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ANALYSIS OF A PRODUCTION INVENTORY MODEL WITH LINEAR TIME PRODUCTION RATE, HOLDING COST AND STOCK DEPENDENT DEMAND.

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ABSTRACT: *This paper focused on the analysis of a production inventory where the production rate and holding cost are linearly dependent, while the demand is stock dependent demand rate. The production inventory model is formulated using system of differential equations and integral calculus including initial boundary/matching conditions and integral calculus were also used to analyse the inventory problem. These differential equations were solved to give the best cycle length that will minimize the inventory cost per unit time. A Mathematical theorem and all its proof is presented to established the convexity of the cost function. A numerical example is also given to demonstrate the applicability of the model developed accompanied by sensitivity analysis to see the effects of the parameter changes.*

KEYWORDS: Linear Holding cost, Production, Inventory, Demand, Stock dependent, Rate.

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INTRODUCTION

In the realm of inventory management, efficient production and storage are critical to maintaining profitability and meeting customer demands. Traditionally inventory models are often assumed the key factors such as production rate, demand and holding costs are independent of each other. However, in many real-world scenariors, these factors are interdependent, requiring a more dynamic approach to inventory control. Misra[6] was the first to developed a production lot size model for deteriorating inventories and considered both constant and variable rate of deterioration and then obtain the approximate expression for the production lot size problem. Shah and Jaiswal [11] derived results that are similar to those of Misra [6] for a constant deterioration and introduced backlogging. They assumed the average carrying inventory to be approximately one half the maximum inventory model and obtain approximate expression for the optimal production lot size, the production cycle time and the inventory cycle time.

Wee [13] developed a production lot size model for deteriorating items with constant production and demand rate with partial backlogging. The demand rate, production rate, and deterioration are assume to be function of time. Liao [4] established a production inventory for deteriorating items under the conditions of the supplier providing the retailer with retail credit and with no shortages. Monad [8] formulated a production inventory model for imperfect production considering the influence of demand in the market with variable production unit cost. Baraula and Sani [1] developed an Economic production quantity (EPQ) model for delayed deteriorating items with stock dependent demand rate and linear time depending holding cost. The holding cost per unit of time was a constant. They also extended the model and considered the demand to be linear level dependent in a linear function form during and after production.The holding cost is linearly dependent on time. The goal of the model was to determine the optimal replenishment cycle-time and to minimize the variable cost.

Cheng [2] studied on Economic production inventory model and considered a two-storage facilities system with detective process for variable production rate. Shiraful Islam and Shanfuddin [12], discussed a production inventory model which considered constant production rate and constant holding cost with little mount of decay. They assumed that the demand is linear level dependent during production and after production. The demand during production is different from the demand after production.

Mohammed et al. [7] discoursed an overtime production inventory model for decaying inventories with nonlinear priceand linear dependent demand. The model assumed the deterioration rate to be constant and Lingo sofware was used to solved the formulated equation to obtain the optimal solution of the system. Prerna et al. [10] constructed an inventory model to handle detriorating items by reworking of the processed items to make the product attractive in themarket. The model considered the demand to be dependent on the price of the items as well as the advertisement of the product. The energy that was released during production and the cost of the emission of carbon was also consideredby the model.

Pankey and Pitus [9] studied an imperfect production system to considered price and time-dependent demand for non-delay decaying items. They noted that all the manufacturing industries aim to manufacture perfect items but after some time it stands to produce some defective or imperfect items due to overuse of the machine. The imperfect items are sold at adiscount rate. Madaki and Sani [5] developed a production inventory model with linear time

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dependent Production rate, Linear level dependent demand and constant holding cost. The model considered a small amount of decay wihout any shortage. Harrion et al. [2] Analysed and inventory model for time - dependent linear demand rate, three levels of production withshortage. They investigate the continous-production inventory problem for a single product at three facing a constant deterioration.

This paper explores an advance production rate where the production rate and the holding cost are function of time. The model also considered small amount of decay during and after production . Production bebins at a buffer stock level.

Assumptions

The production rate $p(t) = p_0 + \beta t$ is a linear funcion of time and always greater than the demand rate. The rate of decay ω is constant and small. The demand rate during production is given by $D(t) = \alpha + bI(t)$ where α and b are constants and satisfy the condition that $p_0 + \beta t > \alpha + bI(t)$.

