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ON COMPARING THE OPTIMALITY CRITERIA PERFORMANCE OF RESOLUTION *IV* AND RESOLUTION *V* FACTORIAL DESIGNS

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ABSTRACT: *This study looks at the optimality criteria performances on the factorial design of Resolution IV and Resolution V. The Comparative studies of Resolution IV and Resolution V design were evaluated using the D, G, and I-optimality criteria. The FDS plots were also used. The results showed that in all the factors k considered, Resolution V has a better factorial design when it comes to D-optimality, G-optimality and I-optimality, but when the interest is on the spread of the scale prediction variance, Resolution IV is preferred. The FDS plots for Resolution IV and V design were relatively the same for factors k = 6 and k = 10.*

KEYWORDS: Optimality criteria, resolution iv and resolution v, factorial designs, D-optimality, I-optimality, G-optimality, Design matrix, Fraction of Design Space (FDS) Plots.

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INTRODUCTION

Experiments are carried out by researchers, to study and model the effects of variables on the responses of interest. The basic idea of response surface methodology (RSM) in any experiment is to model and optimize responses. RSM can be expressed as a collection of statistical methods that are used for modelling and analysis of issues where the response in question is determined by variables and the goal of the analysis is to optimise the response. It may also be explained as a technique that uses intricate calculations throughout the optimisation process (OP). By using the data from the experiment to create an equation with theoretical value, this strategy creates an acceptable experimental design. The equation is the result of a well-planned regression study, which focuses mostly on the controlled values of the independent variable.

In statistics and mathematics, the primary motivation behind RSM according to Box and Wilson (1951) is to use experimental design to obtain an optimization process (OP).

RSM has been embraced by many research institutions whose experiments may be thoroughly explained and whose results are believed to be statistically valid.

A small amount of time is required to test all the factors related to the researcher's assessment by using RSM in the optimization procedure. Furthermore, estimating parameters makes it simple to identify the variables that have the most impact on the model, allowing researchers to concentrate on the factors that affect the end product acceptability. Also, in RSM, the response surfaces are represented graphically and are used to describe the interaction effects of variables and their effects on response.

In response to surface methodology, choosing an appropriate central composite design for an experiment has been centred mostly on the choice of the axial distance or region of operability with little or no emphasis on the factorial points of the design as the number of factors increases.

This study will focus on the choice of factorial point that will help construct a central composite design to help increase the optimum precision of the estimated models of a response surface design.

The main aim of this study is to ascertain between Resolution *V* and Resolution *IV, a* factorial design that will give an optimum prediction variance capability which can be incorporated into the central composited design. This can be achieved with the following objectives

Evaluating the D-, G- and I- optimality criteria values of Resolution *V* and Resolution *IV* design for factors k = 6 to 10 and comparing the variance performance capabilities of Resolution *V* and Resolution *IV* design.

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LITERATURE/THEORETICAL

Optimal designs (OD) are a kind of designs that are optimal in terms of statistical criterion in the design of experiments. It allows parameters to be approximated without bias and with the least amount of variance when estimating statistical models. In practice, optimal experiments can help to cut down on experimental expenditures (Kirstine Smith, 2001).

The statistical model determines the optimality of a design, which is evaluated using a statistical criterion related to the estimator's variance matrix.

Chigbu et al. (2008) compared the prediction variances (PV) of different CCDs in spherical regions in their study. The results reveal that none of the designs are superior across all three comparison criteria, namely the (VDG), I and G optimality criteria, and more. However, it is demonstrated that the CCD is at its optimum under the I optimality. (gives the least amount of prediction variance) at five centre points and three components (k = 3). For the G optimality, the prediction variance (PV) of the CCD grows as the number of centre points (CP) increases. Under the I optimality criterion, the Small composite designs (SCD) behave similarly, however, under the G optimality criterion, it achieves its least variance by employing two centre points. When the number of centre points rises, the variance falls. The Minimum-runs resolution V designs (MinRes.V) are at their optimum with five centre points under the I optimality and deteriorate similarly with CCD and SCD under the G optimality.

Oehlert (2002) carried out research on Small efficient, resolution V fractions of 2k designs and their Application to Central Composite Designs. The study demonstrates that Resolution V column-wise-pairwise (CP) designs offer appealing efficiency when employed as irregular fractions as well as the foundation for central composite designs. They can be built for a far wider variety of sample sizes than ordinary irregular fractions, providing experimenters with greater options in balancing size and the availability of efficient designs.

