SOLVING VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER USING PERTURBATION COLLOCATION METHOD

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ABSTRACT: *This paper is concerned with the formulation of a scheme via construction of Canonical with Shifted-Chebyshev Polynomials (SCP), for the direct solution of fractional order integro-differential equations (FIDEs). Perturbation collocation method (PCB) is the approximate method developed, to handle a special singular class of fractional multi-order Volterra type for approximation. The process involves the incorporation of perturbation variables otherwise known as parameters, to the given mathematical models under consideration. Systems of equations are evolved, and the embedded unknown constants are sought for.*

INTRODUCTION

Perturbation method is a progressive method in the fields of applied and theoretical mechanics, based on the crucial roles it plays in the development of science and technology, most especially in this twentieth century. The method involves the existence of parameters, popularly called perturbation terms, that transform complex emergent models into less complicated mathematical equations. Hence, getting accurate solutions to the mathematical problems analytically is invariably difficult, even with the use of high performance computational soft-wares such as Mathematica, Matlab, Maple, and so on. Numerical methods are barely inefficient in handling problems akin to this nature, especially the nonlinear ones containing points of singularities. However, it has been verified and ascertained that perturbation method is one of the versatile and efficient analytical techniques for handling both linear and nonlinear problems as described in Nayfeh (1985 & 2000), Hinch (1991), Murdock (1985), Bush (1992), Kahn & Zarmi (1998).

Fractional integro-differential equation (FIDEs) as a kind of fractional calculus, plays significant role in the concept of mathematical modeling of physical situations such as: models to reduce the spread of epidemic diseases and behaviour of electrical circuits e.t.c. It can either be linear or nonlinear, particularly in the fields of image processing, visco-elasticity, heat-thermal system, fluid flow mechanism and solid dynamics in applied sciences and engineering as illustrated in Mohammed et al. (2022). And due to its frequent occurrences, it has been extensively solved by varieties methods such as Yang & Hou (2013) and Khosrow et al. (2013), to mention but few.

Numerical methods that are based on orthogonal polynomials have been utilized by several authors such as Okedayo et al. (2018), Owolanke et al. (2017 & 2019) among others, to provide approximate solutions to both calculus and fractional calculus problems, whereby block methods and collocation methods are adopted in the construction of the numerical schemes. Perturbation method is another approximation method whose reliability has been confirmed in handling fractional calculus and calculus problems. With the aid of the method, complex fractional problems had been tackled, simply by breaking down the equations into less complicated ones which could be achieved via adding perturbation terms in order to alter the original equation. For instance in Fatheah et al. (2017), variational iteration method (VIM) alongside homotopy perturbation method (HPM) are used to solve fractional integro-differential equations of the nonlinear type; the solutions are derived via infinite convergent series. A class of fractional integro-differential equations are also solved in Huda et al. (2024), whereby Laplace transform method is merged with the perturbation iteration algorithm. Similarly, perturbation method in combination with the integral transform method solved a Fredholm type integro-differential equation of fractional order in Mohammed et al. (2022), whereby the results generated by the existing method align with that of the exact solutions.

Furthermore, homotopy perturbation method (HPM) is another powerful tool for the solution of FIDEs. Its significance is verified and confirmed in Oyedepo et al. (2019) in which constructed orthogonal polynomials are used as initial approximation; and the emergence of the coefficients of the homotopy parameter is determined. Another numerical scheme that is based on the shifted Chebyshev polynomial and least square method is illustrated in Taiwo et

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al. (2015) such that the approximate solution is meant to reduce FIDEs into system of equations. Thus in this paper, a reliable and very effective alternative method with the aids of constructed canonical polynomials, is developed to further address the challenges of analyzing models involving FIDEs.

METHODOLOGY

Assuming the fractional integro-differential equations of kth-order is of the form

$$I^q y(x) + A(x)y(x) + B(x)y'^{(x)} + C(x)y''(x) + \cdots + N(x)y^k(x) + \int_0^x \frac{y(s)}{(m-x)} ds = g(x), \quad (5)$$

where,

$$I^q y(x) = \begin{cases} \dfrac{1}{(m-q-1)!} \displaystyle\int_0^x (x-q)^{m-q-1} D^m f(\tau) d\tau, & m-1 < q < m \\ \dfrac{d^m f(x)}{dx^m}, \quad q = m, m \in \aleph \end{cases} \quad (6)$$

