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ANALYSIS OF TRANSVERSE DISPLACEMENT AND ROTATION UNDER MOVING LOAD OF PRESTRESSED DAMPED SHEAR BEAM RESTING ON VLASOV FOUNDATION

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ABSTRACT: *This study examines the transverse displacement and rotation of a prestressed damped shear beam supported by a Vlasov foundation when subjected to a moving load traveling at a constant velocity. The governing equations are expressed as coupled second-order partial differential equations. To simplify these equations, the finite Fourier series method was employed, transforming the coupled second-order partial differential equations into a sequence of coupled second-order ordinary differential equations. Subsequently, the simplified equations that describe the motion of the beam-load system were solved using Laplace transformation in conjunction with convolution theory to obtain the solutions. The effects of some pertinent structural parameters on the transverse displacement and rotation of a prestressed shear beam when under the moving load were illustrated in graphs. Notably, the graphs indicate that an increase in these pertinent structural parameters reduces the transverse displacement and rotation of a prestressed shear beam when subjected to the moving load. From a practical standpoint, increase in the values of these structural parameters significantly enhances the stability of the beam and increases the critical speed of the dynamic system, thereby minimizing the risk of resonance and ensuring the safety of the structure's occupants.*

KEYWORDS: Transverse displacement, Rotation, moving load, Prestressed, Vlasov foundation, Critical speed, Resonance.

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INTRODUCTION

The analysis of vibrations, whether linear or non-linear of structural members, has been an exciting topic of investigation in Engineering, Science, and other allied fields. This interest is largely due to its pertinence to a diverse array of significant problems relevant to numerous engineering applications. Notable examples include the analysis and design of highway and railway bridges, cable railways, cableways and overhead cranes. A thorough understanding of the complex dynamic interactions between structural members and the loads they support is crucial for controlling structural vibrations and optimizing the performance of such dynamic systems. Specifically, numerous researchers have examined how the complexity of interactions between beams or beam-like structural elements and the loads moving across them at different velocities influences the dynamic characteristics of these structures [1-7]. The research conducted by Lin and Trethewey [8] focused on the dynamic analysis of an elastic Bernoulli-Euler beam experiencing dynamic loads from the arbitrary movements of a spring-mass-damper system, employing the finite element method (FEM) for their analysis. Similarly, Olsson [9] made significant contributions to the understanding of the moving load problem, providing essential reference data for further studies. Jaiswal and Iyenggar [10] examined the dynamic response of an infinitely long beam that is supported by a finite-depth foundation under the influence of a moving force, investigating how various parameters such as foundation mass, load velocity, damping, and axial force affect the beam. Lee [11] utilized the Bernoulli-Euler beam theory in combination with the assumed mode method to analyze the transverse vibrations of a beam that is constrained at intermediate points while subjected to a moving load.

Early research predominantly concentrated on structures like beams and plates that were not supported by elastic foundations. However, for real-life applications, it is beneficial to investigate structures that are indeed supported by such foundations. For instance, an analysis that incorporates such a foundation can be instrumental in understanding the behavior of plates and beams on roadways or runways. The Winkler approximation model is often cited in academic literature as an elastic foundation model. Worthy of note is the work of Ogunbamike [12] who examined the dynamic behavior of a simply supported Timoshenko beam that rests on a Winkler foundation and subjected to a moving uniformly distributed load. He successfully obtained the analytical solution to the fourth-order partial differential equation through the application of the Fourier Sine Transformation Method. The results revealed that increase in the dynamic load, moving at constant speed leads to higher deflections and bending moments of the beam, while deflection decreases with increasing foundation modulus. Similarly, Clastornik et al. [13] conducted a study on the dynamic analysis of elastic beams supported by a variable Winkler elastic foundation.

