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EXPLORING THE PRIMITIVITY AND REGULARITY OF DIHEDRAL GROUPS OF DEGREE 5P USING NUMERICAL APPROACHES

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ABSTRACT: *This paper delves into exploration of primitivity and regularity of Dihedral groups of degree 5p, where p is prime, focusing on cases where these groups are not p-groups. By utilizing numerical approaches, the properties of these groups are examined to shed light on their structure, behavior, and underlying algebraic characteristics. Key numerical methods are employed to calculate invariants and test conditions for primitivity and regularity in these groups. [20][13]*

KEYWORDS: Dihedral groups, Primitivity, Regularity, Numerical approach, GAP.

ABBREVIATIONS: GAP: Group Algorithm Programming; D_{5p} : Refers to dihedral group of symmetries of a regular polygon with (5p) sides where p is typically prime, this group is composed of rotations and reflections.

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INTRODUCTION

Dihedral groups whose degree is 5p, where p is prime are fundamental in abstract algebra, representing symmetries of regular polygons [11]. The focus here is on investigating the primitivity and regularity of such groups that are not p-groups, meaning their order is not a power of p. Using computational techniques, [14] we address these properties and explore whether numerical analysis can offer deeper insights into their structure. Dihedral Groups are groups that represent the symmetries of polygons and consist of rotations and reflections [3]. For a group of order (n = 5p), they combine geometric interpretations with algebraic formalism, [2].

Primitivity in a group refers to if a group preserves the nontrivial partition of a set [21]. In the context of dihedral groups, understanding primitivity involves analyzing their actions on various sets, [10].

Regularity: A group is regular if every element can be expressed uniquely as a product of generators [6]. Exploring this property involves understanding how elements combine in the dihedral group D_{5p} [19].

Numerical Approach: In this section, numerical techniques such as group element counting, cycle structure analysis, and matrix representations of group actions are used to study the properties of D_{5p} [8].

Main computational tools include:

Cycle Decomposition: Decomposing group elements into cycles to check if the group action is primitive [5].

Testing Regularity: Using numerical algorithms to test if every element can be uniquely written as a product of group generators [15].

These work-study Dihedral groups of Degree 5p for small p. We present numerical simulations for small prime values of p ={7,11}. We calculate the group order, structure, and test for primitivity and regularity for each case [22], [16].

[4] work on a dihedral group of degree 3p that are not p-group he came out with a nice result that the dihedral group of degree 3p are imprimitive and soluble.

Preliminaries

The following definitions are important to this research work:

Permutation Group

A permutation group is a group G whose elements are permutations of a given set X and whose group operation is the composition of functions in G which are a bijection from the set X to itself.

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Symmetric Group

The symmetric group S_n is the group of permutations on a set with n elements. The symmetric group of degree n on a finite set is defined to be the group whose elements are all bijective functions from X to X and whose group operation is that of function composition. Permutations and bijection are two the same operation meaning rearrangement.

Abelian Group

A group G is called abelian if for every $a, b \in G, ab = ba$. Otherwise G is said to be non-abelian.

Dihedral Group

A dihedral group D_n is a symmetric group for an n-sided regular polygon for $n>2$. Dihedral groups are non-abelian permutation groups with group order 2n. We can mathematically write dihedral group $D_n = \{x, y | x^n = y^2 = 1, yx = x^{n-1}y = x^{-1}y\}$

Stabilizer

A kind of dual role is played by the set of elements in G which fix a specified point α. This is called the stabilizer of α in G and is denoted by $G_\alpha := \{\alpha^x = \alpha \}$.

Transitive Group

A group G acting on a set Ω is said to be transitive on Ω if it has one orbit and so $\alpha^G = \Omega$ for all $\alpha \in \Omega$. Equivalently, G is transitive if for every pair of point $\alpha, \delta \in \Omega$ there exists $g \in \Omega$ such that $\alpha^g = \beta$. A group which is not transitive is called intransitive.

If $|\Omega| \geq 2$, we say that the action of G on Ω is doubly transitive if for any $\alpha_1, \alpha_2 \in \Omega$ such that $\alpha_1 \neq \alpha_2$ and $\beta_1, \beta_2 \in \Omega$ such that $\beta_1 \neq \beta_2$ there exist $g \in G$ such that $\alpha_1^g = \beta_1,\ \alpha_2^g = \beta_2$.

The group G is said to be k-transitive (or k-fold transitive) on Ω if for any sequences $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that $\alpha_i \neq \alpha_j$ when $i \neq j$ and $\beta_1, \beta_2, \ldots, \beta_k$ such that $\beta_i \neq \beta_j$ when $i \neq j$ of k element on Ω, there exists $g \in G$ such that $\alpha_1^g = \beta_1\ for\ 1 \leq i \leq k$

Thus,

$G_1 = \{(1), (12), (13), (23), (123), (132)\}$ is transitive and

$G_2 = \{(1), (12), (34), (12)(34)\}$ is intransitive.

Imprimitivity

A subset Δ of Ω is said to be a set of imprimitivity for the action of G on Ω, if for each $g \in G$, either $\Delta^g = \Delta\ or\ \Delta^g$ and Δ are disjoint. In particular, Ω itself, the 1-element subsets of Ω and the empty set are obviously sets of imprimitivity which are called trivial set of imprimitivity.