Then equation (5) becomes

$$I^q y(x) + A(x)y(x) + B(x)y'^{(x)} + C(x)y''(x) + \cdots + N(x)y^k(x) = G(x) \quad (7)$$

Let a differential operator I be defined as

$$A\frac{d^q}{dx^q} + B\frac{d^2}{dx^2} + C\frac{d^3}{dx^3} + \cdots + N\frac{d^n}{dx^n} \equiv I \quad (8)$$

$$Ix^j = \frac{\Gamma(j+1)}{\Gamma(j+1-q)} x^{j-q} + Ax^j + B(j)x^{j-1} + C(j)(j-1)x^{j-2} + \cdots + N(j)(j-1)(j-2)\ldots(j-k+1)x^{j-k} \quad (9)$$

With the aid of Lanczos (1956)

$$Dp_j(x) = x^j, \qquad j = 0,1,2,\ldots \quad (10)$$

equation (9) is transformed to

$$Ix^j = \frac{\Gamma(j+1)}{\Gamma(j+1-q)} Dp_{j-q}(x) + ADp_j(x) + B(j)Dp_{j-1}(x) + C(j)(j-1)Dp_{j-2}(x) + \cdots + N(j)(j-1)(j-2)\ldots(j-k+1)Dp_{j-k}(x) \quad (11)$$

With the existence of the inverse of I in equation (11), it becomes

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$$x^{j}=\frac{\Gamma(j+1)}{\Gamma(j+1-q)}p_{j-q}(x)+Ap_{j}(x)+B(j)p_{j-1}(x)+C(j)(j-1)p_{j-2}(x)+\cdots+N(j)(j-1)(j-2)\ldots(j-k+1)p_{j-k}(x) \tag{12}$$

Equation (12) can equivalently be written as

$$p_{j}(x)=\frac{1}{A}\{\ x^{j}-\left[\frac{\Gamma(j+1)}{\Gamma(j+1-q)}p_{j-q}(x)+Ap_{j}(x)+B(j)p_{j-1}(x)+C(j)(j-1)p_{j-2}(x)+\cdots+N(j)(j-1)(j-2)\ldots(j-k+1)p_{j-k}(x)\right]\} \tag{13}$$

In furtherance,

$$p_{j}(x)=\frac{1}{A}\{\ x^{j}-\left[\frac{\Gamma(j+1)}{\Gamma(j+1-q)}p_{j-q}(x)+B(j)p_{j-1}(x)+C(j)(j-1)p_{j-2}(x)+\cdots+N(j)(j-1)(j-2)\ldots(j-k+1)p_{j-k}(x)\right]\} \tag{14}$$

is the required canonical polynomial of kth order fractional integro-differential equations. It can therefore be deduced that the

first order is represented as

$$p_{j}(x)=\frac{1}{A}\left\{\ x^{j}-\left[\frac{\Gamma(j+1)}{\Gamma(j+1-q)}p_{j-q}(x)+B(j)p_{j-1}(x)\right]\right\},j=0,1,2,\ldots. \tag{15}$$

the second order is represented as

$$p_{j}(x)=\frac{1}{A}\left\{\ x^{j}-\left[\frac{\Gamma(j+1)}{\Gamma(j+1-\alpha)}p_{j-\alpha}(x)+B(j)p_{j-1}(x)+C(j)(j-1)p_{j-2}(x)\right]\right\},\quad j=0,1,2,\ldots. \tag{16}$$

Perturbed Collocation Method (PCM)

In this method, the set of canonical polynomials generated from the scheme developed in equation (14) will be applied as basis function for the approximation of any given nonlinear and linear fractional integro-differential equations. The concept of **PCM** is the addition to equation (5) a perturbation term $H_n(x)$ which causes the equation

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$$I^q y(x) + A(x)y(x) + B(x)y'^{(x)} + C(x)y''(x) + \cdots + N(x)y^k(x) + \int_0^x \frac{y(s)}{(m-x)} ds = g(x) + H_n(x), a \leq x \leq b \leq \quad (17a)$$

subject to the conditions ${y^k}_n(0) = y_k, k = 0,1,2, \ldots, n-1$ (17b)

to have an exact polynomial solution $y_n(x)$ that satisfies exactly the given conditions

