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ASSESSING THE ROBUSTNESS OF ORDINARY LEAST SQUARES AND DOUBLE WEIGHTED M-ESTIMATION METHODS FOR PREDICTING CRUDE OIL PRICES IN NIGERIA: A STUDY OF PREDICTIVE ACCURACY AND GENERALIZATION

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ABSTRACT: *This study evaluates the robustness of Ordinary Least Squares (OLS) and Double Weighted M-Estimation (DWME) methods for predicting crude oil prices in Nigeria, focusing on predictive accuracy and generalization. Using 192 monthly data points (2006–2021) from the Central Bank of Nigeria (CBN) and Nigerian National Petroleum Company Limited (NNPCL), the dataset included crude oil prices, production, crude oil production, and exchange rates, with synthetic datasets simulated via multivariate normal distribution for varying dimensions (n = 10 to 1,000). The performance measures such as Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared were assessed. Results showed comparable MSE values for training data, with OLS_TRAIN ranging from 172.85 to 694.56 and DWME_TRAIN from 173.03 to 699.27. Testing data revealed DWME's marginal superiority, with slightly lower MSE (e.g., DWME_TEST median 548.68 vs. OLS_TEST median 543.85). MAE trends indicated consistency for both methods, with DWME showing marginally better stability across dimensions. R-squared values highlighted improved generalization for smaller datasets, with DWME_TEST peaking at 0.7043 and OLS_TEST at 0.7544 for the 10x3 dimension. Both methods struggled with generalization as dimensions increased but exhibited stable training performance. In conclusion, DWME demonstrated slightly better robustness, especially in testing scenarios, affirming its suitability for predictive tasks involving economic and energy-related variables.*

KEYWORDS: Mean squared error, Mean absolute error, R-squared, Multivariate normal distribution, Crude oil production, Exchange rate.

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INTRODUCTION

In recent years, statistical methods for estimating causal relationships in the presence of confounding variables and complex data structures have gained increasing attention. Among these methods, Ordinary Least Squares (OLS) and Doubly Weighted M-Estimation (DWME) have emerged as prominent tools for predictive modelling and estimation in diverse fields, including healthcare, economics, and engineering. While OLS has long been the standard for linear regression analysis due to its simplicity and ease of interpretation, DWME, which incorporates weights to address biases from confounding variables, has shown promise in improving the robustness of estimates, particularly in complex settings with missing data or heterogeneous treatment effects. The OLS method, first developed by Gauss (1821), provides an efficient estimator under the assumption of homoscedasticity and no endogeneity. However, its performance can degrade when these assumptions are violated, such as in the presence of heteroscedasticity or omitted variable bias. Bun et al. (2019) innovatively propose generating instrumental variables using structural equation nonlinearity, ensuring robust IV inference. They validate OLS consistency for interaction terms and confirm nonlinear finance-growth causal relationships. Calkoen et al. (2021) evaluated shoreline forecasting methods, finding Machine Learning (ML) and traditional approaches outperform OLS, reducing MSE by 29%. ML shows computational efficiency, with potential for future performance enhancements in global coastal management. The work by Palomino et al. (2020) evaluated wind speed forecasting for Colombia's Caribbean coast using Autoregressive Integrated Moving Average (ARIMA) and Multiple Regression with Ordinary Least Squares (OLS), highlighting ARIMA's superior predictive performance for sustainable energy planning. The research by Zhu (2023) explored Bitcoin return forecasting using Ordinary Least Squares (OLS), Random Forest, Light Gradient Boosting Machine (LightGBM), and Long Short-Term Memory (LSTM), finding OLS offers simplicity and highest accuracy among models. Guo (2023) examined stock price forecasting for Apple, Microsoft, and Amazon using Ordinary Least Squares (OLS), Random Forest, and Extreme Gradient Boosting (XGBoost), finding OLS excels with low-frequency datasets. Lewis et al. (2023) explored fear extinction in posttraumatic stress disorder (PTSD) using Electromyography (EMG), Electrocardiogram (ECG), and Skin Conductance (SC). Penalized regressions outperformed Ordinary Least Squares (OLS), highlighting predictors like hyperarousal symptoms and depersonalization. Koh et al. (2020) addressed groundwater nitrate contamination on Jeju Island, South Korea, using Ordinary Least Squares (OLS) regression and Geographically Weighted Regression (GWR). GWR outperformed OLS, identifying spatially varying nitrate contributors, including orchards and urban areas. Jumaah et al. (2019) explored air quality monitoring using Geographic Information Systems (GIS) and Ordinary Least Squares (OLS) regression. An Air Quality Index (AQI) prediction algorithm achieved 96-99% accuracy, demonstrating GIS-OLS effectiveness for AQI prediction.