However, for complex engineering problems, such as the vibration of plates or beams, it is advisable to employ a two-parameter foundation model instead of relying exclusively on the Winkler approximation. The Winkler model has faced criticism for its limitations. This one-parameter model inadequately represents the continuous characteristics of practical foundations, as it overlooks the interactions between lateral springs. Furthermore, it predicts discontinuities in the deflections of the foundation surface at the ends of a finite beam, which is contrary to actual practices. Thus, it is recommended to utilize a more compact and realistic elastic foundation model known as the bi-parametric elastic foundation or Vlasov foundation model. This model takes into account the continuity of surface displacement beyond the load application area. It introduces a second foundation constant, the shear modulus G, in addition

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to the foundation stiffness K. The addition of the shear modulus enhances the accuracy and reliability of the analysis, although it complicates the problem and increases the difficulty of finding a solution. The behavior of moving loads on bi-parametric elastic foundations has been thoroughly examined. Ogunbamike and Oni [14] investigated the dynamic behavior of a non-prismatic Rayleigh beam with classical boundary conditions, which is supported by a Vlasov elastic foundation and subjected to partially distributed moving masses with variable velocities. They employed a methodology based on the Generalized Galerkin's method to derive closed-form solutions for this class of dynamic problems. Jimoh [15] considered the analysis of non-uniformly prestressed tapered beams with exponentially varying thickness resting on Vlasov foundation under variable harmonic load moving with constant velocity. Rajib et al. [16] analyzed the dynamic response of a beam that is influenced by both moving loads and moving masses, with support from a Pasternak foundation. In the same vein, Oni and Ogunbamike [17] conducted an analysis of the dynamic behavior of a non-prismatic Rayleigh beam on a Pasternak foundation, subjected to partially distributed masses that move at varying velocities. Their approach involved expanding the Heaviside function in series, employing the generalized Galerkin method, and modifying Struble's asymptotic technique. The findings indicated that an increase in the rotatory inertia correction factor r0 leads to a reduction in the response amplitude of the Rayleigh beam. Similarly, increase in the values of the foundation stiffness K, shear modulus G and axial force N decreases the deflection of the beam. Similarly, Jimoh and Awelewa [18] examined the dynamic behavior of a non-uniform elastic structure resting on an exponentially decaying Vlasov foundation subjected to repeated rolling concentrated loads.

Moreover, engineers often incorporate artificial stresses into structures prior to load application to ensure that the stresses present under load are more favorable than they would otherwise be. These artificial stresses can exist as forces acting axially or in other directions. When these forces act axially, they are referred to as axial forces. This method of inducing artificial stresses is known as pre-stressing. The primary goal of pre-stressed structures is to reduce tensile stresses, thereby decreasing the likelihood of flexural cracking or bending during operational conditions. Consequently, a significant amount of research has been devoted to exploring the vibrations of pre-stressed beams that are subjected to moving loads. Jimoh, Oni and Ajijola [19] studied the effect of variable axial force on the deflection of thick beams under distributed moving load. The transverse displacement for moving force and moving distributed mass models for the dynamical problem obtained were calculated for various time t and analyzed. Jimoh, Ogunbamike and Ajijola [20] investigated the dynamic response of non-uniformly prestressed thick beams under distributed moving load travelling at varying velocity. They used a technique based on the method of Galerkin's with the series representation of Heaviside function to transform the equations, thereafter the transformed equations were simplified using Strubles asymptotic method and solved by Laplace transformation techniques in conjunction with convolution theory. It is found that the moving distributed force is not an upper bound for the accurate solution of the moving distributed mass problem. They also found that increase in the values of some pertinent structural parameters reduces the response amplitudes of non-uniformly prestressed thick beams under moving distributed loads. The dynamic response of the Timoshenko beam supported by an elastic foundation and subjected to a harmonic moving load was also examined by Ogunbamike [21]. The method of modal analysis (MA) was employed to obtain a closed form solution to this class of dynamical systems. The effects of axial force and foundation parameters on the dynamic characteristics of the beams were studied and described in detail.

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It is pertinent to mention that a vast array of research has been conducted on dynamic problems involving Bernoulli-Euler beams and various other beam types under both lumped and distributed loads [22-25]. However, literature addressing shear beams under moving loads remains limited. The theory of shear beams is a vital aspect of structural engineering, concentrating on beams where shear deformation is significant. Unlike the traditional Euler-Bernoulli beam theory, which assumes that plane sections remain unchanged and perpendicular to the neutral axis, shear beam theory takes into account shear deformations, making it crucial for the analysis of short and deep beams. The shear beam model is typically defined by a pair of coupled partial differential equations that involve two dependent variables: the transverse displacement of the cross-section relative to the neutral axis and the rotation of the cross section measured about the neutral axis. In this present study, an approximate analytical solution of the transverse displacement and rotation of a simply supported prestressed shear beam resting on bi-parametric elastic subgrade when under the action of moving load travelling at a constant velocity is obtained. Effects of some pertinent structural parameters on the transverse displacement and rotation of a prestressed shear beam carrying moving load are investigated. Additionally, the study identifies the conditions under which the beam-load system will experience resonance phenomenon and the corresponding speeds at which these phenomena may occur.

