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DYNAMIC RESPONSE TO MOVING LOAD OF PRESTRESSED DAMPED SHEAR BEAM RESTING ON BI-PARAMETRIC ELASTIC FOUNDATION

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ABSTRACT: *The dynamic behaviour of damped shear beam resting on bi-parametric elastic foundation when traversed by moving load travelling at constant velocity is investigated in this present study. The beam-type structure has a uniform cross-sectional area and it is assumed to be simply supported. The governing equations are coupled second order partial differential equations. The method of integral transformation called Finite Fourier series was first used to reduce the sets of coupled second order partial differential equations governing the motion of this class of dynamical problem to sequence of coupled second order ordinary differential equations. Thereafter, the simplified coupled equations describing the motion of the beam-load system were then solved by Laplace transformation in conjunction with convolution theory to obtain the solution. The closed form solution obtained was analyzed to obtain the conditions under which the beam-load system will experience resonance phenomenon and speeds at which this may occur are also established. The effects of pertinent structural parameters on the response of a damped shear beam when under the action of the moving load were presented in plotted curves. From the graphs, it is interestingly found that increase in the values of vital structural parameters, such as axial force Nf, circular frequency CF, foundation stiffness K and shear modulus G, reduces the transverse displacement of the damped shear beam when under the action of the moving load. Practically speaking, increase in the values of these structural parameters significantly enhances the stability of the beam and increases the critical speed of the dynamical system. Consequently, the resonance risk of the vibrating system is reduced and thus the safety of the occupant of this structural member is guaranteed.*

KEYWORDS: Dynamic response, axial force, moving load, shear beam, elastic foundation, critical speed, resonance.

INTRODUCTION

The analysis of interactions between moving bodies and structural members has been a prominent area of research in structural dynamics for over a century. This enduring interest stems from the extensive applications of structural dynamics across various fields, including Civil, Mechanical, Aerospace, and Structural Engineering, which have inspired numerous engineers, mathematicians, and physicists to explore studies on the vibrations of structures under moving loads. Moving loads can induce considerable vibrations in elastic structures, particularly at high speeds. In contrast to stationary loads or subsystems that cause constant stresses and deformations, moving loads create effects that vary based on the load's position, which is also time-dependent [1]. Practical instances of vibrations induced by moving loads include those experienced in bridges and railways due to vehicles or trains, piping systems affected by two-phase flow, beams subjected to pressure waves, and operations involving high-speed machining. The vibrations of beams resulting from traveling loads have been thoroughly investigated [2-7]. One of the early investigations into the dynamic behavior of elastic beams under traveling loads was carried out by Ayre et al. [8], who analyzed the effects of the load-to-beam weight ratio for a uniformly moving mass load. They successfully derived the solution for the associated partial differential equation using the infinite series method. Worthy of note is the work of Bolotin [9] who applied Galerkin's method to study the dynamics of a concentrated mass traversing a simply supported beam at a constant speed. In more recent work, Lin and Trethewey [10] studied the dynamic analysis of an elastic Bernoulli-Euler beam subjected to dynamic loads resulting from arbitrary movements of a spring-mass-damper system, employing the finite element method (FEM).

Similarly, Olsson [11] contributed essential insights into the moving load problem and provided reference data for more extensive studies. Jaiswal and Iyenggar [12] investigated the dynamic response of an infinitely long beam supported by a finite-depth foundation under the influence of a moving force, examining the impacts of various parameters such as foundation mass, load velocity, damping, and axial force on the beam. Lee [13] utilized the Bernoulli-Euler beam theory in conjunction with the assumed mode method to investigate the transverse vibrations of a beam that is constrained at intermediate points and subjected to a moving load. In the majority of early studies, the focus was primarily on structures such as beams or plates that did not rest on elastic foundations. However, for practical applications, it is useful to examine structures that are supported by elastic foundations. For instance, an analysis that incorporates such a foundation can be instrumental in understanding the behavior of plates and beams on roadways or runways. The Winkler approximation model is frequently referenced in the literature as a foundation model [14,15]. Nevertheless, for significant engineering challenges, such as the vibration of plates or beams, it is advisable to utilize a more compact and accurate two-parameter foundation model rather than relying solely on the Winkler approximation. The Winkler model has been the object of some criticisms. This one-parameter model fails to accurately depict the continuous nature of practical foundations, as it neglects the interaction between lateral springs. Additionally, it predicts discontinuity in the deflections of the foundation surface at the ends of a finite beam, which contradicts real-world observations [16,17]. Therefore, it is recommended to adopt a more compact and realistic elastic foundation model, known as the bi-parametric elastic foundation or Pasternak foundation model, which accounts for the continuity of surface displacement beyond the load area. This model introduces a second foundation constant, the shear modulus G, alongside the foundation stiffness K. The inclusion of the shear modulus enhances the accuracy and reliability of the analysis, although it complicates the problem and makes it more challenging to solve. Notably, the dynamics of