The demand rate after production is $\gamma + \rho I(t)$ where γ and ρ are constants and assumed to be greater than the demand during production. Production starts with a buffer stocks and shortages are not allowed. Invetory is hignest at the end of production and after this point the inventory depletesdue to demand and deterioration.

Notation

$I(t)$ = inventory level at any instant t.

I_{1h} = Holding cost for undercayed inventory for the period $t = 0$ to $t = t_1$

I_{2h} = Holding cost for the decayed inventory for the period $t = 0$ to $t = T_1$

D_1 = Holding cost for deteriorated inventory for the period $t = 0$ to $t = t_1$ D_2 = Holding cost for deteriorated inventory for the period $t = 0$ to $t = T_1$ dt = very small potion of intant t.

K_0 = Set up cost

$H = H_0 + ht$ the Linear holding cost

$TC = TC(T_1)$ = Total average inventory cost per unit time.

t_1 = Time whom inventory gets to the maximum level

T_1 = Total cycle lenght

Q_1^*= Optimal order quantity

t_1^*= Optimal time for a maximun inventory T_1^*= Optimal order onterval.

$TC(T_1)^*$ = Optiamal average inventory coat per unit time. Q and Q_1 are the buffer stock and maximum inventory level respectively.

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Model Formulation

The production starts with a buffer stock from the begining of the cycle at $t = 0$ where the production rate $P_0 + \beta t$ is a linear function of time. The inventory changes (increase) at the rate of $P_0 + \beta t - \alpha - bI(t) - \omega I(t)$ between $t = 0$ to $t = t_1$. The market demand is $\alpha + bI(t)$ and $\omega I(t)$ is the decay of $I(t)$ inventory at any instant t. By consisting the above facts, we can formulate the differential equations as below.

$$I(t+dt) = I(t) + \ p_0 + \beta \ t - \alpha - bI(t) - \omega I(t) \quad (1)$$

$$\lim_{dt\to\ 0} \frac{I(t+dt)-I(t)}{dt} = p_0 + \beta \ t - \alpha - bI(t) - \omega I(t)$$

$$\frac{dI(t)}{dt} + \omega I(t) = P_0 + \beta \ t - \alpha - bI(t)$$

$$\frac{dI(t)}{dt} + (\omega + b)I(t) = P_0 + \beta \ t - \alpha$$

$$I(t) = \frac{1}{e^{(\omega+b)t}}\{(p_0 + \beta \ - \alpha)\frac{e(\omega+b)t}{(\omega+b)} - \beta \ \frac{e^{(\omega+b)t}}{(\omega+b)^2} + A\}$$

$$I(t) = \frac{P_0 + \beta \ t - \alpha}{\omega + b} - \frac{\beta}{(\omega+b)^2} + Ae^{-(\omega+b)t}$$

which is the general solution of the differential equations. Applying the following initial

condition

$I(t) = Q$ and $t = 0$. we get, Therefore

$$I(t) = \frac{p_0 + \beta \ t - \alpha}{\omega + b} - \frac{\beta}{(\omega+b)^2} + (Q - \frac{p_0 - \alpha}{(\omega b)} + \frac{\beta}{(\omega+b)^2})e^{-(\omega+b)t} \quad (2)$$

From other boundary/matchings condition that is at $t = t_1, I(t) = Q_1$ taking up to the first degree of ω, we get

$$Q_1 = \frac{P_0 + \beta \ t_1 - \alpha}{\omega + b} - \frac{\beta}{(\omega+b)^2}\{Q - \frac{P_0 - \alpha}{\omega + b} + \frac{\omega}{(\omega+b)^2}\}\{1 - (\omega+b)t_1\} \quad (3)$$

Usin equation (4) and considering the total inventory in the period $t = 0$ to $t = t_1$ and consisting up to the second degree of ω

$$Q_1 = Q + (-Q(\omega + b))t_1 + (P_0 - \alpha)t_1 \quad (4)$$

$$I_{1h} = \int_0^{t_1}(H_0 + ht)I(t)dt = \int_0^{t_1}(H_0 + ht[\frac{p_0-\alpha}{\omega+b} + \frac{\beta \ t}{(\omega+b)} - \frac{\beta}{(\omega+b)^2} + (Q - \frac{p_0-\alpha}{\omega+b} + \frac{\beta}{(\omega+b)^2})e^{-}(\omega+b)t]dt$$

