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MATHEMATICAL MODELLING OF SINGLE PHASE GAS FLOW IN A PIPELINE USING FLUX VECTOR SPLITTING METHOD.

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ABSTRACT: *Pipe transportation is the process of sending liquid to end users from production point. Many works on pipeline transportation has been in existing for decade but most of the works were concerned with the transportation without giving regard to problem encountered during the transportation. In Nigeria transportation of gas was done through tankers because the use of pipeline has not been used in the gas distributions. This institutional based research proposal will shade more light on pipeline transportation its importance and problems associated with it. In considering the problems a one-dimensional homogeneous model which represents a system of partial differential equations to describe mathematically the transient gas flows in a pipe. The governing equation was solved using Implicit Steger-Warming flux vector splitting method. The method has the capability of taken care of any flow that involves propagation especially pipe flow system. The result of this work will served as a way forward for implementing gas transportation in Nigeria and in due course Zamfara since it is a mineral resource producing state, it is hope that there will be time gas reservoir will be discovered in the state.*

KEYWORDS: Gas-flow, pipeline, flux vector.

INTRODUCTION

Mathematical Modelling in gas industries become paramount tools in determining the problems associated with transporting the product. The gas is transported through pipeline and its simulations are widely used by gas transmission operators that are obliged to ensure that the system is balanced and that deliveries of the gas are maintained. According to EU regulatory framework, the responsibility for the physical balance of the system is imposed on the pipeline operator and the balancing should be carried out on a daily and monthly basis. This decision has many ramifications in the field of flow measurement; among other things is an increased importance of the accuracy of pipeline simulation, which is used for the determination of system line-pack (gas network accumulation) on an hourly and daily basis. Pipeline leak detection system based on volume balance methods is another example where the accuracy of simulation results is an important matter. Physical balancing of the system can be considered as a management of system line-pack. The minimum pipeline line-pack is the amount of gas in the pipeline required to achieve the desired gas flow, and the required delivery pressure. The latter supports the hourly modulation of gas delivery and supply rates and is determined by means of pipeline simulation in order to preclude pressure values above permissible limits. Transportation of gas to end users constitute problems which associated with the environment and pipe conditions. In the early age, most analysis of gas flow in pipe line were based on steady state assumption where the interest is geared only in transporting the fluid without considering the effect of the flow environment (geometry). Authors such as Everdingen and Hurst (1949) were among the early researchers to present a work on the flow of fluid in pipeline which were concerned primarily with the transient condition prevailing in oil reservoirs during the time of transportation. Bruce *et al.* (1953) studied the work of Everdigen but consider the radial effect on the pipe. They presented that flowing of fluid through pipe line is essentially a transient process due to the pressure drop and temperature difference of the flowing fluid and the flow environment.

Gonzales et al., (2008) have used MATLAB-Simulink and prepared some S-functions to simulate transient flow in gas pipeline networks. At their work, two simplified models have derived containing Crank-Nicolson algorithm and method of characteristics.

Mbaya, (2018), developed a model to determining the effect of inflow performance for flow of gases that flows through a pipe and discover that the major challenges in the pipeline is the mass balance between the pipe and the environment fluid which depend on the inflow performance. Ali et al, (2017), studied pressure build up test for the period of 24 hours and used the pressure data obtained to analyze the flow process characteristics for the inflow performance relationship (IPR) of pipeline. Guo and Nie, (2013) developed a model that can modeled the non-linear transient flow behavior of pipeline transportation that is buried in in an underground formation based on Darcy's law and material balance equations for two phase flow gas and liquid (oil or water). Juiping *et al* (2012), presents a model of coupled differential equations concerning pressure, temperature density and velocity for pipeline inserted in a well with the aim of transporting the gas from reservoir to wellhead according to the conservation of mass, momentum and energy assuming the flow to be at steady state which is solved using fourth-order Runge Kutta method.

Many algorithms and numerical methods such as implicit and explicit finite differences, method of characteristics and so on, have been applied by several researchers for transient flow in gas pipelines (Kiuchi, 1994) but unfortunately, almost all of these conventional schemes are

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time consuming especially for gas network analysis. Toro (2008) has described Steger-Warming Implicit Vector Splitting Method to be a scheme that is suitable for transient situation of a flow of gas in a pipeline and has been applied by (Babahni Najad, 2008) in gas transportation in pipeline network. The present study will focus on transportation of single phase gas flow using flux vector splitting method. The method was chosen because it is unconditionally stable (Toro, 2008).

