) = m+1\right] = E\left[m \frac{1}{\lambda}\frac{R_Q}{2}\right] = \frac{1}{\lambda}\frac{R_Q}{2} E[m] \quad\right] = \frac{1}{\lambda}\frac{R_Q}{2} E\left[N_2\left(R_Q\right) - 1\right]$$

Since $E\left[N_2\left(R_Q\right)\right] = \frac{R_Q}{\mu+1}$, the expected inventory holding cost at the retailer in a delivery cycle is estimated as

$$h_R R_Q E[T_i] E\left[N_2\left(R_Q\right) - 1\right] - h_R \frac{1}{\lambda}\frac{R_q}{2} E\left[N_2\left(R_Q\right) - 1\right]$$

$$= h_R \left(R_Q \frac{1}{\lambda}\left(\frac{R_Q}{\mu} + 1\right) - \frac{1}{\lambda}\frac{R_Q}{2}\frac{R_Q}{\mu}\right)$$

$$= \frac{h_R R_Q (R_Q + 2\mu)}{2\lambda\mu}$$

Expected Inventory Holding Cost at the Supplier in a Replenishment Cycle

The inventory level of the supplier in a replenishment cycle is expressed as

$$I_S(t) = \begin{cases} S_Q & if\ 0 \le t < F_{L^j(R_Q)} \\ S_Q - D_{L^j(R_Q)} & if\ F_{L^j(R_Q)} \le t < \sum_{j=1}^{2} F_{L^j(R_Q)} \\ S_Q - \sum_{j=1}^{2} D_{L^j(R_Q)} & if\ \sum_{j=1}^{2} F_{L^j(R_Q)} \le t < \sum_{j=1}^{3} F_{L^j(R_Q)} \\ \vdots & \\ S_Q - \sum_{j=1}^{K-1} D_{L^j(R_Q)} & if\ \sum_{j=1}^{K-1} F_{L^j(R_Q)} \le t \le \sum_{j=1}^{K} F_{L^j(R_Q)} \end{cases}$$

The expected inventory holding cost at the supplier in a replenishment cycle is given as

$$h_s E\left[S_S F_{L^j(R_Q)}\right] + \left(S_Q - D_{L^j(R_Q)}\right) F_{L^2(R_Q)} + \cdots + \left(S_Q - \sum_{j=1}^{K-1} D_{L^j(R_Q)}\right) F_{L^K(R_Q)}$$

$$= h_S E\left[S_Q \sum_{j=1}^{K} F_{L^j(R_Q)} - \sum_{i=2}^{K}\left\{F_{L^j(R_Q)} \sum_{j=1}^{2} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{L^j(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\} / K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[F_{N_2(R_Q)}\right] E\left[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)} / K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E[F_{N_2(R_Q)}] E\left[E[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)} / K = k]\right]$$

But

$$E\left[E[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)} / K = k]\right] = E\left[E[\sum_{j=1}^{k-1} (k-j) D_{L^j(R_Q)} / K = k\right]$$

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$$= E\left[D_{N_{2(R_Q)}}\right] E\left[E\left[\sum_{j=1}^{k-1}(k-j)/K=k\right]\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] E\left[\frac{K^2-K}{2}\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] \frac{Var[K]+E[K]^2-E[K]}{2}$$

Hence, the expected inventory holding cost at the supplier in a cycle is given as

$$h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[F_{N_2(R_Q)}\right] E\left[D_{L^j(R_Q)}\right] \frac{Var[K]+E[K]^2-E[K]}{2}$$

$$= \frac{h_S(S_Q+1)(S_Q+\mu)(4\mu S_Q+4\mu R_Q+2\mu^2+2\sigma^2-8\mu)}{4\lambda\mu(2\mu R_Q+\mu^2+\sigma^2)} \tag{10}$$

Hence the total average long-run cost is obtained by adding equation (3), (5), (8), (9), and (10) divided by the expected Replenishment Cycle Length (7). That is, substituting the total sum of equation, (3), (5), (7), (8), (9), and (10) in replenishment cycle cost in equation (1) to give

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} + \frac{h_S(\sigma^2-\mu^2-6\mu)}{4\mu} + \lambda\mu(C_R+C_D) \tag{11}$$

Since all demands at the planned period will be eventually satisfied through the replenishment and delivery processes, the cost terms related to the unit replenishment cost (C_R) and the unit delivery cost (C_D) are not affected by the decision variables (i.e., the order-up- to-levels). This implies that the same quantity should be replenished and delivered regardless of the order-up-to levels.