$$h_{1h} = \frac{H_0(P_0-\alpha)t}{\omega+b} + \frac{H\beta \ t_1^2}{2\omega+b} - \frac{H\beta \ t_1}{(\omega+b)^2} + H_0(Q - \frac{P_0-\alpha}{\omega+b}\frac{\beta}{(\omega+b)^2})\{t_1 - \frac{(\omega+b)t_1^2}{2}\} + \frac{h(p_0-\alpha)t_1^2}{2(\omega+b)} + \frac{\beta \ t_1^3}{3(\omega+b)} - \frac{\beta \ t_1^2}{2(\omega+b)^2} + (Q - \frac{P_0-\alpha}{\omega+b} + \frac{\beta}{(\omega+b)^2})\{ht_1[t_1 - \frac{(\omega+b)t^2}{2}] - h[\frac{-t_1}{\omega+b} + \frac{t_1^2}{2}]\}$$

$$I_{1h} = H_0Qt_1 - \frac{H_0Q(\omega+b)t_1^2}{2} - \frac{H_0(p_0-\alpha)t_1^2}{2} + \frac{\beta \ t_1^3}{3(\omega+b)} + \frac{hQt_1^2}{2} - \frac{hQ(\omega+b)t_1^2}{2} + \frac{hQt_1}{(\omega+b)} + \frac{h(p_0-\alpha)t_1^3}{2} - \frac{h(p_0-\alpha)t_1}{(\omega+b)^2} - \frac{\beta \ t_1^3}{2(\omega+b)^2} + \frac{\beta \ t_1}{(\omega+b)^3} \quad (5)$$

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Also, on the other hand this inventory changes at the rate of $\gamma + \rho I(t) + \omega I(t)$. The demand after production is assumed to be greater than the demand during production. The inventory depletes to the level of buffer stock due to demand and deterioration. The differential equations is obtained as usual

$(t + dt) = I(t) + \{-\gamma - \rho I(t) - \omega I(t)\}dt$

$(t + dt) - I(t) = \{-\gamma - \rho I(t) - \omega I(t)\}dt$

$\lim_{dt\to 0} \frac{I(t+dt-It)}{dt} = -\gamma - \rho I(t) - \omega I(t)$

$$I(t) = \frac{-\gamma}{\omega+\rho} + (Q + \frac{\gamma}{\omega+\rho})e^{(\omega+\rho)(T_1-t)} \qquad (6)$$

Using initial/matching condition $I(t) = Q_1$ when $t = t_1$ considering to first term at ω to obtain

$$Q_1 = \frac{-\gamma}{\omega+\rho} + (Q + \frac{\gamma}{\omega+\rho})e^{(\omega+\rho)(T_1-t_1)} \qquad (7)$$

$$Q_1 = Q + \{\gamma + Q(\omega + \rho)\}(T_1 - t_1) \qquad (8)$$

Using equation (9) to get to get the total holding cost for the inventory during $t = t_1$ to

$t = T_1$

$$I_{2h} = \int_{t_1}^{T_1}(H_0 + ht)I(t)dt = \int_{t_1}^{T_1}(H_0 + ht)[\tfrac{-\gamma}{\omega+\rho} + \{Q + \tfrac{\gamma}{\omega+\rho}\}e^{(\omega+\rho)(T_1-t_1)}]dt.$$

$$I_{2h} = [-\tfrac{(H_0\gamma)t}{\omega+\rho} + H_0(Q+\tfrac{\gamma}{\omega+\rho})(\tfrac{e^{(\omega+\rho)(T_1-t_1)}}{\omega+\rho}) + \tfrac{h(-\gamma)t^2}{2(\omega+\rho)} + (Q+\tfrac{\gamma}{\omega+\rho})\{\tfrac{hte^{(\omega+\rho)(T_1-t_1)}}{-(\omega+\rho)} - \tfrac{he^{(\omega+\rho)(T_1-t))}}{(\omega+\rho)^2}\}]$$

$$I_{2h} = [\tfrac{[-H_0\omega\gamma t}{\omega+\rho} + H_0(Q+\tfrac{\gamma}{\omega+\rho})(e^{(\omega+\rho)(T_1-t)}) + \tfrac{h(\gamma)t^2}{2(\omega+\rho)} + (Q+\tfrac{\gamma}{\omega+\rho})\{\tfrac{hte^{(\omega+\rho)(T_1-t)}}{-(\omega+\rho)} - \tfrac{h(e^{(\omega+\rho)(T_1-t)}}{(\omega+\rho)^2}\}]_{t_1}^{T_1}$$