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moving loads on bi-parametric elastic foundations have been thoroughly investigated [18-21].

Engineers frequently incorporate artificial stresses in structures prior to applying loads, so that the stresses which then exist in the structures under the load are more favourable than would otherwise be the case. These artificial stresses can exist as forces acting axially or in other directions. When these forces act axially, they are referred to as axial forces. This process of inducing artificial stresses is known as pre-stressing. The primary objective of pre-stressed structures is to mitigate tensile stresses, thereby reducing the likelihood of flexural cracking or bending during operational conditions. As a result, a substantial body of literature has been dedicated to the investigation of the vibrations of pre-stressed beams subjected to moving loads [22-24]. It should be noted at this juncture that a tremendous amount of work has been done on the dynamical problems involving Bernoulli-Euler and other beam types under moving loads, lumped or distributed [25-31]. However, studies involving shear beams under moving loads are scanty in the literature. Shear beam theory is a fundamental aspect of structural engineering, focusing on beams where shear deformation plays a significant role. Unlike the classical Euler-Bernoulli beam theory, which assumes that plane sections remain plane and perpendicular to the neutral axis, shear beam theory accounts for shear deformations, making it crucial for analyzing short and deep beams. The shear beam model is commonly characterized by two coupled partial differential equations in terms of two dependent variables, namely transverse displacement of the cross-section measured about the neutral axis and the rotation of the cross-section measured about the neutral axis. In this present study, an approximate analytical solution of the transverse response of a simply supported damped shear beam resting on bi-parametric elastic subgrade when under the action of moving load travelling at constant velocity is obtained. Both the beam and elastic foundation models were assumed to be homogeneous. Effects of axial force, shear modulus, foundation stiffness and some other vital structural parameters on dynamic behaviour of a beam-like structural member carrying moving load are investigated. The conditions under which the beam-load system will experience resonance phenomenon and speeds at which this may occur are also established.

Problem Statement

The equations governing transverse displacement (deflection) of shear beam on elastic foundation and under the action of moving load are based on the following assumptions:

(a) The material is linearly elastic and the beam is homogeneous at any cross-section (prismatic)

(b) The x - y plane is the principal plane.

(c) There is an axis of the beam that undergoes no extension or contraction. The x-axis is located along this neutral axis.

(d) Plane section remains plain after bending but is no longer normal to the longitudinal axis.

(e) The effect of shear deformation is considered.

(f) The beam is simply supported (Pin-Pin ends).

(g) The applied moving load is concentrated.

(h) The damping, prestressed and foundation parameters are all linear.

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MATHEMATICAL MODEL

The shear beam investigated in the present study is finite and uniform. The governing equations of motion describing the transverse displacement $V(x,t)$ and rotation of the cross-section, $W(x,t)$ of a finite damped shear beam resting on bi-parametric elastic foundation and subjected to moving load travelling at constant velocity are second order simultaneous partial differential equations given by

$$\frac{\partial}{\partial x}\left[K^*G^*A\left(W(x,t)-\frac{\partial V(x,t)}{\partial x}\right)\right]+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-Nf\frac{\partial^2 V(x,t)}{\partial x^2}-C\frac{\partial V(x,t)}{\partial t}+F_r(x,t)=P(x,t) \quad (1)$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial W(x,t)}{\partial x}\right)-K^*G^*A\left(W(x,t)-\frac{\partial V(x,t)}{\partial x}\right)=0 \quad (2)$$