Asumptions

In developing the governing equation the following asumptions were made

a. The gas is single phase no other liquid involved

b. The pipe is horizontal, the effect of inclination is neglecble

c. Effect of viscusity is neglected

d. The control volume consists of single phase when it moves and mass its varies with time

Governing Equation

We present a set of partial differential equations that described general one dimensional compressible gas flow dynamics through a pipeline constituting conservation of mass, momentum and equation of state relating the pressure, density and celerity wave.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2 + P)}{\partial x} + \frac{f \rho v |v|}{2D} = 0 \tag{2}$$

$$P = \rho c^2 \tag{3}$$

$$\rho = \frac{P}{ZRT} \tag{4}$$

where ρ is the density of flowing fluid, f is coefficient of pipe friction, P is the pressure, v is the velocity of the fluid, D is the pipeline diameter and c is a celerity wave. Equations (1) and (2) form a system of two nonlinear partial differential equations of hyperbolic type in which the pressure P and the velocity v are considered as main variables of the flow. To solve numerically these equations, we must express the density ρ according to the fluid pressure.

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Equation (1) to (3) can be written in convective form as

$$\frac{\partial Q}{\partial t} = -\frac{\partial E(Q)}{\partial x} + H(Q) \tag{5}$$

where $Q = \begin{pmatrix} \rho \\ \rho u \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$, $E(Q) = \begin{pmatrix} q_2 \\ \frac{q_2^2}{q_1} + c^2 q_1 \end{pmatrix}$ and $H(Q) = \begin{pmatrix} 0 \\ -\frac{q_2 f|q_2|}{2q_1 D} \end{pmatrix}$ (6a, b, c)

Steger Warming Flux Vector Splitting

The Steger-Warming flux vector Splitting scheme method (FSM) has been considered in this work as the numerical scheme because literature has shown that it does not have the problem of numerical instability. In delta formulation, the finite difference form of the method is

$$\begin{aligned} -\left(\frac{\Delta t}{\Delta x} A_{j-1}^{+}\right)\Delta Q_{j-1} + \left(I + \frac{\Delta t}{\Delta x}\left(A_j^{+} - A_j^{-}\right) - \Delta t B_j\right)\Delta Q_j + \left(\frac{\Delta t}{\Delta x} A_{j+1}^{+}\right)\Delta Q_{j+1} \\ = \frac{\Delta t}{\Delta x}\left(E_j^{+} - E_{j-1}^{+} + E_{j+1}^{-} - E_j^{-}\right) + \Delta t H_j \end{aligned} \tag{7}$$

The subscript j indicate the spatial grid point while the superscript indicates the time level and

$$\Delta Q = Q^{n+1} - Q^{n} \tag{8}$$

In equation (6) I is an identity matrix and A and B are Jacobian matrix defined by

$$A = \frac{\partial E}{\partial W}, \qquad B = \frac{\partial H}{\partial W} \tag{9a, b}$$

And A^{+}, A^{-} are positive and negative parts of the Jacobian matrix A defined as follows.

$$A^{+} = \begin{bmatrix} \frac{a^2 - u^2}{2a} & \frac{u+a}{2a} \\ \frac{(u+a)^2 (a-u)}{2a} & \frac{(u+a)^2}{2a} \end{bmatrix} \qquad A^{-} = \begin{bmatrix} \frac{u^2 - a^2}{2a} & \frac{a-u}{2a} \\ \frac{(u+a)(a-u)^2}{2a} & -\frac{(a-u)^2}{2a} \end{bmatrix} \tag{10}$$

In (7) also E^{+} and E^{-} are the positive and negative part of E defined as

$$E^{+} = \begin{bmatrix} \frac{w_1(u+a)}{2} \\ \frac{w_1(u+a)^2}{2} \end{bmatrix} \qquad E^{-} = \begin{bmatrix} \frac{w_1(u-a)}{2} \\ \frac{w_1(u-a)^2}{2} \end{bmatrix} \tag{10}$$

Applying equation (7) to each grid point, a block tridiagonal system is formed. The equation is then solve at each time step which resulted to ΔQ.