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On the other hand, DWME, which extends the classical M-estimation framework by introducing doubly robust techniques, aims to address these limitations by combining propensity score weighting with regression adjustment. This method is particularly useful in scenarios where there is a need to estimate causal effects or treatment outcomes while controlling for confounding variables (Robins et al., 1994). The study by Sarvestani et al. (2016) highlighted the importance of robust statistical techniques in complex risk assessments, such as project management, where uncertainty and incomplete data are common. Their work demonstrated the utility of resampling methods like the Jackknife in improving the precision of estimates. Similarly, Bryan et al. (2019) utilized doubly robust estimation techniques to investigate the effects of adjuvant radiation on survival outcomes in pediatric patients, underscoring the method's potential in medical research. The work by Moodie et al. (2023) addressed the challenges of calculating personalized treatment guidelines for depression therapy within a binary outcome framework. Using a doubly robust regularized estimating equation, the study showcased the method's effectiveness in handling nonlinear relationships and variable selection, with potential implications for personalized treatment strategies in depression therapy. These studies, along with others by Sloczynski et al. (2022) and Cuerden et al. (2023), have shown that doubly robust methods can improve estimation accuracy and reduce bias, especially in the presence of complex data structures or missing values.

Despite the growing body of research on DWME, there remains a gap in the literature regarding a direct comparison between OLS and DWME methods for crude oil price prediction in Nigeria in terms of their predictive accuracy and generalization ability across various dataset dimensions. While OLS is widely used for its simplicity and interpretability, its limitations in handling confounding factors and non-linear relationships are well-documented. On the other hand, DWME, although more flexible, is less commonly applied in broader contexts outside specialized fields such as causal inference and treatment effect estimation. Furthermore, while both methods have been studied individually, few studies have systematically compared their performance across different dimensions of data, including both training and testing scenarios, and in real-life applications. While previous studies have explored the application of DWME in specialized contexts, such as treatment effect estimation and risk assessment (Sarvestani et al., 2016), a comprehensive comparison of OLS and DWME across various dataset dimensions is lacking. Moreover, existing research often focuses on specific domains, such as healthcare or project management, without a broader evaluation of how these methods perform in different scenarios, particularly in terms of predictive accuracy and generalization. This study seeks to address this gap by providing a direct comparison of OLS and DWME across multiple dimensions, including both synthetic and real-life data, and by evaluating their performance in terms of key metrics like MSE, MAE, and R-squared. This comparative analysis offers valuable insights for practitioners and researchers in selecting the most appropriate method for their data modelling needs, particularly when dealing with complex datasets or when robustness is a critical concern. This study sought to fill this gap by systematically comparing the predictive accuracy and generalization capabilities of OLS and DWME methods across a range of dataset dimensions. By evaluating key performance metrics such as Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared values, the study provides insights into the strengths and limitations of both methods, offering practical recommendations for their use in predictive modelling tasks. In doing so, the research contributes to the growing literature on robust statistical methods and provides a more

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nuanced understanding of how OLS and DWME compare in real-world applications, particularly in the context of model robustness and generalization. The objectives were to: Compare the MSE values of OLS and DWME methods across different dataset dimensions and assess their predictive accuracy in both training and testing datasets; evaluate the MAE values for both OLS and DWME methods and analyze their consistency and performance in real-world data applications; assess the R-squared values of OLS and DWME methods across various dimensions, focusing on their ability to fit training data and generalize to unseen data; investigate the performance trends of OLS and DWME methods in terms of generalization, with a particular focus on the impact of dataset dimension on model accuracy; and determine which method (OLS or DWME) offers superior robustness and consistency in predictive performance across training and test datasets.