PROBLEM STATEMENT

The equations governing transverse displacement and rotation of shear beam on elastic foundation and under the action of moving load are based on the following assumptions:

(i) The material is linearly elastic and the beam is homogeneous at any cross-section (prismatic)

(ii) The x - y plane is the principal plane.

(iii) There is an axis of the beam that undergoes no extension or contraction. The x-axis is located along this neutral axis.

(iv) Plane section remains plain after bending but is no longer normal to the longitudinal axis.

(v) The effect of shear deformation is considered.

(vi) The beam is simply supported (Pin-Pin ends).

(vii) The applied moving load is concentrated.

(viii) The damping, prestressed and foundation parameters are all linear.

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MATHEMATICAL MODEL

The governing equations of motion describing the transverse displacement $V(x,t)$ and rotation $\phi(x,t)$ of a finite damped shear beam resting on Vlasov foundation and subjected to moving load travelling at a constant velocity are formulated as coupled second order partial differential equations given by

$$M\frac{\partial^2 V(x,t)}{\partial t^2} + \frac{\partial}{\partial x}\left[K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] - N\frac{\partial^2 V(x,t)}{\partial x^2} - C\frac{\partial V(x,t)}{\partial t} + F(x,t) = P(x,t) \quad (1)$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \phi(x,t)}{\partial x}\right) - K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right) = 0 \quad (2)$$

where M is the mass per unit length of the beam, K^* is the shear correction factor, G^* is the shear parameter of the beam, A is the cross-sectional area of the beam, N is the axial force, C is the coefficient of viscous damping per unit length of the beam, E is the Young modulus of elasticity of the beam material, I is the moment of inertia, EI is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate, $F(x,t)$ is the foundation reaction and $P(x,t)$ is the moving load acting on the beam per unit length.

The relationship between the foundation reaction $F(x,t)$ and lateral deflection $V(x,t)$ is given by

$$F(x,t) = KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} \quad (3)$$

where K and G are two parameters of the foundation model. Specifically, K is the Foundation Stiffness

and G is the Shear Modulus.

In this study, it is assumed that the load function $P(x,t)$ is given in the form

$$P(x,t) = P_0\delta(x - ut). \quad (4)$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property.

$$\int_b^a \partial(x-ut)f(x)dx = \{\ 0,\ \ for\ ut < a < b,\ \ f(ut),\ \ for\ a < ut < b,\ \ 1,\ \ for\ a < b < ut. \quad (5)$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x = 0$ and $x = L$. Thus, boundary conditions are given as

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$$V(0,t) = V(L,t) = 0, \quad \frac{\partial V(0,t)}{\partial x} = \frac{\partial V(L,t)}{\partial x} = 0 \quad W(0,t) = W(L,t) = 0, \quad \frac{\partial W(0,t)}{\partial x} = \frac{\partial W(L,t)}{\partial x} = 0 \tag{6}$$

and the initial conditions are given as

$$V(0,x) = 0 = \frac{\partial V(x,0)}{\partial t} W(0,x) = \ 0 = \frac{\partial W(x,0)}{\partial t} \tag{7}$$

Substituting (3) and (4) into (1), after some simplifications and re-arrangements, Equations (1) and (2) become

$$\frac{\partial}{\partial x}\left[K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] + \ M\frac{\partial^2 V(x,t)}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} - C\frac{\partial V(x,t)}{\partial t} + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} = P_0\delta(x - vt) \tag{8}$$

and

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \phi(x,t)}{\partial x}\right) - K^*G^*A\left(\phi(x,t) \ - \frac{\partial V(x,t)}{\partial x}\right) = 0 \tag{9}$$

(8) and (9) are the second order partial differential equations governing the flexural motion of the structurally damped shear beam resting on Vlasov elastic foundation and subjected to moving load travelling at constant velocity.

