



CONJUGATE IMPACTS OF LINEAR VARIABLE THERMAL CONDUCTIVITY AND THERMAL RADIATION ON NONLINEAR ELECTRICALLY CONDUCTING FLUID

Zayyanu S. Y.^{1*}, Hussaini A.¹, and Isah B. Y.²

¹Department of Mathematics, Sokoto State University, Sokoto, Nigeria.

²Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria

*Corresponding Author's Email: zayyanusahabi12@gmail.com; Tel.: 07032968681

Cite this article:

Zayyanu, S. Y., Hussaini, A., Isah, B. Y. (2024), Conjugate Impacts of Linear Variable Thermal Conductivity and Thermal Radiation on Nonlinear Electrically Conducting Fluid. African Journal of Mathematics and Statistics Studies 7(4), 122-133. DOI: 10.52589/AJMSS-EBCFFU7A

Manuscript History

Received: 13 Aug 2024

Accepted: 21 Aug 2024

Published: 28 Oct 2024

Copyright © 2024 The Author(s).

This is an Open Access article distributed under the terms of Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0), which permits anyone to share, use, reproduce and redistribute in any medium, provided the original author and source are credited.

ABSTRACT: *This study investigates the conjugate impacts of variable thermal conductivity and thermal radiation on nonlinear heat conducting fluid moving vertically through parallel channels. The dimensional governing equations were reduced to dimensionless partial differential equations and consequently transformed to ordinary differential equations. The resulting ordinary differential equations were solved using the homotopy perturbation method. The study aimed at discovering the possible effects of thermal radiation parameter (R), thermal conductivity parameter (β), temperature difference parameter (C_T) and Magnetic parameter (M) through the help of line graphs. It was found that, Velocity and temperature distributions were increasing functions of R , while Velocity and temperature distributions are decreasing functions of β*

KEYWORDS: Thermal conductivity, Homotopy perturbation, thermal conductivity, electrically conducting fluid, MHD.



INTRODUCTION

Thermal conductivity is a fundamental property of materials that plays a crucial role in various fields of science and engineering. It quantifies the ability of a material to conduct heat. It is useful in designing efficient heat exchangers, insulating materials and electronic devices. Thermal radiation is the electromagnetic radiation emitted by a body due to its temperature. It is a fundamental mode of heat transfer and plays a vital role in various scientific and engineering applications. Understanding the principles of thermal radiation is crucial for designing energy-efficient systems and optimizing heat exchange processes. Virtually all the fluids of industrial and engineering applications are useful in energy dissipation in a thermal system, for instance, the temperature is known to alter the fluid thermal conductivity significantly, and in control systems, it is well known that interference with magnetic fields alters the flow behavior of most electrically conducting fluids. This control strategy plays an important role in skin friction and the rate of heat transfer. Other important applications are witnessed in many geophysical situations where MHD problems arise from the origin of the Earth's magnetic field to the prediction of space weather, the damping of turbulent fluctuations in semiconductor melts during crystal growth and even in the measurement of the flow rates of beverages in the food industry. Other interesting applications of MHD are not limited to spraying in metallurgical engineering, electrochemistry and electroplating processing, and other surface occurrences. In view of these wide applications, several researchers have worked on various aspects of Magnetohydrodynamic in the boundary layer region. For instance, recent studies have explored the complex dynamics of heat in various fluid systems with variable thermal conductivity and thermal radiation on heat conducting fluids with implications for various engineering and industrial applications. These studies emphasized the importance of variable thermal conductivity in enhancing transport phenomena. These investigations contribute valuable insights to thermal engineering applications, such as heat exchangers, power generation and electronic cooling systems.

The quality of the final product depends on thermal management of the system and therefore cooling procedures have to be controlled. Variable thermal conductivity has been shown to increase temperature and thermal boundary layer thickness in Williamson fluid flow over a stretching sheet [1]. Unsteady MHD nanofluid flow with variable thermal conductivity, thermal radiation and viscous dissipation has been investigated for both Blasius and Sakiadis flows [2]. The effects of thermal radiation stratification and Joule heating on MHD Sutter by nanofluid flow have been studied, revealing that variable thermal conductivity decreases temperature while radiation decreases it [3]. Zubair *et al.* [4] investigated Williamson hybrid nanofluids with variable thermal conductivity and radiation, finding that hybrid nanoparticles enhance energy production compared to single nanostructures. Rehman *et al.* [5] examined combined convection and thermal radiation in nanofluids with temperature-dependent viscosity and thermal conductivity, observing increased temperature fluctuations with rising thermal conductivity. [6] studied MHD flow of Williamson nanofluid over a curved surface with variable thermal conductivity, considering factors such as activation energy and radiation. [7] analyzed radiative aspects of MHD flow in Williamson nanofluid with variable thickness and thermal conductivity, incorporating melting heat flow.