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Example

The group of symmetry $D_4 = (1), (1234), (13)(24)(1432), (13), (24), (12)(34), (14)(23)\}$ with vertices 1,2,3,4 is not primitive. For take $G_1 = \{(1), (24)\}$ = reflection in the

line joining vertices 1 and 3 = stabilizer of the point 1, and $H = \{(1), (24), (13)$ = reflection in m the line joining vertices 2 and 4, (13)(24) = rotation in 180^0, $H = \{(1), (24), (13), (13)(24)\}$. Thus $G_1 < H < G$.

Primitive

A permutation group G acting on a nonempty set Ω is called primitive if G acts transitively on Ω and G preserves no non trivial partition of Ω. Where non trivial partition means a partition that is not a partition into singleton set or partition into one set Ω. In other word, a group G is said to be primitive on a set Ω if the only sets of imprimitivity are trivial ones otherwise G is imprimitive on Ω, example the group (e ∗) For each $g \in G$, $\Delta^g = \Delta$, $\Delta^g \cap \Delta \neq \emptyset$. Thus $G = S_3 = \{\ .\ .\ .\ \}$ is primitive.

p-subgroup

Let G be a group. Let H be a subgroup of G. If H is a p-group, then H is a p-subgroup of G. Thus, A p-subgroup of G is a subgroup whose order is some power of p.

Sylow p-subgroup

A Sylow p-subgroup of G is a subgroup whose order is p^k, where k is the largest natural number for which p^k divides |G|.

Normal group

A subgroup N of a group G is normal in G if the left and right cosets are the same, that is if $gH = Hg$ for every g∈G and a subgroup H of G.

Semi-regular and Regular Group

A permutation group G is called semi-regular if one is the only element of G which fixes each point. In other words, G is semi-regular when G_α=1 for each α∈G. A transitive semi-regular is called a regular group. Thus, the group G={(1),(12)(34),(14)(23),(13)(24)} is a regular group.

Clearly subgroups of semi-group are semi-regular; 1 is semi-regular. As we get that in a semi –regular group G, orbits have the same size, namely $|G|$, and hence, the order of G divides the degree of G. Furthermore, in a regular group G we have that $|G| = |\alpha^G| = |\Omega|$, $\alpha \epsilon \Omega$ and so the order and the degree of G coincide.

METHODOLOGY

Introduction

In this work, acknowledgment of the basic facts from both the theory of abstract finite groups and the theory of permutation will be assumed throughout. Relevant theorems and results are

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given and quoted with examples where necessary, in order to enhance proper understanding of the subject matter.

Theorem [7]

Any finite group G is isomorphic to a subgroup of the symmetric group S_n of degree n, where $n = |G|$.

Proof:

Let G act on itself by right multiplication $g^h = gh$ for all $g, h \in G$. If $g^h = g$ then $gh = g$ and so $h = 1$. That is, the kernel of the action is $\{1\}$. The mapping $f: G \to sym(G)$ define by $f_g \to f_g$ where $\alpha f_g = \alpha^g$ for any $\alpha \in G$ is a homomorphism. Then $\frac{G}{Kerf} \cong im\, f$ but $kerf = \{1\}$ and $imf \leq sym(G) = S_n$.

Accordingly, $G \leq S_n$. In general, we have that if G acts on Ω with k kernel of the action then $G/K \leq sym(\Omega)$.

Fundamental Counting Lemma or Orbit formula [9]

Let G act on Ω and α ∈ Ω. If G is finite then $|G| = |G_\alpha||\alpha^G|$.

Proof:

We determine the length $|\alpha^G|$ of the α^G, we have that $\alpha^x = \alpha^y$ if and only if $\alpha^{xy\text{-}1} = \alpha$ if and only if $\alpha^{xy} \in G_\alpha$ if and only if $G_\alpha x = G_\alpha y$. Thus there is one to one correspondence given by the mapping $G_\alpha x \to \alpha^x$ between the set of right cosets G_α and the G-orbit α^G in Ω. Accordingly, as G is finite we have that $|G:G_\alpha| = |\alpha^G|$ and so $|G| = |G_\alpha||\alpha^G|$.

Theorem [23]

Let G be a finite group. If $|G| = p^r m(p,m) = 1$, then

1. There is at least one Sylow p-subgroup H of G.

2. If B is any p-subgroup of G, then $B \subseteq x^{-1}Hx$ for some$x \in G$.

3. If K is any Sylow p-subgroup of G, K=$g^{-1}Hg \;\; for\ some\ g \in G$

4. If n is the number of Sylow p-subgroups of G, then n divides m and n ≡1modp.

Corollary [23]

Let G be a finite group and H a Slow p-subgroup of G. Then H is the only Sylow p-group of G if and only if H is normal in G.

Proof

By Sylow theorem, the Sylow p-Subgroups of G are the elements of the sets $g^{-1}Hg|g \in G$ and this reduces to a singleton set if and only if $g^{-1}Hg = H \;\; for\ all \;\; g \in G$; that is precisely when H is normal in G;

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Corollary [18]

Let p be prime. If $H \trianglelefteq$ G and G/H = p or p^2 then G/H is abelian. That is, every group of order p or p^2 is abelian.