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Boundary Condition

The flow boundary depends on the flow time and the distance the transportation will cover

$$\rho(0,t)=\rho_0(t), \quad \frac{\partial u(0,t)}{\partial x}=u_0(t), \quad T(0,t)=T_0(t), \quad P(0,t)=P_0(t) \qquad \text{(7a, b, c, d)}$$

where ρ, x, T_f, k_e, T_0, $\dot{Q}_0$ are the inlet gas density, depth (m), temperature of the flowing fluid, temperature of flow environment (tubing) and thermal conductivity of the earth formation

RESULTS AND DISCUSSIONS

Comparison was carried out on flow Characteristics on gas transportation through pipelines system with work of Bai et al., (2013) and the implicit Steger-Warming flux vector splitting respectively. At this point the inlet is at 0 m distance to outlet a distance of 3000 m. Figure 1 which shows the present method is in good agreement with the existing work with small flow rate of 2.213 kg/s.

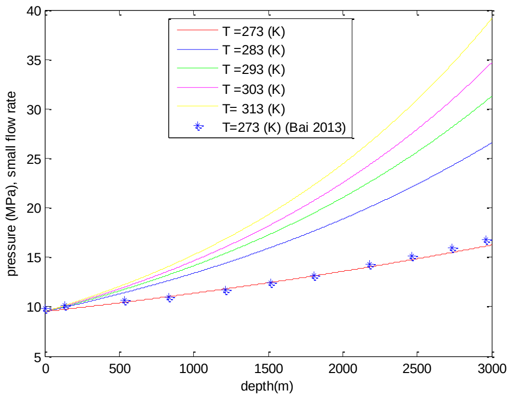

Figure 1: Pressure Compares

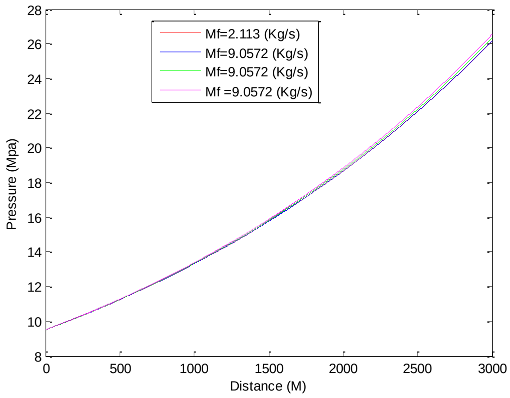

Figure 2: Pressure at different flow rate

Temperature was calculated at the inlet point was observed to be stable but increases when the pipe is exposed to different environment as shown in figure 3.

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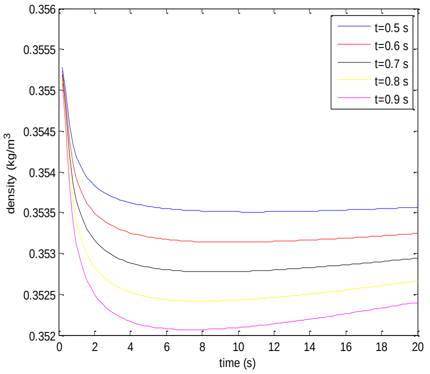

Figure 3: Temperature profile of the flowing fluid

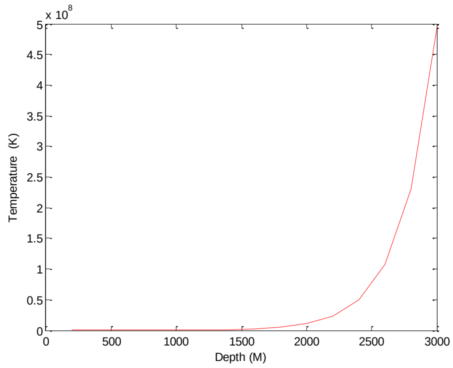

Figure 4: Density profile at different time.

During gas transportation, as time increases, density profile decreases simultaneously, it is observed that at different time and the density drop very fast but later stabilize while for velocity it increases as time increases due to decrease in pressure profile.

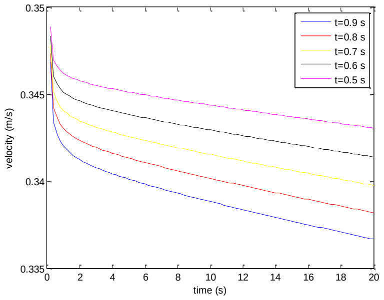

Figure 5: Velocity profile at different time

CONCLUSION

We developed a model for single phase flow and compare our work with the work of (Bai*et al.*, 2013) and the results are in good agreement.

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