where μ is the mass per unit length of the beam, K^* is the shear correction factor, G^* is the shear parameter of the beam, A is the cross-sectional area of the beam, Nf is the axial force, C is the coefficient of viscous damping per unit length of the beam, E is the Young modulus of elasticity of the beam material, I is the moment of inertia, EI is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate, $F_r(x,t)$ is the foundation reaction and $P(x,t)$ is the moving load acting on the beam per unit length. The relationship between the foundation reaction $F_r(x,t)$ and lateral deflection $V(x,t)$ is given by

$$F_r(x,t)=KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2} \quad (3)$$

where K and G are two parameters of the foundation model. Specifically, K is the Foundation Stiffness

and G is the Shear Modulus.

In this study, it is assumed that the load function $P(x,t)$ is given in the form

$$P(x,t)=P_0\delta(x-vt). \quad (4)$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property.

$$\int_b^a \partial(x-vt)f(x)dx=\{\ 0,\quad for\ vt<a<b,\quad f(vt),\quad for\ a<vt<b,\quad 1,\quad for\ a<b<vt. \quad (5)$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x=0$ and $x=L$. Thus, boundary conditions are given as

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$$V(0,t) = V(L,t) = 0, \quad \frac{\partial V(0,t)}{\partial x} = \frac{\partial V(L,t)}{\partial x} = 0 \quad W(0,t) = W(L,t) = 0, \quad \frac{\partial W(0,t)}{\partial x} = \frac{\partial W(L,t)}{\partial x} = 0 \tag{6}$$

and the initial conditions are given as

$$V(0,x) = 0 = \frac{\partial V(x,0)}{\partial t} W(0,x) = \ 0 = \frac{\partial W(x,0)}{\partial t} \tag{7}$$

Substituting (3) and (4) into (1), after some simplifications and re-arrangements, Equations (1) and (2) become

$$\frac{\partial}{\partial x}\left[K^* G^* A\left(W(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] + \mu \frac{\partial^2 V(x,t)}{\partial t^2} - Nf \frac{\partial^2 V(x,t)}{\partial x^2} - C \frac{\partial V(x,t)}{\partial t} + KV(x,t) - G \frac{\partial^2 V(x,t)}{\partial x^2} = P_0 \delta(x - vt) \tag{8}$$

and

$$\frac{\partial}{\partial x}\left(EI \frac{\partial W(x,t)}{\partial x}\right) - K^* G^* A \left(W(x,t) \ - \ \frac{\partial V(x,t)}{\partial x}\right) = 0 \tag{9}$$

$(8)\ and (9)$ are the second order partial differential equations governing the flexural motion of the structurally damped shear beam resting on bi-parametric elastic foundation and subjected to moving load travelling at constant velocity.

SOLUTION PROCEDURES

To find the analytical solution of the initial boundary value problem in (8) and (9), the finite Fourier transformation method is employed alongside the Laplace Transform. Hence, we provide the following definition and theorem:

Definition 1: The finite Fourier sine transform $u(n,t)$ of a function $U(x,t)$ is defined as

$$u(n,t) = \int_0^l U(x,t)\, sin\, sin\, \frac{n\pi x}{l}\, dx \tag{10}$$

and the inverse transform is

$$U(x,t) = \frac{2}{l} \sum_{n=1}^{\infty} u(n,t)\, sin\, sin\, \frac{n\pi x}{l}\, dx. \tag{11}$$

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Definition 2: The finite Fourier cosine transform $u_0(n,t)$ of a function $U_0(x,t)$ is defined as

$$u_0(n,t) = \int_0^l U_0(x,t) \cos\cos \frac{n\pi x}{l}\, dx \tag{12}$$

and the inverse transform is

$$U_0(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} u_0(n,t) \cos\cos \frac{n\pi x}{l}\, dx. \tag{13}$$

Definition 3: The Laplace transform $F(s)$ of a function $f(t)$ is defined as

$$L\big(f(t)\big) = F(s) = \int_0^{\infty} f(t)\, e^{-st} dt\, . \tag{14}$$

Theorem 1: The convolution theorem states that

$$L^{-1}\{F(s)G(s)\} = F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{15}$$

where $F(s)$ and $G(s)$ are the Laplace transforms of $f(t)$ and $g(t)$ respectively.