SOLUTION PROCEDURES

The shear beam investigated in the present study is finite and uniform. To find the analytical solution of the initial boundary value problem in (8) and (9), the finite Fourier transformation method is employed alongside the Laplace Transform. Hence, we provide the following definition and theorem:

Definition 1: The finite Fourier sine transform $u(n,t)$ of a function $U(x,t)$ is defined as

$$u(n,t) = \int_0^l U(x,t)\ sin\ sin\ \frac{n\pi x}{l}\ dx \tag{10}$$

and the inverse transform is

$$U(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} u(n,t)\ sin\ sin\ \frac{n\pi x}{l}\ dx. \tag{11}$$

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Definition 2: The finite Fourier cosine transform $u_0(n,t)$ of a function $U_0(x,t)$ is defined as

$$u_0(n,t) = \int_0^l U_0(x,t) \cos\cos \frac{n\pi x}{l} \, dx \tag{12}$$

and the inverse transform is

$$U_0(x,t) = \frac{2}{l} \sum_{n=1}^{\infty} u_0(n,t) \cos\cos \frac{n\pi x}{l} \, dx. \tag{13}$$

Definition 3: The Laplace transform $F(s)$ of a function $f(t)$ is defined as

$$L\big(f(t)\big) = F(s) = \int_0^{\infty} f(t)\, e^{-st} dt\,. \tag{14}$$

Theorem 1: The convolution theorem states that

$$L^{-1}\{F(s)G(s)\} = F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{15}$$

where $F(s)$ and $G(s)$ are the Laplace transforms of $f(t)$ and $g(t)$ respectively.

Thus, applying (10) and (12) to the governing equations (8) and (9) respectively, in conjunction with the Dirac delta function property in (5), we obtain

$$\frac{\partial^2 V(n,t)}{\partial t^2} + \left[\left(\frac{n\pi}{ML}\right)^2 (N+G) - \frac{K}{M}\right] V(n,t) - \left(\frac{n\pi}{ML}\right) K^* G^* A \frac{\partial \phi(n,t)}{\partial x} - \frac{C}{M}\frac{\partial V(n,t)}{\partial t} = \frac{P_0}{M} \sin \theta_n t \tag{16}$$

and

$$-EI\left(\frac{n\pi}{L}\right)^2 \phi(n,t) + K^* GA\left(\frac{n\pi}{L} V(n,t) - \phi(n,t)\right) = 0 \tag{17}$$

where

$$\theta_n = \frac{n\pi u}{L}$$

Then from Equation (17), we have

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$$\phi(n,t) = \alpha_0 V(n,t) \tag{18}$$

where

$$\alpha_0 = \frac{\frac{n\pi}{L} K^* G^* A}{EI\left(\frac{n\pi}{L}\right)^2 + K^* G^* A} \tag{19}$$

Now substituting (18) into (16), we have

$$\frac{\partial^2 V(n,t)}{\partial t^2} - \mu_1 \frac{\partial V(n,t)}{\partial t} + \left[\left(\frac{n\pi}{ML}\right)^2 (N+G) - \frac{K}{M}\right] V(n,t) - \left(\frac{n\pi}{ML}\right) K^* G^* A \frac{\partial}{\partial x}\left(\alpha_0 V(n,t)\right) = \frac{P_0}{M} \sin\theta_n t \tag{20}$$

The term involving the derivative with respect to x in (19) vanishes as $V(n,t)$ is a function of t alone and after some simplifications and re-arrangements, we obtain

$$V_{tt}(n,t) + \mu_1 V_t(n,t) + \mu_2 V(n,t) = Q_1 \sin\theta_n t \tag{21}$$

where

$\mu_1 = -\frac{C}{M}, \quad \mu_2 = \left(\frac{n\pi}{ML}\right)^2 (N+G) - \frac{K}{M} - \alpha_0 \left(\frac{n\pi}{ML}\right) K^* G^* A, \quad Q_1 = \frac{P_0}{M}.$

Next, we subject (21) to Laplace transformation (14), namely

$$\mathcal{L}\big(f(t)\big) = \mathrm{F}(s) = \int_0^\infty f(t)\, e^{-st} dt \tag{22}$$

where s is the Laplace parameter. In view of (22), (21) becomes

$$s^2 \tilde{V}(n,s) + \mu_1 s \tilde{V}(n,s) + \mu_2 \tilde{V}(n,s) = Q_1 \left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{23}$$