Our objective here is to minimize the average long-run cost and the minimization problem is given by

$$TC(S_Q, R_Q)$$

$$Subject\ to\ S_Q, R_Q \geq 0$$

We then give a cost analysis of the quantity-based model. The optimal values of ${S_Q}^*$ and ${R_Q}^*$ can be obtained in analytical form. We obtain the optimal solution for the best lower bound of the average long-run cost.

The optimal solution for the average long-run cost $TC({S_Q}^*, {R_Q}^*)$ in (35) is given as follows; from (11) we have

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} \tag{12}$$

and

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$$\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} \quad (13)$$

We note that the cost function $TC(S_Q, R_Q)$ is strictly convex for any positive S_Q and R_Q. Thus the unique global minimum for any positive S_Q and R_Q can be obtained by solving

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} = 0 \text{ and}$$

$$\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} = 0$$

That is for $\frac{\partial C(S_Q,R_Q)}{\partial S_S} = 0$, we get

$$0 = -\frac{\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2}$$

$$\frac{h_S}{2} = \frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)}$$

$$(S_Q+1)^2 = \frac{2\lambda\mu A_R(R_Q+\mu) + \lambda(\sigma^2-\mu^2)A_R}{2h_S(R_Q+\mu)}$$

$$= \frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}$$

$$S_Q = \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1 \quad (14)$$

For value of R_Q, $\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = 0$

$$0 = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2}$$

$$0 = \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) - \lambda(\sigma^2-\mu^2)A_R + (h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q}{2(S_S+1)(S_R+\mu)^2}$$

$$(h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q = 2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R$$

$$(R_Q+\mu)^2 = \frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}$$

$$R_Q = \sqrt{\frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu \quad (15)$$

The optimal pair is then given by

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$$\left(R_Q{}^*, S_Q{}^*\right) = \left(\sqrt{\frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu, \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1\right) \tag{16}$$

Note the demand is compound Poisson and the demand quantities follow an exponential distribution. Thus, the approximated cost function is derived from equation (11) by letting $\mu^2 = \sigma^2$. Therefore,

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\mu^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)} + \frac{\lambda(\mu^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} + \frac{h_S(\mu^2-\mu^2-6\mu)}{4\mu} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} - \frac{3h_S}{2} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} - \frac{h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{h_S(R_Q+\mu)}{2} + \frac{h_R(R_Q+\mu)}{2} - \frac{3h_S}{2} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} + h_S\left\{\frac{S_Q+R_Q+\mu}{2} - 1\right\} + \frac{h_R}{2}\left\{R_Q + \mu - \frac{\mu^2}{R_Q+\mu}\right\} + \lambda\mu(C_R+C_D) \tag{17}$$

The optimal pair is then given by

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = 0, \text{ we ge } -\frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{h_S}{2} = 0$$

$$\frac{\lambda\mu A_R}{(S_Q+1)^2} = \frac{h_S}{2} \Rightarrow (S_Q+1)^2 = \frac{2\lambda\mu A_R}{h_S}$$

$$S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 \tag{18}$$

For value of R_Q,

$$0 = \frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)^2} + \frac{(h_S+h_R)}{2} = 0$$

$$\frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)^2} = \frac{(h_S+h_R)}{2} \Rightarrow (R_Q+\mu)^2 = \frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}$$

$$R_Q{}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu \tag{19}$$

$$\left(R_Q{}^*, S_Q{}^*\right) = \left(\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu, \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1\right) \tag{20}$$

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The corresponding optimal costs is

$$TC(R_Q,S_Q)=\frac{\lambda\mu A_R}{\left(\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+1\right)}+\frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu\right)}+h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu}{2}-1\right\}+$$

$$\frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu-\frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu}\right\}+\lambda\mu(C_R+C_D)$$

$$TC(R_Q,S_Q)=\frac{\lambda\mu A_R}{\left(\sqrt{\frac{2\lambda\mu A_R}{h_S}}\right)}+\frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}\right)}+h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}}{2}-1\right\}+$$

$$\frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}}\right\}+\lambda\mu(C_R+C_D)$$

$$TC(R_Q{}^*,S_Q{}^*)=\frac{\sqrt{2\lambda\mu A_R h_S}}{2}+\lambda\mu A_R\frac{\sqrt{(2\lambda\mu A_D-h_R\mu^2)(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2}+h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-3}{2}\right\}$$

$$+\frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\frac{\mu^2\sqrt{2\lambda\mu A_D-h_R\mu^2(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2}\right\}+\lambda\mu(C_R+C_D) \qquad (21)$$