A year after, [8] investigated Magneto nanofluid flow over a cylinder, considering thermal radiation and variable thermal conductivity; they found that fluid velocity decreases with higher Weissenberg number. [9] examined mixed convective-radiative fluid flow in a channel, considering temperature-dependent thermal conductivity. They analyzed the effects of various parameters on Temperature and Velocity fields. [10] studied the interaction of variable diffusion coefficients with electro kinetically modulated peristaltic motion of a radiative Carreau-Yasuda nanofluid. They observed that temperature profile increase due to joule heating and manipulation of radiation and thermal conductivity parameters can improve thermal efficiency. [11] analyzed viscoelastic fluid flow, considering variable thermal conductivity using mathematical models and numerical methods. [12] conducted comparative analysis of thermo-physical aspects in Magnetized non-Newtonian fluid flow examining the impacts of mixed convection, heat generation and thermal radiation. [13] explored second-grade fluid flow with Soret and Dufour and thermal radiation. [14] studied a note on hydromagnetic Blasius flow with variable thermal conductivity. [15] studied heat transfer of squeezing unsteady nanofluid flow under the effects of magnetic field and variable thermal conductivity. [16] studied a similarity solution for natural convection flow near a vertical plate with thermal radiation.

The purpose of the present study is to investigate/examine the conjugate effects of variable thermal conductivity and thermal radiation on non-linear electrically conducting fluid. The study will contribute to the broader field of materials science, enabling researchers to develop new materials with enhanced thermal properties, improving the efficiency of energy conversion and utilization; it will also allow engineers and researchers to make more informed decisions in designing a wide range of systems and devices by considering the complex interplay between the variable thermal conductivity and thermal radiation.

MATHEMATICAL ANALYSIS

Consider an infinite vertical channel formed by two parallel plates kept h distance apart. The channel is filled with an electrically conducting fluid at the expense of radiative heat flux of intensity q_r which is absorbed by the plates and transferred to the fluid. Assuming negligible effects of viscous dissipation and assuming all the physical properties of the fluid are constant except for thermal conductivity and thermal radiation. It is also assumed that the flow is at steady-state such a way that y' is dependent on y' .

The x' - axis is taken along the channel in the vertically upward direction being the direction of the flow, while the y' - axis is taken normal to it. The temperature of the plate kept at $y' = 0$ rise to T_w and maintained impulsively at uniform velocity $u = 0$.while the other plate at $y' = h$

remains

a

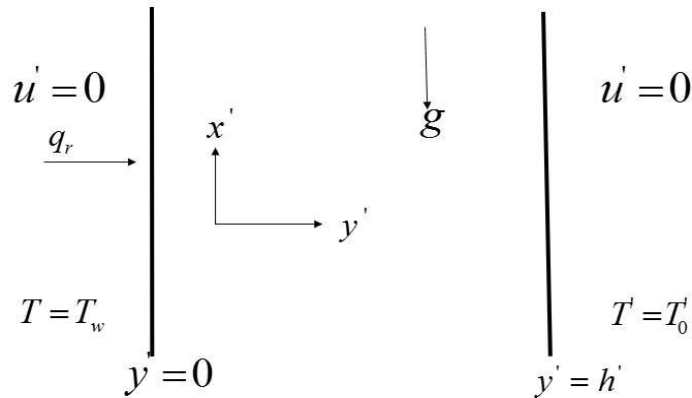


Figure 1: Diagram of the Problem

Momentum equation

$$\nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_0) - \frac{\sigma\beta_0^2}{\rho} u' = 0 \tag{1}$$

Heat transfer equation

$$\frac{1}{\rho C_p} \left(\frac{\partial}{\partial y'} \left(k \frac{\partial T'}{\partial y'} \right) - \frac{\partial q_r}{\partial y'} \right) = 0 \tag{2}$$

$$\left. \begin{aligned} u' = 0, T = T_w & \quad \text{at} \quad y' = T_w \\ u' = 0, T = T_0 & \quad \text{at} \quad y' = T_0 \end{aligned} \right\} \tag{3}$$