Thus, applying (10) and (12) to the governing equations (8) and (9) respectively, in conjunction with the Dirac delta function property in (5), we obtain

$$\mu \frac{\partial^2 V_n(t)}{\partial t^2} + \left[\left(\frac{n\pi}{L}\right)^2 (Nf + G) - K\right] V_n(t) - \left(\frac{n\pi}{L}\right) K^* G^* A \frac{\partial W_n(t)}{\partial x} - C\frac{\partial V_n(t)}{\partial t} = P_0 \sin\sin \theta_n\, t \tag{16}$$

and

$$-EI\left(\frac{n\pi}{L}\right)^2 W_n(t) + K^* GA\left(\frac{n\pi}{L} V_n(t) - W_n(t)\right) = 0 \tag{17}$$

where

$$\theta_n = \frac{n\pi v}{L}$$

Then from Equation (17), we have

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$$W_n(t) = \frac{\frac{n\pi}{L} K^* GA}{EI\left(\frac{n\pi}{L}\right)^2 + K^* G^* A} V_n(t) \tag{18}$$

Now substituting (18) into (16), we have

$$\mu \frac{\partial^2 V_n(t)}{\partial t^2} + \left[\left(\frac{n\pi}{L}\right)^2 (Nf + G) - K\right] V_n(t) - \left(\frac{n\pi}{L}\right) K^* G^* A \frac{\partial}{\partial x}\left(\gamma_n V_n(t)\right) - C \frac{\partial V_n(t)}{\partial t} = P_0 \sin \sin \theta_n t \tag{19}$$

where

$$\gamma_n = \frac{\frac{n\pi}{L} K^* G^* A}{EI\left(\frac{n\pi}{L}\right)^2 + K^* G^* A} \tag{20}$$

The term involving the derivative with respect to x in (19) vanishes as $V_n(t)$ is a function of t alone and after some simplifications and re-arrangements, we obtain

$$\mu \ddot{V}_n(t) + \left[\left(\frac{n\pi}{L}\right)^2 (N_0 + G_0) - K_0 - \gamma_n \left(\frac{n\pi}{L}\right) K^* GA\right] V_n(t) - C\dot{V}_n(t) = R_0 \sin \sin \theta_n t \tag{21}$$

Thus, (21) simplifies to

$$\ddot{V}_n(t) + \alpha_{a1}\dot{V}_n(t) + \alpha_{a4} V_n(t) = R_0 \sin \sin \theta_n t \tag{22}$$

where

$$\alpha_{a1} = -\frac{C}{\mu}, \qquad \alpha_{a4} = \frac{\left(\frac{n\pi}{L}\right)^2 (N_0 + G_0) - K_0 - \gamma_n \left(\frac{n\pi}{L}\right) K^* GA}{\mu}, \qquad R_0 = \frac{P_0}{\mu}$$

Next, we subject (22) to Laplace transformation (14), namely

$$L\left(f(t)\right) = F(s) = \int_0^\infty f(t)\, e^{-st} dt \tag{23}$$

where s is the Laplace parameter. In view of (23), (22) becomes

$$s^2 \tilde{V}(n,s) + \alpha_{a1} s \tilde{V}(n,s) + \alpha_{a4} \tilde{V}(n,s) = R_0 \left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{24}$$

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After simplification and rearrangement, we obtain the simple algebraic equation given by

$$\tilde{V}(n,s) = R_0\left[\frac{1}{s^2+\alpha_{a1}s+\alpha_{a4}}\right]\left[\frac{\theta_n}{s^2+{\theta_n}^2}\right] \tag{25}$$

which is further simplified to give

$$\tilde{V}(n,s) = R_0\left[\frac{1}{\left(s+\frac{\alpha_{a1}}{2}\right)^2+p^2}\right]\left[\frac{\theta_n}{s^2+{\theta_n}^2}\right] \tag{26}$$

where

$$p^2 = \alpha_{a4} - \left(\frac{\alpha_{a1}}{2}\right)^2. \tag{27}$$

In order to obtain the Laplace inversions of (26), we set

$$F(s) = \left[\frac{1}{\left(s+\frac{\alpha_{a1}}{2}\right)^2+p^2}\right]$$

and

$$G(s) = \left[\frac{\theta_n}{s^2+{\theta_n}^2}\right]$$

so that the Laplace inversion of (26) is the convolution of $F(s)$ and $G(s)$ defined by (15)

