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A SINE LOMAX-EXPONENTIAL DISTRIBUTION: ITS PROPERTIES, SIMULATION AND APPLICATIONS TO SURVIVAL DATA.

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ABSTRACT: *The process of introducing extra parameters or mathematical constants into existing distribution to increase its flexibility and performance has yielded good results in the area of statistical theory and applications. The trigonometric-based Sine-G family, a method of developing new distributions, is one of the most efficient methods proposed and studied for introducing skewness and flexibility into probability distributions. In this paper, the method has been used to add flexibility to the Lomax-exponential distribution resulting in a new continuous distribution known as "Sine Lomax-Exponential distribution". The properties, estimation of parameters and simulation study of the new distribution using the method of maximum likelihood estimation with real life applications are presented and discussed in this paper. The new model has been applied to two survival datasets and the result is being compared among the fitted distributions using some information criteria.*

KEYWORDS: Lomax-Exponential distribution, Sine-G family, Properties, Maximum Likelihood Estimation, Simulation study, Application.

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INTRODUCTION

Probability distributions have great and significant roles in analyzing the random phenomena in every sector of life. However, more useful efforts are still needed to look for more flexible probability distributions for data modelling in several areas such as the medicine/biological sector, engineering (modelling reliability phenomena), economics (modelling and predicting import and export), genetics, agronomy; see Rao and Aslam (2021), Strzelecki (2021), Tung *et al*. (2021), Reynolds *et al*. (2021), and Prataviera (2022). Thus, researchers aim to develop new probability models that are able to provide the best description of the phenomenon under consideration, so we (or new researchers) can have a better and easier understanding of the factors involved.

In order to make probability distributions more flexible, most of the developed methods (or probability distributions) involve a number of extra parameters. Hence, more computational work is required to obtain the estimates and distributional properties of such distributions; see Souza et al. (2019). Hence, it is desirable to develop new methodologies or new probability models that have a small number of additional parameters and provide greater distributional flexibility in data modelling with a large degree of best fitting; see Thach and Bris (2020), Starling *et al*. (2021), and Giles (2021).

In order to reach the above aim, a number of researchers have introduced new methodologies for developing new probability distributions. However, few researchers have focused on using trigonometric functions to develop new distributions. Following recent improvements in the use of trigonometric functions, Kumar *et al*. (2015) introduced a family of distributions based on the sine function referred to as the sine G family of probability distributions, (Sine-G family) for generating new probability models. This method is found to be appropriate and has been used by researchers to develop new probability distributions which include, Sine-exponential distribution by Isa *et al*., (2022a), sine-Lomax distribution by Mustapha et al. (2023), the sine-modified Lindley distribution by Tomy *et al*., (2021) and sine Burr XII distribution by Isa *et al*., (2022b). This family of distributions is tractable and simplified since it does not introduce any new or extra parameters for estimation.

The exponential distribution used in Poisson processes describes the time between events. Many of its applications are carried out in life-testing experiments. It has memoryless property with a constant failure rate making it unfit for real life situations and then creating a problem in statistical modeling and applications.

In order to make the exponential distribution better, Keller and Kamath (1982) proposed a modified version of the Exponential distribution, called the inverse exponential distribution and it has also been studied in some detail by Lin et al. (1989). The inverse exponential distribution was found adequate for modelling datasets with inverted bathtub failure rates (Keller and Kamath, 1982) but it also has a limitation which is its inability to efficiently analyze datasets that are highly skewed (either positively or negatively) (Aboummoh and Alshingiti, 2009). This therefore makes it necessary for introducing skewness and flexibility

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into the inverse exponential distribution to enable it to adequately model heavily skewed datasets.

It is worthy to note that there are many generalizations of the exponential or inverse exponential distribution using differently proposed families of continuous probability distributions and some of these recent studies include the odd Lindley inverse exponential distribution Ieren and Abdullahi (2020), the Exponential Inverse Exponential distribution Oguntunde at al. (2017a), the Kumaraswamy Inverse Exponential distribution Oguntunde at al. (2017b), the exponentiated generalized Inverse Exponential distribution Oguntunde at al. (2017c), a new Lindley-Exponential distribution Oguntunde at al. (2016), the Lomax-exponential distribution Ieren and Kuhe (2018), the transmuted odd generalized exponential-exponential distribution Abdullahi et al. (2018), the transmuted exponential distribution Owoloko et al. (2015), transmuted inverse exponential distribution Oguntunde and Adejumo (2015), the odd generalized exponential-exponential distribution Maiti and Pramanik (2015), the transmuted Weibull-exponential distribution Yahaya and Ieren (2017) and the Weibull-exponential distribution Oguntunde at al. (2015). Following these recent publications and considering our desire to improve the flexibility of the Lomax-Exponential distribution which was found to be an improvement over the Exponential distribution, this article proposes a new distribution called the Sine Lomax-Exponential distribution.

The probability density function (pdf) of the Lomax-Exponential distribution (LED) according to Ieren and Kuhe (2018) is defined by

$$g(x) = \alpha\beta^{\alpha}\lambda(\beta + \lambda x)^{-(\alpha+1)} \tag{1}$$

The corresponding cumulative distribution function (CDF) of LED is given by

$$G(x) = 1 - \beta^{\alpha}(\beta + \lambda x)^{-\alpha} \tag{2}$$

where, $x > 0, \lambda > 0, \alpha > 0, \beta > 0$; α and β are shape parameters and λ is a scale parameter. According to Ieren and Kuhe (2018), the distribution is found to be better than the Weibull-exponential distribution and exponential distribution after considering three applications to real life data.

The aim of this paper is to introduce a new continuous distribution called the Sine Lomax-Exponential distribution (SLED) using the Sine-G family. This paper is organized in different sections as follows: definition of the new distribution with its plots is provided in section 2. Section 3 derived some properties of the proposed distribution. The estimation of parameters using maximum likelihood estimation (MLE) and simulation study is presented in section 4. An application of the new model with other existing distributions to real life data is done in section 5 and a conclusion is given in section 6.

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The Sine Lomax-Exponential distribution (SLED)

According to Kumar et al. (2015), the cumulative distribution function (CDF) of the Sine-G family is expressed by:

$$F(x,\eta)=\sin\left[\frac{\pi}{2}G(x,\eta)\right]$$
(3)

And the corresponding probability density function (pdf) (for $x>0$) is defined by

$$f(x,\eta)=\frac{\pi}{2}g(x,\eta)\cos\left[\frac{\pi}{2}G(x,\eta)\right]$$
(4)

respectively, where $g(x)$ and $G(x)$ represent the *pdf* and the *cdf* of the continuous distribution to be modified respectively.