${y^i}_N(0) = \alpha_i, i = 0,1,2, \ldots, n-1$ (18)

where $H_n(x)$ is defined as follow

$H_n(x) = \sum_{k=1}^{N} \tau_k T_{N-k+1}(x), x\epsilon[a,b], (18a)$

$$\alpha_j; \tau_k;\ j = 0,1, \ldots, N; k = 1, \ldots, n, (18b)$$

The choice of $T_{N-k}(x)$ is the set of shifted Chebyshev polynomials defined for $x\epsilon[a,b]$ that characterized the Lanczos tau method (Lanczos, 1956). A form of collocation method used in Owolanke (2019), involving collocating equation (17) at (N+1) equally spaced interior points on $[a,b]$ in addition to the $(n-1)$ conditions of equation (18) which is to be satisfied by $y_n(x)$. These collocation equations lead to (N+n+1) algebraic equations for the unique determination of (N+n+1) parameters.

Solving equation (17)

Shifted Chebyshev Polynomials (SCP)

Chebyshev polynomials belong to a family of orthogonal polynomials in the interval $[-1,1]$. They are widely useful for their good properties in the approximation of functions. The Chebyshev polynomial of the first kind of degree n denoted by T_n is defined as follows

$$T_k(x) = cosk\theta \quad (18)$$

$$\theta = cos^{-1}x \quad (19)$$

The recurrence relation is given by

$$T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x), k \geq 1 \quad (20)$$

Given a fractional order integro-differential equation of the form

$$I^q y(x) + A(x)y(x) + B(x)y'^{(x)} + C(x)y''(x) + \cdots + N(x)y^k(x) + \int_0^x \frac{y(s)}{(m-x)} ds = g(x) + H_z(x), m \leq x \leq n \quad (21)$$

However, for the purpose of this paper, the Chebyshev polynomials valid in the interval $[m,n]$ defined as

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$$T_k(x) = \cos\left(k\cos^{-1}\left[\frac{2x-m+n}{m-n}\right]\right), k \geq 0 \qquad (22)$$

The recurrence relation is given as

$$T_{k+1}(x) = 2\left(\frac{2x-m+n}{m-n}\right)T_k(x) - T_{k-1}(x), k \geq 0 \qquad (23)$$

shifting $[-1,1]$ on $y = ux + v$ to $[m,n]$ gives

$$m = -u + v$$

and

$$n = u + v$$

Hence, $v = \frac{m+n}{2}$, $u = \frac{n-m}{2}$, and y= $(\frac{n-m}{2})x + (\frac{m+n}{2})$

Thus, it implies that the shifted function for $y = ux + v$ is y= $(\frac{n-m}{2})x + (\frac{m+n}{2})$

In furtherance, using the interval $[0,1]$ accordingly from equation (23), the following terms are obtained recursively

$$T_1(x) = 2x - 1$$

$$T_2(x) = 8x^2 - 8x + 1$$
$$T_3(x) = 32x^3 - 48x^2 + 18x - 1$$

$$T_4(x) = 128x^4 - 256x^3 + 160x^2 - 32x + 1$$

The above polynomials are called Shifted Chebyshev polynomials.

Furthermore, to solve equations (17a) and (17b), the equations are collocated at points $x = x_i$, where

$x_i = a + \frac{(b-a)i}{N-n+1}$, i = 1,2, ..., N − n (24)

Hence, (N +n) algebraic equations are obtained in (N + n + 1) unknown constants $(a_i, i = 0,1,2,\ldots,n;\ \tau_k, k = 1,2,\ldots,n)$. Extra equations are obtained from the given conditions. Altogether, $(n + 2N)$ algebraic equations in $(n + 2N)$ unknowns are determined.

Numerical Examples

The following examples with respect to the exact solutions are verified with the new method

1. $\frac{d^2y}{dx^2} + \frac{d^{3/2}y}{dx^{3/2}} - \int_0^x \frac{y(t)}{\sqrt{x-t}}dt = 2 + 12x^2 + 2.256758334x^{0.5} + 7.221626669x^{2.5} - \frac{16}{15}x^{5/2} - \frac{256}{315}x^{9/2}, y'(0) = 0, y(0) = 0$

Exact solution : $y(x) = x^4 + x^2$

Solution:

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The constructed canonical polynomials using equation (16) is