For n = 22, K= 6, the CP method produces the same design as D-optimality, however for n = 64, K= 9 produces a design with slightly lower A-efficiency criteria than the one produced by maximizing D-efficiency.

Iwundu (2016) examined the effectiveness of numerous second-order N-Point spherical response surface method designs. In the investigation, equiradial designs were examined in modelling second-order response functions as an alternative spherical design to rotatable central composite designs (CCDs) and D-optimal precise designs. These designs are comparable to traditional second-order response surface approaches. The simple equiradial designs appear to have some attractive optimality qualities. An evaluation of the D-efficiency numbers demonstrates that equiradial designs are not always inferior. In fact, they seem to be more optimum than certain regularly used second-order response surface approach designs.

Ukaegbu et al. (2020) explored the prediction variance aspects of 3 to 10 element rotatable central composite designs (RCCD). They investigated the prediction variance features of the CCD with rotatable by recreating the CCD's cube and star components. The designs were assessed using three design optimality criteria: D, G, and V. FDS charts for scaled and unscaled prediction variances (PV) are used to analyse the performance characteristics of design prediction variances over the design region. According to the findings, cube-replicated CCDs are D-efficient for k = 3 to 10 factors and three centre points.

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None of the design alternatives, cube- and star-replicated, consistently outperformed the others in terms of G-efficiency, V-criterion, and FDS plots for any of the k factors tested.

Nwanya and Dozie (2020) investigated the best Prediction Variance Capabilities (PVC) of Inscribed Central Composite Designs (ICCD). They evaluated the effect of replication on the factorial and axial sections of an inscribed central composite design (ICCD) using the G-optimal, I-optimal, and FDS plots. The ICCD with repeated factorial and axial sections, according to their findings, has an increased maximum scaled prediction variance (SPV) for factors k = 2 to 4. At 5 and 6-factor levels, the Inscribe central composite design with replicated factorial portions provides a higher maximum and average SPV. The FDS graphs indicate that the ICCD outperforms both the inscribe central composite design (ICCD) and the inscribe central composite design (ICCD) with replicated factorial portion and the inscribe central composite design (ICCD) with replicated axial portion from 0.0 to 0.5 of the design space, while the ICCD with replicated factorial portion is superior to both the inscribe central composite design without replicated factorial and axial portion and the inscribe central composite design (ICCD) with identical.

Factorial Design

A factorial design (FD) is a sort of structured experiment that allows you to investigate the impact of several factors on a response. When doing an investigation, instead of adjusting the levels of each element one at a time, you may explore the interactions between them by varying the levels of all factors at once. In this study, the prediction variance performances of factorial design (Resolution IV and Resolution V) were compared using the optimality criteria.

To execute a 2^{k-p} fractional factorial design, p-independent generators must be chosen, and a technique of creating a resolution R design of m -factors in n runs is given.

Let X be the n by m design matrix, with +1 and -1 denoting high and low values of a factor, respectively. A whole 2m-1 factorial design is written down to generate a one-half fraction, and then the mth factor is added by identifying its plus and minus levels with the signs of GHI...(M - 1). M= GHI... (M - 1), meaning that I=GHI... M, where G, H, I..., M=x1, x2, x3,..., xm, respectively. Generators are produced when more components are introduced to interactions. The set of different words created by all conceivable products of any subset of the factors involving p

Generators provide the defining relation, which has 2p terms, including the identity term I. We have IW=WI=W and W2=I for a set of generators W=W1, W2,..., Wp.

Another method is to divide the runs into two blocks using the highest-order interaction GHI... M.

Resolution IV Designs

In resolution IV design, 2-factor interactions are typically aliased with one another. That is, two-factor interactions are mixed together. The smallest resolution IV design can be produced by beginning with a full 2^3. A 2^{6-2} R.IV design with I = GHIK, I = GHJL and I = IJKL as the generators is shown in Table 2.1.