Putting equations (1) and (2) into equations (3) and (4) and simplifying, we obtain the cdf and pdf of the SLED given in equations (5) and (6) respectively as follows:

$$F(x,\lambda,\alpha,\beta)=\sin\left[\frac{\pi}{2}\left(1-\beta^{\alpha}(\beta+\lambda x)^{-\alpha}\right)\right] \tag{5}$$

and

$$f(x)=\frac{\pi}{2}\alpha\beta^{\alpha}\lambda(\beta+\lambda x)^{-(\alpha+1)}\cos\left[\frac{\pi}{2}\left(1-\beta^{\alpha}(\beta+\lambda x)^{-\alpha}\right)\right] \tag{6}$$

where, $x>0,\lambda>0,\alpha>0,\beta>0$; α and β are shape parameters and λ is a scale parameter.

Plots of the pdf and cdf of the SLED using some parameter values are presented in **Figure 1** as follows.

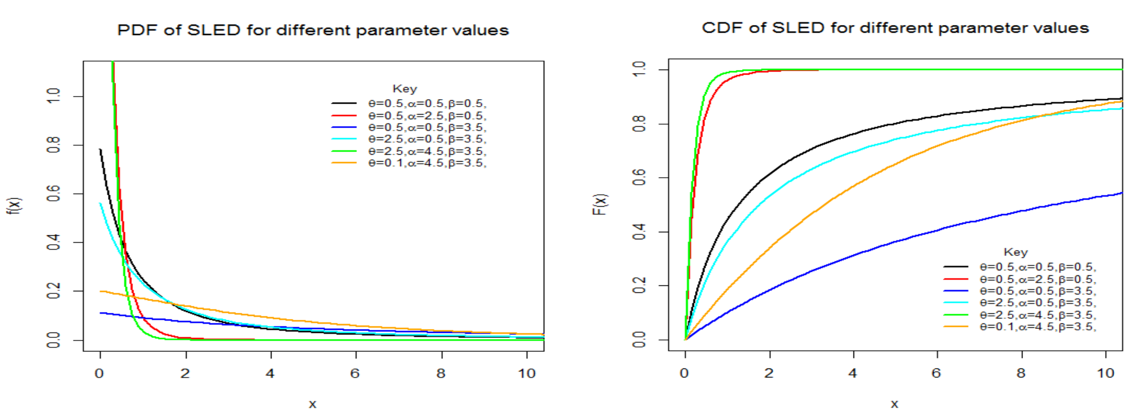

Fig. 1: PDF and CDF of the SLED for different values of the parameters.

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From the figure above, it can be seen that the pdf SLED is positively skewed and takes various shapes depending on the parameter values. Also, from the above plot of the cdf, it is clear that the cdf equals one when X approaches infinity and equals zero when X tends to zero as normally expected.

Some Properties of SLED

In this section, some properties of the SLED distribution are derived and discussed as follows:

Quantile Function

According to Hyndman and Fan (1996), the quantile function for any distribution is defined in the form $Q(u) = X_q = F^{-1}(u)$ where $Q(u)$ is the quantile function of $F(x)$ for $0 < u < 1$

Taking *F(x)* to be the *cdf* of the SLED and inverting it as above will give us the quantile function as follows:

$$F(x,\lambda,\alpha,\beta) = \sin\left[\frac{\pi}{2}\left(1-\beta^{\alpha}(\beta+\lambda x)^{-\alpha}\right)\right] = u \tag{7}$$

Simplifying equation (7) above and solving for X presents the quantile function of the SLED as:

$$Q(u) = \frac{\beta}{\lambda}\left\{\left[1-\frac{2\alpha\sin(u)}{\pi}\right]^{\frac{-1}{\alpha}} - 1\right\} \tag{8}$$

This function is used for obtaining some moments like skewness and kurtosis as well as the median and for the generation of random variables from the distribution in question.

Skewness and Kurtosis

This paper presents the quantile-based measures of skewness and kurtosis due to the non-existence of the classical measures in some cases.

According to Kenney and Keeping (1962), Bowley's measure of skewness based on quartiles is given by:

$$SK = \frac{Q\left(3/4\right) - 2Q\left(1/2\right) + Q\left(1/4\right)}{Q\left(3/4\right) - Q\left(1/4\right)} \tag{9}$$

Also, the Moors kurtosis based on octiles proposed by Moors (1988) and is given by;

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$$KT = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + \left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{1}{8}\right)} \tag{10}$$

Where $Q(.)$ is obtainable with the help of equation (8).

Reliability analysis of the SLED

A derivation and study of the survival function and the hazard rate function is presented in this section.

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \tag{11}$$

Applying the cdf of the SLED in (11), the survival function for the SLED is obtained as:

$$S(x) = \left\{1 - \sin\left[\frac{\pi}{2}\left(1 - \beta^{\alpha}(\beta + \lambda x)^{-\alpha}\right)\right]\right\} \tag{12}$$

The plot for the survival function of the SLED using different parameter values is shown in Figure 3.3.**1** below:

A hazard function is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \tag{13}$$

Meanwhile, the expression for the hazard rate of the SLED is given by:

$$h(x) = \frac{\pi\alpha\beta^{\alpha}\lambda(\beta + \lambda x)^{-(\alpha+1)}\cos\left[\frac{\pi}{2}\left(1 - \beta^{\alpha}(\beta + \lambda x)^{-\alpha}\right)\right]}{2\left(1 - \sin\left[\frac{\pi}{2}\left(1 - \beta^{\alpha}(\beta + \lambda x)^{-\alpha}\right)\right]\right)} \tag{14}$$

where $\lambda, \alpha, \beta > 0$.

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A plot of the hazard function for arbitrary parameter values is presented in Figure **3.3.1** as follows

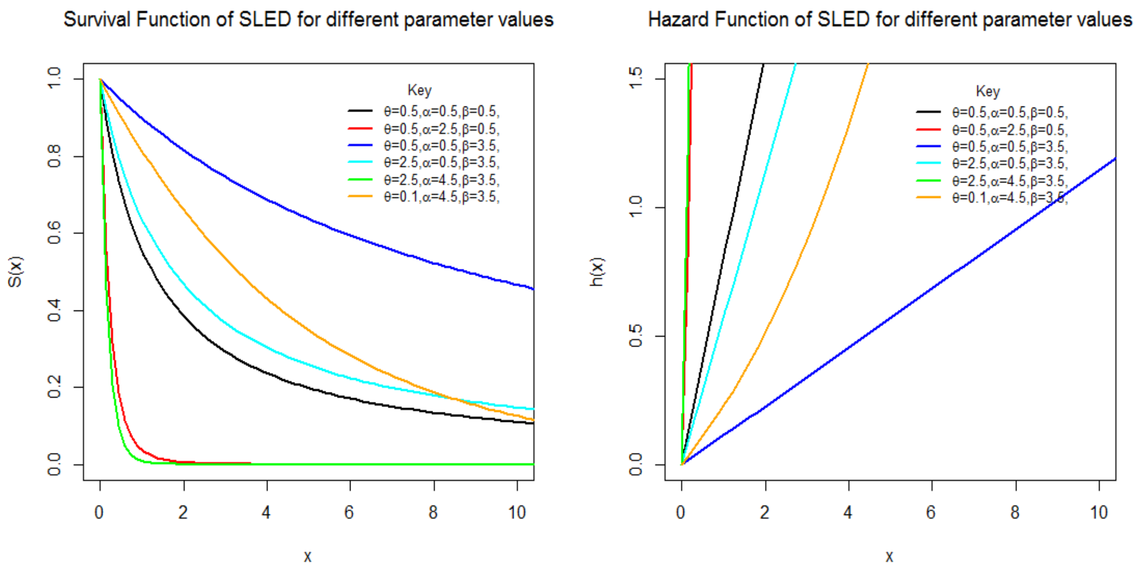

Fig. 3.1: The Survival Function and Hazard Function of SLED.