Using Rosseland approximation

$$q_r = -\frac{4\sigma}{3k'} \frac{\partial T^4}{\partial y'} \tag{4}$$

$$k = k_0(1 + \beta\theta)$$



$$u = \frac{u'}{U}, \quad y = \frac{y'}{h}, \quad U = \frac{1}{g\beta(T_w - T_0)}, \quad \theta = \frac{T' - T_0}{T_w - T_0}, \quad C_T = \frac{T_w}{T_w - T_0}, \quad R = \frac{4\sigma(T_w - T_0)^3}{3k^*k_0}, \quad k = k_0(1 + \beta\theta) \quad (5)$$

After necessary analysis, the dimensionless equations reduced to

Dimensionless Momentum equation

$$\frac{d^2u}{dy^2} + \theta - M^2u = 0 \quad (6)$$

Dimensionless Heat transfer equation

$$\left(1 + \beta\theta + \frac{4}{3}R(C_T + \theta)^3\right) \frac{d^2\theta}{dy^2} + 4R(C_T + \theta)^3 \left(\frac{d\theta}{dy}\right)^2 = 0 \quad (7)$$

$$\left. \begin{aligned} u = 0, \theta = 1 & \quad \text{at} \quad y = 0 \\ u = 0, \theta = 0 & \quad \text{at} \quad y = 1 \end{aligned} \right\} \quad (8)$$

Using homotopy perturbation on equation (7), we have

$$(1-p) \frac{d^2\theta}{dy^2} + p \left(\frac{d^2\theta}{dy^2} + \left(\beta\theta + \frac{4}{3}R(C_T + \theta)^3 \right) \frac{d^2\theta}{dy^2} + 4R(C_T + \theta)^3 \left(\frac{d\theta}{dy} \right)^2 \right) = 0 \quad (9)$$

$$(1-p) \frac{d^2u}{dy^2} + p \left(\frac{d^2u}{dy^2} + \theta - M^2u \right) = 0 \quad (10)$$

But

$$(C_T + \theta)^2 = \theta^2 + 2C_T\theta + C_T^2 \quad (11)$$

$$(C_T + \theta)^3 = \theta^3 + 3C_T\theta^2 + 3C_T^2\theta + C_T^3 \quad (12)$$

Also

$$\theta = \theta + P\theta_1 + P^2\theta_2 + P^3\theta_3 + \dots \quad (13)$$

$$\theta^2 = \theta_0^2 + 2P\theta_0\theta_1 + 2P^2\theta_0\theta_2 + P^2\theta_1^2 + \dots \quad (14)$$



$$\theta^3 = \theta_0^3 + 3P\theta_0^2\theta_1 + 2P^2\theta_0^2\theta_2 + 2P^2\theta_0\theta_1^2 + \dots$$

$$\left(\frac{d\theta}{dy}\right)^2 = \left(\frac{d\theta_0}{dy}\right)^2 + 2P\frac{d\theta_0}{dy}\frac{d\theta_1}{dy} + 2P^2\frac{d\theta_0}{dy}\frac{d\theta_2}{dy} + P^2\left(\frac{d\theta_1}{dy}\right)^2 + \dots \tag{15}$$

Neglecting higher powers of P and substituting Equations (10-14) into Equation (6), we have

$$P^0 : \begin{cases} \frac{d^2\theta_0}{dy^2} = 0 \\ \frac{d^2u_0}{dy^2} = 0 \end{cases} \tag{16}$$

$$P^1 : \begin{cases} \frac{d^2\theta_1}{dy^2} + 4R\theta_0^3\left(\frac{d\theta_0}{dy}\right)^2 + 12RC_T\theta_0^2\left(\frac{d\theta_0}{dy}\right)^2 + 12RC_T^2\theta_0\left(\frac{d\theta_0}{dy}\right)^2 = 0 \\ \frac{d^2u_1}{dy^2} - M^2u_0 = -\theta_0 \end{cases} \tag{17}$$