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Table 2.1: 2_{IV}^{6-2} **Design with I = GHIK, I = GHJL and I = IJKL as the generators**

Factor G	Factor H	Factor I	Factor J	K= GHI	L= GHJ	Treatment combination
-	-	-	-	_	-	(1)
+	-	-	-	+	+	GKL
-	+	-	-	+	+	HKL
+	+	-	-	-	-	GH
_	-	+	-	+	-	IK
+	-	+	-	-	+	GIL
_	+	+	-	-	+	HIL
+	+	+	-	+	-	GHIK
_	-	-	+	-	+	JL
+	-	-	+	+	-	JK
_	+	-	+	+	-	HJK
+	+	-	+	-	+	GHJL
_	-	+	+	+	+	IJKL
+	-	+	+	-	-	GIJ
_	+	+	+	-	-	HIJ
+	+	+	+	+	+	GHIJKL

Resolution *V* Designs

In resolution v design, two-factor interactions are aliased using three-factor interactions, but no major impact or two-factor interaction is aliased with any other major effect or two-factor interaction. The smallest Resolution V design 2^{5-1} may likewise be built in the same way as the earlier mentioned Resolution IV design.

The resolution of a design is often equal to the least number of letters in any term appearing in the defining relation (DR). The words in the defining relation (DR) are made up of the initial generators and all of their generalized interactions. The entire defining relation (DR) for the design described in Table 2.1 is I = GHIK= GHJL = IJKL.

When fractionating a complete factorial, one issue that may emerge is that two or more effects may have the same number. The effects in this scenario are known as aliases, and the researcher must ensure that factors thought to be relevant are not aliased with one other. Any factor's aliases may be formed by combining it with all of the terms in the defining relation.

Table 2.2 depicts the alias structure for the Resolution IV design with the defining relation I = GHIK, GHJL, IJK

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Table 2.2: Alias Structure for 2_{IV}^{6-2} Design

I	GHIK	GHJF	IJKL
G	HIK	HJL	GIJKL
H	GIK	GJL	HIJKL
I	GHK	GHIJK	JKL
J	GHIJK	GHL	IKL
K	GHI	GHJKL	IJL
L	GHIKL	GHJ	IJK
GH	IK	JL	GHJKL
GI	HK	HIJL	GJKL
GJ	HIJK	HL	GIKL
GK	HI	HJK	GIJL
GL	HIKL	HJ	GIJK
IJ	GHJK	GHIL	KL
IL	GHKL	GHIJ	JK
GIJ	HJK	HIL	GKL
GKJ	HIJ	HKL	ACF GIL

METHODS OF OPTIMALITY CRITERIA

Optimal designs (OD) are test designs that are created by applying a certain optimality criterion to a single statistical model. When fitting second-order models, optimum design approaches require a single criterion in order to create designs for RSM. Kiefer (1959) introduced the principle of optimal designs.

An optimum design (OP) maximises or reduces an optimality criterion, which is a measure of how excellent a design is. There are several optimality-criteria methodologies for dealing with forecast variance. This study will concentrate on the D-, G-, and I- optimality criteria.

D-optimality (determinant)

The most commonly utilised optimality criterion for selecting designs is the estimation-oriented D-optimality criterion, which seeks designs that maximize the information matrix of determinants while reducing the volume of the confidence ellipsoid regarding unknown model parameters. M = X′X is the information matrix for accurate designs, where X is the np model matrix, n is the number of runs, and p is the number of terms in the model. For accurate designs, the information matrix is calculated as M = X′X, where X is the np model matrix, n is the number of runs and p is the number of terms in the model.

G-optimality

The practitioner's objective may be to have a strong prediction at a given location in the design space. The scaled prediction variance (SPV) was developed by Box and Hunter (1957).

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The SPV is defined as

$$\frac{N\,\mathrm{var}\left[\hat{y}(x)\right]}{\sigma^2} = Nf^T(x)\left(X^T X\right)^{-1} f(\mathrm{x}) \tag{2.1}$$

where $f(x)$ is the coordinate vector of a location in the region of interest extended to model form.

That is $f^T(\mathrm{x}) = \left[1, x_1, ..., x_k, x_1^2, ..., x_k^2, x_1 x_2, ..., x_{k-1} x_k\right]$, N is the total sample size penalizing the larger designs, X is the design matrix and σ^2 is the process variance which is assumed to be 1. At every point in the design space, the SPV offers a measure of the projected response's accuracy. The greatest SPV over the experimental design zone is minimised in a G-optimal design. This is how it is written:

$$Min\left\{\max N \,\mathrm{var}\, \hat{y}(x)\right\} = \min\left\{N \max\ f^T(x)(\mathrm{X}^T\ X)^{-1} f(x)\right\} \tag{2.2}$$

G-optimality shields the experimenter from having to deal with the worst-case scenario. The lower restriction for the maximum SPV is equivalent to p, the number of model parameters, which is an intriguing and useful conclusion (Myers and Montgomery, 2002).