The plot in Figure **3.1** shows that the probability of survival is always sure at an initial time or early age and it decreases as time increases up to zero (0) at infinity.

The figure above revealed that the SLED has an increasing as well as constant failure rate which implies that the probability of failure for any random variable following a SLED increases as time increases, that is, the probability of failure or death increases with age.

Estimation of Parameters and Simulation Study

Estimation of Unknown Parameters of the SLED

In this section, the estimation of the parameters of the SLED is done by using the method of maximum likelihood estimation (MLE). Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the SLED with unknown parameters α, λ and β defined previously.

The likelihood function of the SLED using the pdf in equation (6) is given by;

$$f(x) = \frac{\pi}{2}\alpha\beta^{\alpha}\lambda(\beta+\lambda x)^{-(\alpha+1)}\cos\left[\frac{\pi}{2}\left(1-\beta^{\alpha}(\beta+\lambda x)^{-\alpha}\right)\right]$$

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$$L\left(\underline{X}/\alpha,\lambda,\beta\right)=\left(\alpha\lambda\beta^{\alpha}\right)^{n}\prod_{i=1}^{n}\left(\left(\beta+\lambda x\right)^{-\alpha-1}\right)\prod_{i=1}^{n}\left[\cos\left[\frac{\pi}{2}\left(1-\beta^{\alpha}\left(\beta+\lambda x\right)^{-\alpha}\right)\right]\right] \tag{19}$$

Let the natural logarithm of the likelihood function be, $l=\log L\left(\underline{X}\mid\alpha,\lambda,\beta\right)$, therefore, taking the natural logarithm of the function above gives:

$$l=n\log\left(\frac{\pi}{2}\right)+n\log\alpha+\alpha n\log\beta+n\log\lambda-\left(\alpha+1\right)\sum_{i=1}^{n}\log\left(\beta+\lambda x\right)+\sum_{i=1}^{n}\log\left[\cos\left[\frac{\pi}{2}\left(1-\beta^{\alpha}\left(\beta+\lambda x\right)^{-\alpha}\right)\right]\right]$$
(20)

Differentiating l partially with respect to α,λ and β respectively gives the maximum likelihood estimates of the parameters respectively. However, these solutions cannot be obtained manually except numerically with the aid of a suitable computer application such as *R* programming language.

Simulation study for Sine Lomax-Exponential distribution (SLED)

In this section, a simulation study is conducted for three different combinations of λ, α and β. These combination values are given by (i) $\lambda=0.5,\ \alpha=0.5\ and\ \beta=0.5$, (ii) $\lambda=2.5,\ \alpha=0.5\ and\ \beta=0.5$ (iii) $\lambda=0.5,\ \alpha=2.5\ and\ \beta=0.5$. and (iv) $\lambda=0.5,\ \alpha=0.5\ and\ \beta=2.5$. The judgment about the performances of $\hat{\theta}_{MLE}$, $\hat{\lambda}_{MLE}$ and $\hat{\alpha}_{MLE}$ is made by considering two evaluation criteria. These criteria are the Mean square error (MSE) and Bias. For every sample size, the average MLEs, mean square errors (MSE) and Absolute biases were computed. The results obtained after performing the MC simulation are provided in Tables 4.2.1-4.2.4 and displayed graphically in Figures 4.2.1–4.2.4.

Table 4.2.1: Simulation results for the SLED for $\lambda=0.5,\ \alpha=0.5\ and\ \beta=0.5$

n	Measures /Criteria	Parameters			n	Measure s/Criteria	Parameters		
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$			$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
n=25	MLEs	0.5227	0.7080	0.7384	n=125	MLEs	0.5192	0.5226	0.5439
	Biases	0.0227	0.2080	0.2384		Biases	0.0192	0.0226	0.0439
	MSEs	0.0243	0.5208	0.5121		MSEs	0.0067	0.0121	0.0190
n=50	MLEs	0.5131	0.5880	0.6231	n=200	MLEs	0.5162	0.5143	0.5296
	Biases	0.0131	0.0880	0.1231		Biases	0.0162	0.0143	0.0296
	MSEs	0.0163	0.0672	0.0995		MSEs	0.0036	0.0045	0.0075
n=75	MLEs	0.5211	0.5429	0.5672	n=300	MLEs	0.5098	0.5117	0.5236
	Biases	0.0211	0.0429	0.0672		Biases	0.0098	0.0117	0.0236
	MSEs	0.0111	0.0216	0.0322		MSEs	0.0021	0.0037	0.0057
n=100	MLEs	0.5207	0.5237	0.5500	n=400	MLEs	0.5107	0.5059	0.5184

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	Biases	0.0207	0.0237	0.0500	Biases	0.0107	0.0059	0.0184
	MSEs	0.0068	0.0122	0.0195	MSEs	0.0021	0.0026	0.0035