$$P^2 : \begin{cases} \frac{d^2\theta_2}{dy^2} + \beta\theta_0\frac{d^2\theta_1}{dy^2} + \frac{4RC_T^2}{3}\frac{d^2\theta_1}{dy^2} + \frac{4R\theta_0^2}{3}\frac{d^2\theta_1}{dy^2} + \frac{8R}{3}\theta_0\theta_1\frac{d^2\theta_1}{dy^2} + \frac{8R}{3}\theta_0C_T\frac{d^2\theta_1}{dy^2} + 8R\theta_0^3\frac{d\theta_0}{dy}\frac{d\theta_1}{dy} + \\ 24RC_T\theta_0^2\frac{d\theta_0}{dy}\frac{d\theta_1}{dy} + 24R\theta_0\theta_1C_T\left(\frac{d\theta_0}{dy}\right)^2 + 24RC_T^2\theta_0\frac{d\theta_0}{dy}\frac{d\theta_1}{dy} + 12RC_T^2\theta_1\left(\frac{d\theta_0}{dy}\right)^2 = 0 \\ \frac{d^2u_2}{dy^2} - M^2u_1 = -\theta_1 \end{cases} \tag{18}$$

The corresponding boundary conditions are

$$\left. \begin{aligned} u_0 = u_1 = u_2 = u_3 = 0 & \quad \text{at} \quad y = 0 \\ \theta_0 = 1, \theta_1 = \theta_2 = \theta_3 = 0 & \quad \text{at} \quad y = 1 \end{aligned} \right\} \tag{19}$$

Now, solving the equations (16-18) and applying the corresponding boundary conditions in (19), we have:

$$\theta_0 = 1 + \alpha y \tag{20}$$

$$\theta_1 = K_5(y^2 - y)\alpha^2 + K_6(y^3 - y)\alpha^3 + K_7(y^4 - y)\alpha^4 + K_8(y^5 - y)\alpha^5 \tag{21}$$



$$\theta_2 = C_{21}y^{12} + C_{20}y^{11} + C_{19}y^{10} + C_{18}y^9 + C_{17}y^8 + C_{16}y^7 + C_{15}y^6 + C_{14}y^5 + C_{13}y^4 + C_{12}y^3 + C_{11}y^2 + C_{10}y + C_9 \quad (22)$$

$$u_0 = 0 \quad (23)$$

$$u_1 = \frac{y^3}{6} - \frac{y^2}{2} + \frac{y}{3} \quad (24)$$

$$u_2 = \frac{K_8 y^7}{42} - \frac{K_7 y^6}{30} + \frac{1}{20} \left(K_6 + \frac{M^2}{6} \right) y^5 - \frac{1}{12} \left(K_5 + \frac{M^2}{2} \right) y^4 - \frac{z_1 y^3}{6} + h_1 y + \frac{M^2 y^3}{18} \quad (25)$$

$$u_3 = L_1 y^{14} + L_2 y^{13} + L_3 y^{12} + L_4 y^{11} + L_5 y^{10} + L_6 y^9 + L_7 y^8 + L_8 y^7 + L_9 y^6 + L_{10} y^5 + L_{11} y^4 + L_{12} y^3 + L_{13} y$$

(26)

RESULT AND DISCUSSION

The problem of Non-linear heat conducting fluid due to interaction of thermal radiation and linear variable thermal conductivity have been considered. The dimensional partial differential equations are transformed into a system of ordinary differential equations via a Homotopy perturbation technique. The final dimensionless equations are solved analytically and the obtained closed form solution was integrated into Matlab. The default values of pertinent parameters embedded in the flow are Prandtl number ($Pr = 0.72$), thermal radiation ($R = 0.5$), temperature difference parameter (C_T), Magnetic parameter ($M = 1.00$) and thermal conduction parameter ($\beta = 0.10$).

Fig. 2 exhibits the impacts of thermal radiation (R) on temperature distribution. It is observed that increase in R gives rise to temperature distribution for both Fig. 2a and Fig. 2b. It is also noted that the impact of R is more pronounced in Fig. 2b when thermal conduction is absent; this is expected as thermal conduction reduces, the fluid acquires more heat through radiation which in turn increases the temperature of the fluid. Fig. 3 portrays the impact of temperature difference parameter (C_T) on temperature distribution, it is evident that the presence of C_T is inversely proportional to temperature distribution, it is interesting to note that C_T has no effect on temperature when thermal conduction and thermal radiation are absent.