Proof

Let $|G| = p^2$. If $|Z(G)| = p^2$, then certainly G is abelian, so suppose that $|Z(G)| = p$. Then $G/Z(G)$ is a cyclic group of order p, generated by the coset $Z(G)a$; then every element of G has the form za^i , where $z \in Z(G)$ and $i = 0,1,..., p -1$. By inspection, these elements commute.

Thus, G is abelian.

Lemma [24]

Let G be a dihedral group of any order, then G is transitive.

Proof:

For given α_i, α_j as any two vertices of the regular polygon with $i< j$, we readily see that

$(\alpha_1,\alpha_2,\ldots,\alpha_i,\ldots,\alpha_j,\ldots,\alpha_n)^{j-I}$ is the rotation about the center of the polygon through angle $\frac{2\pi^c}{n}$, (where n is the number of edges of the polygon) which take α_i to α_j. As such G is transitive.

Theorem [24]

Let G be a non-trivial transitive permutation group on Ω. Then G is primitive iff G_α, $(\alpha\in\Omega)$ is a maximal subgroup of G or equivalently, G is imprimitivity if and only if there is a subgroup H of G properly lying between G_α, $(\alpha\in\Omega)$ and G.

Proof:

Suppose G is imprimitive and ψ a non-trivial subset of imprimitivity of G.

Let $H = \{\psi^g = \psi\}$.

Clearly H is a subgroup of G and a proper subgroup of G because $\psi \subset \Omega$ and G is transitive.

Now choose $\alpha \in \psi$. If g∈G then $\alpha^g = \alpha$, showing that $\alpha \in \psi \cap \psi^g$ and so $\psi = \psi^g$.

Hence $G \leq H$.

Hence, $G_\alpha \leq H \leq G$.

Since $|\psi| = 1$, choose $\beta \in \psi$ *such that* $\beta \neq \alpha$. By transitivity of G, there exist some $h \in G$ with $\alpha^h = \beta$ *so that* $h \in G_\alpha$. Now $\beta \in \psi \cap \psi^h$ so $\psi = \psi^g$ and $h \in H - G_h$. Thus, $H \neq G_\alpha$ Hence G_α is not a maximal subgroup.

Conversely, suppose that $G_\alpha \leq H \leq G$ for some subgroup H.

Let $\psi = \alpha^H$.

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Since $H > G_\alpha$, $|\psi| \neq 1$.

Now If $\psi = \Omega$, then H is transitive on Ω and hence $\Omega = |G: G_\alpha| = |H: G_\alpha|$ showing that H = G, a contradiction. Hence, $\psi = \Omega\psi = \Omega$. Now we shall show that ψ is a subset of imprimitivity of G.

Let $h \in G$ and $\beta \in \psi \cap \psi^g$ then $\beta = a^h = a^{hg}$ for some $h, h' \in H$.

Hence $\alpha_{hgh^{-1}} = \alpha$. So $hgh^{-1} \in G_\alpha < H$.

Thus $\psi = \psi^g$. Hence ψ is a non-trivial subset of imprimitivity. So G is imprimitive.

Theorem [24]

Let G be a transitive permutation group of prime degree on Ω. Then G is primitive.

Proof

Now since G is transitive, it permutes the sets of imprimitivity bodily and all the sets have the same size. But $\Omega = \cup |\Omega_i|$, Ω_i being the sets of imprimitivity. As |Ω| is prime we Have that either each $|\Omega_i|=1$ or Ω is the set of imprimitivity. So G is primitive.

Lemma [24]

Let *G* be a dihedral group of any order, then *G* is transitive.

Proof

For given α_i, α_j as any two vertices of the regular polygon with i < j, we readily see that $(\alpha_1\alpha_2 \ldots \alpha_i \ldots \alpha_j \ldots \alpha_n)^{j-i}$ is the rotation about the center of the polygon through angle $2\pi^c/n$, (where n is the number of edges of the polygon) which takes α_i to α_j. As such *G* is transitive.

Theorem [1]

Let G be a transitive abelian group. Then, G is regular.

Proof:

Fix $\alpha \in \Omega$. If $\beta \in \Omega$ such that $\exists\ g \in G$ with $\alpha^G = \beta$. Now $G_\alpha = G_\alpha^g = (G_\alpha)^g = g^{-1}(G_\alpha)g = G_\alpha$ (since G is abelian). As α, β are arbitrary, we get that $G_\alpha = 1$ since G is transitive, it is regular.

Proposition [17]

A transitive group is regular if and only if its order and degree are equal

Proof:

Let G be a regular on Ω. of degree n since $|\alpha^G| = |G|$ and G is transitive Hence $|G| = n$, conversely, by transitivity of G it follows that, $n|G_\alpha| = |G|$. Hence $G_\alpha = 1$, since $|G| = n$ by assumption Hence G is semi-regular, but G is transitive so G is regular

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Proposition [17]

An intransitive group is irregular if and only if its order and degree are not equal

Proof:

Let G be an irregular group on Ω. of degree n, since $|\alpha^G| \neq |G|$ and G is intransitive Hence $|G| = n$. Conversely by transitivity of G it follows that, $n|G_\alpha| = |G|$. Hence $|G_\alpha| \neq 1$, since $|G| = n$ by assumption. Hence G is Semi-regular, but G is intransitive so G is irregular.

Regularity Property [9]

A group *G* is called regular if every subgroup of *G* is normal.