$$F(s)*G(s) = \int_0^t f(t-u)g(u)du. \tag{28}$$

Noting that

$$L^{-1}[F(s)] = \frac{1}{p}\ exp\ exp\ \left(-\frac{\alpha_{a1}}{2}t\right)\ sin\ sin\ (pt) \tag{29}$$

and

$$L^{-1}[G(s)] = sin\ sin\ (\theta_n t) \tag{30}$$

Now using (29) and (30) in (28), (26) becomes

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$$V(n,t) = \left\{\beta_2 \left[P \exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \sin\sin(\theta_n t) - \theta_n \sin\sin Pt\right] + \beta_0 \left[P \exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \sin\sin(\theta_n t) + \theta_n \sin\sin(Pt)\right] - \alpha_{a1} P \theta_n \left[\exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \cos\cos(\theta_n t) - \cos\cos(Pt)\right]\right\} \times \frac{R_0 \exp\exp\left(-\frac{\alpha_{a1}}{2} t\right)}{P(\beta_1 - \beta_0)(\beta_2 - \beta_0)} \qquad (31)$$

where

$$\beta_1 = (P + \theta_n)^2, \quad \beta_2 = (P - \theta_n)^2, \quad \beta_0 = -\left(\frac{\alpha_{a1}}{2}\right)^2.$$

Thus, in view of (11), one obtains

$$V(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \left\{\beta_2 \left[P \exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \sin\sin\theta_n t - \theta_n \sin\sin Pt\right] + \beta_0 \left[P \exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \sin\sin(\theta_n t) + \theta_n \sin\sin(Pt)\right] - \alpha_{a1} P \theta_n \left[\exp\exp \left(\frac{\alpha_{a1}}{2} t\right) \cos\cos(\theta_n t) - \cos\cos(Pt)\right]\right\} \times \frac{R_0 \exp\exp\left(-\frac{\alpha_{a1}}{2} t\right)}{P(\beta_1 - \beta_0)(\beta_2 - \beta_0)} \sin\sin\frac{n\pi x}{L} \qquad (32)$$

which represents the transverse displacement to the moving load of prestressed damped shear beam resting on bi-parametric elastic foundation.

DISCUSSION OF THE CLOSED-FORM SOLUTION

Resonance in a dynamical system is of great concern in design engineering and engineering analysis; hence, it is pertinent to establish the condition under which resonance occurs. Resonance takes place when the motion of the vibrating system becomes unbounded, that is, the point at which transverse displacement of an elastic beam increases without limit. In actual practice, when this happens, the structure would collapse as the intensive vibration causes cracks or permanent deformation in the vibrating structures. It is clearly seen from Equation (32) that the simply supported uniform damped shear beam resting on bi-parametric elastic foundation and traversed by moving load considered in this study reaches a state of resonance whenever

$$\beta_1 = \beta_0 \quad or \quad \beta_2 = \beta_0 \qquad (33)$$

The velocity at which resonance may occur, termed the critical velocity associated with the conditions (33) respectively, are given as

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$$V_{cr}^{1} = \frac{L}{n\pi}\left[\sqrt{\beta_0} - \sqrt{\alpha_{a5} + \beta_0}\,\right] \tag{34}$$

and

$$V_{cr}^{2} = \frac{L}{n\pi}\left[\sqrt{\alpha_{a5} + \beta_0}\, - \sqrt{\beta_0}\right]. \tag{35}$$

NUMERICAL RESULT AND DISCUSSION

The uniform damped shear beam of length $L = 12.192\, m$ is considered in order to illustrate the analysis presented in this study. The load is assumed to travel along the beam with constant velocity $V = 8.128\, m/s$, Young modulus of elasticity $E = 2.10924 \times 109\, Kg/m$, moment of inertia I = 2.87698×10^{-3}, $\pi = 22/7$, the damping coefficient $C = 3000$ and the mass per unit length of the beam $\mu = 2758.291 kg/m$. The values of foundation stiffness K and shear modulus G are varied between $0\, N/m^3$ and $4 \times 10^7\, N/m^3$. Also, the values of axial force Nf are varied between $0N$ and $4 \times 10^8\, N$. The transverse displacement V of the beam is calculated and plotted against time t for various values of axial force Nf, foundation stiffness K, shear modulus G, load position Lp, circular frequency CF and the load velocity V. The results are as shown on the various graphs given below.