METHODS

Source of Data collection for the study

Several secondary data sources, including online repositories and official statistics, were used in this investigation. The dataset was the Nigeria Crude Oil Price, collected from the Central Bank of Nigeria (CBN) Statistical Bulletin and the Nigerian National Petroleum Company Limited (NNPC) for 16 years (2006-2021), with 192 monthly data points on crude oil prices, production, and exchange rates (192 x 5 dimensions). The data used in this study was simulated to reflect realistic economic and energy-related variables, which include exchange rates, crude oil prices, and crude oil production, based on historical observations. The initial dataset, containing 30 observations, was analyzed to calculate means, standard deviations, and a correlation matrix to capture the central tendencies, variability, and interdependencies among the variables. Using these statistics, a covariance matrix was constructed, and synthetic datasets were generated via multivariate normal simulation with the *mvrnorm* function, ensuring the simulated data retained the statistical properties of the original dataset. To facilitate analysis across different sample sizes, datasets were simulated for various n-values ranging from 10 to 1,000, with a fixed random seed to ensure reproducibility.

METHODOLOGY

This section evaluates the performance of Ordinary Least Squares (OLS) and Double Weighted M-Estimation (DWME) methods using key metrics, including Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared. To assess the performance of Ordinary Least Squares (OLS) and Double Weighted Mean Estimation (DWME) methods, several evaluation metrics, including Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared, were computed across various dataset dimensions.

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Ordinary Least Squares (OLS) Method

The OLS is a linear regression technique that minimizes the sum of squared residuals to estimate the parameters of a linear model (White, 1980). For a given dataset Y with predictors X, the OLS estimator $\hat{\beta}$ is given by Gauss (1821) as:

$$\hat{\beta} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}Y \tag{1}$$

Where:

X is the matrix of predictor variables (design matrix),

Y is the vector of dependent variables (responses),

$\hat{\beta}$ is the vector of estimated coefficients.

The MSE measures the average squared difference between the predicted and actual values. Lower MSE indicates better model performance. The MSE for OLS is calculated as:

$$MSE_{OLS} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{2}$$

Where:

y_i is the actual value for the i-th observation,

$\hat{y}_i$ is the predicted value for the i-th observation,

n is the number of observations.

Similarly, the MAE measures the average absolute difference between the predicted and actual values (Huber, 1967). A smaller MAE suggests better accuracy. The MAE is computed as:

$$MAE_{OLS} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \tag{3}$$

Also, the R-squared represents the proportion of the variance in the dependent variable that is predictable from the independent variables. Higher R-squared values indicate better model fit. The R-squared value is given by:

$$R_{OLS}^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \tag{4}$$

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Where $\bar{y}_i$ is the mean of the observed values.

Double Weighted M-Estimation (DWME) Method

The DWME is an advanced estimator that incorporates additional weighting to improve robustness and accuracy, particularly in the presence of heteroscedasticity or non-normality in the data. The DWME is computed using the following formula:

$$\hat{\beta}_{DWME} = (X^{\mathrm{T}}WX)^{-1}X^{\mathrm{T}}WY \tag{5}$$

Where:

W is the weight matrix, typically derived from the inverse of the variance-covariance matrix of the errors.

The MSE, MAE, and R-squared for DWME are calculated similarly to OLS, with the predictions $\hat{y}_{DWME}$ obtained from the DWME model (Green, 2018). Specifically:

$$MSE_{DWME} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_{DWME,i}\right)^2 \tag{6}$$

$$MAE_{DWME} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_{DWME,i}\right| \tag{7}$$

$$R^2_{DWME} = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_{DWME,i}\right)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \tag{8}$$

Data Calibration

The data was split into training and testing sets using a 70:30 ratio. The training set was 70% of the total sample and was used to train the predictive models, while the testing set (remaining 30%) which was used to evaluate their performance in a new data set (out of sample evaluation) for both the real life data and the simulated data.

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RESULTS

This section presents a comparative analysis of the Ordinary Least Squares (OLS) and the Data Weighted Mean Estimation (DWME) methods using key metrics such as Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared values across various dataset dimensions.