Proof:

The wreath product $H \wr G$ is defined as the semi-direct product $HX \rtimes G$, where HX is the direct product of X copies of H. The elements of $H \wr G$ can be represented as pairs (f, g), where $f{:}X \to H$ is a function and $g \in G$. Multiplication in $H \wr G$ is defined component-wise as $(f1,g1)\cdot(f2,g2)=(f1\,f2,g1g2)$, where $f1f2$ is the pointwise product of functions. Consider a subgroup K of $H \wr G$. Since HX is a normal subgroup of $H \wr G$, the projection of K onto HX is also a normal subgroup. The projection map $\pi{:}H \wr G \to HX$ is defined as $(f, g) \mapsto f$. This is a group homomorphism, and its kernel is the set of elements of the form $(1,g)$ for all $g \in G$, which is isomorphic to G. Therefore, HX is a normal subgroup of $H \wr G$. Let K be a subgroup of $H \wr G$, and let $a \in H \wr G$. We need to show that $aKa{-}1{=}K$ for all $a \in H \wr G$. Consider an element $a = (fa, ga) \in H \wr G$. The conjugate of K by a is given by $aKa^{-1}=\{(fakfa^{-1},ga) \mid k \in K\}$. Since HX is a normal subgroup, the conjugation $fakfa^{-1}$ lies in HX. Therefore, $aKa^{-1} \subseteq HX \times G$. The subgroup HX is normal in $H \wr G$, and $K \cap HX$ is normal in HX. As HX is normal in $H \wr G$ and $K \cap HX$ is normal in HX, the subgroup K is normal in $H \wr G$.

Thus, the wreath product $H \wr G$ is regular, as every subgroup is normal.

RESULTS AND DISCUSSIONS

Introduction

In this section, we constructed Dihedral groups of degree 5p. We stated a proposition and provided its proofs using the concepts of Group Theory. Throughout the letter p is an odd prime number.

Primitivity and Regularity of Dihedral Group of Degree 5p.

Our main result on the dihedral group of degree 5p is as below.

Proposition

Let *G* be the dihedral group of degree 5p. Then *G* is imprimitive and irregular.

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Proof:

$|G| = 2\times5p$ and $\Omega = \{1, 2, 3, \ldots, 5p\}$. That G is transitive follows easily from Lemma 3.1.6 Now, name the vertices of G as 1,2,3,…, 5p and let l be the line of symmetry joining the middle of the vertices 1 and 2p with the middle of the vertices $\frac{5p+1}{2}$ *and* $\frac{5p+3}{2}$ *so that*

$$\alpha = (2, 5p)(3, 5p-1)(4, 5p-2) \ldots \left(\frac{5p+1}{2}, \frac{5p+3}{2}\right)$$

is the reflection in l.

Then $G_1 = \left\{(1), (2, 5p)(3, 5p-1)(4, 5p-2) \ldots \left(\frac{5p+1}{2}, \frac{5p+3}{2}\right)\right\}$ is the stabilizer of point 1. We readily see that G_1 is a non-identity proper subgroup of G which has

$$H = \left\{(1), (2, 5p), (3, 5p-1), (4, 5p-2), \ldots, \left(\frac{5p+1}{2}, \frac{5p+3}{2}\right), \alpha\right\}$$

as a subgroup properly lying between G_1 and G. That is, $G_1 < H < G$. Thus, by virtue of Theorem 3.1.7 G is imprimitive and G is transitive and imprimitive.

Now $|G| = 2\times5p$. Therefore, G has Sylow 2-subgroups ($Syl_2(G)$) of order 2, Sylow 5-subgroups ($Syl_5(G)$) of order 5 and Sylow p-subgroups ($Syl_p(G)$) of order p.

Let n_p denote the number of Sylow p-subgroups in G of order p. Then, by Sylow Theorem $n_p \equiv 1$ modulo p and $n_p | 10 \Rightarrow n_p = \{1,10\}$ (for $p > 3$)

However, there exists a subgroup of G, say $K = Syl_2(G)$ which is not normal in G since $n_2 = \{1, 5p\}$. Thus, G is irregular by Theorem 3.2.3

Also, from (Orbit-formula) theorem 3.1.2 $|\alpha^G||G_\alpha| = |G|$ we have that, $|G_\alpha| = 2 \neq 1$ and again, $|G| = 10p \neq 5p = |\Omega|$, by theorem 3.2.0 and Theorem 3.2.1

Therefore, G is irregular as required.

Example 1

We shall now construct dihedral groups of degree 5p regular polygon and discuss whether they are transitive, primitive and regular using the concepts of Sylow p-subgroups and group actions to validate our proposition. [12] We use GAP to generate the group as follows.

```
┌───────┐   GAP 4.11.1 of 2021-03-02
│  GAP  │   https://www.gap-system.org
└───────┘   Architecture: x86_64-pc-cygwin-default64-kv7
 Configuration:  gmp 6.2.0, GASMAN, readline
Loading the library and packages ...
```


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Packages:

gap> # DihedralGroup of Degree 5p P=7#

gap> G :=DihedralGroup(IsGroup,70);

Group([(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35),(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)])

gap> Order(G);

70

gap> Elements(G);;

gap> SG :=AllSubgroups(G);;

gap> Size(SG);

52

gap> NG :=Normal Subgroups(G);