I-optimality (integrated)

The average prediction variance (APV) is minimised via an I-optimal design.

$$Average\,Variance = \frac{\int_R f^T(x)\mathrm{M}^{-1} f(x)dx}{\int_R dx} \tag{2.3}$$

over the experimental region χ. To calculate this average variance, we exploit the fact that, when calculating the trace of a matrix product, we can cyclically permute the matrices. Therefore, we can rewrite the formula for the APV as

$$Average\,Variance = \frac{1}{\int_R dx} tr\left[M^{-1}B\right] \tag{2.4}$$

Where

$$B = \int_R f(x)\ f^T(x)\,\mathrm{dx} \tag{2.5}$$

This matrix is known as the moment matrix because its members are proportional to uniform distribution moments on the experimental region χ.

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Fraction of Design Space (FDS) Plots

The peculiarities of the design's prediction variance are not entirely reflected by single-number criteria like D, A, and G-efficiency, or the I-criterion. However, a design that performs well by one optimality criteria (O P) could not be as effective by another. Borkowski (1995) states that by reducing a design's attributes to a single value, considerable information about a design's potential performance is lost when its qualities are reduced to a single value.

As an alternative to single-value criteria, the FDS plot presented by Zahran, Anderson-Cook, and Myers (2003) overcomes this shortcoming by illustrating the properties of prediction variance throughout the whole design space. On a single two-dimensional graph, the graphic likewise depicts scaled prediction variance (SPV) properties throughout a multidimensional region, at this point using a single curve. The FDS graphic depicts the proportion of the design space at or below a particular SPV value. It is generated by collecting a large number of values, for n, from all across the design space and computing all of the associated SPV values, which are then ordered and plotted against the quantiles (1/n, 2/n, and so on). The x-axis indicates design space (DS) from 0 to 1, while the y-axis displays SPV values. We can calculate what proportion of the entire design space has SPV values that are equal or lower to the given value at a given point on the curve. The FDS plot can be created in a number of ways. The SPV values for the graphs were first calculated analytically with the Mathematical software application. Design Expert version 13 was the software program utilized in this investigation. While this strategy was computationally possible for models up to second order in fewer dimensions of up to 5 elements, it became too cumbersome and slow in higher dimensions. As a result, additional options were investigated.

The charted line in an FDS plot should be as low and flat as feasible since this implies that the SPVs are as little as possible and that, as the proportion of design space covered grows, the SPV does not increase dramatically.

METHODOLOGY

Analytical Approach

To identify the correlation between the response variable and the design variable $x_1, x_2, \ldots, x_k$ in numerous experimental circumstances, an N-run experiment is performed to collect data, which is then fitted into a response surface model or a matrix form

$$Y = X\beta + \varepsilon \qquad (3.1)$$

where Y is an N x 1 response vector, X and is an N x p(square matrix) extended design matrix formed using the N x k design matrix D. Each row of D provides an experimental point, and each column contains the experimental settings of the K design variables. β Is a vector coefficient, and ε is the error.

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Table 3.1. Design matrix of Factorial Design (Resolution iv) for k= 6

x_0	x_1	x_2	x_3	x_4	x_1^2	x_2^2	x_3^2	x_4^2	$x_1x_2x_3$	$x_1x_2x_4$
1	-1	-1	-1	-1	1	1	1	1	-1	-1
1	+1	-1	-1	-1	1	1	1	1	+1	+1
1	-1	+1	-1	-1	1	1	1	1	+1	+1
1	+1	+1	-1	-1	1	1	1	1	-1	-1
1	-1	-1	+1	-1	1	1	1	1	+1	-1
1	+1	-1	+1	-1	1	1	1	1	-1	+1
1	-1	+1	+1	-1	1	1	1	1	-1	+1
1	+1	+1	+1	-1	1	1	1	1	+1	-1
1	-1	-1	-1	+1	1	1	1	1	-1	+1
1	+1	-1	-1	+1	1	1	1	1	+1	-1
1	-1	+1	-1	+1	1	1	1	1	+1	-1
1	+1	+1	-1	+1	1	1	1	1	-1	+1
1	-1	-1	+1	+1	1	1	1	1	+1	+1
1	+1	-1	+1	+1	1	1	1	1	-1	-1
1	-1	+1	+1	+1	1	1	1	1	-1	-1
1	+1	+1	+1	+1	1	1	1	1	+1	+1