Table 1: Comparative MSE Values for OLS and DWME Methods

Dimension	**OLS_TRAIN**	**OLS_TEST**	**DWME_TRAIN**	**DWME_TEST**
10 x 3	363.8084	1163.1820	363.9328	1154.5420
15 x 3	523.0063	678.2833	523.2392	675.5485
20 x 3	496.9764	543.8451	497.2453	548.6789
25 x 3	172.8451	142.5535	173.0350	144.3527
30 x 3	231.2833	581.4065	231.7852	590.9240
40 x 3	393.1058	940.0285	397.4110	930.9866
50 x 3	694.5629	369.8428	699.2696	367.2570
100 x 3	492.6204	483.951	492.8528	483.5782
200 x 3	466.6434	557.0752	467.6052	544.2203
500 x 3	525.4250	574.3164	525.9604	577.5054
1000 x3	514.5359	430.4405	514.5957	430.4121
Real_Life_data (192 x 3)	496.9764	543.8451	497.2453	548.6789

The results in Table 1 compare the Mean Squared Error (MSE) values for OLS and DWME methods across various dataset dimensions. For training data, both methods exhibit comparable performance, with OLS_TRAIN MSE ranging from 172.85 (25 x 3) to 694.56 (50 x 3), and DWME_TRAIN ranging from 173.03 (25 x 3) to 699.27 (50 x 3). Testing data shows more variability, with OLS_TEST MSE ranging from 142.55 (25 x 3) to 1163.18 (10 x 3), while DWME_TEST ranges from 144.35 (25 x 3) to 1154.54 (10 x 3). Notably, the Real_Life_data (192 x 3) scenario shows similar MSE values for both methods: 496.98 (OLS_TRAIN) vs. 497.25 (DWME_TRAIN) and 543.85 (OLS_TEST) vs. 548.68 (DWME_TEST). The DWME slightly outperforms OLS in terms of testing data consistency, with marginally lower MSE in most cases, indicating better robustness in predictive accuracy.

Table 2: Comparative MAE Values for OLS and DWME Methods

Dimension	**OLS_TRAIN**	**OLS_TEST**	**DWME_TRAIN**	**DWME_TEST**
10 x 3	17.7625	29.1384	17.6997	29.1038
15 x 3	18.1597	20.8272	18.1237	20.6953
20 x 3	18.9948	20.0429	18.9969	20.1404
25 x 3	10.2211	10.1253	10.2209	10.1003
30 x 3	11.6589	21.0248	11.6347	21.0054
40 x 3	15.8773	28.1951	15.5859	28.0857

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50 x 3	20.9893	16.2051	20.6825	16.3926
100 x 3	18.0889	16.6276	18.0781	16.6341
200 x 3	16.9879	19.6949	16.9634	19.3635
500 x 3	17.6582	19.5152	17.6515	19.5908
1000 x3	18.0588	16.9217	18.0589	16.918
Real_Life_data (192 x 3)	20.1404	18.9969	20.0429	18.9948

The result presented in Table 2 shows the Mean Absolute Error (MAE) values for the OLS and DWME methods across different dimensions. The MAE values for OLS and DWME methods are relatively similar across the training and testing datasets, with slight variations depending on the dimension. For instance, in the 10x3 dimension, the MAE for OLS_TRAIN is 17.7625, while DWME_TRAIN is 17.6997, indicating minimal difference. Similarly, in the 100x3 dimension, both OLS_TEST (16.6276) and DWME_TEST (16.6341) show close values. Notably, the MAE values tend to decrease for both methods as the dimension increases, with a peak at the 50x3 dimension, where OLS_TEST has a value of 16.2051, and DWME_TEST has a value of 16.3926. The real-life data (192x3) shows similar trends, with OLS_TEST (18.9969) and DWME_TEST (18.9948) values being nearly identical, suggesting that both methods perform comparably well in real-world applications.

Table 3: Comparative R-Square Values for OLS and DWME Methods

Dimension	OLS_TRAIN	OLS_TEST	DWME_TRAIN	DWME_TEST
10 x 3	0.3539	0.7544	0.3535	0.7043
15 x 3	0.2832	0.2142	0.2828	0.2093
20 x 3	0.3067	0.0733	0.3064	0.0651
25 x 3	0.4953	0.0524	0.4947	0.0404
30 x 3	0.5275	0.0212	0.5265	0.0379
40 x 3	0.3775	0.0707	0.3707	0.0604
50 x 3	0.2814	0.4399	0.2765	0.4438
100 x 3	0.3157	0.1101	0.3154	0.1108
200 x 3	0.3065	0.2226	0.3051	0.2405
500 x 3	0.1805	0.3032	0.1797	0.2993
1000 x3	0.2518	0.2172	0.2517	0.2172
Real_Life_data (192 x 3)	0.3067	0.0733	0.3064	0.0651