$$I_{2h} = -\tfrac{H_0\gamma(T_1-t_1)}{\omega+\rho} + H_0(Q+\tfrac{\gamma}{\omega+\rho})\{\tfrac{e^{(\omega+\rho)(T_1-T_1)} - e^{(\omega+\rho)(T_1-t_1)}}{-(\omega+\rho)}\} + h\tfrac{(-\gamma)}{(\omega+\rho)}\tfrac{(T_1^2-t_1^2)}{(2)} + (Q+\tfrac{\gamma}{\omega+\rho}\{h(T_1 - t_1)(\tfrac{e^{(\omega+\rho)(T_1-t_1)} - e^{(\omega+\rho)(T_1-t_1)}}{-(\omega+\rho)})\} - h(\tfrac{e^{(\omega+\rho)(T_1-t_1)} - e^{(\omega+\rho)(T_1-t_1)}}{-(\omega+\rho)^2})\}$$

$$I_{2h} = -\frac{H_0\gamma(T_1-t_1)}{\omega+\rho} + H_0Q(T_1-t_1) + H_0(\frac{\gamma}{\omega+\rho})(T_1-t_1) - h(\frac{\gamma}{\omega+\rho})\frac{(T_1^2-t_1^2)}{2} + hQ(T_1^2 - 2T_1t_1 + t_1^2) + \frac{hQ(T_1-t_1)}{\omega+\rho} + h(\frac{\gamma}{\omega+\rho})(T_1^2 - 2T_1t_1 + t_1^2) + \frac{h\gamma(T_1-t_1)}{(\omega+\rho)^2} \qquad (9)$$

we equate equation (5) and (8)

$Q + (-Q(\omega + b))t_1 + (P_o - \alpha)t_1 = Q + \{\gamma + (\omega + \rho)\}(T_1 - t_1)$

$$t_1 = \frac{(\gamma + Q(\omega+\rho))T_1}{P_0 - \alpha + \gamma + Q(-(\omega+b) + (\omega+\rho))} \qquad (10)$$

$$V = \frac{(\gamma + Q(\omega+\rho))}{P_0 - \alpha + \gamma + Q(-(\omega+b) + (\omega+\rho))}$$

Let (11)

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Therefore,

$$t_1 = V\,T_1 \tag{12}$$

The total average cost unit cost is giving as

$$TC(T_1) = \frac{K_0 + I_{1h} + I_{2h}}{T_1} \tag{13}$$

we now substitute(5),(9) in to equation (13) yields

$$TC(T_1) = \frac{K_0}{T_1}[H_0Qt_1 - \frac{H_0Q(\omega+b)t_1^2}{2} - \frac{H_0Q(P_0-\alpha)t_1^2}{2} + \frac{\beta\ t_1^3}{3(\omega+b)} + \frac{hQt_1^2}{2} - \frac{hQ(\omega+b)t_1^2}{2} + \frac{hQt_1}{\omega+b} + \frac{h(P_0-\alpha)t_1^2}{2} - \frac{h(P_0-\alpha)t_1}{(\omega+b)^2} - \frac{\beta\ t_1^3}{2(\omega+b)^2} + \frac{\beta\ t_1}{(\omega+b)^3} - \frac{H_0\gamma(T_1-t_1)}{\omega+\rho} H_0Q(T_1-t_1) + H_0(\frac{\gamma}{(\omega+\rho)}(T_1-t_1) + h(-\frac{\gamma}{\omega+\rho})(\frac{T_1^2-t_1^2}{2}) + hQ(T_1^2-2T_1t_1+t_1^2) + \frac{hQ(T_1-t_1)}{\omega+\rho} + h(\frac{\gamma}{\omega+\rho})(T_1^2-2T_1t_1+t_1^2) + \frac{h\gamma(T_1-t_1)}{(\omega+\rho)^2}]$$