The structure of the matrix X in Table 1, X^TX for a CCD is determined by matrix multiplication and the outcome block matrix form is

$$X^TX = \begin{bmatrix} N & 0 & (f)J_q & 0 \\ 0 & diag(d_i) & 0 & 0 \\ (f).J_q & 0 & (f).J_qJ_q & 0 \\ 0 & 0 & 0 & (f).J_qJ_q \end{bmatrix} \quad (3.2)$$

Where **0**’s are zero matrices of appropriate sizes, J_q is a q x 1 unit column vector and where $diag(d_i)$ are diagonal matrices such that $d_i \;\; = f$

The first entry N is a scalar quantity and is the total number of runs given as $N = f$

$$\text{D-efficiency} = 100\frac{\left|\mathrm{X}^T\mathrm{X}\right|^{\frac{1}{p}}}{\mathrm{N}} \quad (3.3)$$

$$\text{G-efficiency} = \frac{100p}{N\hat{\sigma}^2_{\max}} \quad (3.4)$$

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I- criterion = $N\sigma_{ave}^{2}$
(3.5)

Where N is the design size, σ_{ave}^{2} is the average of $f^{T}(x)\left(X^{T}X\right)^{-1}f(x)$ over the design space, P is the number of model parameters, and $\hat{\sigma}_{\max}^{2}$ is the minimum of the maximum of $f^{T}(\mathrm{x})\left(X^{T}X\right)^{-1}f(x)$ approximated over the set of candidate points.

RESULTS/FINDINGS

Optimality Criteria Results

Table 4.1 is the result of the analysis of the factorial design of resolution *IV* and Resolution *V* Design.

Table 4.1: Result of optimality criteria for the factorial design

Factorial Design								
Resolution V					Resolution IV			
K	*N*	*D-opt.*	*G-opt.*	*I-opt.*	*N*	*D-opt.*	*G-opt.*	*I-opt.*
6	25	33.01	24.91	22.02	17	26.1	17.00	14.00
7	33	43.46	33.00	28.94	19	22.31	17.63	14.99
8	41	55.19	41.00	36.98	21	27.28	20.79	16.99
9	49	68.85	49.00	45.86	23	35.74	22.29	19.99
10	59	84.08	59.00	55.99	25	37.45	25.00	20.00

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From Table 4.1, at factor k = 6, the number of runs for Resolution *V*is is 25. The D-, G-, and I-optimality criteria are 33.01, 24.91, and 22.02 respectively while the D-optimality, G-optimality and I-optimality criteria for Resolution *IV* are 26.1, 17.00 and 14.00 respectively with 17 numbers of runs.

At factor k = 7, the number of runs for Resolution *V* is 33. The D-optimality, G-optimality and I-optimality criteria are 43.46, 33.00, and 28.94 respectively while the D-optimality, G-optimality and I-optimality criteria for Resolution *IV* are 22.31, 17.63 and 14.99 respectively with 19 number of runs.

At factor k = 8, the number of runs for Resolution *V* is 41. The D-optimality, G-optimality and I-optimality criteria are 55.19, 41.00, and 36.98 respectively while the D-optimality, G-optimality and I-optimality criteria for Resolution *IV* are 27.28, 20.79 and 16.99 respectively with 21 number of runs.

At factor k = 9, the number of runs for Resolution *V* is 49. The D-optimality, G-optimality and I-optimality criteria are 68.85, 49.00, and 45.86 respectively while the D-optimality, G-optimality and I-optimality criteria for Resolution IV are 35.74, 22.29 and 19.99 respectively with 23 numbers of runs.

At factor k = 10, the number of runs for Resolution *V* is 59. The D-optimality, G-optimality and I-optimality criteria are 84.08, 59.00, and 55.99 respectively while the D-optimality, G-optimality and I-optimality criteria for Resolution *IV* are 37.45, 25.00 and 20.00 respectively with 25 numbers of runs.

Fraction of Design Space (FDS) Plots

The values for the fractional of design space (FDS) were generated using the Design Expert software version. The values of each of the k factors were inputted into the software and the values generated are in Appendix