[Group([(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35),(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)]),Group([(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35),(1,6,11,16,21,26,31)(2,7,12,17,22,27,32)(3,8,13,18,23,28,33)(4,9,14,19,24,29,34)(5,10,15,20,25,30,35)]),

Group([(1,22,8,29,15)(2,23,9,30,16)(3,24,10,31,17)(4,25,11,32,18)(5,26,12,33,19)(6,27,13,34,20)(7,28,14,35,21)]),Group([(1,6,11,16,21,26,31)(2,7,12,17,22,27,32)(3,8,13,18,23,28,33)(4,9,14,19,24,29,34)(5,10,15,20,25,30,35)]),Group(())]

gap> Size(NG);

5

gap> IsTransitive(G);

true

gap> IsPrimitive(G);

false

gap> IsAbelian(G);

false

gap> IsRegular(G);

false

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```
gap> S2 :=SylowSubgroup(G,2);

Group([(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(
15,22)(16,21)(17,20)(18,19)])

gap> Order(S2);

2

gap> List(S2);

[(),(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,2
2)(16,21)(17,20)(18,19)]

gap> IsNormal(G,S2);

false

gap> IsTrantive(S2);

Error, Variable: 'IsTrantive' must have a value

not in any function at *stdin*:16

gap> IsTransitive(S2);

false

gap> IsPrimitive(S2);

false

gap> IsAbelian(S2);

true

gap> IsRegular(S2);

false

gap> S5 :=SylowSubgroup(G,5);

Group([(1,29,22,15,8)(2,30,23,16,9)(3,31,24,17,10)(4,32,25,18,11)(5,33,26,19,12)(6,34,27,2
0,13)(7,35,28,21,14)])

gap> Order(S5);

5

gap> List(S5);

[(),(1,8,15,22,29)(2,9,16,23,30)(3,10,17,24,31)(4,11,18,25,32)(5,12,19,26,33)(6,13,20,27,34)(
7,14,21,28,35),(1,15,29,8,22)(2,16,30,9,23)(3,17,31,10,24)(4,18,32,11,25)(5,19,33,12,26)(6,2
0,34,13,27)(7,21,35,14,28),(1,22,8,29,15)(2,23,9,30,16)(3,24,10,31,17)(4,25,11,32,18)(5,26,1
```


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2,33,19)(6,27,13,34,20)(7,28,14,35,21),(1,29,22,15,8)(2,30,23,16,9)(3,31,24,17,10)(4,32,25,18,11)(5,33,26,19,12)(6,34,27,20,13)(7,35,28,21,14)]

gap> IsNormal(G,S5);

true

gap> IsTransitive(S5);

false

gap> IsPrimitive(S5);

false

gap> IsAbelian(S5);

true

gap> IsRegular(S5);

false

gap> S7 :=SylowSubgroup(G,7);

Group([(1,26,16,6,31,21,11)(2,27,17,7,32,22,12)(3,28,18,8,33,23,13)(4,29,19,9,34,24,14)(5,30,20,10,35,25,15)])

gap> Order(S7);

7

gap> List(S7);

[(),(1,11,21,31,6,16,26)(2,12,22,32,7,17,27)(3,13,23,33,8,18,28)(4,14,24,34,9,19,29)(5,15,25,35,10,20,30),(1,21,6,26,11,31,16)(2,22,7,27,12,32,17)(3,23,8,28,13,33,18)(4,24,9,29,14,34,19)(5,25,10,30,15,35,20),(1,31,26,21,16,11,6)(2,32,27,22,17,12,7)(3,33,28,23,18,13,8)(4,34,29,24,19,14,9)(5,35,30,25,20,15,10),(1,6,11,16,21,26,31)(2,7,12,17,22,27,32)(3,8,13,18,23,28,33)(4,9,14,19,24,29,34)(5,10,15,20,25,30,35),(1,16,31,11,26,6,21)(2,17,32,12,27,7,22)(3,18,33,13,28,8,23)(4,19,34,14,29,9,24)(5,20,35,15,30,10,25),(1,26,16,6,31,21,11)(2,27,17,7,32,22,12)(3,28,18,8,33,23,13)(4,29,19,9,34,24,14)(5,30,20,10,35,25,15)]

gap> IsTransitive(S7);

false

gap> IsPrimitive(S7);

false

gap> IsAbelian(S7);

true

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gap> IsRegular(S7);

false

gap> IsNormal(G,S7);

true

gap> S1 :=Stabilizer(G,1);

Group([(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)])

gap> Order(S1);

2

gap> Elements(S1);

[(),(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)]

gap> S5 :=Stabilizer(G,5);

Group([(1,9)(2,8)(3,7)(4,6)(10,35)(11,34)(12,33)(13,32)(14,31)(15,30)(16,29)(17,28)(18,27)(19,26)(20,25)(21,24)(22,23)])

gap> Order(S5);

2

gap> Elements(S5);

[(),(1,9)(2,8)(3,7)(4,6)(10,35)(11,34)(12,33)(13,32)(14,31)(15,30)(16,29)(17,28)(18,27)(19,26)(20,25)(21,24)(22,23)](1,31,26,21,16,11,6)(2,32,27,22,17,12,7)(3,33,28,23,18,13,8)(4,34,29,24,19,14,9)(5,35,30,25,20,15,10)]

gap> S7 :=Stabilizer(G,7);

Group([(1,13)(2,12)(3,11)(4,10)(5,9)(6,8)(14,35)(15,34)(16,33)(17,32)(18,31)(19,30)(20,29)(21,28)(22,27)(23,26)(24,25)])

gap> Order(S7);

2

gap> Elements(S7);

[(),(1,13)(2,12)(3,11)(4,10)(5,9)(6,8)(14,35)(15,34)(16,33)(17,32)(18,31)(19,30)(20,29)(21,28)(22,27)(23,26)(24,25)]

gap>

The group of symmetry $G = D_{70}$ of the regular polygon viz:

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D_{70} = {[Element as listed above]}

Since from above result $|D_{70}| = 70$ and the order of Sylow 7-Subgroup $|S7|=7$, the stabilizer $|S7_{(1)}| = 2$, as a subgroup properly lying between $|S7_{(1)}|$ and $|D_{70}|$. that is, $|S7_{(1)}| < |S7| < |D_{70}|$. Thus, by virtue of Theorem 3.1.7 *G* is transitive and imprimitive.

Now $|D_{70}| = 70 = 2.5.7$ by Sylow's theorem 3.1.3

(a) The Sylow 2-subgroups of D_{70} have order 2. The number is congruent to 1 modulo 2 and it divides 35. So $n_2 = \{1,5,7,35\}$, hence, not normal in D_{70}.

(b) The Sylow 5-subgroup of D_{70} has order 5 and it divides 14. We readily see that D_{70} has a Sylow 5-subgroup given by $n_5 =\{1\}$ Hence unique therefore normal by corollary 3.1.4, it is not normal in D_{70} .

(c) The Sylow 7-Subgroup of D_{70} has order 7 and it divides 10. We readily see that D_{70} has a unique Sylow 7-subgroup given by $n_7 = \{1\}$ hence unique therefore normal by corollary 3.1.4

Now the stabilizer of the point 1 in D_{70} is given by $D_{70(1)}$=[(2,35)(3,34)(4,33)(5,32)(6,31)(7,30)(8,29)(9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)])}. Thus $|D_{70(1)}| = 2$,

$\forall\ \alpha \in \Omega$ where $\Omega = \{1, 2,...,35\}$, $|D_{70(1)}| = 2 \neq 1$. Therefore, as D_{70} is transitive and not semi regular, then D_{70} = G is irregular. By theorems 3.2.0 and 3.2.1

Example 2

We shall now construct another dihedral groups of degree 5p regular polygon and discuss whether they are transitive, primitive and regular using the concepts of Sylow p-subgroups and group actions just to validate our proposition. [12] We use GAP to generate the group as follows.