By substituting $t_1 = V\,T_1$

$$TC(T_1) = \frac{K_0}{T_1} + \frac{H_0QVT_1}{T_1} - \frac{H_0Q(\omega+b)V^2T_1^2}{2T_1} + \frac{H_0(P_0-\alpha)V^2T_1^2}{2T_1} + \frac{\beta\ V^3T_1^3}{3(\omega+b)T_1} + \frac{hQV^2T_1^2}{(2T_1)} - \frac{hQ(\omega+b)V^2T_1^2)}{2T_1} + \frac{hQVT_1}{(\omega+b)T_1} + \frac{h(P_0-\alpha)V^3T_1^3}{2T_1} - \frac{h(P_0-\alpha)VT_1}{(\omega+b)^2T_1} - \frac{\beta\ V^3T_1^3}{2(\omega+b)^2T_1} + \frac{\beta\ VT_1}{(\omega+b)^3} - \frac{H_0\gamma(1-V)T_1}{(\omega+\rho)T_1} + \frac{H_0Q(1-V)T_1}{T_1} + \frac{H)_0\gamma(1-V)T_1}{(\omega+\rho)T_1} + \frac{h(-\gamma)(1-V^2)T_1}{2(\omega+\rho)T_1}\frac{hQ(1-2V+V^2)T_1^2}{T_1} + \frac{h\gamma(1-V)T_1}{(\omega+\rho)^2} \tag{14}$$

$$TC(T_1) = \frac{K_0}{T_1} + H_0QV - \frac{H_0Q(\omega+b)V^2T_1}{2} + \frac{H_0(P_0-\alpha)V^2T_1}{2} + \frac{\beta\ V^3T_1^2}{3(\omega+b)} - \frac{hQV^2T_1}{2} + hQV^2T_1 - \frac{hQ(\omega+b)V^3T_1^2}{2} + \frac{hQV}{\omega+b} + \frac{h(P_0-\alpha)V^3T_1^2}{2} - \frac{h(P_0-\alpha)V}{(\omega+b)^2} - \frac{\beta\ V^3T_1^2}{2(\omega+b)^2} + \frac{\beta\ V}{(\omega+b)^3} + H_0Q(1-V) - \frac{h\gamma}{2(\omega+\rho)}(1-V^2)T_1 + HQ(1-2V+V^2)T_1 + \frac{h\gamma(1-V)}{(\omega+\rho)^2} + \frac{h\gamma}{\omega+\rho}(1-2V+V^2)T_1 + \frac{hQ(1-V)}{\omega+\rho} \tag{15}$$

The main objective is to find the value of T_1 which minimize the inventory cost per unit time. The suitable conditions to reduce $TC(T_1)$ is

$$(i)\frac{dTC(T_1)}{dT_1} = 0 (ii)\frac{d^2TC(T_1)}{dT_1^2} > 0 \tag{16}$$

$$\frac{dTC(T_1)}{dT_1} = \frac{-K_0}{T_1^2} - \frac{H_0Q(\omega+b)V^2}{2} + \frac{H_0(P_0-\alpha)V^2}{2}\frac{2\beta\ V^3T_1}{3(\omega+b)} - \frac{hQv^2}{2} - hQV^2 - hQ(\omega+b)V^3T_1 + h(P_0-\alpha)V^3T_1 - \frac{\beta\ V^3T_1}{(\omega+b)^2} - \frac{h\gamma}{2(\omega+\rho)}(1-V^2) + hQ(1-2V+V^2) \tag{17}$$

Equation (17) is equated to zero so as to obstain the value of T_1 that will minimise the cost function.

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Theorem

If$Q(\omega+b) < (p_0-\alpha) - \frac{\beta}{(\omega+b)^2} + \frac{\beta}{3(\omega+b)}$ then the cost function is convext.

Proof.

From equation (16) we take the second derivative

$$\frac{d^2TC(T_1)}{dT_1^2} = \frac{2k_0}{T_1^3} - hQ(\omega+b)V^3 + h(p_0-\alpha)V^3 - \frac{\beta V^3}{(\omega+b)^2} + \frac{2\beta V^3}{3(3\omega+b)} \quad (18)$$

Therfore, $\frac{d^2TC(T_1)}{dT_1^2}$ >0, provided$hQ(\omega+b)V^3 < h(p_0-\alpha)V^3 - \frac{\beta V^3}{(\omega+b)^2} + \frac{2\beta V^3}{3(\omega+b)}$. Therefoe

equation (14) shows that there is convex in T_1, then there is optimality in T_1

Demonsrtration of the model

A numerical example in provided to demonstrate the developed model. The values of various parameter are as follows $k_0 = 100, \beta = 2p_0 = 50, Q = 10, H_0 = 5, h = 2, b = 0.4, \rho = 0.8, \omega = 0.01, \alpha = 5, p = 22, \gamma = 5.5$. We substitute the value of variou above parameter in to 1n equation (14) to compute the values of T_1 using excell, the solution of T_1 obtain from equation (14) is now put in to equations (4), (11) and (12) to obtain the optimal solution as

