African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 1, 2025 (pp. 167-186)
www.abjournals.org

QUEUING THEORY AND ITS APPLICATION TO THE OPTIMUM NUMBER OF ATM MACHINES NEEDED TO REDUCE WAITING TIME OF CUSTOMERS IN THE QUEUE

Leo Tanyam Encho[1,3*], Abraham Okolo[2], Terrence Ayendoh Sama[1],

O. C. Asogwa[3], and C. C. Christopher[4]

[1]Department of Mathematics and Computer Science, The University of Bamenda, North West Region, Cameroon.

[2]Department of Statistics and Operations Research, Modibbo Adama University of Technology, Yola, Nigeria

[3]Department of Mathematics and Statistics, Alex Ekweme Federal University Ndufu-Alike, Nigeria.

[4]Department of Management and Information Systems, Cowan University, Perth, Western Australia.

[*]Corresponding Author's Email: encholeo2008@yahoo.co.uk

Cite this article:

L. T. Encho, A. Okolo, A. T. Sama, O. C. Asogwa (2025), Queuing Theory and its Application to the Optimum Number of ATM Machines Needed to Reduce Waiting Time of Customers in the Queue. African Journal of Mathematics and Statistics Studies 8(1), 167-186. DOI: 10.52589/AJMSS-IU8RVUTA

Manuscript History

Received: 14 Jan 2025

Accepted: 20 Jan 2025

Published: 11 Mar 2025

ABSTRACT: *Time spent by customers to access a service from banks with single ATM facility is increasingly a major source of concern and justification on the rationale of such waiting. This also imposes a potential threat to customers' services. In Cameroon, most commercial banks having a single ATM machine with large number of customers have cases where customers may not be attended to on time. The consequences of keeping customers in a queue for too long in order to get service can seriously affect business growth. In this study, the single server queuing models was used to analyze service efficiency of the Credit Communautaire D'Afrique (Community Credit of Africa) (CCA) bank at commercial Avenue Bamenda, Cameroon. Primary data was collected through observation and questionnaire methods at the bank over a ten days period to determine how to minimize the waiting time, expected service time, inter-arrival time and traffic intensity of customers in a queue. An M/M/1 method was considered where the arrival and service time of the customer were both exponentially distributed and with the implementation of a first come first serve. The result showed that increasing the number of ATM machines will reduce the waiting time of customers, overutilization of ATM and provide an optimal satisfaction of customers.*

KEYWORDS: Queue theory, M/M/1, waiting time, arrival rate service time, ATM and Customer Satisfaction.

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

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INTRODUCTION

Waiting for services is part of human daily life. We wait to eat at restaurants, queue up at the check-out counters in grocery stores and line up for service in post offices, banks and petrol filling stations. The waiting phenomenon is not an experience limited to human beings only; jobs wait to be processed on a machine, and cars stop at traffic lights. Unfortunately, waiting without incurring inordinate expenses cannot be eliminated. There is the need to reduce the adverse impact of waiting to acceptable levels (Taha, 1997).

Banks play a crucial role in the financial lives of customers by providing a range of services that help them manage their money, make transections and achieve their financial objectives. However, the major problem is the inability of banks to match their service facilities to the needs of customers without much delay. The problem is the time spent by customers to access a service is increasingly becoming a major source of concern due to lack of facilities and justification on the rationale of such waiting. Waiting a long time before service may be due to lack of enough manpower which results to the loss of many customers and properties. For instance, when the perception of waiting increases, customer satisfaction tends to decrease (Katz et al., 1991), escalate into anger and dissatisfaction (Duggirala et al., 2008), and causes inconveniences to the customers (Obamiro, 2010).

The length of wait directly affects service evaluation and gives a negative relationship between actual or perceived time spent waiting and service quality (Taylor, 1994). Consequently, the goals of efficient resource utilization and provision of high quality services which is one of the most important operational issues in public service providers are not met (Pierskalla & Wilson, 1989). This has continue to increase the movement of customers from one bank to the other where they can obtain banking services without much delay

In this situation, it is necessary to employ queuing techniques to achieve serenity and discipline in customer service. However, the act of queuing is associated with waiting time, which is an inevitable part of modern life (James, 2014).

The introduction of an Automated Teller Machine (ATM) by many banks which was an attempt to minimize waiting line problem has not yielded the much needed desired result due to frequent breakdown of such computerization and networking arrangements. Hence, long queue persist in many banks and customers are still waiting for service.

A queuing system can be simply described as customers arriving and waiting for service if it is not immediate and if having waited for service, leaving the system after being served. The term “customers” is used in a general sense and does not necessarily imply a human customer. For example, a customer could be a ball bearing waiting to be polished, an airplane waiting in line to take off, a computer program waiting to be run, or a telephone call waiting to be answered (Taha, 2001).

Queuing theory is the formal study of waiting in line and is an entire discipline in operation management. Queuing theory utilizes mathematical models and performance measures to assess and hopefully improves the flow of customers through a waiting line (Bunday, 1996; Prabhu, 1997 and Gorney, 1981).