Fig. 4 depicts the effect of thermal conduction parameter (β) on the fluid temperature; interestingly, thermal boundary layer thickness is retarded and the fluid temperature diminishes with a boost in thermal conductivity. The same phenomena is observed in Fig. 5. As thermal conduction parameter (β) increases, the velocity distribution of the flow is decaying and vice-



versa. While Fig. 6. showcases the role of temperature difference parameter (C_T) On velocity distribution, the fluid velocity near the plate is seen to decrease as the temperature difference gets larger, but the velocity far from the plate increases significantly.

Fig. 7 is prepared to show the role of magnetic parameter (M) on velocity distribution, it is noted from the Figure that M is directly proportional to the fluid velocity and vice-versa. The same trend is observed in Fig. 8. An increase in thermal radiation parameter (R) lead to more flow formation which in turn increases the velocity of the fluid.

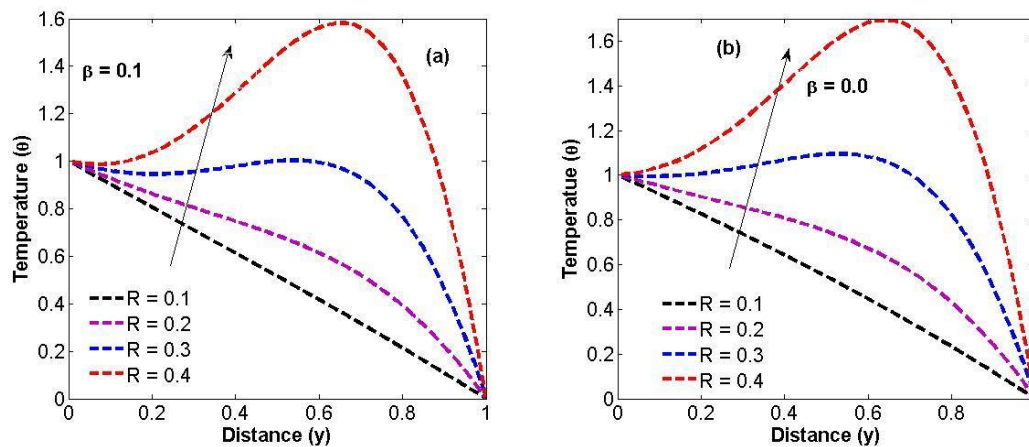


Fig. 2: Effects of Radiation Parameter (R) on Temperature Profile

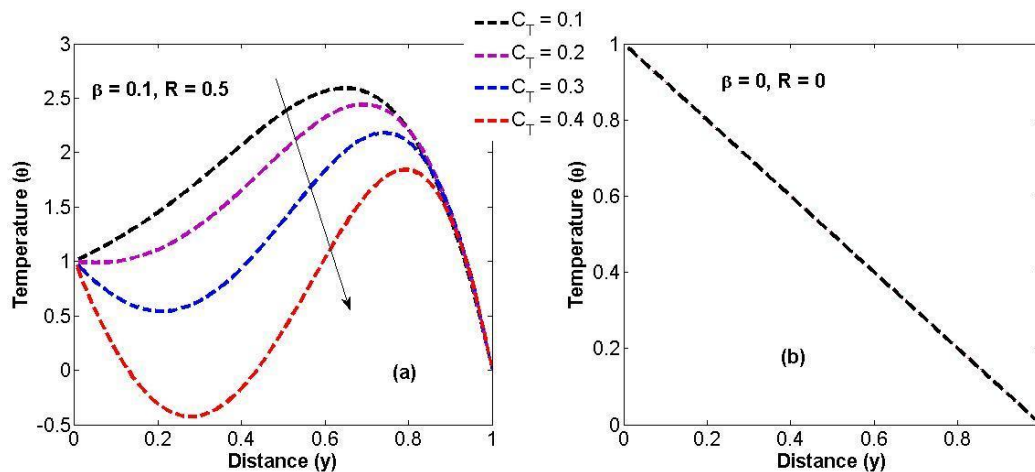


Fig. 3: Effects of Temperature Difference Parameter (C_T) on Temperature Profile

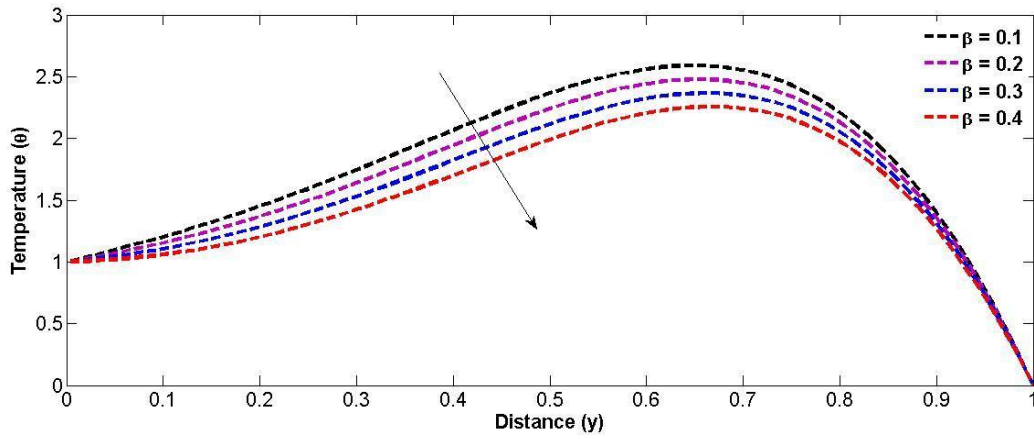


Fig. 4: Effects of Variable Thermal Conductivity Parameter (β) on Temperature Profile

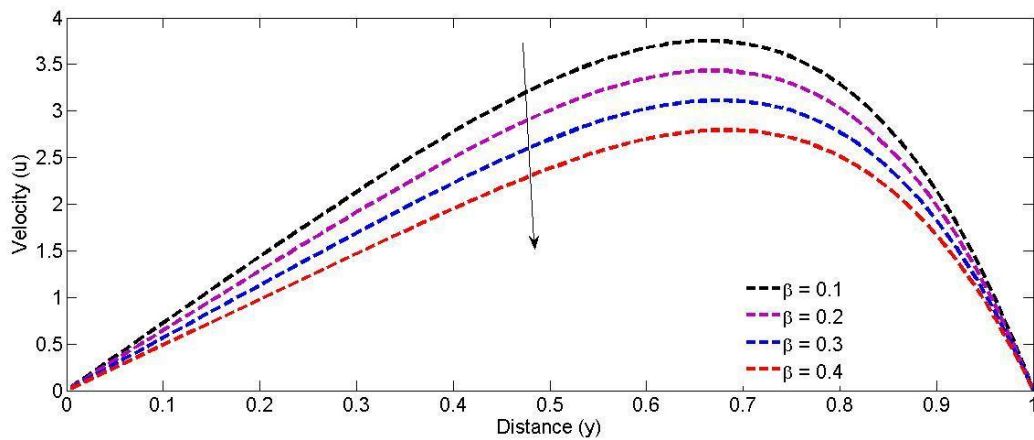


Fig. 5: Effects of Variable Thermal Conductivity Parameter (β) on Velocity Profile