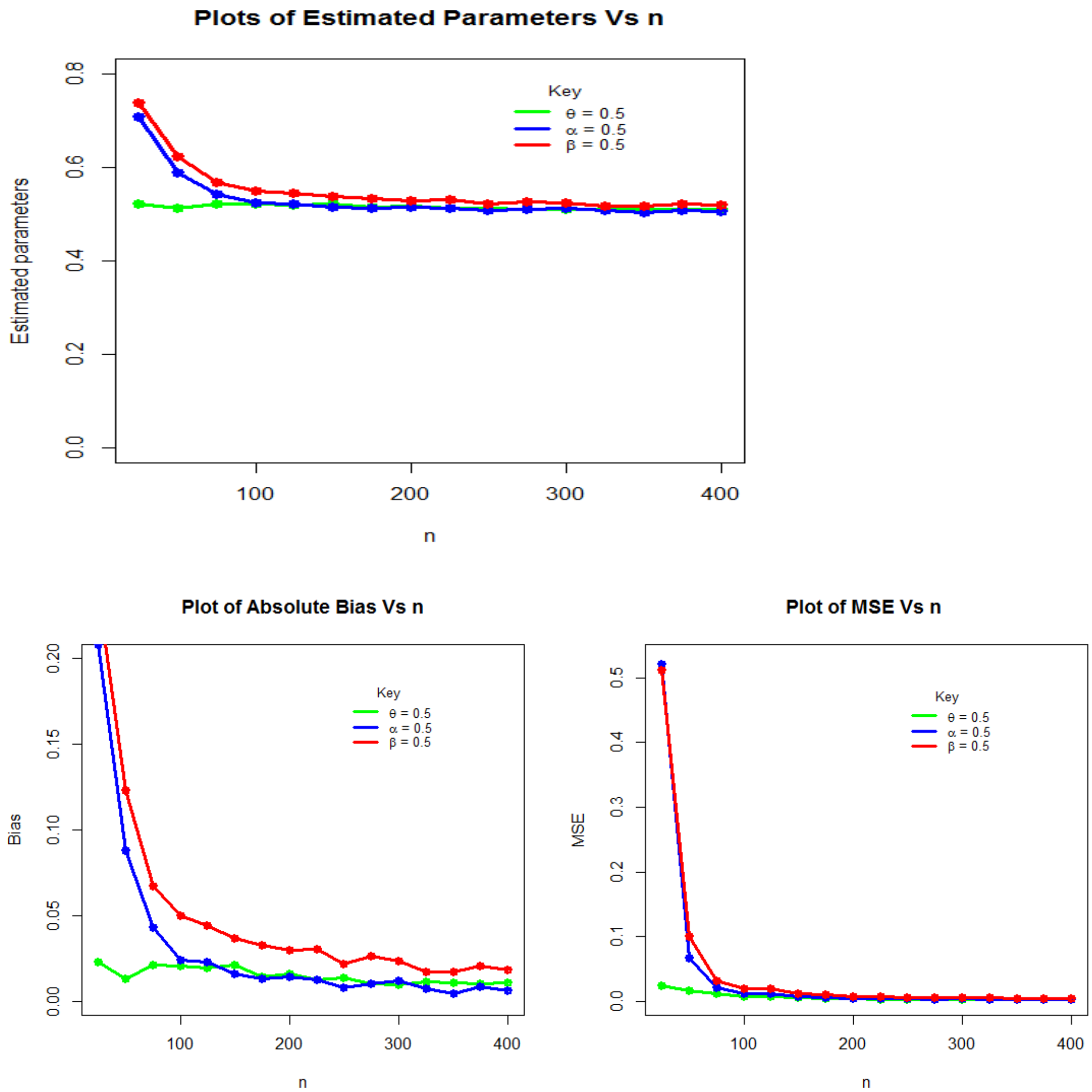

Fig. 4.2.1: Plots of MLEs, Absolute Biases and MSEs of the SLED for $\lambda = 0.5, \ \alpha = 0.5 \ \textit{and} \ \beta = 0.5$

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Table 4.2.2: Simulation results for the SLED for $\lambda = 2.5, \quad \alpha = 0.5 \; and \quad \beta = 0.5$

n	Measures /Criteria	Parameters			n	Measure s/Criteria	Parameters		
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$			$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
n=25	MLEs	2.4163	0.7774	0.8708	n=125	MLEs	2.4949	0.5218	0.5375
	Biases	0.0837	0.2774	0.3708		Biases	0.0051	0.0218	0.0375
	MSEs	0.0901	0.6668	0.9441		MSEs	0.0081	0.0114	0.0290
n=50	MLEs	2.4698	0.5655	0.6136	n=200	MLEs	2.5032	0.5136	0.5239
	Biases	0.0302	0.0655	0.1136		Biases	0.0032	0.0136	0.0239
	MSEs	0.0177	0.0390	0.1064		MSEs	0.0066	0.0048	0.0128
n=75	MLEs	2.5009	0.5405	0.5692	n=300	MLEs	2.5033	0.5067	0.5140
	Biases	0.0009	0.0405	0.0692		Biases	0.0033	0.0067	0.0140
	MSEs	0.0254	0.0252	0.0659		MSEs	0.0048	0.0025	0.0069
n=100	MLEs	2.4983	0.5242	0.5457	n=400	MLEs	2.5016	0.5075	0.5152
	Biases	0.0017	0.0242	0.0457		Biases	0.0016	0.0075	0.0152
	MSEs	0.0120	0.0120	0.0331		MSEs	0.0040	0.0019	0.0054

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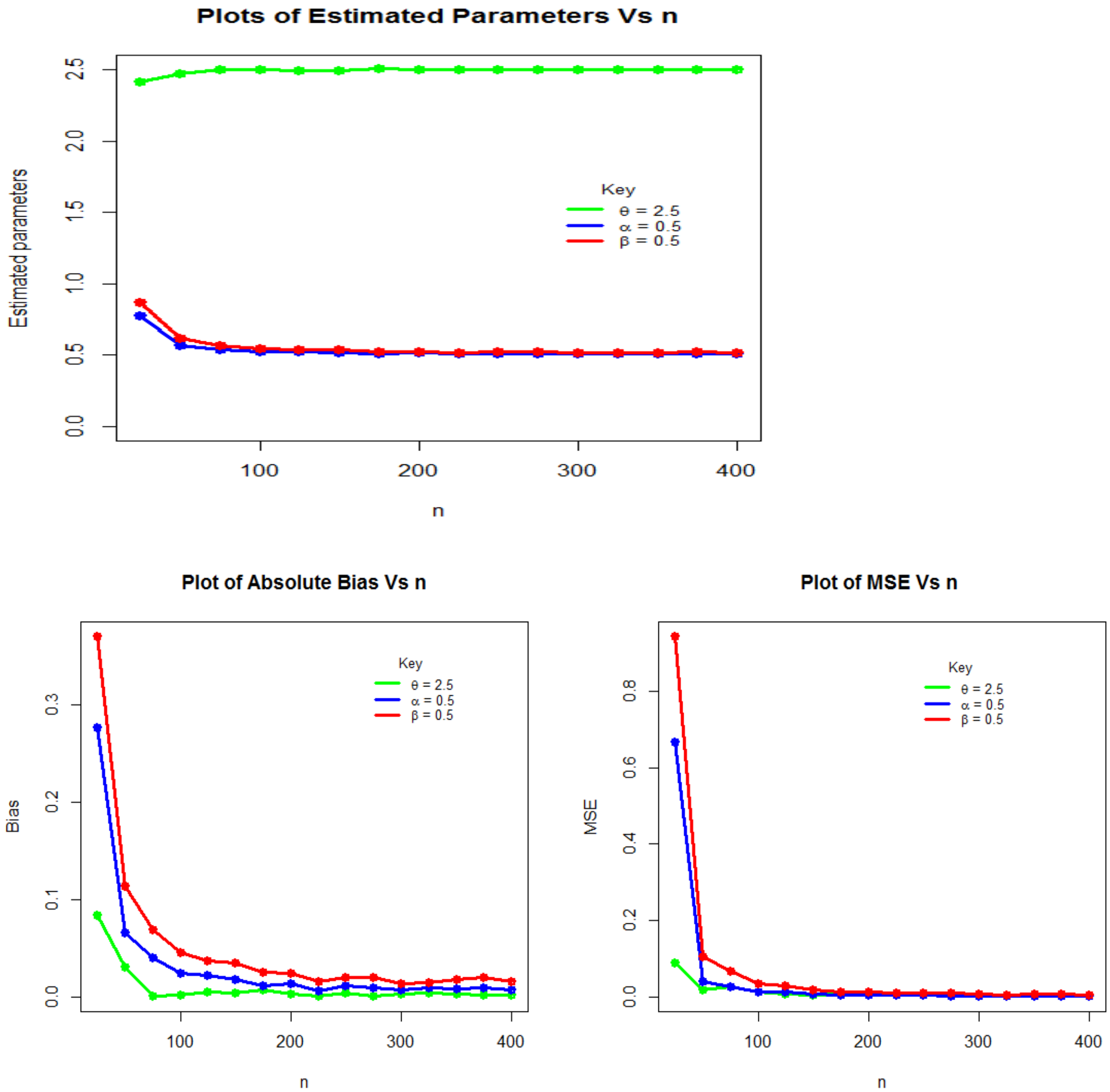