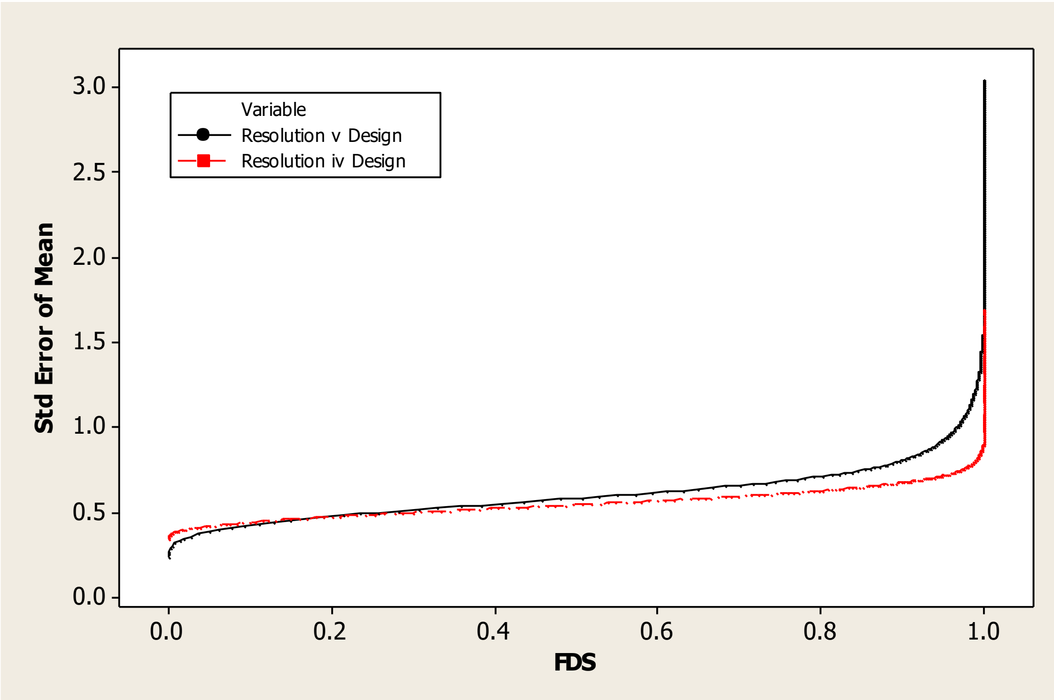

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Figure 4.1: FDS plot of Factorial Design for k = 6

Figure 4.1 is the fraction of design space (FDS) Plot for factor k = 6. The figure shows that the FDS plot for Resolution V and Resolution IV are relatively the same from 0 to 0.5 in the space plot. Also, the convergence rate to one between Resolution IV and V, are relatively the same.

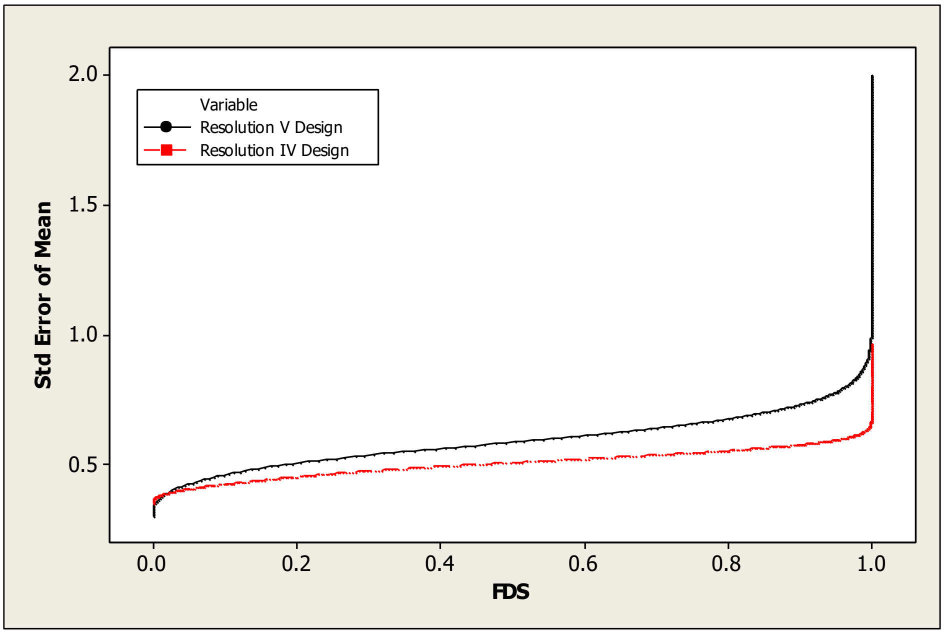

Figure 4. 2: FDS plot of Factorial Design for k = 7

Figure 4.2. is the fraction of design space (FDS) Plot for factor k = 7. The figure shows that the FDS plot for Resolution IV is better than Resolution *V* in the design space plot. This is because Resolution *IV* has lower FDS values compared to Resolution V. The Resolution *V* design rate converges to one is quicker than Resolution IV.