After simplification and rearrangement, we obtain the simple algebraic equation given by

$$\tilde{V}(n,s) = Q_1 \left[\frac{1}{s^2 + \mu_1 s + \mu_2}\right]\left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{24}$$

which is further simplified to give

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$$\tilde{V}(n,s) = Q_1 \left[\frac{1}{\left(s + \frac{\mu_1}{2}\right)^2 + K^2} \right] \left[\frac{\theta_n}{s^2 + {\theta_n}^2} \right] \tag{25}$$

Where

$$K^2 = \mu_2 - \left(\frac{\mu_1}{2}\right)^2. \tag{26}$$

At this juncture, in order to obtain the Laplace inversions of (25), we set

$$F(\mathrm{s}) = \left[\frac{1}{\left(s + \frac{\mu_1}{2}\right)^2 + K^2} \right]$$

and

$$G(\mathrm{s}) = \left[\frac{\theta_n}{s^2 + {\theta_n}^2} \right]$$

so that the Laplace inversion of (25) is the convolution of $F(\mathrm{s})$ and $G(\mathrm{s})$ defined by (14) namely

$$F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{28}$$

Noting that

$$\mathcal{L}^{-1}[F(\mathrm{s})] = \frac{1}{p} \exp\left(-\frac{\mu_1}{2}t\right) \sin(Kt) \tag{29}$$

and

$$\mathcal{L}^{-1}[G(\mathrm{s})] = \sin(\theta_n t) \tag{30}$$

Now using (29) and (30) in (28), (25) becomes

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$$V(n,t) = \frac{Q_1 e^{-\frac{\mu_1}{2}t}}{K(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\left\{\omega_2\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t - \theta_n \sin K t\right] + \omega_0\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t + \theta_n \sin K t\right] - \mu_1 K\theta_n\left[e^{\frac{\mu_1}{2}t}\cos\theta_n t - \cos K t\right]\right\} \tag{31}$$

where,

$$\omega_1 = (K + \theta_n)^2, \quad \omega_2 = (K - \theta_n)^2, \quad \omega_0 = -\left(\frac{\mu_1}{2}\right)^2$$

Thus, in view of (11), one obtains

$$V(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{Q_1 e^{-\frac{\mu_1}{2}t}}{K(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\left\{\omega_2\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t - \theta_n \sin K t\right] + \omega_0\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t + \theta_n \sin K t\right] - \mu_1 K\theta_n\left[e^{\frac{\mu_1}{2}t}\cos\theta_n t - \cos K t\right]\right\}\sin\frac{n\pi x}{l} \tag{32}$$

which represents the transverse displacement to the moving load of prestressed damped shear beam resting on Vlasov elastic foundation.

Now, using (31) in (18), we have

$$\phi(n,t) = \frac{\alpha_0 Q_1 e^{-\frac{\mu_1}{2}t}}{K(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\left\{\omega_2\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t - \theta_n \sin K t\right] + \omega_0\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t + \theta_n \sin K t\right] - \mu_1 K\theta_n\left[e^{\frac{\mu_1}{2}t}\cos\theta_n t - \cos K t\right]\right\} \tag{33}$$

Similarly, in view of (13), one obtains

$$\phi(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{\alpha_0 Q_1 e^{-\frac{\mu_1}{2}t}}{K(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\left\{\omega_2\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t - \theta_n \sin K t\right] + \omega_0\left[Ke^{\frac{\mu_1}{2}t}\sin\theta_n t + \theta_n \sin K t\right] - \mu_1 K\theta_n\left[e^{\frac{\mu_1}{2}t}\cos\theta_n t - \cos K t\right]\right\}\cos\frac{n\pi x}{l} \tag{34}$$

which represents the rotation to the moving load of prestressed damped shear beam resting on

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Vlasov elastic foundation.