Fig. 4.2.2: Plots of MLEs, Absolute Biases & MSEs of the SLED for $\lambda = 2.5, \quad \alpha = 0.5 \ \ and \ \ \beta = 0.5$

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Table 4.2.3: Simulation results for the SLED for $\lambda = 0.5, \quad \alpha = 2.5 \ \ and \quad \beta = 0.5$

n	Measures /Criteria	Parameters			n	Measure s/Criteria	Parameters		
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$			$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
n=25	MLEs	0.5469	3.4772	0.6806	n=125	MLEs	0.5188	2.9953	0.5782
	Biases	0.0469	0.9772	0.1806		Biases	0.0188	0.4953	0.0782
	MSEs	0.0674	3.5476	0.2115		MSEs	0.0382	1.8181	0.1091
n=50	MLEs	0.5392	3.2628	0.6574	n=200	MLEs	0.4937	2.9822	0.5702
	Biases	0.0392	0.7628	0.1574		Biases	0.0063	0.4822	0.0702
	MSEs	0.0468	2.7292	0.1954		MSEs	0.0256	1.3778	0.0845
n=75	MLEs	0.5176	3.0897	0.6002	n=300	MLEs	0.5014	2.7991	0.5542
	Biases	0.0176	0.5897	0.1002		Biases	0.0014	0.2991	0.0542
	MSEs	0.0407	2.2060	0.1392		MSEs	0.0175	0.8332	0.0566
n=100	MLEs	0.5321	3.0704	0.6251	n=400	MLEs	0.5036	2.7455	0.5411
	Biases	0.0321	0.5704	0.1251		Biases	0.0036	0.2455	0.0411
	MSEs	0.0350	2.0799	0.1436		MSEs	0.0168	0.6875	0.0458

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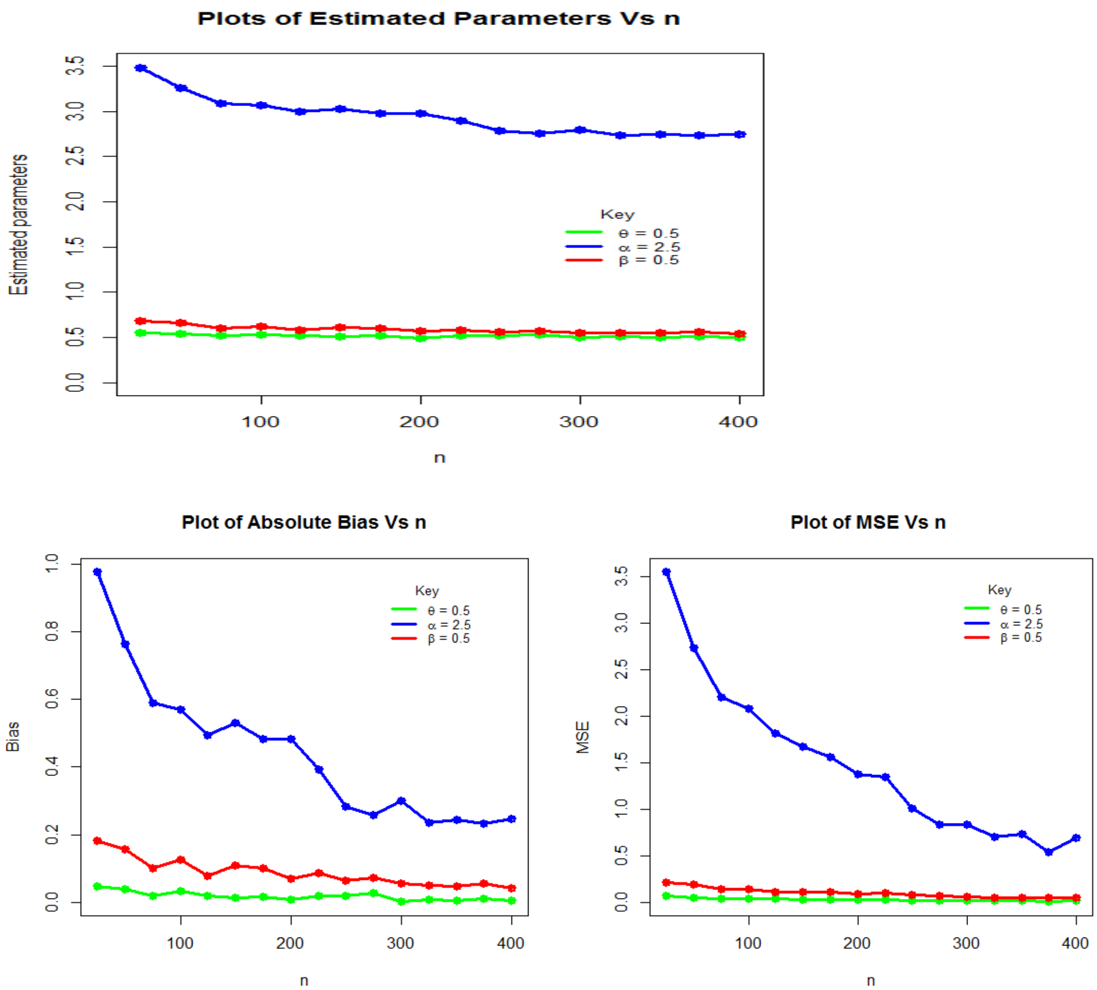

Fig. 4.2.3: Plots of MLEs, Absolute Biases & MSEs of the SLED for $\lambda = 0.5, \quad \alpha = 2.5 \ \ and \quad \beta = 0.5$