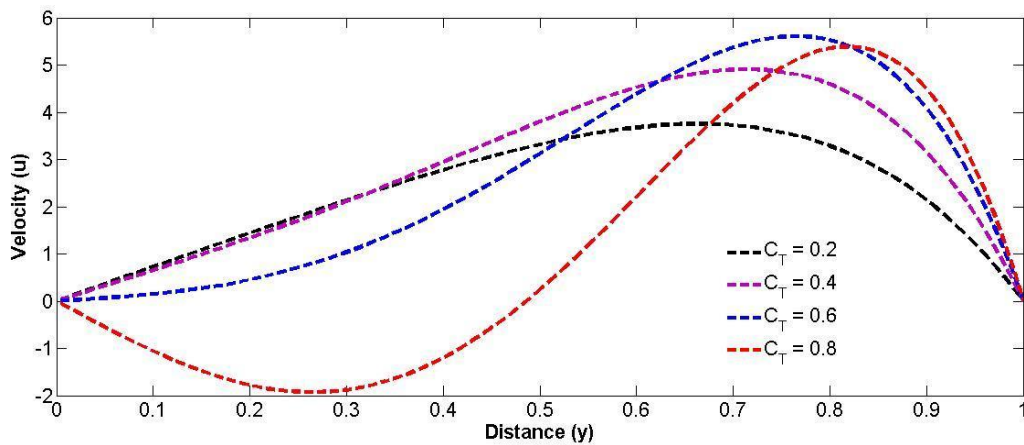


Fig. 6: Effects of Temperature Difference Parameter (C_T) on Velocity Profile

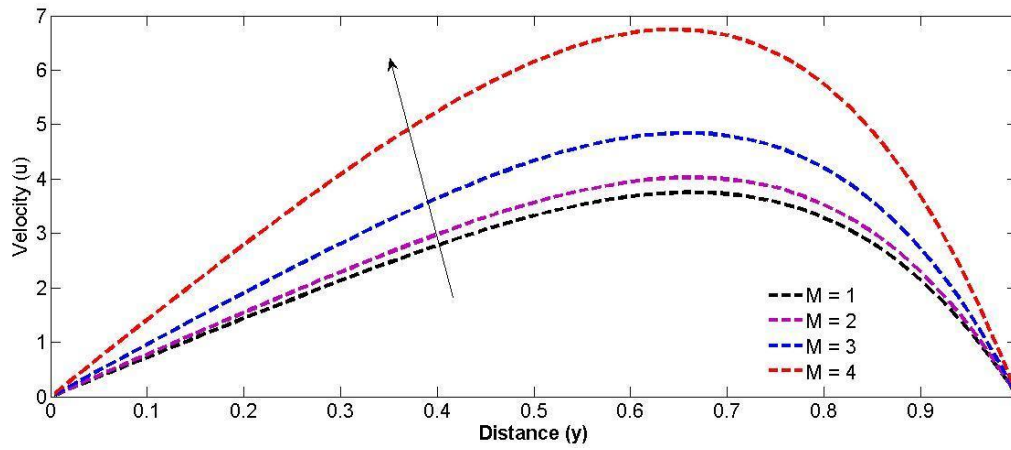


Fig. 7: Effects of Magnetic Parameter (M) on Velocity Profile

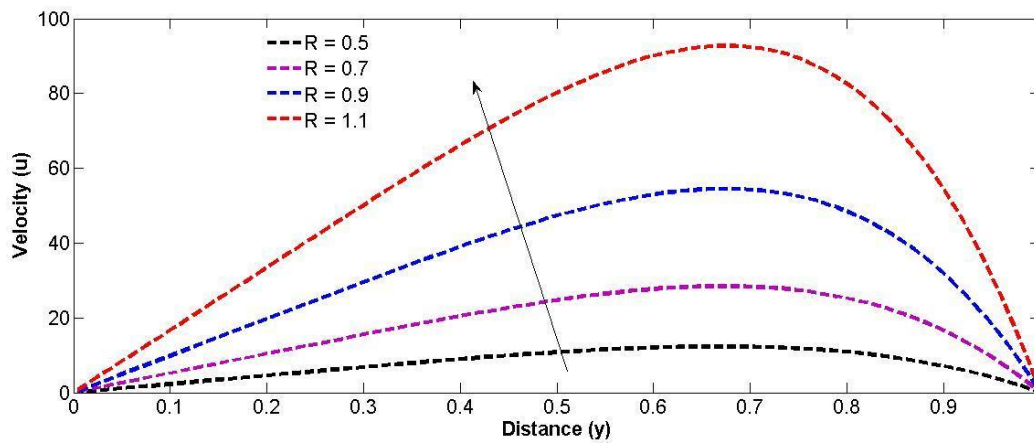


Fig. 8: Effects of Radiation Parameter (R) on Velocity Profile



CONCLUSIONS

From the findings of this study, it could be concluded the following situations holds:

- Temperature distribution attain a peak value through an increase in thermal radiation parameter (R) when thermal conduction is absent.
- The presence of temperature difference parameter (C_T) retard the temperature distribution and velocity distribution at some point.
- Thermal conduction parameter (β) decreases the temperature and velocity distributions.
- Velocity distribution is an increasing function of magnetic (M) and thermal radiation parameters (R).