$T_1^* = 2.210959(808days), t_1^* = 0.551726, Q_1^* = 32.56557$ and $TC(T_1)^* = 55.03861$

Effect of the parameter on the decision variables

We carefully examine the effects of each parameter $K_0, P_0, Q, H_0, h, b, \rho, \omega, \gamma, \beta$ and α on the optimal time for the maximum inventory. t_1^*, optimal cycle lengthT_1^*, optimal Quality $Q_1^*$and the total average inventory cost $TC(T_1)^*$. The sensitivity analysis is carried out by changing each of the parameters by 50%, 25%, 10%, 5%, -5%, -10%, -25%, -50%, taking one parameter at a time and leaving other parameter unchange.

Sensitivity Analysis on the numerical example to see the changes in the values of $T_1^*, t_1^*, Q_1^*$and $TC(T_1)^*$

% change in parameter K_0	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	2.676712(978days)	0.66795	37.31916	75.50748
25 %	2.457534(898days)	0.613256	35.08218	65.75997
10 %	2.312329(845days)	0.577021	33.60018	59.46424

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5 %	2.263014(827days)	0.564715	33.09686	57.27643
0 %	2.210959(808days)	0.551727	32.56557	55.03861
-5%	2.158904(789days)	0.538736	32.03429	52.74712
-10%	2.10411(769days)	0.525062	31.47504	50.39752
-25%	1.926027(704days)	0.480623	29.6575	42.94765
-50%	1.589041(581days)	0.396531	26.21813	28.71601

% change in parameter P_0	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	2.123288(776days)	0.363229	33.9368	48.20302
25 %	2.156164(788days)	0.437669	33.37153	51.01549
10 %	2.18630(299days)	0.499726	32.93742	53.24175
5 %	2.19726(803days)	0.524259	32.75282	54.10434
0 %	2.210959(808days)	0.5517265	32.56557	55.03861
-5%	2.224658(813days)	0.581834	32.34241	56.05246
-10%	2.243836(820days)	0.616488	32.13193	57.15494
-25%	2.312329(845days)	0.748754	31.26462	61.08748
-50%	2.632877(962days)	1.213801	29.29943	69.67315
% change in parameter Q				
50 %	2.961644(717days)	0.612797	38.80715	75.308844
25 %	2.068493(756days)	0.582346	35.72105	65.20834

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10 %	2.147945(785days)	0.563787	33.82772	59.12312
5 %	2.180822(797days)	0.558362	33.22255	57.08468
0 %	2.210959(808days)	0.551726	32.56557	55.030861
-5%	2.246575(821days)	0.545922	31.94012	52.98389
-10%	2.282192(834days)	0.539542	31.28849	50.91843
-25%	2.410959(881days)	0.521623	29.36906	44.63778
-50%	2.70137(987days)	0.491392	26.10529	33.71298

% change in parameter *h*	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	1.923288(703days)	0.47994	29.62953	26.79933
25 %	2.052055(750days)	0.512072	30.94376	41.1697
10 %	2.139726(782days)	0.53395	31.83855	49.55891
5 %	2.175342(795days)	0.542838	32.20206	52.31088
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.249315(822days)	0.561297	32.95705	57.74085
-10%	2.284932(835ays)	0.570185	33.32056	60.4159
-25%	2.419178(884days)	0.603685	34.69071	68.26274
-50%	2.706849(989days)	0.675471	37.62675	80.61138
% change in parameter γ				
50 %	2.13247(780days)	0.609518	34.9293	45.26671

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25 %	2.172603(794days)	0.582277	33.81513	50.05648
10 %	2.194521(802days)	0.564077	33.07077	53.02224
5 %	2.20274(805days)	0.557974	32.82113	54.02648
0 %	2.21095(808days)	0.551726	32.56557	55.03861
-5%	2.219178(811days)	0.545331	32.30402	56.05864
-10%	2.22397(814days)	0.538787	32.03637	57.08662
-25%	2.254795(824days)	0.518868	31.2217	60.21860
-50%	2.30137(841days)	0.482509	29.73464	65.59985