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Table 4.2.4: Simulation results for the SLED for $\lambda = 0.5, \quad \alpha = 0.5 \ \ and \quad \beta = 2.5$

n	Measures /Criteria	Parameters			n	Measure s/Criteria	Parameters		
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$			$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
n=25	MLEs	0.5244	0.7619	2.5631	n=125	MLEs	0.5092	0.5242	2.5426
	Biases	0.0244	0.2619	0.0631		Biases	0.0092	0.0242	0.0426
	MSEs	0.1513	0.6442	0.0948		MSEs	0.0200	0.0107	0.0218
n=50	MLEs	0.5024	0.5828	2.5641	n=200	MLEs	0.5009	0.5186	2.5293
	Biases	0.0024	0.0828	0.0641		Biases	0.0009	0.0186	0.0293
	MSEs	0.0516	0.0522	0.0511		MSEs	0.0104	0.0058	0.0124
n=75	MLEs	0.5000	0.5535	2.5583	n=300	MLEs	0.5078	0.5088	2.5216
	Biases	0.0000	0.0535	0.0583		Biases	0.0078	0.0088	0.0216
	MSEs	0.0330	0.0279	0.0380		MSEs	0.0083	0.0036	0.0080
n=100	MLEs	0.5040	0.5334	2.5541	n=400	MLEs	0.4989	0.5099	2.5286
	Biases	0.0040	0.0334	0.0541		Biases	0.0011	0.0099	0.0286
	MSEs	0.0244	0.0141	0.0299		MSEs	0.0049	0.0021	0.0082

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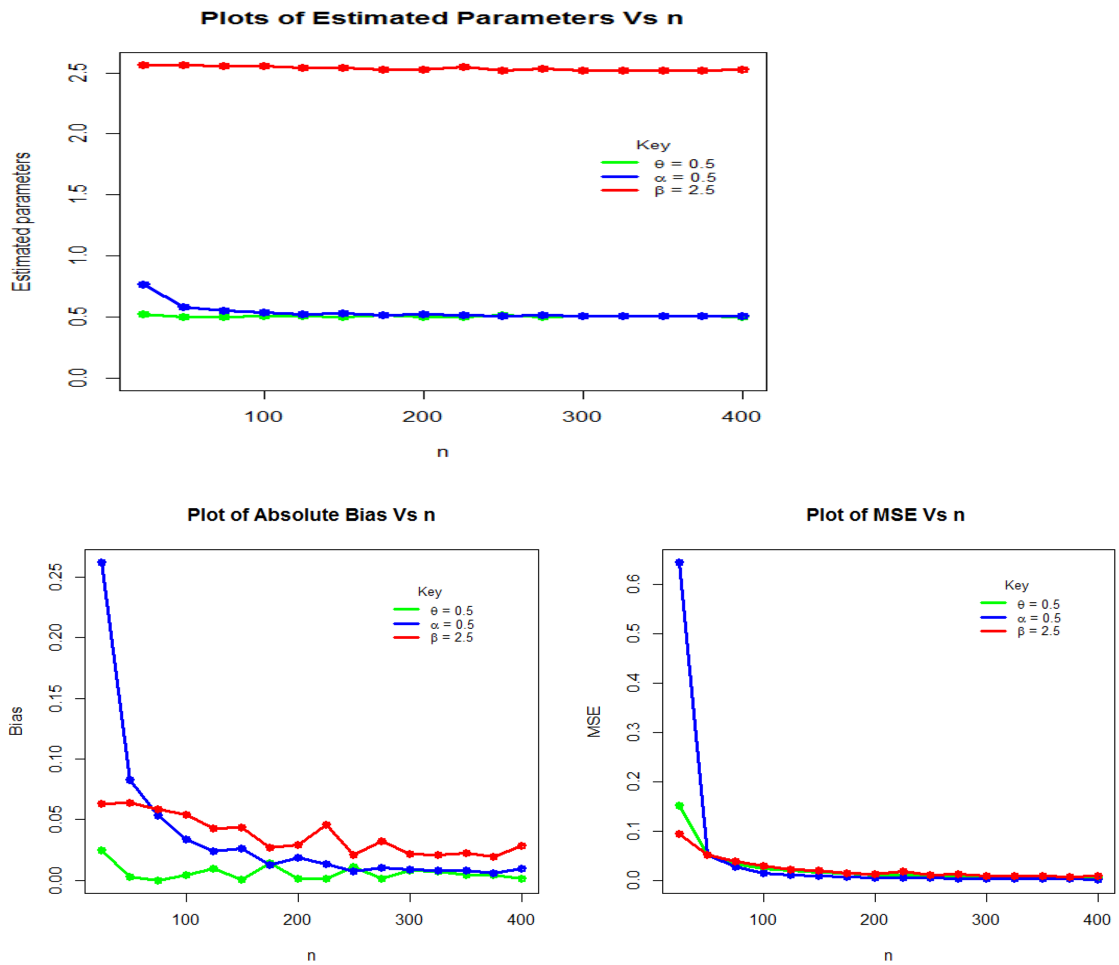

Fig. 4.2.4: Plots of MLEs, Absolute Biases & MSEs of the SLED for $\lambda = 0.5, \quad \alpha = 0.5 \ \ and \quad \beta = 2.5$

The results in Tables 4.2.1-4.2.3 and Figures 4.2.1-4.2.3 show the averages of the MLEs (Mean), their biases (Absolute Bias) and mean square errors (MSEs) for the parameters of the SLED. Based on the values from the tables and the visual illustration in the figures, it is seen that the average estimates get closer to the true parameters values when sample size increases and the biases and mean square errors all tend to zero as sample size increases which is expected for every normal parameter.

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APPLICATIONS

This section presents two applications of the proposed distribution to real life data. The MLEs of the model parameters are determined and some goodness-of-fit statistics for this distribution are compared with other competitive models. For all the datasets, the fits of the sine Lomax-exponential distribution (SLED) is compared with those of the Lomax-exponential distribution (LED), Lomax distribution (LD) and the conventional exponential distribution (ED).

The model selection is carried out based upon the value of the log-likelihood function evaluated at the MLEs, ℓ, Akaike Information Criterion, *AIC*, Consistent Akaike Information Criterion, *CAIC*, Bayesian Information Criterion, *BIC*, Hannan Quin Information Criterion, *HQIC*, Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. The details about the statistics A*, W* and K-S are discussed in Chen and Balakrishna (1995). Meanwhile, the smaller these statistics are, the better the fit of the distribution is. The required computations are carried out using the R package "AdequacyModel" which is freely available from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf.

Application 1: Based on Dataset I

This data represents the survival times of a group of patients suffering from head and neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT) (Efron (1988), Shanker *et al*. (2015), Oguntunde *et al*. (2017d), Ieren *et al*., (2020), Abdulkadir *et al*., (2020)). The observations are as follows:12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776. The summary is given as follows:

Table 5.1: Descriptive Statistics for Dataset I.

parameter	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	44	12.20	67.21	128.5	219.0	223.48	1776.00	93286.4	3.38382	13.5596

From the parameters in Table 5.1 above, it is discovered that the real life dataset (dataset I) is negatively skewed, that is, skewed to the left and hence a suitable choice for the proposed distribution.