Figure 1 depicts the displacement response of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of axial force Nf and for fixed values of foundation stiffness K=400000, $shear\ modulus\ G = 40000000$ and the damping coefficient $C = 3000$. The graph shows that as the value of axial force Nf increases, the deflection of the beam decreases noticeably.

The deflection profile of a simply supported uniform damped shear beam subjected to moving load travelling at constant velocity for various values of foundation stiffness K is presented in Figure 2. It is observed that for fixed values of axial force $Nf = 40000000$, $shear\ modulus\ G = 40000000$ and the damping coefficient $C = 3000$, a higher value of foundation stiffness K reduces the transverse displacement of the vibrating beam considerably.

In the same vein, a similar graph is plotted against various values of time t in Figure 3 for a simply supported uniform damped shear beam under moving load travelling at constant velocity for various values of shear modulus G and for fixed values of axial force Nf =40000000, $foundation\ stiffness\ K = 40000000$, and the damping coefficient $C = 3000$. It is clearly noted that the deflection of the beam decreases significantly with an increase in the value shear modulus G.

The response of a simply supported uniform damped shear beam subjected to moving load travelling at constant velocity for various values of the load position coordinate Lp and for fixed values of other parameters is displayed in Figure 4. It is deduced from the figure that the dynamic deflection at the mid-span of the beam is very large compared to other load positions.

For various values of circular frequency CF and for fixed values of other parameters, Figure 5

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shows the deflection profile of the vibrating damped shear beam. It is evident from the curve that the higher the value of the circular frequency, the lower the deflection of the beam.

The response amplitude of the uniform damped shear beam to the travelling load for various load velocities is presented in Figure 6. It is clearly seen from the figure that the higher the speed of the travelling load the larger, the deflection of a simply supported uniform damped shear beam.

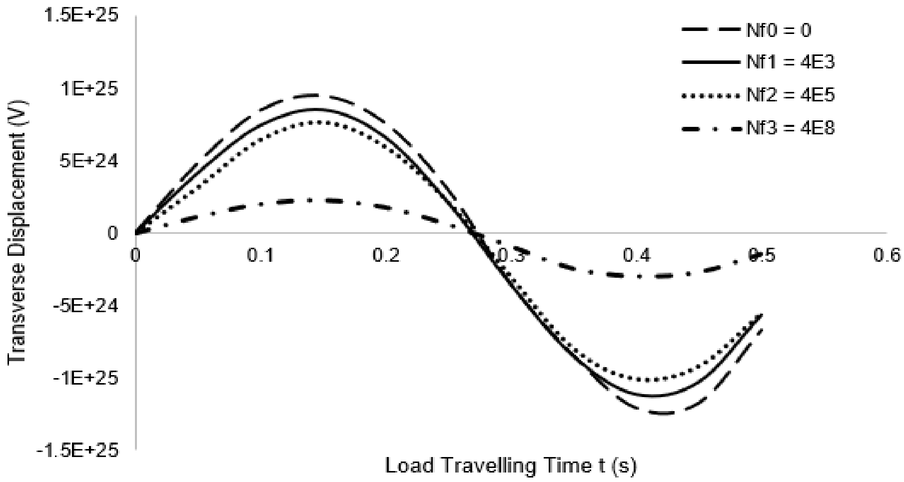

Figure 1: The displacement response of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of axial force N and for fixed values of K = $400000, G = 40000000$ and $Co = 3000$.