$\sum_{i=0}^{n} a_i p_i(x)\,, n = 14$

And the perturbation in terms of the shifted Chebyshev polynomials from equation (18, 18a, 18b) is

$$H_n(x) = \sum_{k=1}^{N} \tau_k T_{N-k+1}(x), x\epsilon[a,b]$$

produces

$$\begin{aligned} H(x) = \tau_1 - 392\tau_1 x + 25480\tau_1 x^2 - 652288\tau_1 x^3 + 8712704\tau_1 x^4 - 69701632\tau_1 x^5 \\ + 361181184\tau_1 x^6 - 1270087680\tau_1 x^7 + 3111714816\tau_1 x^8 \\ - 5369233408\tau_1 x^9 + 64995983336\tau_1 x^{10} - 5402263552\tau_1 x^{11} \\ + 293612800\tau_1 x^{12} - 939524096\tau_1 x^{13} + 134217728\tau_1 x^{14} - \tau_2 \\ + 338\tau_2 x - 18928\tau_2 x^2 + 416416\tau_2 x^3 - 4759040\tau_2 x^4 + 32361472\tau_2 x^5 \\ - 141213696\tau_2 x^6 + 412778496\tau_2 x^7 - 825556992\tau_2 x^8 \\ + 1133117440\tau_2 x^9 - 1049624576\tau_2 x^{10} + 627048448\tau_2 x^{11} \\ - 218103808\tau_2 x^{12} + 33554432\tau_2 x^{13} \end{aligned}$$

where,

τ's and a's are constants to be determined;

$N = 14$ of the shifted Chebyshev polynomial,

And the fractional derivative for $\frac{d^\alpha y}{dx^\alpha}$ is obtained using the Caputo method

$$\frac{1}{\Gamma(n-\alpha)} \int_0^x (x-u)^{n-\alpha-1} \frac{d^n}{du^n} f(u)du$$

Here follows the table of results and the graph in comparison with the exact solution.

Table 1:

x	Exact	Approximate	Absolute Error
0.10	0.01010000	0.00938269	7.1731e-04
0.20	0.04160000	0.04021944	1.3806e-03
0.30	0.09810000	0.09611383	1.9862e-03
0.40	0.18560000	0.18304166	2.5583e-03
0.50	0.31250000	0.30938891	3.1111e-03

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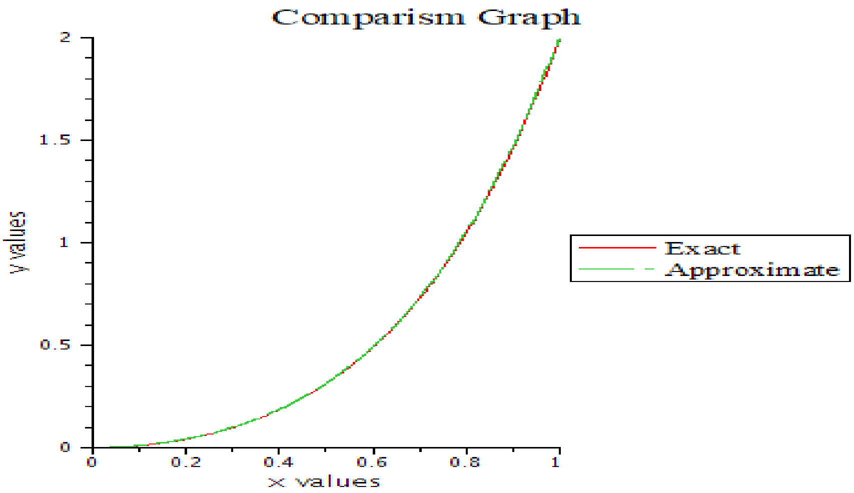
0.60	0.48960000	0.48594463	3.6554e-03
0.70	0.73010000	0.72589770	4.2023e-03
0.80	1.04960000	1.04482976	4.7702e-03
0.90	1.46610000	1.46068896	5.4110e-03
1.00	2.00000000	1.99372382	6.2762e-03

Graph 1



2. Here, a fourth order fractional integro differential equation is considered

$$\frac{d^4y}{dx^4}+\frac{d^{7/2}y}{dx^{7/2}}+\frac{d^3y}{dx^3}+\frac{1}{x^2}\frac{d^2y}{dx^2}-x\frac{dy}{dx}+\int_0^x\frac{y(t)}{\sqrt{x-t}}dt=\frac{256}{315}x^{9/2}-5x^4+27.08110001x^{0.5}+24x+26\,,y'(0)=0,y(0)=0$$

Exact solution : $y(x)=x^4$

Solution:

The constructed canonical polynomials using equation (16) is

$\sum_{i=0}^{n} a_i p_i(x)\,,n=13$

While the perturbation in terms of the shifted Chebyshev polynomials from equation (18, 18a, 18b) is

$$H_n(x)=\sum_{k=1}^{N}\tau_k T_{N-k+1}(x),x\epsilon[a,b]$$

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produces

$$\begin{aligned}H(x) = &-\tau_1 + 338\tau_1 x - 18928\tau_1 x^2 + 416416\tau_1 x^3 - 4759040\tau_1 x^4 + 32361472\tau_1 x^5 \\ &- 141213696\tau_1 x^6 + 412778496\tau_1 x^7 - 825556992\tau_1 x^8 \\ &+ 1133117440\tau_1 x^9 - 1049624576\tau_1 x^{10} + 627048448\tau_1 x^{11} \\ &- 218103808\tau_1 x^{12} + 33554432\tau_1 x^{13} + \tau_2 - 288\tau_2 x + 13728\tau_2 x^2 \\ &- 256256\tau_2 x^3 + 2471040\tau_2 x^4 - 14057472\tau_2 x^5 \\ &+ 50692096\tau_2 x^6 - 120324096\tau_2 x^7 + 190513152\tau_2 x^8 - 199229440\tau_2 x^9 \\ &+ 132120576\tau_2 x^{10} - 50331648\tau_2 x^{11} + 8388608\tau_2 x^{12} - \tau_3 + 242\tau_3 x \\ &- 9680\tau_3 x^2 + 151008\tau_3 x^3 - 1208064\tau_3 x^4 + 5637632\tau_3 x^5 \\ &- 16400384\tau_3 x^6 + 30638080\tau_3 x^7 - 36765696\tau_3 x^8 + 27394048\tau_3 x^9 \\ &- 11534336\tau_3 x^{10} + 2097152\tau_3 x^{11} + \tau_4 - 200\tau_4 x + 6600\tau_4 x^2 \\ &- 84480\tau_4 x^3 + 549120\tau_4 x^4 - 2050048\tau_4 x^5 + 4659200\tau_4 x^6 \\ &- 6553600\tau_4 x^7 + 5570560\tau_4 x^8 - 2621440\tau_4 x^9 + 524288\tau_4 x^{10}\end{aligned}$$

where,

τ's and a's are constants to be determined

$N = 13$ of the shifted Chebyshev polynomial.

And the fractional derivative for $\frac{d^\alpha y}{dx^\alpha}$ is obtained using the Caputo method

$$\frac{1}{\Gamma(n-\alpha)}\int_0^x (x-u)^{n-\alpha-1}\frac{d^n}{du^n}f(u)du$$

Here follows the table of results and the graph in comparison with the exact solution.

Table 2:

x	Exact	Approximate	Absolute Error
0.10	0.0001	0.00001010	8.9899e-05
0.20	0.0016	0.00064144	9.5856e-04
0.30	0.0081	0.00533917	2.7608e-03
0.40	0.0256	0.01990924	5.6908e-03
0.50	0.0625	0.05114524	1.1355e-02
0.60	0.1296	0.10990912	1.9691e-02
0.70	0.2401	0.21091470	2.9185e-02

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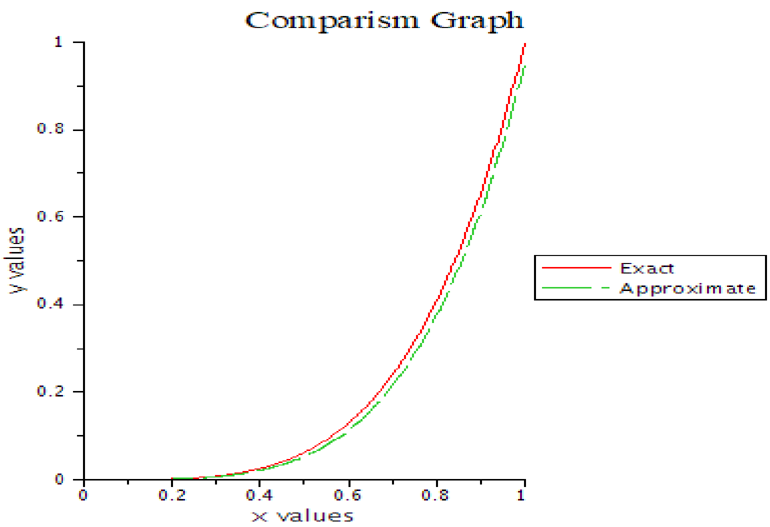
0.80	0.4096	0.36917560	4.0424e-02
0.90	0.6561	0.60437480	5.1725e-02
1.00	1.0000	0.94483765	5.5162e-02

Graph 2:



CONCLUSION

The study is focused to reveal the novelty in the significance use of perturbation collocation method at solving fractional integro-differential equations. The tasks involves shifting an interval agreeing with the Chebyshev polynomials to another interval in order to obtain a set of polynomials known as Shifted Chebyshev polynomials. As some fractional mathematical problems are investigated to evidently prove the efficacy of the method, satisfactory outcomes are yielded in form of empirical tables and graphs, when compared with exact solutions.