This is a lower bound of $\left(R_Q{}^*,S_Q{}^*\right)$ for any positive values of R_Q and S_Q,

i.e $TC(R_Q{}^*,S_Q{}^*)\geq\frac{\sqrt{2\lambda\mu A_R h_S}}{2}+\lambda\mu A_R\frac{\sqrt{(2\lambda\mu A_D-h_R\mu^2)(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2}+h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-3}{2}\right\}$

$$+\frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\frac{\mu^2\sqrt{2\lambda\mu A_D-h_R\mu^2(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2}\right\}+\lambda\mu(C_R+C_D) \qquad (22)$$

for any $R_Q,S_Q\geq 0$.

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RESULTS AND DISCUSSIONS

From the models, we developed under inventory replenishment with quantity-based dispatchable model, the optimal values of S_Q and R_Q that minimized the expected long-run average cost can be obtained as follows.

(a) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$, ${S_Q}^* = 0$ and ${R_Q}^* = 0$.

Since the cost for replenishing and delivering products is less than the cost of holding inventories, both the supplier and the retailer use a policy in which they satisfy the requirement from downstream members of the supply chain without carrying inventory but with immediate replenishments from upstream members.

b) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D \geq \mu(h_S + 2h_R)$, ${S_Q}^* = 0$ and ${R_Q}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$

The supplier does not hold inventory since the cost of holding inventories at the supplier is greater than the cost of replenishing products from the outside supplier. In this case, there is a single delivery cycle within a replenishment cycle.

c) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D \geq \mu(h_S + 2h_R)$, ${S_Q}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and ${R_Q}^* = 0$.

The retailer does not hold inventory since the cost of holding inventories at the retailer is greater than the cost of delivering products from the supplier to the retailer, when needed. In this case, there may be multiple delivery cycles within a replenishment cycle, that is, replenishment occurs when the cumulative demand exceeds the order-up-to level of the supplier while delivery occurs when there is demand at retailer.

d) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$, ${S_Q}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and

${R_Q}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$.

Both members hold inventories, since the cost of holding inventories is less than the cost of replenishment or delivery. If the order-up-to level of the supplier is smaller than that of the retailer, there is a single delivery cycle within a replenishment cycle. Otherwise, they may be multiple delivery cycles within a replenishment cycle.

We used the numerical variables and constant values to evaluate the performance of the model developed. We let fixed replenishment cost $(A_R) = 200\ and\ 400$ fixed delivery cost

$(A_D) = 10, 20, 30\ and\ 40$, the unit inventory holding cost for the retailer

$(h_R) = 1, 2, 3, 4\ and\ 6$, the unit inventory holding cost at the supplier $(h_S) = 1,2,3,4\ and\ 5$, arrival rate, $(\lambda) = 1,3,4,5\ and\ 6$, and the mean of the demand size, $(\mu) = 1, 2, 3, 4, 5\ and\ 6$.

We set the unit replenishment cost (C_R), and unit delivery cost (C_D) to be 1 and $\lambda = \mu = h_R = h_S = 1$, $A_R = 200$ and $A_D = 10$ to compute $\left({R_Q}^*, {S_Q}^*\right)$ in equation (42) and (43) and obtained

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${R_Q}^* = 2.08$ and ${S_Q}^* = 19.00$, respectively. The total minimum cost, $TC({R_Q}^*, {S_Q}^*)$ is obtained from Equation (45) as 1257.53. We vary one parameter at a time while keeping other at based values. We rounded the values of ${R_Q}^*$, ${S_Q}^*$ and $TC({R_Q}^*, {S_Q}^*)$ to the nearest two decimal places. The 9th, 10th columns indicate the near-optimal set of $({R_Q}^*, {S_Q}^*)$. The calculated results are shown in Table 1.

Table 1: Variation of the optimality of replenishment quantity of the retailer and supplier and total relevant cost