REFERENCES

- [1] Dada, M. S. and Onwubuoya, C. (2020). Variable viscosity and thermal conductivity effects on Williamson fluid flow over a cylinder ring stretching sheet. *World Journal of Engineering*, 17(3), 357-371. DOI: 10.1108/wje-08-2019-0222.
- [2] Shaayeti, S., and Muzara, H. (2020). Unsteady MHD Blasius and Sakiadis flows with variable thermal conductivity in the presence of thermal radiation and viscous dissipation. *Frontiers in Heat and Mass Transfer (FHMT)*, 14. DOI:10.5098/hmt.1418
- [3] Sohail, M., Nazir, U., Chu, Y. M., Alrabaiah, H., Al-Kouz, W. and Thounthng, P. (2020). Computational exploration for radiative flow of Sutterby nanofluid with variable thermal conductivity and diffusion coefficient. *Open Physics*, 18(1), 1073-1083. DOI: 10.1515/phys-2020-0216
- [4] Zubair, T., Usman, M., Hamid, M., Sohail, M., Nazir, U., Nisar, K. S., and Vijayakumar, V. (2021). Computational analysis of radiative Williamson hybrid nanofluid comprising variable thermal conductivity. *Japanese Journal of Applied Physics*, 60(8), 087004. DOI: 10.35848/1347-4065/ac1388
- [5] Rehman, A., Hussain, A., and Nadeem, S. (2021). Physical aspects of convective and radiative molecular theory of liquid originated in nanofluid flow in the existence of variable properties. *Physica Scripta*, 96(3), 085219. DOI: 10.1088/1402-4896/abd790
- [6] Ahmed, K., Akbar, T., Muhammad, T., and Alghamdi, M. (2021). Heat transfer characteristics of MHD flow of Williamson nanofluid over an exponential permeable stretching curved surface with variable thermal conductivity. *Case studies in Thermal Engineering*, 28, 101544. DOI: 10.1016/j.csite.2021.101544
- [7] Islam, S., Ur Rasheed, H., Nisar, K. S., Alshehri, N. A., and Zakarya, M. (2021). Numerical simulation of heat mass transfer effects on MHD flow of Williamson nanofluid by a stretching surface with thermal conductivity and variable thickness. *Coatings*, 11(6), 684. DOI: 10.3390/COATINGS11060684



- [8] Bilal, M., Siddique, I., Borawski, A., Raza, A., Nadeem, M., and Sallah, M. (2022). Williamson magneto-nanofluid flow over a partially slip and convective cylinder with thermal radiation and variable conductivity. *Scientific report*, 12(1), 12727. DOI: 10.1038/s41598-022-16268-2
- [9] Kumar, T. S., DN, P. K., and Reddy. A. S. (2022). Study of mixed convective-radiative fluid flow in a channel with temperature-dependent thermal conductivity, *Partial Differential Equations In Applied Mathematics*, 5, 100344. DOI: 10.1016/j.padiff.2022.100344
- [10] Akbar, Y., Huang, S., Ashraf, M. U., Nisar, K. S. and Alam, M. M. (2023). Interaction of variable diffusion coefficients with electro-kinetically regulated peristalsis of Carreau-Yasuda nanofluid. *Case studies in Thermal Engineering*, 45, 102962. DOI: 10.1016/j.csite.2023.102962
- [11] Anwar, M. S., Alghamdi, M., Muhammad, M., Hussain, M. and Puneeth, V.(2024). Analysis of nonlinear convection and diffusion in viscoelastic fluid flow with variable time-dependent thermal conductivity and thermal radiations. *Modern Physics Letters B*. 38(22), 2450146. DOI: 10.1142/s021798492450146x
- [12] Rehman, K. U., Shatanawi, W. and Firdous, U. (2023). A comparative thermal case study on thermophysical aspects in thermally magnetized flow regime with variable thermal conductivity. *Case studies in Thermal Engineering*, 44, 102839. DOI: 10.1016/j.csite.2023.102839
- [13] Abbas, T., Saeed, M., Ghachem, K., Alshammari, B. M. Khan, S. U. and Kolsi, L. (2023). Thermal conductivity and mixed convection influence the flow of viscoelastic fluid due to the inclined cylinders. *Journal of Magnetism*, 28(1), 24-33. DOI: 10.4283/jmag.2023.28.1.024
- [14] Makinde, O. D., Adesanya, S. O., and Ferdows, M. (2020). A note on the hydromagnetic Blasius flow with variable thermal conductivity. *Journal of Applied and Computational Mechanics*. DOI: 10.22055/JACM.2020.32638.2051.
- [15] Lehmar, S., Kezzar, M., Eid, M. R., and Sari, M. R. (2020). Heat transfer of squeezing unsteady nanofluid flow under the effects of an inclined magnetic field and variable thermal conductivity. *Physica A: Statistical Mechanics and its Applications*, 540(2020), 123138. DOI: 10.1016/j.physa.2019.123138
- [16] Jha, B. K. and Samaila, G. (2020). A similarity solution for natural convection flow near a vertical plate with thermal radiation. *Microgravity Science and Technology*, 32(6), 1031-1038. DOI: 10.1007/s12217-020-09830-y