REFERENCES

1. Ayodele Olakiitan Owolanke, Ohi Uwaheren and Friday Ogbenerukevwe Obarhua (2017). An eight order two step Taylor series algorithm for the numerical solutions of initial value problems of second order ordinary differential equations. Open Access Library Journal. Volume 4, e3486.
2. Bush, A.W. (1992). Perturbation methods for Engineers and Scientist. CRC Press Library of Engineering Mathematics. CRC Press, Boca Raton, Florida.
3. Fatheah Alhendi, Wafa Shammackh, Hind Al-Badrani (2017). Numerical solutions for quadratic integro-differential equations fractional orders. Open Journal of Applied sciences, Vol. (7): 157-170.

Article DOI: 10.52589/AJMSS-F6NAYRAE
DOI URL: https://doi.org/10.52589/AJMSS-F6NAYRAE

4. Hinch, E.J. (1991). Perturbation methods, Cambridge texts in Applied Mathematics. Cambridge University Press, Cambridge.
5. Huda A. Salim, Bashaer M. Abdali, Fajir A. Abdulkhaleq, Osama H. Mohammed (2024). Perturbation iteration transform method for solving fractional order integro-differential equations. Partial differential equations in Applied mathematics, 11.
6. Kahn, B.P. and Zarmi, Y. Nonlinear dynamics (1998). Exploration through normed forms. John Wiley and Sons, Inc., New York.
7. Khosrow, M., Monireh, N.S. and Azadeh, O (2013). Numerical solutions of fractional integro differential equations using Cubic B-spline wavelets. Proceedings of the world congress on Engineering London U.K. Vol. (1).
8. Mohammed, F., Kazeem and Ali Al-Fayadh (2022). Solving Fredholm integro-differential equations fractional order using Sawi homotopy perturbation method. Journal of Physics. 2322 (2022) 012056.
9. Murdock, J.A. (1985). Perturbation theory and methods. John Wiley and Sons, New York.
10. Nayfeh, A.H. (1985). Introduction to perturbation techniques. John Wiley and Sons, New York.
11. Nayfeh, A.H. (1985). Problems in perturbation. John Wiley and Sons, New York.
12. Nayfeh, A.H. (2000). Perturbation method. John Wiley and Sons, New York.
13. Owolanke, A.O., Uweheren, O.A, and Taiwo, O.A. (2019). Exponentially fitted collocation method for solving singular multi-order fractional integro-differential equations. Journal of Research and Development, Vol.6, No,1&2; pp.10-23.
14. Oyedepo T., Uwaheren O.A., Okperhie E.p. and Peter O.J. (2019). Solution of fractional integro-differential equations modified homotopy perturbation technique and constructed orthogonal polynomials as basis functions. Journal of Science Technology and Education, 7(3): 157-164.
15. Taiwo, O. Adebayo and Fesojaye, M. Olalekan (2015). Perturbation least squares Chebyshev method for solving fractional order integro-differential equations. Journal of Theoretical Mathematics and Applications, Vol. 5(4): 37-47.
16. Toyin Gideon Okedayo, Ayodele Olakiitan Owolanke, Olaseni Taiwo Amumeji and Muyiwa Philip Adesuyi (2018). Modified Legendre collocation block method for solving initial value problems of first order ordinary differential equations. Open Access Library Journal, Volume 5, e4565.
17. T.G. Okedayo, O.T. Amumeji and A.O. Owolanke (2018). Modified Laguerre collocation block method for solving initial value problems of first order ordinary differential equations. Journal of Advances in Mathematics and Computer Science. 29(2):1-13.
18. Yang, C. and Hou J. (2013). Numerical solution of integro-differential equation of fractional order by Laplace decomposition method. WSEAS Transaction on Mathematics. 12(12): 2224-2880.

Article DOI: 10.52589/AJMSS-F6NAYRAE
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