```
┌───────┐   GAP 4.12.2 built on 2022-12-19 10:30:03+0000
│  GAP  │   https://www.gap-system.org
└───────┘   Architecture: x86_64-pc-cygwin-default64-kv8
Configuration:  gmp 6.2.1, GASMAN, readline
Loading the library and packages ...
Try '??help' for help. See also '?copyright', '?cite' and '?authors'
gap>
gap> # DihedralGroup of Degree 5p  p = 11#
gap>
```


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```
gap> G := DihedralGroup(IsGroup, 110);

<permutation group with 2 generators>

gap> Order(G);

110

gap> Elements(G);;

gap> SG := AllSubgroups(G);;

gap> Size(SG);

76

gap> NG := Normal Subgroups(G);

[ <permutation group of size 110 with 2 generators>, <permutation group of size 55 with 2 generators>,

Group([(1,12,23,34,45)(2,13,24,35,46)(3,14,25,36,47)(4,15,26,37,48)(5,16,27,38,49)(6,17,28,39,50)(7,18,29,40,51)(8,19,30,41,52)(9,20,31,42,53)(10,21,32,43,54)(11,22,33,44,55)]),Group([(1,6,11,16,21,26,31,36,41,46,51)(2,7,12,17,22,27,32,37,42,47,52)(3,8,13,18,23,28,33,38,43,48,53)(4,9,14,19,24,29,34,39,44,49,54)(5,10,15,20,25,30,35,40,45,50,55)]), Group(()) ]

gap> Size(NG);

5

gap> IsTransitive(G);

true

gap> IsPrimitive(G);

false

gap> IsAbelian(G);

false

gap> IsRegular(G);

false

gap> S2 := SylowSubgroup(G,2);

Group([(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29) ])

gap> Order(S2);
```


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2

gap> List(S2);

[(),(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)]

gap> IsNormal(G,S2);

false

gap> IsTransitive(S2);

false

gap> IsPrimitive(S2);

false

gap> IsAbelian(S2);

true

gap> IsRegular(S2);

false

gap> S5 := SylowSubgroup(G,5);

Group([(1,34,12,45,23)(2,35,13,46,24)(3,36,14,47,25)(4,37,15,48,26)(5,38,16,49,27)(6,39,17,50,28)(7,40,18,51,29)(8,41,19,52,30)(9,42,20,53,31)(10,43,21,54,32)(11,44,22,55,33)])

gap> Order(S5);

5

gap> List(S5);