The R-squared values for both OLS and DWME methods across different dimensions in Table 3 reveal distinct performance patterns. In the OLS_TRAIN dataset, the R-squared values range from 0.1805 to 0.5275, with the highest value observed at the 30x3 dimension (0.5275), indicating a relatively better fit of the model to the training data in larger dimensions. However, the OLS_TEST dataset shows a wider range, from 0.0212 to 0.7544, with the 10x3 dimension achieving the highest test R-squared value (0.7544), suggesting that the model's ability to generalize to unseen data is

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more variable. For the DWME_TRAIN dataset, R-squared values range from 0.1797 to 0.5265, with values closely mirroring the OLS_TRAIN results, indicating stable performance across training sets. In the DWME_TEST dataset, R-squared values range from 0.0379 to 0.7043, with the 10x3 dimension again showing the highest value (0.7043), but the model's performance on test data is generally lower than on training data. This suggests that both methods, especially DWME, struggle with generalization as the dimensions increase, with smaller dimensions performing better in terms of R-squared values.

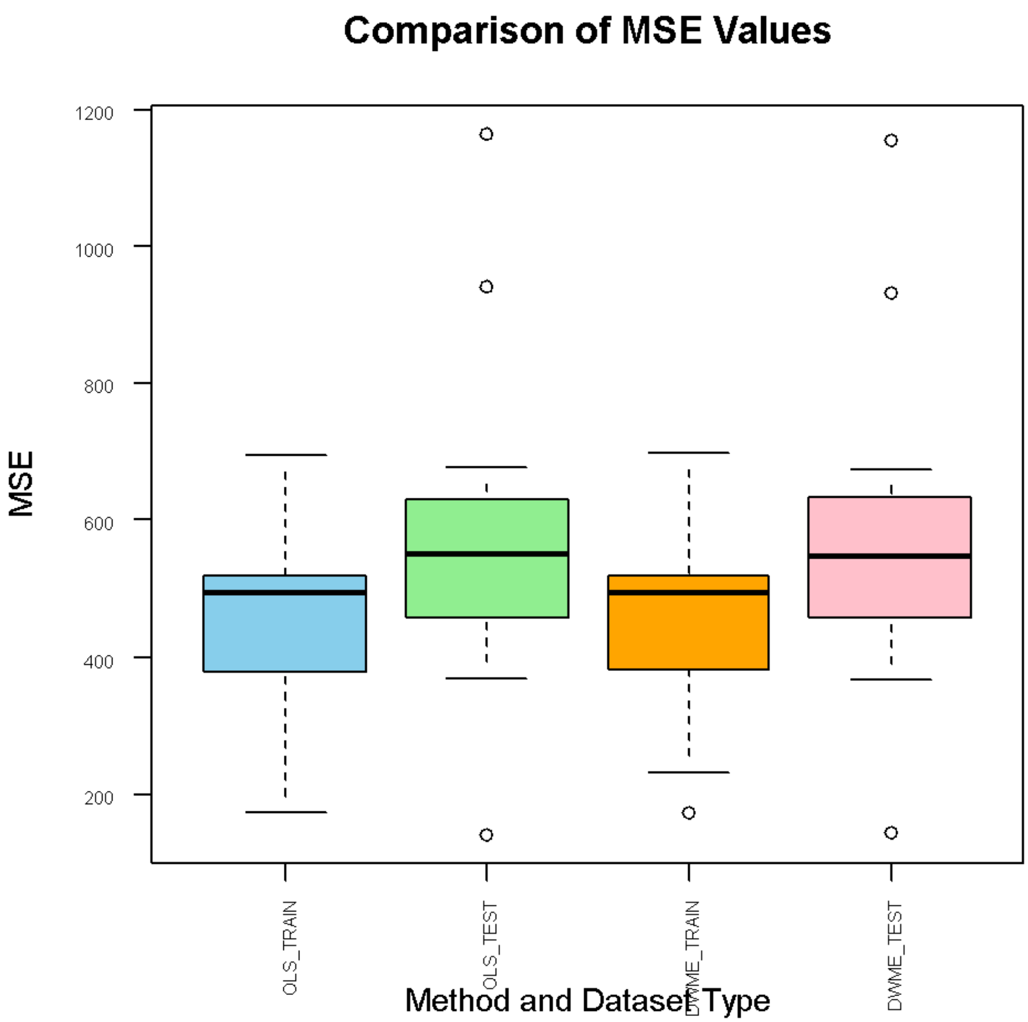

Figure 2: The Boxplot comparing the MSE values of the methods

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The result presented in Figure 1 reveals the performance of OLS and DWME methods in training and testing scenarios based on MSE. For OLS_TRAIN, the MSE ranges from 172.85 to 694.56 with a median of approximately 496.98, showing moderate variability. OLS_TEST exhibits a wider range (142.55 to 1163.18) and a higher median of 543.85, indicating greater sensitivity to test data. Similarly, DWME_TRAIN has an MSE range of 173.03 to 699.27 with a median of 497.25, closely resembling OLS_TRAIN. DWME_TEST shows a range of 144.35 to 1154.54 and a median of 548.68, slightly outperforming OLS_TEST in consistency. Hence, the DWME demonstrates marginally better performance, especially during testing, as reflected by its slightly lower median and comparable range. These indicate the relative robustness of DWME over OLS in this context.