Comments on the Closed-form Solution

Resonance in a dynamical system is of great concern in design engineering and engineering analysis, hence it is pertinent to establish the condition under which resonance occurs. Resonance takes place when the motion of the vibrating system becomes unbounded. That is, the point at which transverse displacement of an elastic beam increases without limit. In actual practice, when this happens, the structure would collapse as the intensive vibration causes cracks or permanent deformation in the vibrating structures. It is clearly seen from equation (32) that the simply supported Uniform damped Shear beam resting on bi-parametric elastic foundation and traversed by moving load considered in this study reaches a state of resonance whenever

$$\omega_1 = \omega_0, \quad (35)$$

$$\omega_2 = \omega_0 \quad (36)$$

The velocity at which resonance may occur termed the critical velocity associated with the conditions (35) and (36) respectively are given as

$$u_{cr1} = \frac{L\left[\sqrt{\omega_0} - \sqrt{\mu_2 + \omega_0}\right]}{n\pi}$$

and

$$u_{cr2} = \frac{L\left[\sqrt{\mu_2 + \omega_0} - \sqrt{\omega_0}\right]}{n\pi}$$

Numerical Simulation and Discussion of Result

The uniform prestressed damped shear beam of length $L = 12.192\ m$ is considered in order to illustrate the analysis presented in this study. The load is assumed to travel along the beam with constant velocity $u = 8.128\ m/s$, Young modulus of elasticity $E = 2.10924 \times 109\ Kg/m$, moment of inertia I = 2.87698×10^{-3}, $\pi = 22/7$, the damping coefficient $Co = 3000$ and the mass per unit length of the beam $M = 2758.291 kg/m$. The values of foundation stiffness K and shear modulus G are varied between $0\ N/m^3\ and\ 4 \times 10^7\ N/m^3$. Also, the values of axial force N are varied between $0\ N\ and\ 4 \times 10^8\ N$. The transverse displacement V and rotation ϕ of the beam are calculated and plotted against time t for various values of axial force N, foundation stiffness K and shear modulus G. The results are shown on the various graphs given below.

Figure 1 describes the transverse displacement and rotation of a simply supported prestressed damped shear beam under the action of moving load travelling at constant velocity for various values of axial force N and for fixed values of K=400000, G=40000000 and Co=3000. The graph shows that as the value of axial force N increases, the transverse displacement and rotation of the beam decrease noticeably. This implies that as the value of axial force N increases, the tensile stresses present in the vibrating beam reduce significantly. This makes

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the beam to becomes more inflexible and stable, thus the likelihood of flexural cracking or bending of the beam system is minimized.

Similarly, the response amplitude profile of a simply supported uniform damped shear beam subjected to moving load travelling at constant velocity for various values of foundation stiffness K is presented in Figure 2. It is observed that for fixed values of axial force N =$40000000,$ shear modulus G $= 40000000$ and the damping coefficient $Co = 3000,$ increase in the value of foundation stiffness K reduces the transverse displacement and rotation of the vibrating beam considerably. Consequently, the transverse displacement and rotation of the vibrating beam decrease with increasing values of foundation stiffness K because it provides resistance against torsion of the beam, thus making the system to become more rigid. Hence the existence of foundation stiffness increases the overall rigidity of the beam system.

Also, for various values of shear modulus G and for fixed values of other parameters, figure 3 clearly shows the response amplitude profile of the vibrating damped shear beam. It is evident from the curve that the higher the value of the shear modulus G the lower the transverse displacement and rotation of the beam. Practically speaking, this indicates that increase in the value of the shear modulus G increases the beam's rigidity and its resistance to deformation due to shear.

Finally, comparison of the effects of Winkler and Vlasov foundations on the transverse displacement and rotation of a simply supported prestressed damped shear beam traversed by moving load traveling at a constant velocity is presented in Figure 4. It is clearly seen from the figures that the transverse displacement and rotation of a prestressed damped shear beam supported by Winkler foundation is higher than that of a prestressed damped shear beam supported by Vlasov foundation. This justifies the claim that for engineering practices, Vlasov foundation is more adequate, efficient and reliable than Winkler foundation.