No	A_R	A_D	h_R	h_S	λ	μ	${R_Q}^*$	${S_Q}^*$	$TC({R_Q}^*, {S_Q}^*)$
1	200	10	1	1	1	1	2.08	19.00	1257.53
2	200	20	1	1	1	1	3.42	19.00	1790.99
3	200	30	1	1	1	1	4.43	19.00	2197.20
4	200	40	1	1	1	1	5.28	19.00	2538.60
5	200	10	1	2	1	3	1.12	23.50	7470.92
6	200	20	1	2	1	4	2.93	27.28	16684.87
7	200	30	1	2	1	5	4.57	30.62	28787.06
8	200	40	1	2	1	6	6.17	33.64	43866.62
9	200	10	2	1	3	2	4.11	47.99	22056.83
10	200	20	2	2	3	4	6.58	47.99	101686.7
11	200	30	2	2	3	5	9.58	53.77	175025.5
12	200	40	2	1	3	6	15.35	83.85	230729.8
13	200	10	3	1	1	3	4.30	68.28	70130.64
14	200	20	3	2	4	5	7.04	62.25	240925.2
15	400	30	3	3	6	4	11.23	79.00	877486.6
16	400	40	3	4	4	2	7.47	39.00	212234.6
17	400	10	4	3	5	4	2.93	72.03	388129.6
18	400	20	3	2	3	6	5.06	83.85	398420.4
19	400	30	6	3	4	2	5.12	45.189	204968.2
20	400	40	4	5	6	5	10.99	68.28	1726590

Table 1 shows the computed values of the optimal replenishment quantity of the retailer and supplier and minimum total relevant cost of the supply chain. From the computed values of the retailer and supplier optimal replenishment quantity and minimum total cost of the supply chain, we observed that the optimality replenishment quantity of retailer, supplier and minimum total cost of the supply chain increases with increase in the parameters. As the individual parameters such as the fixed replenishment cost (A_R), increases from 200 to 400, fixed delivery cost (A_D), from 10 to 40 the unit inventory cost the retailer (h_R), from 1 to 6, the unit inventory cost at the supplier (h_S), from 1 to 5, arrival rate, (λ), from 1 to 6 and the mean of the demand size, (μ) from 1 to 6, the retailer's and supplier's optimal replenishment quantity and minimum relevant total cost of the supply chain increases which account for the carrying cost. At fixed replenishment cost (A_R) of 200, fixed delivery cost (A_D) of 10, unit replenishment cost (C_R), and unit delivery cost (C_D) to be 1 and $\lambda = \mu = h_R = h_S = 1$The value of the minimum relevant cost of the supply chain is 257.53 which agrees with the minimum total cost of the supply chain as given in literature (Shao-Fu *et al.*, 2006). This minimum total average long-run cost 1257.53 is at fixed replenishment cost$(A_R) = 200$, fixed delivery cost $(A_D) =$

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10, the unit inventory holding cost for the retailer $(h_R) = 1$, arrival rate, $(\lambda) = 1$, mean of the demand size, $(\mu) = 5$, optimal replenishment quantity of retailer $\left(R_Q^*\right) = 2.08$ and optimal replenishment quantity of retailer $\left(S_Q^*\right) = 19.00$.

The Table also shows the analysis of the variation of total relevant cost of the supply chain with respect to the simultaneous variation of retailer's and supplier's replenishment quantity. From Table 1, it is evident that there is a general increase in the optimal order quantity of the retailer, supplier and total relevant cost of the supply chain if there is an increase in the arrival rate, (λ), the mean of the demand size, (μ), fixed replenishment cost (A_R) and fixed delivery cost (A_D).

CONCLUSION

We considered a supply chain consisting of a single supplier and a single retailer, in which the supplier ordered her quantity from the manufacturer and released part to the retailer on an order. We proposed an integrated inventory replenishment and shipment delivery planning model for a case of compound Poisson demands with distribution-free demand quantity using the renewal theory. After developing several properties for obtaining a closed-form expression for approximated long-run average cost, we determined the order-up-to level of each member of the supply chain that minimizes the long-run average cost.

The result shows that the total relevant cost of the supply chain is optimally increasing at an increasing optimal value of replenishment quantity of the retailer and a constant replenishment quantity of the supplier. Also, keeping the replenishment cost, the unit inventory cost of the retailer, arrival rate and the mean of the demand constant while increasing the unit inventory cost of the supplier and varying the fixed delivery cost, the total relevant cost of the supply chain and an optimal order quantity value of the retailer increases while an optimal order quantity of the supplier is constant. Also, there is the variation of total relevant cost of the supply chain with respect to the simultaneous variation of retailer's and supplier's replenishment quantity. There is also a general increase in the optimal order quantity of the retailer, supplier and total relevant cost of the supply chain if there is an increase in the arrival rate, (λ), and the mean of the demand size, (μ), and also when there is an increase in the fixed replenishment cost (A_R), fixed delivery cost (A_D), the value of k, the retailer inventory holding cost and the supplier inventory holding cost, respectively.