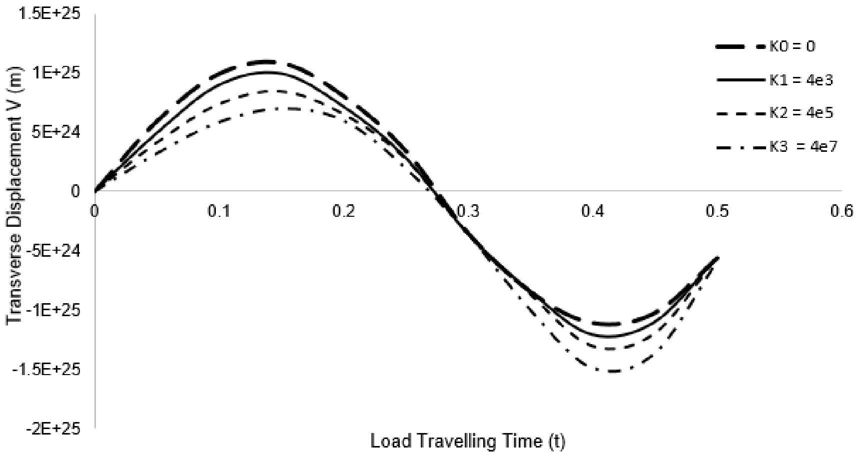

Figure 2: The displacement response of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of foundation stiffness K and for fixed values of Nf = $40000000,\ G = 40000000$ and $C = 3000$.

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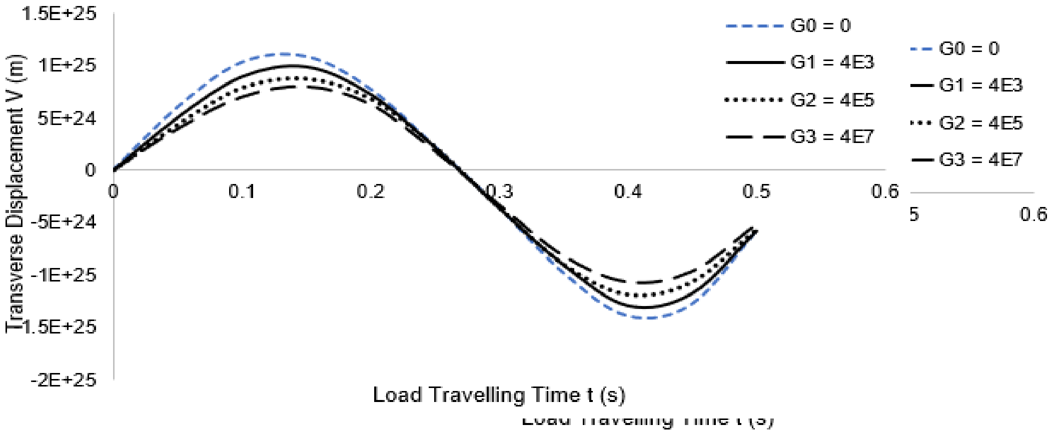

Figure 3: The displacement response of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of $shear\ modulus\ G$ and for fixed values of $Nf = 40000000, K = 40000000$ and $C = 3000$.

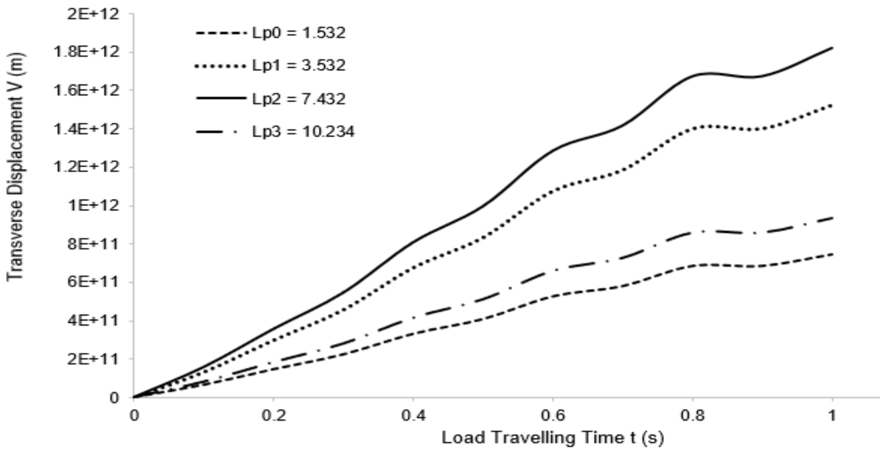

Figure 4: The response amplitude of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of load position Lp and for fixed values of $Nf = 40000000$, $K = 40000000$, $G = 40000000$ and $C = 3000$.