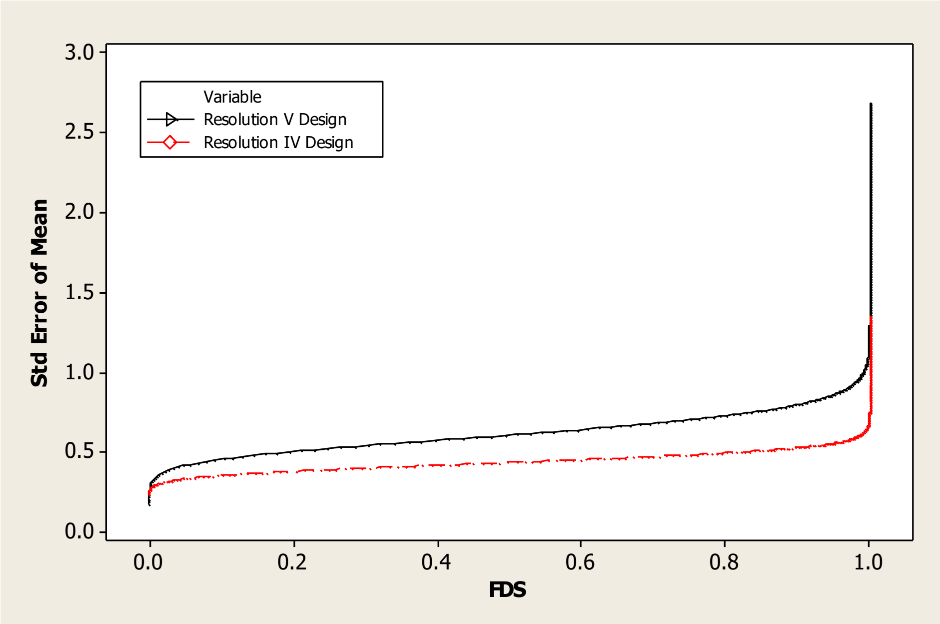

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Figure 4.3: FDS Plot of Factorial Design for k = 8

Figure 4.3 is the fraction of design space (FDS) Plot for factor k = 8. The figure shows that the FDS plot for Resolution IV is better than Resolution V in the design space plot because Resolution IV has low FDS values compared to Resolution V. The Resolution *V* design rate converges to one is quicker than Resolution IV.

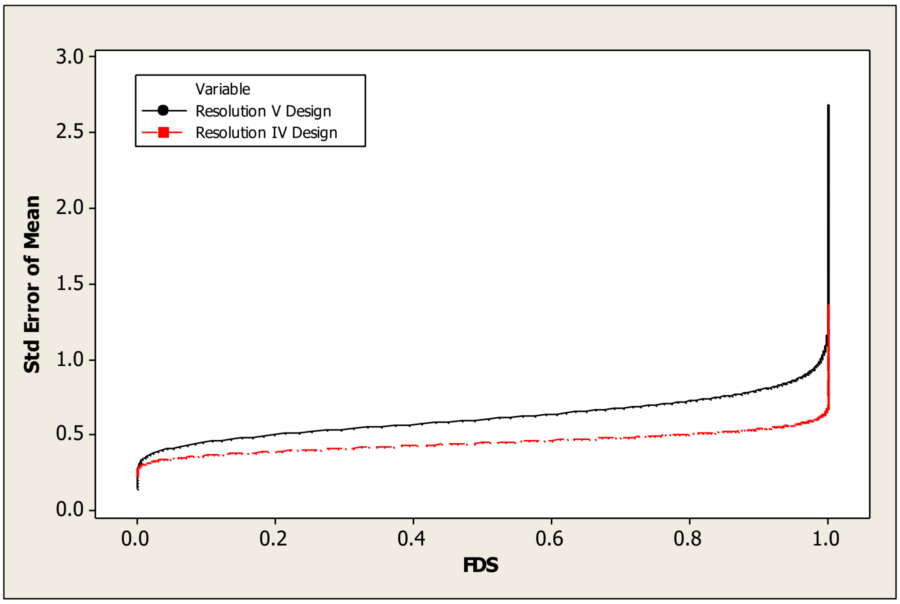

Figure 4.4: FDS plot of Factorial Design for k = 9

Figure 4.4 is the fraction of design space (FDS) Plot for factor k = 9. The figure shows that the FDS plot for Resolution IV is better than Resolution V in the design space plot because Resolution IV has low FDS values compared to Resolution V. The Resolution V design rate converges to one is quicker than Resolution IV.