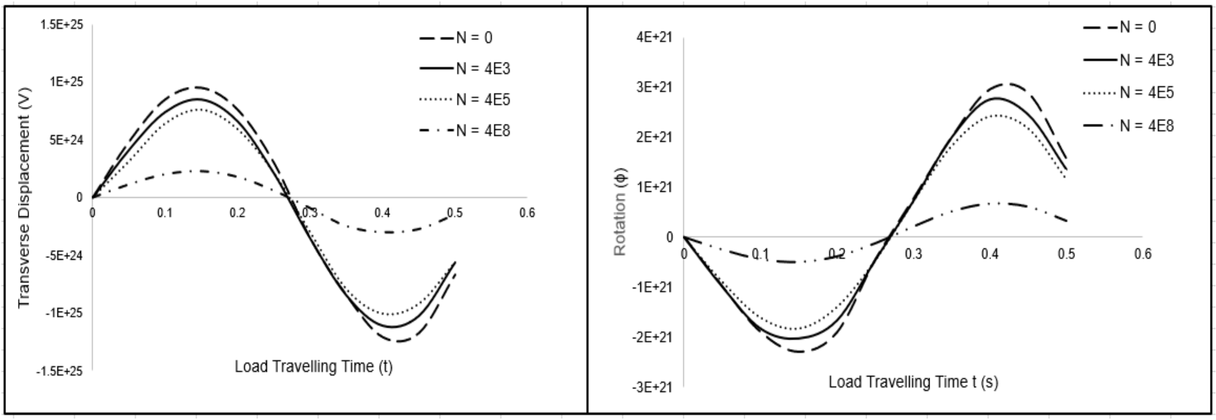

Figure 1: Transverse displacement and rotation of a simply supported prestressed damped shear Beam under the action of moving load travelling at constant velocity for various values of axial force N and for fixed values of K=400000, G=40000000 and Co=3000.

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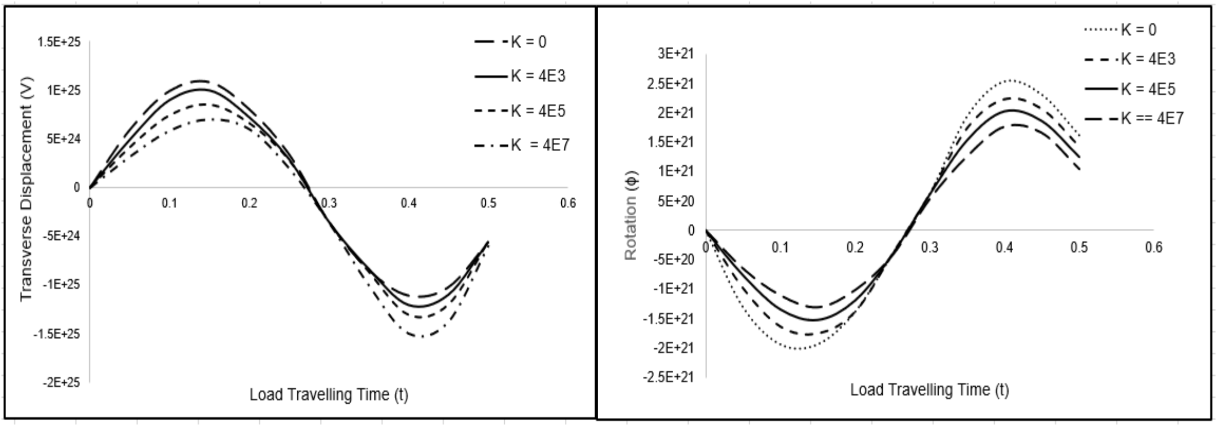

Figure 1: Transverse displacement and rotation of a simply supported prestressed damped shear Beam under the action of moving load travelling at constant velocity for various of foundation stiffness K and for fixed values of $N = 40000000$, $G = 40000000$ *and* $Co = 3000$.