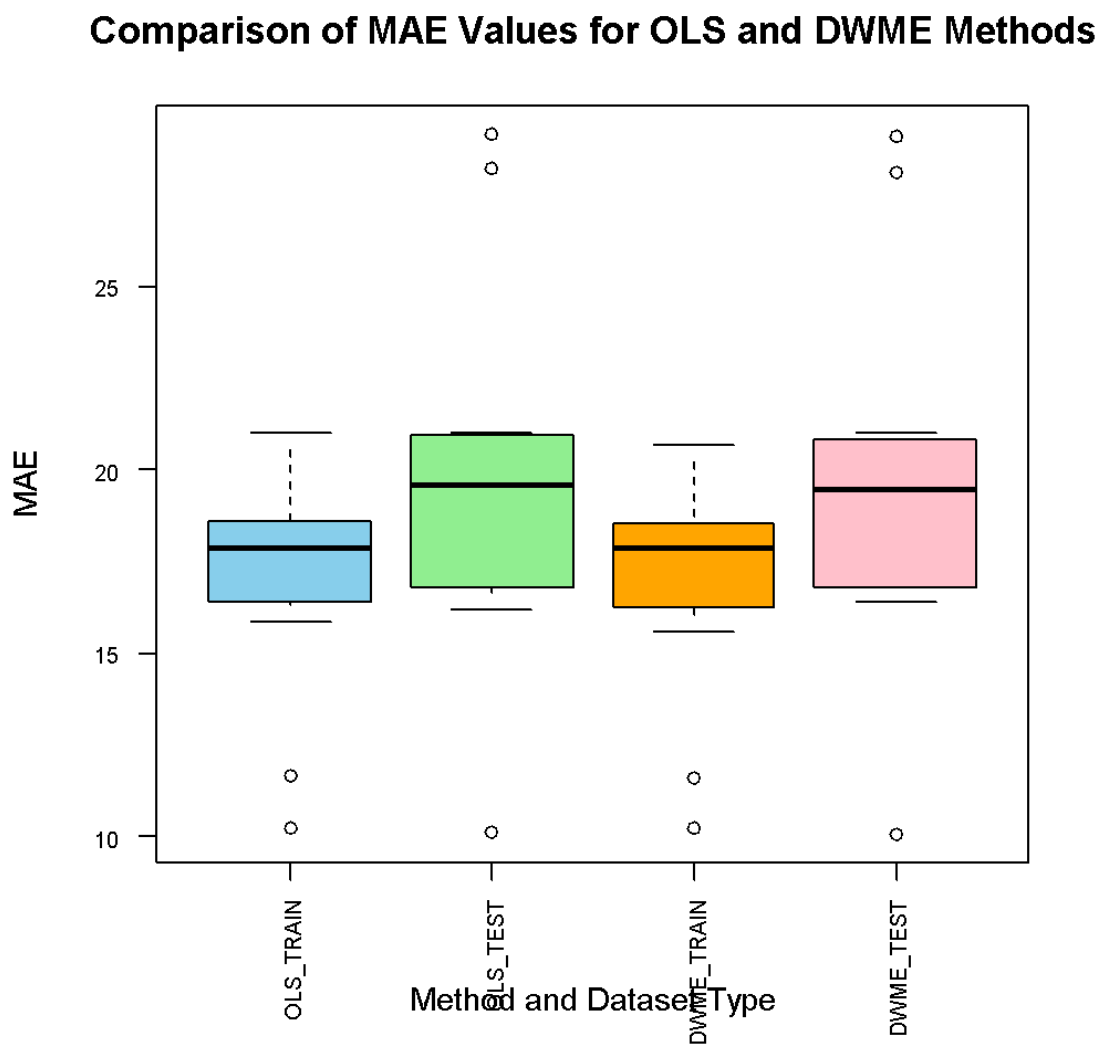

Figure 2: The Boxplot comparing the MAE values of the methods

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The results in Figure 2 show the Mean Absolute Error (MAE) values for both OLS and DWME methods across training and testing datasets. For OLS_TRAIN, the MAE values range from 10.2211 to 20.9893, with a general tendency for values to be lower in the earlier dimensions and higher in the later ones, indicating varying model performance across different dimensions. The OLS_TEST values range from 10.1253 to 29.1384, with the highest value observed in the 10x3 dimension, which indicates that the model's testing performance fluctuates more significantly than its training performance. Similarly, for DWME_TRAIN, the MAE values range from 10.2209 to 20.6825, with relatively consistent results across dimensions, indicating stable performance. In the DWME_TEST dataset, the MAE values range from 10.1003 to 29.1038, showing a similar trend to OLS_TEST, where higher values are observed in the smaller dimensions. Hence, both methods exhibit similar performance in terms of MAE across training and testing sets, with slight differences between the two, particularly in the test datasets.