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Table 5.2: **Maximum Likelihood Parameter Estimates for dataset I**

Distribution	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
SLED	0.05518717	0.42554575	5.01404503
LED	0.04993597	1.73387225	8.25033106
LD	-	0.3554058	8.4945709
ED	0.01314657	-	-

Table 5.3: The statistics ℓ, AIC, CAIC, BIC and HQIC for dataset I

Distribution	$\hat{\ell}$	*AIC*	*CAIC*	*BIC*	*HQIC*	Ranks
SLED	286.4479	578.8957	579.4957	584.2483	580.8807	1st
LED	283.74	573.48	574.08	578.8326	575.465	2nd
LD	307.058	618.1159	618.4086	621.6843	619.4393	3rd
ED	319.8602	641.7204	641.8157	643.5046	642.3821	4th

Table 5.4: **The A^*, W^*, K-S statistic and P-values for dataset I**

Distribution	A^*	W^*	*K-S*	*P-Value (K-S)*	Ranks
SLED	0.117518	0.01695631	0.16429	0.1661	1st
LED	0.1649455	0.02513087	0.21277	0.0316	2nd
LD	0.1780573	0.02771925	0.35351	2.007e-05	3rd
ED	1.157283	0.2004441	0.35543	1.766e-05	4th

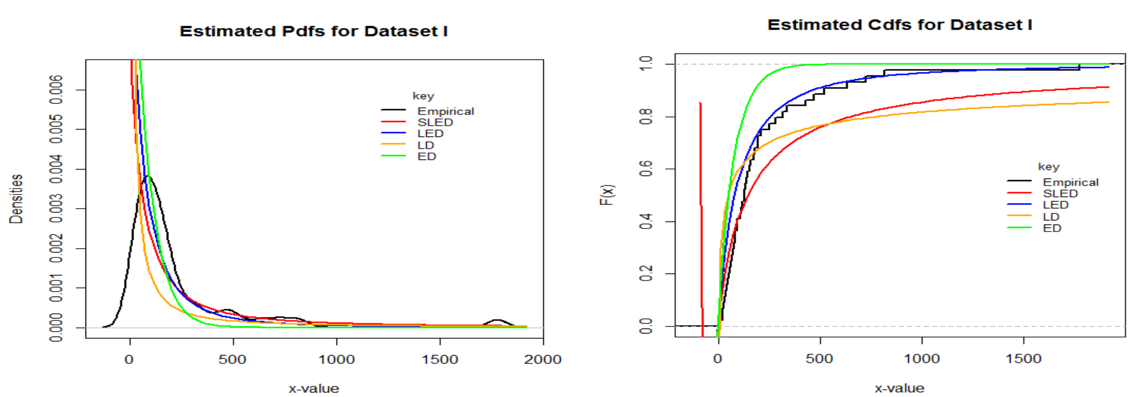

Figure 5.1: Histogram and plots of the estimated densities and cdfs of the SLED and other fitted distributions to dataset I.

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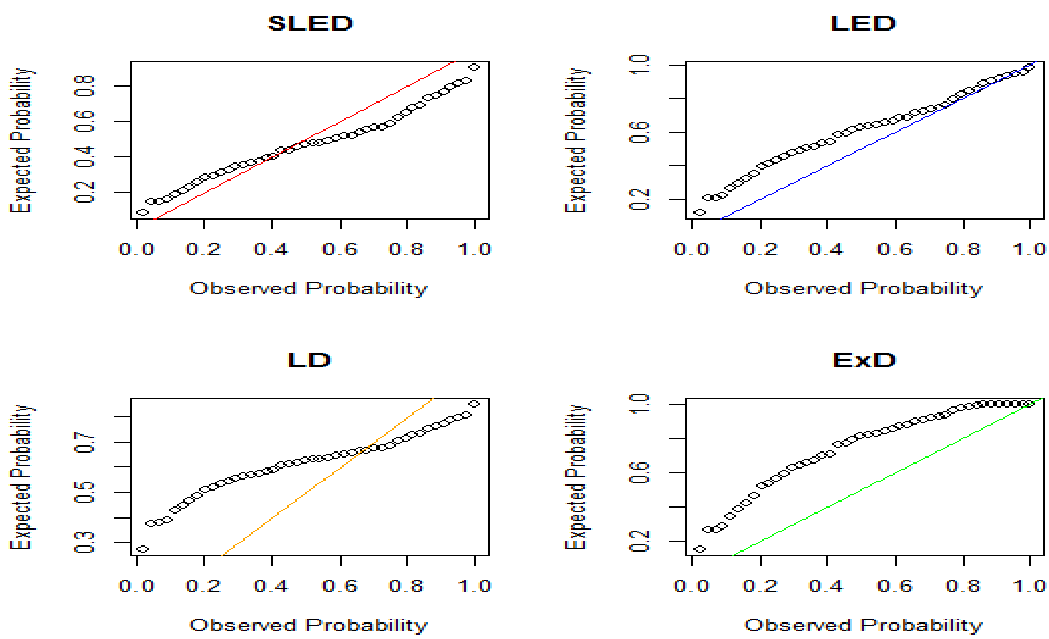

Figure 5.2: Probability plots for the fit of the SLED and other fitted models based on dataset I.

Application 1: Based on Dataset II

This first real dataset is a subset of the data reported by Bekker *et al*. (2000), which corresponds to the survival times (in years) of a group of patients given chemotherapy treatment alone. The data consisting of survival times (in years) or 45 patients is given as follows. 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.033, 4.033. Its summary is given as follows:

Table 5.5: Descriptive Statistics for Dataset II

n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
45	0.047	0.395	0.841	2.178	1.342	4.033	1.55766	0.97473	-0.3287

Based on the descriptive statistics in Table 5.5 above, it is revealed that the real life dataset (dataset II) is positively skewed, that is, skewed to the right and therefore suitable for flexible distributions.

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Table 5.6: Maximum Likelihood Parameter Estimates for Dataset II

Distribution	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$
SLED	0.4410354	6.1295139	5.8789476
LED	0.7667651	8.3604183	7.9665180
LD	-	8.150103	9.762677
ED	0.7451938	-	-

Table 5.7: The statistics ℓ, AIC, CAIC, BIC and HQIC for dataset II

Distribution	$\hat{\ell}$	*AIC*	*CAIC*	*BIC*	*HQIC*	Ranks
SLED	58.5507	123.1014	123.6868	128.5214	125.1219	1st
LED	58.75086	123.5017	124.0871	128.9217	125.5222	2nd
LD	58.7665	121.533	121.8187	125.1463	122.88	3rd
ED	58.24097	118.4819	118.575	120.2886	119.1555	4th

Table 5.8: The A^*, W^*, K-S statistic and P-values for dataset II

Distribution	A^*	W^*	*K-S*	*P-Value (K-S)*	Ranks
SLED	0.5422417	0.08117884	0.089326	0.8653	1st
LED	0.4933702	0.07301163	0.076901	0.9529	2nd
LD	0.4939053	0.07309115	0.072939	0.9704	3rd
ED	0.5269096	0.0786838	0.090932	0.8508	4th