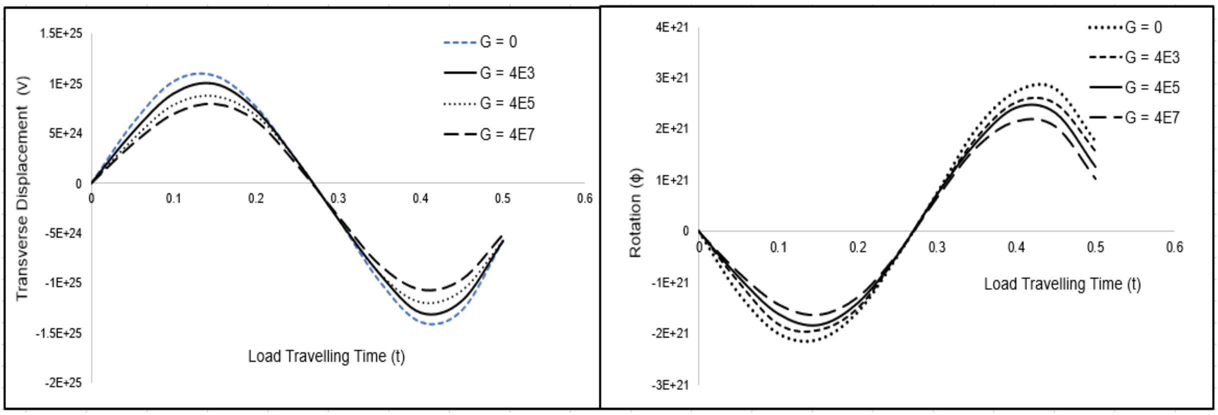

Figure 2: Transverse displacement and rotation of a simply supported prestressed damped shear beam under the action of moving load travelling at constant velocity for various of shear modulus G *and for fixed values of* $N = 40000000, K = 40000000$ *and* $Co = 3000$.

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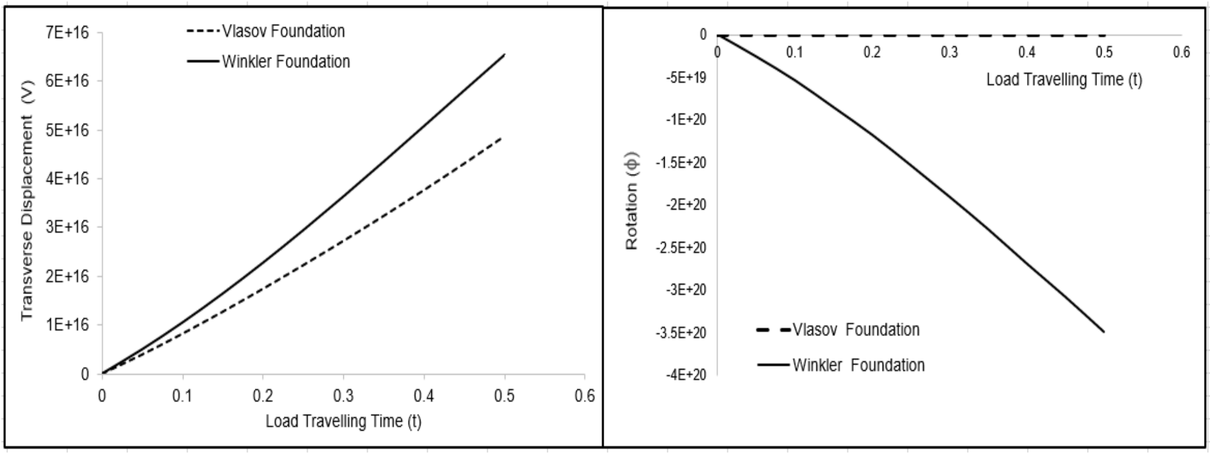

Figure 4: Comparison of the effects of Winkler foundation and Vlasov foundation on the transverse displacement and rotation of a simply supported prestressed damped shear beam when under the action of moving load travelling at constant velocity.

CONCLUDING REMARKS

This paper examines the dynamic behavior of a prestressed damped shear beam supported by a Vlasov foundation when subjected to a moving load. The governing equations are coupled second-order partial differential equations. A solution methodology involving finite Fourier transform techniques and Laplace transformation, along with convolution theory, is employed to derive the solution for the coupled second-order partial differential equations that characterize the motion of the beam-load system. Comprehensive analyses are conducted to assess the influence of key structural parameters, including axial force N, foundation stiffness K, and shear modulus G, on the transverse displacement and rotation of the beam. The graphical representations clearly indicate that these structural parameters significantly enhance the stability of the beam when subjected to the moving load. The findings reveal that the transverse displacement and rotation of the beam are markedly reduced with increasing axial force, shear modulus, and foundation stiffness. Additionally, the study compares the effects of Winkler and Vlasov foundation on the transverse displacement and rotation of a simply supported prestressed damped shear beam under the influence of a moving load traveling at a constant velocity. Furthermore, the research identifies the conditions under which the beam-load system may experience resonance phenomena and the velocities at which such occurrences may take place.

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