[(),(1,23,45,12,34)(2,24,46,13,35)(3,25,47,14,36)(4,26,48,15,37)(5,27,49,16,38)(6,28,50,17,39)(7,29,51,18,40)(8,30,52,19,41)(9,31,53,20,42)(10,32,54,21,43)(11,33,55,22,44),(1,45,34,23,12)(2,46,35,24,13)(3,47,36,25,14)(4,48,37,26,15)(5,49,38,27,16)(6,50,39,28,17)(7,51,40,29,18)(8,52,41,30,19)(9,53,42,31,20)(10,54,43,32,21)(11,55,44,33,22),(1,12,23,34,45)(2,13,24,35,46)(3,14,25,36,47)(4,15,26,37,48)(5,16,27,38,49)(6,17,28,39,50)(7,18,29,40,51)(8,19,30,41,52)(9,20,31,42,53)(10,21,32,43,54)(11,22,33,44,55),(1,34,12,45,23)(2,35,13,46,24)(3,36,14,47,25)(4,37,15,48,26)(5,38,16,49,27)(6,39,17,50,28)(7,40,18,51,29)(8,41,19,52,30)(9,42,20,53,31)(10,43,21,54,32)(11,44,22,55,33)]

gap> IsNormal(G,S5);

true

gap> IsTransitive(S5);

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false

gap> IsPrimitive(S5);

false

gap> IsAbelian(S5);

true

gap> IsRegular(S5);

false

gap> S1 := Stabilizer(G,1);

Group([(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)])

gap> Order(S1);

2

gap> Elements(S1);

[(),(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)]

gap> S5 := Stabilizer(G,5);

Group([(1,9)(2,8)(3,7)(4,6)(10,55)(11,54)(12,53)(13,52)(14,51)(15,50)(16,49)(17,48)(18,47)(19,46)(20,45)(21,44)(22,43)(23,42)(24,41)(25,40)(26,39)(27,38)(28,37)(29,36)(30,35)(31,34)(32,33)])

gap> Order(S5);

2

gap> Elements(S5);

[(),(1,9)(2,8)(3,7)(4,6)(10,55)(11,54)(12,53)(13,52)(14,51)(15,50)(16,49)(17,48)(18,47)(19,46)(20,45)(21,44)(22,43)(23,42)(24,41)(25,40)(26,39)(27,38)(28,37)(29,36)(30,35)(31,34)(32,33)]

gap> S11 := SylowSubgroup(G,11);

Group([(1,46,36,26,16,6,51,41,31,21,11)(2,47,37,27,17,7,52,42,32,22,12)(3,48,38,28,18,8,53,43,33,23,13)(4,49,39,29,19,9,54,44,34,24,14)(5,50,40,30,20,10,55,45,35,25,15)])

gap> Order(S11);

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11

gap> List(S11);

[(),(1,11,21,31,41,51,6,16,26,36,46)(2,12,22,32,42,52,7,17,27,37,47)(3,13,23,33,43,53,8,18,28,38,48)(4,14,24,34,44,54,9,19,29,39,49)(5,15,25,35,45,55,10,20,30,40,50),(1,21,41,6,26,46,11,31,51,16,36)(2,22,42,7,27,47,12,32,52,17,37)(3,23,43,8,28,48,13,33,53,18,38)(4,24,44,9,29,49,14,34,54,19,39)(5,25,45,10,30,50,15,35,55,20,40),(1,31,6,36,11,41,16,46,21,51,26)(2,32,7,37,12,42,17,47,22,52,27)(3,33,8,38,13,43,18,48,23,53,28)(4,34,9,39,14,44,19,49,24,54,29)(5,35,10,40,15,45,20,50,25,55,30),(1,41,26,11,51,36,21,6,46,31,16)(2,42,27,12,52,37,22,7,47,32,17)(3,43,28,13,53,38,23,8,48,33,18)(4,44,29,14,54,39,24,9,49,34,19)(5,45,30,15,55,40,25,10,50,35,20),(1,51,46,41,36,31,26,21,16,11,6)(2,52,47,42,37,32,27,22,17,12,7)(3,53,48,43,38,33,28,23,18,13,8)(4,54,49,44,39,34,29,24,19,14,9)(5,55,50,45,40,35,30,25,20,15,10),(1,6,11,16,21,26,31,36,41,46,51)(2,7,12,17,22,27,32,37,42,47,52)(3,8,13,18,23,28,33,38,43,48,53)(4,9,14,19,24,29,34,39,44,49,54)(5,10,15,20,25,30,35,40,45,50,55),(1,16,31,46,6,21,36,51,11,26,41)(2,17,32,47,7,22,37,52,12,27,42)(3,18,33,48,8,23,38,53,13,28,43)(4,19,34,49,9,24,39,54,14,29,44)(5,20,35,50,10,25,40,55,15,30,45),(1,26,51,21,46,16,41,11,36,6,31)(2,27,52,22,47,17,42,12,37,7,32)(3,28,53,23,48,18,43,13,38,8,33)(4,29,54,24,49,19,44,14,39,9,34)(5,30,55,25,50,20,45,15,40,10,35),(1,36,16,51,31,11,46,26,6,41,21)(2,37,17,52,32,12,47,27,7,42,22)(3,38,18,53,33,13,48,28,8,43,23)(4,39,19,54,34,14,49,29,9,44,24)(5,40,20,55,35,15,50,30,10,45,25),(1,46,36,26,16,6,51,41,31,21,11)(2,47,37,27,17,7,52,42,32,22,12)(3,48,38,28,18,8,53,43,33,23,13)(4,49,39,29,19,9,54,44,34,24,14)(5,50,40,30,20,10,55,45,35,25,15)]

gap> IsNormal(G,S11);

true

gap> IsTransitive(S11);

false

gap> IsPrimitive(S11);

false

gap> IsAbelian(S11);

true

gap> IsRegular(S11);

false

gap> S1 := Stabilizer(G,1);