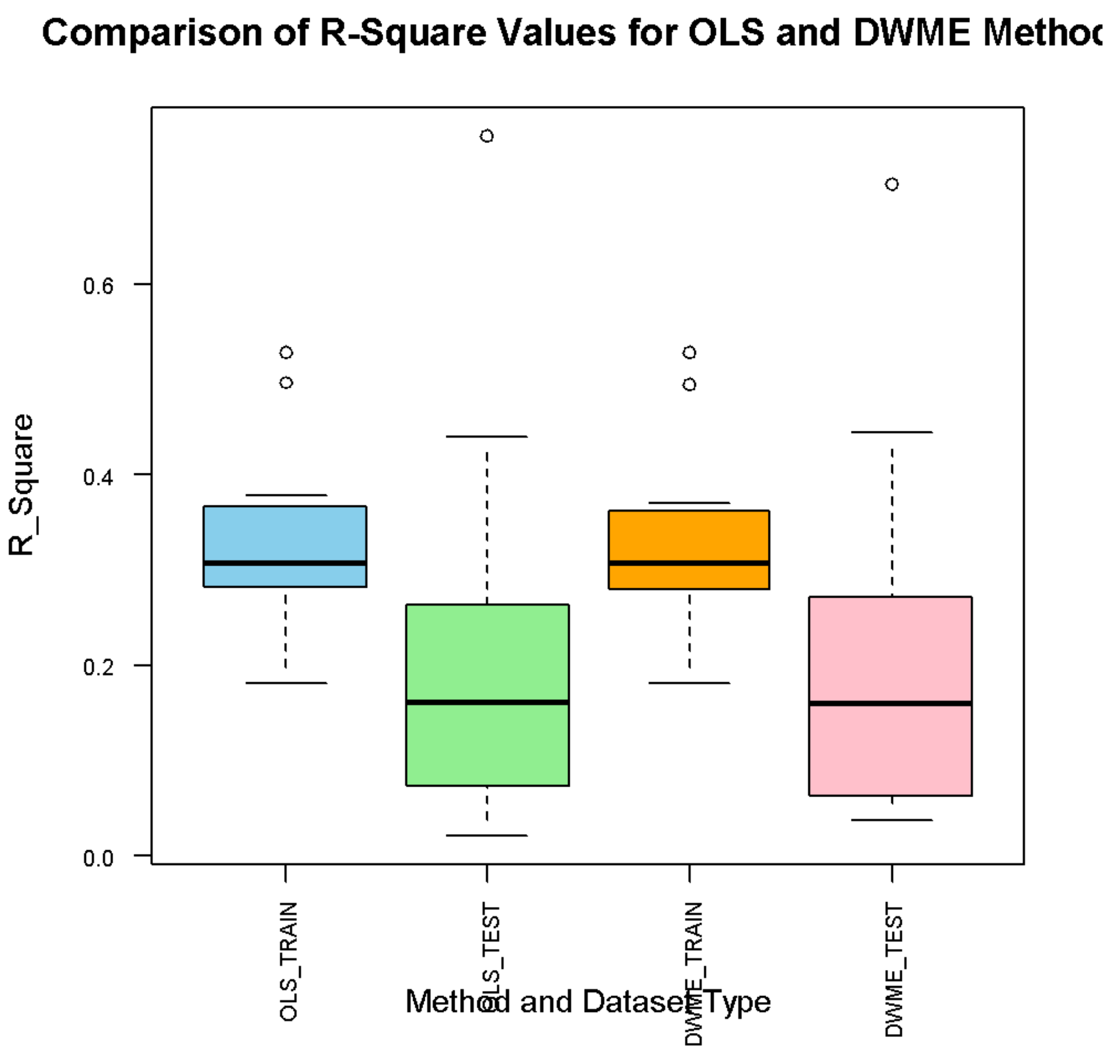

Figure 3: The Boxplot comparing the R-Square values of the methods

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The boxplot values for the R-squared measures of both OLS and DWME methods across training and test datasets in Figure 3 reveal notable trends. For OLS_TRAIN, the R-squared values range from 0.1805 to 0.5275, with the highest value at the 30x3 dimension (0.5275), indicating a relatively better fit for the training data at this dimension. In contrast, the OLS_TEST values show considerable variability, ranging from 0.0212 to 0.7544, with the highest value at the 10x3 dimension (0.7544), suggesting that the model performs better on smaller dimensions for unseen data. Similarly, the DWME_TRAIN values range from 0.1797 to 0.5265, with the best performance again at the 30x3 dimension (0.5265), showing a similar trend to OLS_TRAIN. However, for DWME_TEST, R-squared values range from 0.0379 to 0.7043, with the 10x3 dimension achieving the highest value (0.7043), reflecting better generalization for smaller datasets. These results indicate that both methods show improved performance with smaller dimensions in terms of generalization (test data), but the overall fit is more stable in training data, especially for larger dimensions.

CONCLUSION

This study evaluated the robustness and predictive accuracy of Ordinary Least Squares (OLS) and Double Weighted M-Estimation (DWME) methods for predicting crude oil prices in Nigeria, focusing on their performance across various dataset dimensions. The comparative analysis revealed that both OLS and DWME exhibit comparable MSE values for training data, with DWME slightly outperforming OLS in testing scenarios. This suggests that DWME is marginally