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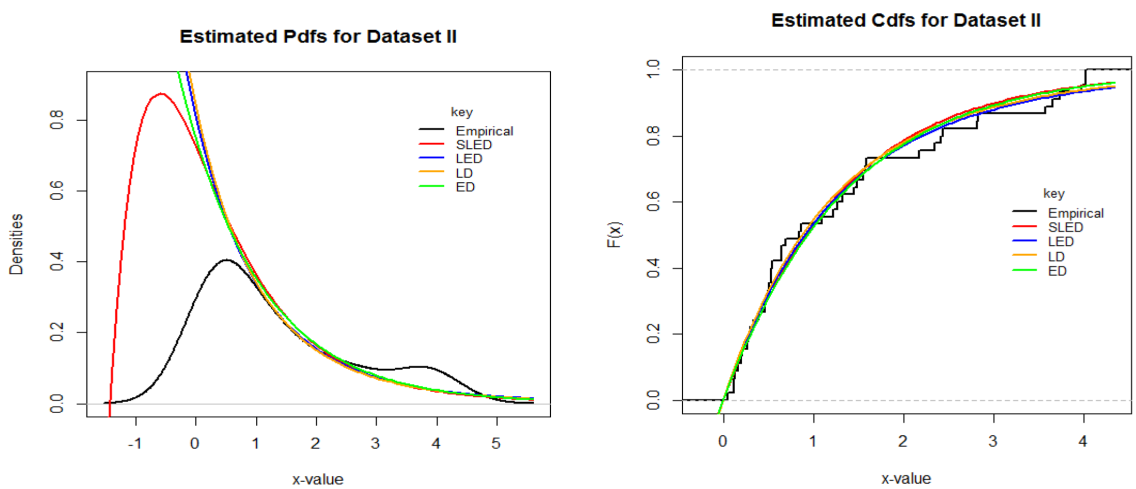

Figure 5.3: Histogram and plots of the estimated densities and cdfs of the SLED and other fitted distributions to dataset II.

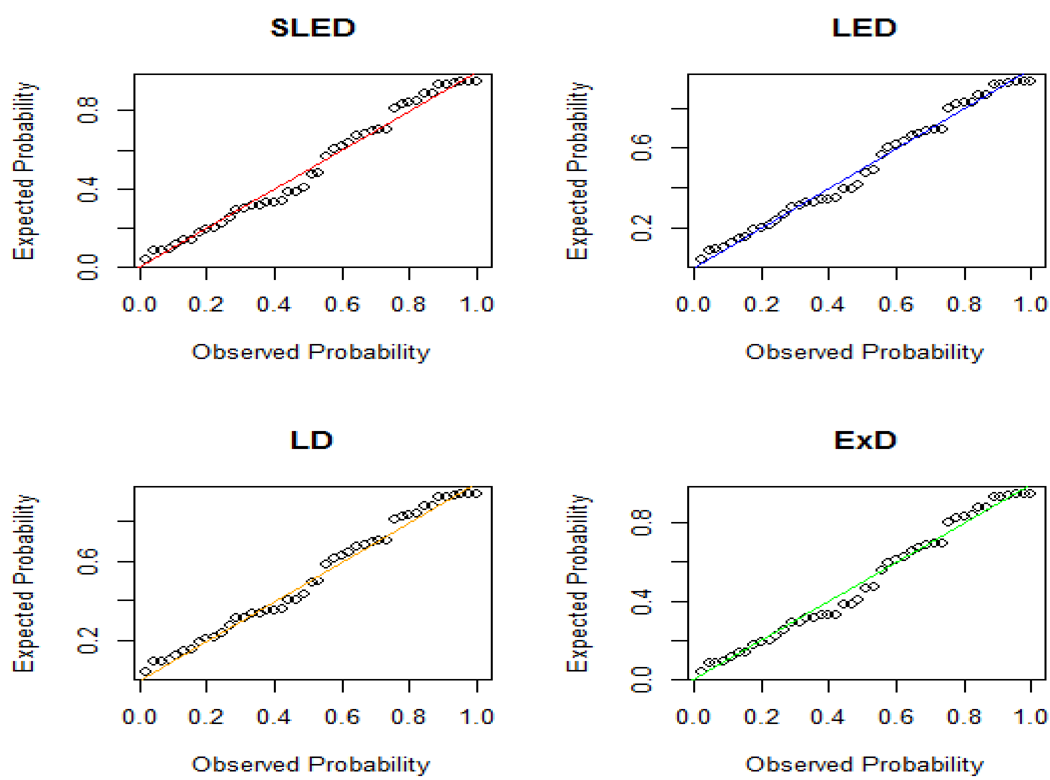

Figure 5.4: Probability plots for the fit of the SLED and other fitted models based on dataset II.

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Tables 5.2, and 5.6 list the values of the MLEs of the model parameters for dataset I and dataset II respectively, whereas the values of AIC, CAIC, BIC and HQIC are listed in Tables 5.3 and 5.7 for datasets I and dataset II respectively. Also, the values of A*, W* and K-S for datasets I and II are presented in Tables 5.4 and 5.8 respectively.

The plots of the fitted SLED density and cumulative distribution with those of competing distributions for datasets I and II are displayed in Figure 5.1 and Figure 5.3 respectively. The PP plots of the fitted distributions are given in Figures 5.2 and 5.4 for datasets I and II respectively. For both datasets I and II which are positively skewed, all the measures above show that the SLED has the best fit and is considered a very good and flexible model compared to the other distributions applied to these datasets including the Lomax-exponential distribution (LED), Lomax distribution (LD) and the conventional exponential distribution (ED).

The results above are a backup to the fact that most compound distributions are more flexible and perform better than their standard counterparts as also discovered by other studies in this area of research. It is therefore essential that the proposed distribution can be used for many applications in the areas of data science, data analytics, machine learning and other real life scenarios, etc.

SUMMARY AND CONCLUSION

This article proposed a new extension of the exponential distribution called “Sine Lomax-Exponential distribution”. The article briefly discussed some properties of the proposed distribution. The maximum likelihood method was used to estimate the model parameters and a simulation study on the parameters was also carried out. Plots of the pdf of the distribution generated with arbitrary parameter values show that it is skewed and flexible and that its shape depends on the values of the parameters. Also, the plots of the survival function show that it is monotone decreasing, where the probability of survival decreases over time. Also, the plot of the hazard rate of the distribution shows that it is increasing for all parameter values and this shape is proof that the proposed model would be appropriate for analyzing events whose failure rate increases with time. Based on the values from the tables and the visual illustration in the figures from the simulation study, it was discovered that the average estimates of the parameters get closer to the true parameters values when sample size increases and the biases and mean square errors all decrease towards zero as sample size increases which is expected for every normal parameter, a proof that the asymptotic property is obeyed. The proposed model was fitted to two real life datasets to demonstrate its capability over some existing distributions. Some model selection criteria were considered to see which distribution has a better fit to these datasets. The results obtained clearly show that the proposed model, Sine Lomax-exponential distribution has a better fit to the data than the other competing models for all the datasets used. Hence, the proposed distribution and its generated properties will attract wider applications in several areas of statistical theory and applications due to its ability and flexibility.

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