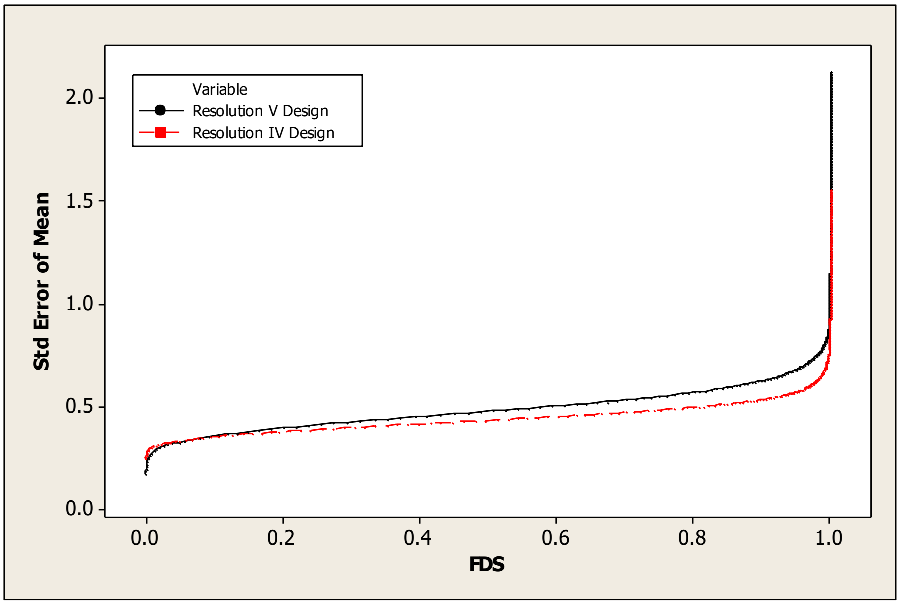

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Figure 4.5 is the fraction of design space (FDS) Plot for factor k = 10. The figure shows that the FDS plots for Resolution V and Resolution IV are relatively the same from 0 to 0.48 in the space plot. Also, the convergence rate to one between Resolution IV and V, are relatively the same.

Comparison of Resolution IV and Resolution V design

Comparing the Resolution *IV* and Resolution *V* designs at factors *k* = 6, 7, 8, 9, and 10 shows that Resolution *V* has better optimality criteria for D-, G-, and I- optimality. This is because of high optimality criteria values.

On the number of runs for any experiment, Resolution IV provides less number of runs and is economical for any design of the experiment.

On the fraction of design space (FDS), the plots reveal that the resolution IV design has a better-scaled prediction variance over the resolution v design in most of the factors considered.

DISCUSSION

This study looks at the prediction variance performances on the factorial design of Resolution IV and Resolution V. The Comparative studies of Resolution IV and Resolution V design were evaluated using the D-, G-, and I- optimality criteria and the fraction of design space plots. The results showed that in all the factors k considered, Resolution V has a better factorial design when it comes to D, I and G- optimality criteria whereas on the number of runs for any experiment, Resolution IV provides less number of runs which proves to be economical for any design of experiment

The FDS Plot for factors k = 6 and 10 shows that Resolution V and Resolution IV are relatively the same from 0 to 0.5 in the space plot. Also, the convergence rate between Resolution IV and V, are relatively the same. While the FDS Plot for factors k = 7, 8, and 9 follows the same pattern, and the convergence rate between Resolution IV and V, are relatively the same.

IMPLICATION TO RESEARCH AND PRACTICE

For researchers whose interests are on point estimates of the determinant, minimum variance and average variance of a design, this work recommends the use of a Resolution V factorial design. While Resolution IV factorial design is recommended for researchers whose interest is on the spread of the scale prediction variance for factors k = 6 to 10.

CONCLUSION

This study focuses on the choice of the factorial point that will help construct a central composite design to help increase the optimum precision of the estimated models of a response surface design.

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In all the factors k considered, Resolution V has a better factorial design when it comes to D-optimality, G-optimality and I-optimality, but when the interest is on the spread of the scale prediction variance, Resolution IV is preferred.

The result also shows that Resolution IV has a low fraction of design space (FDS) values when compared to Resolution V for factors k = 7, 8, 9 while Resolution IV and V are relatively the same for factors k = 6 and 10.

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