Group([(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)])

gap> Order(S1);

2

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gap> Elements(S1);

[(),(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)]

gap> S5 := Stabilizer(G,5);

Group([(1,9)(2,8)(3,7)(4,6)(10,55)(11,54)(12,53)(13,52)(14,51)(15,50)(16,49)(17,48)(18,47)(19,46)(20,45)(21,44)(22,43)(23,42)(24,41)(25,40)(26,39)(27,38)(28,37)(29,36)(30,35)(31,34)(32,33)])

gap> Order(S5);

2

gap> Elements(S5);

[(),(1,9)(2,8)(3,7)(4,6)(10,55)(11,54)(12,53)(13,52)(14,51)(15,50)(16,49)(17,48)(18,47)(19,46)(20,45)(21,44)(22,43)(23,42)(24,41)(25,40)(26,39)(27,38)(28,37)(29,36)(30,35)(31,34)(32,33)]

gap> S11 := Stabilizer(G,11);

Group([(1,21)(2,20)(3,19)(4,18)(5,17)(6,16)(7,15)(8,14)(9,13)(10,12)(22,55)(23,54)(24,53)(25,52)(26,51),(27,50)(28,49)(29,48)(30,47)(31,46)(32,45)(33,44)(34,43)(35,42)(36,41)(37,40)(38,39)])

gap> Order(S11);

2

gap> Elements(S11);

[(),(1,21)(2,20)(3,19)(4,18)(5,17)(6,16)(7,15)(8,14)(9,13)(10,12)(22,55)(23,54)(24,53)(25,52)(26,51)(27,50)(28,49)(29,48)(30,47)(31,46)(32,45)(33,44)(34,43)(35,42)(36,41)(37,40)(38,39)]

gap>

The group of symmetry $G = D_{110}$ of the regular polygon viz:

D_{110} = {[Elements as listed above]}

Since from above result $|D_{110}| = 110$ and the order of Sylow 11-Subgroup $|S11|=11$, the stabilizer $\left|S11_{(1)}\right| = 2$, as a subgroup properly lying between $\left|S11_{(1)}\right|$ and $|D_{110}|$. that is, $\left|S11_{(1)}\right| < |S11| < |D_{110}|$. Thus, by virtue of Theorem 3.1.7 G is transitive and imprimitive.

Now $|D_{110}| = 110 = 2.5.11$ by Sylow's theorem 3.1.3.

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(a) The Sylow 2-subgroups of D_{110} have order 2. The number is congruent to 1 modulo 2 and it divides 55. So $n_2 = \{1,5,11,55\}$, hence, not normal in D_{110}.

(b) The Sylow 5-subgroup of D_{110} has order 5 and it divides 22. We readily see that D_{110} has a Sylow 5-subgroup given by $n_5 =\{1,11\}$ Hence, not normal by corollary 3.1.4, it is not normal in D_{110} .

(c) The Sylow 11-Subgroup of D_{110} has order 11 and it divides 10. We readily see that D_{110} has a unique Sylow 11-subgroup given by $n_{11} = \{1\}$ hence unique therefore normal by corollary 3.1.4.

Now the stabilizer of the point 1 in D_{110} is given by $D_{110(1)}$=[(),(2,55)(3,54)(4,53)(5,52)(6,51)(7,50)(8,49)(9,48)(10,47)(11,46)(12,45)(13,44)(14,43)(15,42)(16,41)(17,40)(18,39)(19,38)(20,37)(21,36)(22,35)(23,34)(24,33)(25,32)(26,31)(27,30)(28,29)])}. Thus $|D_{110(1)}| = 2$,

$\forall\ \alpha \in \Omega$ where $\Omega = \{1, 2,...,55\}$, $|\ D_{110(1)}| = 2 \neq 1$. Therefore, as D_{110} is transitive and not semi regular, then $D_{110} = G$ is irregular. By theorems 3.2.0 and 3.2.1

SUMMARY AND CONCLUSION

The results of the numerical experiments were discussed, particularly focusing on which values of p lead to non-primitive actions. The conditions under which (D_{5p}) fails to be regular, due to patterns observed in the group structures and element behavior. We summarize the findings and highlight how numerical approaches provide useful insights into the primitivity and regularity of Dihedral groups of degree 5p. These methods, while computationally intensive, offer a clear pathway to understanding these fundamental group-theoretic properties as clearly shown by our results that the Dihedral groups of degree 5p where p is odd prime number are Imprimitive and irregular

Contributions to Knowledge

1. We generated a new group of degrees 5p using dihedral method procedures, which shed more light to the Primitive and Regularity nature of the groups generated as our contribution.

2. We proved that dihedral groups of degree 5p that are not p-groups are imprimitive and irregular. A proposition was formulated and proved to back up our claims while a standard program namely Groups, Algorithms and Programming (GAP) version 4.11.1 [12], was employed to compare the results. These findings cannot be found in any text of higher learning.

RECOMMENDATION

For further research, one can check other theoretic properties such as nilpotency, solubility etc. of the same algebraic structure.

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