more robust in handling unseen data, providing better consistency and generalization in predictive accuracy. The MAE values for both methods were similar across training and testing datasets, with minimal differences. However, both methods show reduced MAE as dataset dimensions increase, indicating improved predictive performance with larger datasets.

Also, both methods demonstrate stable performance in training datasets, with R-squared values peaking at intermediate dimensions (e.g., 30x3). In testing datasets, OLS achieves higher variability in R-squared values, while DWME exhibits slightly better generalization for smaller dimensions. This indicates that both methods perform well in capturing the variance in training data but face challenges in generalizing to test data, particularly as dataset dimensions increase. While both OLS and DWME are effective for predicting crude oil prices, DWME shows a marginal advantage in testing scenarios, indicating better robustness in predictive accuracy. The methods are comparable in terms of MAE and MSE, but DWME demonstrates slightly better consistency across dimensions, particularly for real-life data applications. Hence, DWME offers a marginally more robust alternative to OLS for crude oil price prediction, especially in scenarios requiring higher generalization. Both methods, however, benefit from larger datasets, which enhance their predictive accuracy and stability. These findings provide valuable insights for policymakers and industry stakeholders seeking reliable models for crude oil price forecasting in Nigeria. Future research could explore hybrid methods or alternative weighting schemes to further improve predictive performance.

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