% change in parameter *b*	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	2.20274(805days)	0.570614	32.1969	116.8011
25 %	2.20274(805days)	0.559949	32.34196	94.86015
10 %	2.208219(807days)	0.555116	32.4822	74.1294
5 %	2.208219(807days)	0.553071	32.51001	65.23981
0 %	2.210959(808days)	0.551726	32.56557	75.50748
-5%	2.213699(809days)	0.550389	32.62101	55.03861
-10%	2.216438(810days)	0.549063	32.6763	43.25826
-25%	2.219178(811days)	0.543799	32.78516	-28.0948
-50%	2.221918(812days)	0.534833	32.94435	-248.256.50748

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% change in parameter ρ				
50 %	2.123288(776days)	0.638801	36.12696	19.33957
25 %	2.178082(796days)	0.601382	34.59652	34.54703
10 %	2.2(804days)	0.572875	33.4306	45.97519
5 %	2.205479(806days)	0.562917	33.00287	50.33381
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.216438(810days)	0.540795	32.1185	60.14785
-10%	2.216438(810days)	0.528313	31.608	65.73405
-25%	2.208219(807days)	0.487911	29.955557	86.42609
-50%	2.131507(779days)	0.405197	26.57257	148.4579

% change in parameter α	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	2.224658(813days)	0.581834	32.34241	56.05246
25 %	2.219178(811days)	0.566776	32.47267	55.53513
10 %	2.213699(809days)	0.557524	32.52397	55.23474
5 %	2.213699(809days)	0.554955	32.55892	55.13635
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.205479(806days)	0.547845	32.54383	54.9417
-10%	2.205479(806days)	0.545355	32.57769	54.84559
-25%	2.20274(805days)	0.53735	32.6493	54.562

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-50%	2.194521(802days)	0.523605	32.72445	54.10432
% change in parameter ω				
50 %	2.205479(806days)	0.550358	32.48211	57.54618
25 %	2.208219(807days)	0.551041	32.52381	56.30446
10 %	2.208219(807days)	0.551041	32.53208	55.54792
5 %	2.208219(807days)	0.551041	32.53483	55.29379
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.208219(807days)	0.551041	32.54034	54.78258
-10%	2.208219(807days)	0.551041	32.5431	54.52548
-25%	2.208219(807days)	0.551041	32.55136	53.74818
-50%	2.208219(807days)	0.551041	32.56514	52.43234

% change in parameter H_0	T_1^*	t_1^*	Q_1^*	$TC(T_1)^*$
50 %	2.060274(753days)	0.514123	31.02762	86.82731
25 %	2.131507(779days)	0.531898	31.75464	70.99155
10 %	2.172603(974days)	0.542153	32.17408	61.4348
5 %	2.191781(801days)	0.546939	32.36981	58.2392
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.230137(815days)	0.556511	32.76128	51.83325
-10%	2.243836(820days)	0.559929	32.9011	48.62204

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-25%	2.293151(838days)	0.572235	33.40442	38.95616
-50%	2.391781(874days)	0.596847	34.41106	22.72912
% change in parameter β				
50 %	2.224658(813days)	0.555143	32.70536	58.32946
25 %	2.213699(809days)	0.552409	32.59351	56.6847
10 %	2.208219(807days)	0.551041	32.53759	55.69731
5 %	2.210959(808days)	0.551725	32.56555	55.36979
0 %	2.210959(808days)	0.551726	32.56557	55.03861
-5%	2.208219(807days)	0.551041	32.53759	54.70937
-10%	2.205479(806days)	0.550358	32.50962	54.37998
-25%	2.2(804days)	0.54899	32.4537	53.3916
-50%	2.194521(802days)	0.547623	32.39777	51.74341

DISCUSSION OF RESULTS

From the results obtain in the table 1, it can be deduced as follows:

(i) With increase in the value of the parameter P_0 (set up cost), the values of $T_1^*,\ t_1^*,\ Q_1$, and $TC(T_1)^*$, all increases, this implies that increase in set up cost will result in the increase of the optional time for maximum inventory t_1^*,, optional cycle time T_1^*, optional production quantity Q_1^*, and total average inventory cost per unit time $TC(T_1)^*$,. This is clearly expected since excess stocking is encourage as a result of high set up cost. The total average inventory cost per unit time $TC(T_1)^*$ is therefore expected to increase due to increase in stocking cost. The values of $T_1^*,\ t_1^*$ and Q_1^*, all increase due to high set up cost as well as stock holding cost.

(ii) With increase in the value of the parameter p_0 (constant aspect of the production rate), the values of $T_1^*,\ t_1^*$ and $TC(T_1)^*$, decreases while the value of Q_1^*, increases. This is expected because high production rate leads to shorter cycle time T_1^* and t_1^*, the time of

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maximum inventory. This will inturn reduce $TC(T_1)^*$. Q_1^* increases since production rate increases.