There is also an evident that the variation in optimal values of replenishment quantities and total relevant costs of the supplier, retailer and the supply chain is linear with respect to the arrival rate and the mean of the demand respectively. We observed an increase in decision variables and objective function is linear with respect to the arrival rate and the mean of the demand. This is due to the fact that demand increases linearly at the retailer point as the value of the arrival rate and the mean of the demand increases. Consequently, retailers order a greater number of items which in turn increases the total relevant cost of the supply chain.

REFERENCES

Article DOI: 10.52589/AJMSS-UAUZSSPU
DOI URL: https://doi.org/10.52589/AJMSS-UAUZSSPU

Allen H. T (2010) An inventory model for group-buying auction, Department of Applied Mathematics, *The Hong Kong Polytechnic University, Hung Hom*, Hong Kong.

Akaah. I. P., and G. Jackson (1988), Frequency distribution of customer orders in physical distribution system, *Journal of business logistics*, Vol. 9 no. 2 pp. 155-164.

Bookbinder. J.H. Qishu. C. and Q-M. He (2011), Shipment consolidation by private carrier: The discrete time and discrete quantity case, Department of management science, *University of Waterloo, Ontario*, Canada.

Çetinkaya. S. (2004), Coordination of inventory and shipment consolidation decision: A review of promises, models, and justification, chapter1 in *Application of Supply Chain Management and E-commerce Research in industry*. New York: Springer.

Çetinkaya. S. and J. H. Bookbinder, (2003), Stochastic models for the dispatch of consolidated shipment, *Transportation Research* B, VOl.37 pp747-768.

Jac-Hun K. (2010), Inventory control policies in multi-level supply chains under vendor-managed inventory contracts. Department of Industrial & Systems Engineering

Gupta. Y. P. and P. K. Bagchi, (1987), Inbound freight consolidation under Just-In-Time procurement; application of clearing models, Journal *of business logistics,* Vol. 8, no.2, pp. 74-94.

Higginson. J. K. (1993), Modeling shipper costs in physical distribution analysis, *Transportation Research*, Vol. 27, no.2. pp 113-124.

Higginson. J. K. (1995), Recurrent decision approaches to shipment-release timing in freight consolidation, *International Journal of Physical Distribution*

Higginson. J. K.,and J. H.Bookbinder (1994), Policy recommendation for shipment consolidation program, *Journal of Business Logistics*, Vol, 15, pp. 87-112.

Jackson, G. C (1981), Evaluating order consolidation strategies using simulation, Journal of business logistics, Vol, 2, pp. 110-138.

Jackson. G. C. (1985), A survey of freight consolidation practices, *Journal of Business Logistics* Vol.6, pp. 13-34.

Jaruphonga. W., S. Çetinkaya, and C-Y. Lee (2007). Outbound shipment model considerations for integrated inventory and delivery lot-sizing decisions,*Operation Research Letters*, vol.35: pp. 813-822.

Mütlü, F., S. Çetinkaya and J. H. Bookbinder, (2010). An analytical model for computing the optimal time-and-quantity-based policy for consolidated shipments, *IIE Transaction.* Vol. 42, no. 5, pp. 367-377.

Nikos, T.,(2007). The Effect of Operational Performance and Focus on Profitability: A Longitudinal Study of the U.S. Airline Industry. Management Science and Operations Department, London Business School, Regent's Park, London

O'Byrne, Rob. (Nov 28, 2016). 4 Best-in-Class Supply Chains To Watch and Learn From. *Case Studies, Supply Chain.* Retrieved from *http//www.logisticsbureau.com/4-best in class* supply-chains-to-watch-and-learn-from/.

Pooley. J., A. J. Stenger (1992), Modeling and evaluating shipment consolidation in a logistics system, *Journal of Business Logistics*, Vol.13 pp. 153-174.

Qishu Cai. (2011), Shipment consolidation in discrete time and discrete quantity: Matrix-Analytic method. *Department of management Sciences University of Waterloo*, Ontario, Canada.

Ross, S. M, (1996). Stochastic Process. John Wiley York.

Rothschild, M. (2006). Shareholders pay for ROA. Strategic Finance, 88 (5), 26-32.

Simon. M., (2012), Supply chain management practices and profitability of Kenolkobil limited, Thesis, *Business Administration, School of Business*, University of Nairobi

Ülkü. M. A, (2009), Comparison of typical shipment consolidation programs: Structural results. *Management Science and Engineering*, Vol. 3, no. 4.

Article DOI: 10.52589/AJMSS-UAUZSSPU
DOI URL: https://doi.org/10.52589/AJMSS-UAUZSSPU