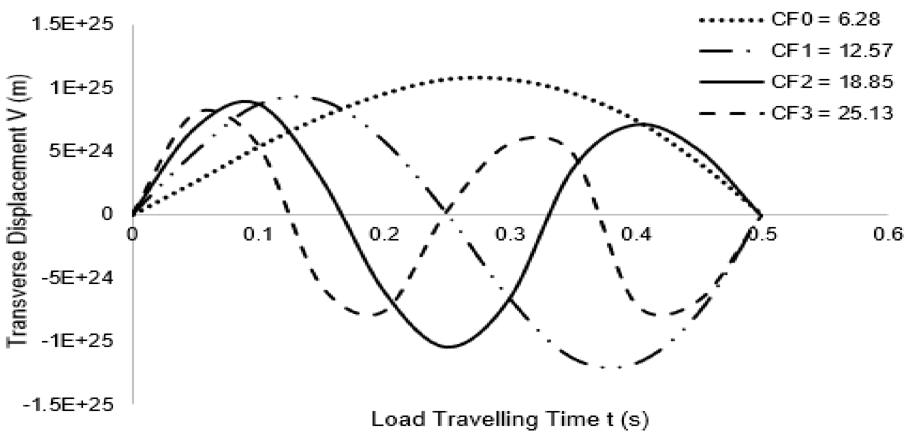

Figure 5: The response amplitude of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of circular frequency

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Cf and for fixed values of $Nf = 40000000,\ K = 40000000,\ G = 40000000$ and $Co = 3000$.

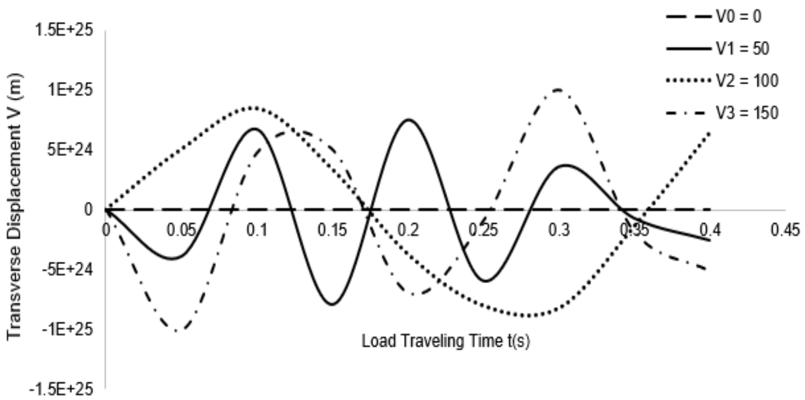

Figure 6: The response amplitude of a simply supported uniform damped shear beam under the action of moving load travelling at constant velocity for various values of load velocity V and for fixed values of $N = 40000000,\ K = 40000000,\ G = 40000000$ and $C = 3000$.

CONCLUSION

This paper investigates the dynamic behaviour of a damped shear beam resting on bi-parametric elastic foundation when under the moving load. The governing equations are coupled second order partial differential equations. Solution procedure, involving finite Fourier transform technique and Laplace transformation in conjunction with convolution theory is used to obtain the solution of the coupled second order partial differential equations describing the motion of the beam-load system. Detailed analyses are performed to investigate the effect of some pertinent structural parameters such as axial force Nf, foundation stiffness K and shear modulus G on dynamic deflection of the beam. It is evident from the plotted curves that the presence of these structural parameters contributes immensely to the stability of the beam when traversed by the travelling load. The study shows that the deflection of the beam reduces significantly with increased axial force, shear modulus and stiffness of the foundation. Also, it shows that the dynamic deflection at the mid-span of the beam is very large compare to other load positions. The study reveals that the higher the value of the circular frequency, the lower the deflection of the beam. The study further reveals that the higher the speed of the travelling loads, the larger the deflection of a simply supported uniform damped shear beam. Consequently, the study further established the conditions under which the beam-load system will experience resonance phenomenon and the speeds at which this may occur.

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