(iii) With increase in the value of *Q* (buffer stock), the values of T_1^* decreases while the values of t_1^*, Q_1^* and $TC(T_1)^*$ inceases. This is because inventory produced takes shorter time to finish hence the optimal cycle T_1^* decrease. On the other hand, the optimal time for maximum inventory t_1^*, and optimal production quantity Q_1^*, increase probably because *Q* is much. The total average inventory cost is increased due to increase in the holding cost for the buffer stock.

(iv) With increase in the value of the parameter H_0 (constant part of the holding cost), the values of T_1^*, t_1^*, and Q_1^*, remain unstable while $TC(T_1)^*$ increases. This is because increase in the holding cost of items will also increase the total average inventory cost per unit time $TC(T_1)^*$. Increase in stocking holding cost, encourages higher number of set ups. The value of the optimal production quality Q_1^*, Is expected to be unstable due to increase in set ups. The values of both t_1^* and T_1^* remains unstable to unstable the value of Q_1^*, therefore the inventory will finish earlier.

(v) With increase in the value of the parameter *h*(time dependent part of the holding cost) the values of T_1^*, t_1^*, Q_1^* and $TC(T_1)^*$, all decreased. This is expected since if the time dependent part of the holding cost is higher, the model will force a reduction in the value of the optimal stock Q_1^*, therefore, T_1^*, t_1^* and Q_1^* will all decrease and this will in turn cause $TC(T_1)^*$ to decrease.

(v) With increase in the value of the parameter α (constant part of the demanding rate during production). The values of T_1^*, t_1^* and $TC(T_1)^*$ increase while the value of Q_1^* decreases. This is expected since if α is higher, the demand rate is higher during production and this will decrease the value of Q_1^*. However the values of T_1^*, and t_1^*, increase probably because the model was trying to reduce cost.

(vii) With increase in the value of the parameter β (stock dependent part of the demand during production) the values of T_1^*, t_1^* and $TC(T_1)^*$ increase while the value of Q_1^* decrease. Increasing the value of β, demand will be high and this will in turn decrease the value of optimal production quantity Q_1^*,. This is supposed to reduce the values of t_1^* and T_1^* however the values of t_1^*, and T_1^*, increase probably because the model was trying to reduce cost.

(viii) With increase in the value of the parameter γ (constant part of the demand after production the values of T_1^* and $(TCT_1)^*$ decrease while the values of t_1^*, and Q_1^* increase, this is expected since if γ increases the demand rate increases so T_1^*, decreases. However, the values of t_1^* and Q_1^* increase since the demand after production does not affect them. The total average inventory cost per unit time will reduce because T_1^* decreases.

(ix) With increase in the value of the parameter ρ (stock dependent demand rate after production) t_1^*, and Q_1^*, increase while the value of $TC(T_1)^*$, decrease. The values of T_1^* are however unstable. This is because if ρ is higher, the demand rate after production is higher and this will increase the optimal cycle length T_1^* though in our case T_1^* is unstable.

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The time for maximum inventoryt_1^*, as well as the optimal reduction quantityT_1^* increase because the demand after production does not affect them. $TC(T_1)^*$, decrease becauseT_1^* is unstable.

(x) With increase in the value of the parameter ω (deterioration rate), the values of $T_1^*$and t_1^* remain unchanged while the values ofQ_1^*, and $TC(T_1)^*$ increases. The values of $T_1^*$and t_1^* are unstable. The instability maybe as a result of the increase in the deterioration rate. The value ofQ_1^*, increases probably because $TC(T_1)^*$, increases.

(xi) With increase in the value β(stock dependent part of the production rate), the values of T_1^* and t_1^* are unstable while the values of Q_1^* and $TC(T_1)^*$, increases. This is because the stock production dependent rate increase will result to an increase in the value ofQ_1^*, which will also cause the total average inventory cost per unit time to be high.

CONCLUSION

This paper presents a mathematical model of a production inventory which linear time production rate and holding cost. The demand is assured to be linear level dependent during and after production. The objective of the model is to obtain the best cycle length that will give the optimal solution of the system. The cost function has been shown to be convext and a numerical example is given to illustrate the model developed. Sensitivity analysis has been carried to see the effects on the parameter on the decision variables.

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