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Agbadudu (1996) also defines queuing theory as a set of tools and techniques for analyzing problems, concerned with providing service to customers so as to have a balance of the cost of waiting and cost of servicing customers in a line.

Customer waiting time for service typically represents the primary or first direct interaction connecting customers and service delivery process (Davis & Heineke, 1993). It is the total elapsed time between issuance of a customer order and satisfaction of that order (Qfinance dictionary, 2009).

Bielen and Demoulin (2007) argued that customer perception of waiting time influence their satisfaction with the service that they receive. The unpleasant experience and its management affects the level of customer satisfaction (Alemu, 2019).

Generally, a queuing or waiting-line, problem arises whenever the demand for customer service cannot perfectly be matched by a set of well-defined service facilities. That is, there is more demand for service than there is facility for service available. This may be adduced to shortage of available services, or limitation to the amount of service that can be provided.

Generally, these limitations can be removed with the expenditure of capital. For adequate service availability, there is the need to know how long a customer will wait and the number of persons in the line. Queuing theory applies a detailed mathematical analysis in resolving service delivery issues (Gross and Harris, 1998).

Daw and Thet (2019) applied a single channel multiple server model to minimize the waiting time of customers in a fuel station. Abdulaziz, et al. (2022) in a multiple servers queuing models applied the regression model to analyze service efficiency in Ahmadu Bello University Teaching Hospital Zaria. Their results revealed that all the dimensions of service quality have significant positive relationship with the patient's satisfaction.

Sumaiyah (2004), applied queue theory to investigate the impact of waiting time perspective towards customer satisfaction before being served at the service counter. The result shows that satisfaction as functions of disconfirmation and perception are important factors to determine the customer's satisfaction with waiting time.

Victor (2015) applied simulation to analyze commercial banks teller counter and ATM services. The study revealed that if the numbers of queues are increased, the customers will not have to wait longer at the counter tellers or ATM services. Jelle (2017) used a simulation model to investigate the impact of waiting on customer behavior and resulting revenue. It was found that within the boundaries of the current capacity, revenue could be increased to a maximum percent if more flexible rules were used to allocate customers to positions.

Maqsood and Hina (2018) used questionnaires to investigate the delays in services and customer service evaluation of family dining restaurants. The results revealed that if good environment is provided at waiting area of restaurant lobbies, then waiting customer behavior can be managed and consequently customer satisfaction achieved. Mittal (2016) studied the influence of waiting time satisfaction on customer loyalty by using a regression test complemented by a correlation test towards multi-stage services in a full-service restaurant. The results suggest that waiting time does indeed impact customer loyalty. Kai et al (2011) applied structural equation modeling to test the theoretical model on the effects of passengers' perceived waiting experience and service guarantee on their satisfaction and repurchase

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intentions. Their results showed that the effects of perceived waiting time and service guarantee on satisfaction and repurchase intentions were significant.

Seigha (2017) evaluated the queuing system in Blue Meadows restaurant using the M/M/s model, the arrival rate, service rate, utilization rate, waiting time in the queue and the probability of customers likely balking and realized that the model improved the quality of service operations within the waiting line.

Chung-Te Ting, (2019) explored the importance of pre-processing service in the context of possible restaurant service crises and constructed a restaurant service recovery model for willingness to pay (WTP) through the contingent valuation method (CVM) to measure the effects before and after implementing service recovery. Mohammad (2018) applied queuing theory on randomly selected customers from an establishment to perceive the relationship between waiting time and customer satisfaction and realized that there is a positive significant relationship between perceived waiting time and service quality on waiting time satisfaction.

Peter and Sivasamy (2019) determined the capacity levels needed to experience demands in a more efficient way and realized that it can be used in identifying other opportunities for service improvement.

The motivation behind this study lies in the importance of customer satisfaction in the banking sector and the potential benefits of implementing efficient queuing systems. By addressing this issue, banks can enhance their customer service levels, retain customers, and improve their overall performance.

This study seeks to address the problem of long queues in banks ,why the bank sectors find it difficult in managing customer flow efficiently to reduce long queues and its effect on customer satisfaction and service delivery by banks. The study will therefore provide answers to the following research questions in an attempt to provide solution to the above problem:

1. How can queuing theory be applied to optimize customer waiting times and enhance satisfaction in the banking industry?
2. What are the key factors influencing customer satisfaction in bank queues?
3. What are the potential challenges and limitations of implementing queuing theory in banks?

The aim of this paper is to explore how queuing theory can be utilized as a model to improve customer satisfaction in the banking industry by reducing waiting times and enhancing service efficiency. This aim shall be achieved by considering the following objectives:

1. Implement queuing theory to optimize customer waiting times in a pilot study within a bank.
2. Assess the impact of queuing theory on customer satisfaction levels.
3. Identify recommendations for enhancing queuing systems in banks.

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STRUCTURE OF QUEUES

The structure of queuing model is separated into input and output queuing system which include queue that must obey a queuing rule and service mechanism as in Figure 1 (Hillier and Liberman, 2005).

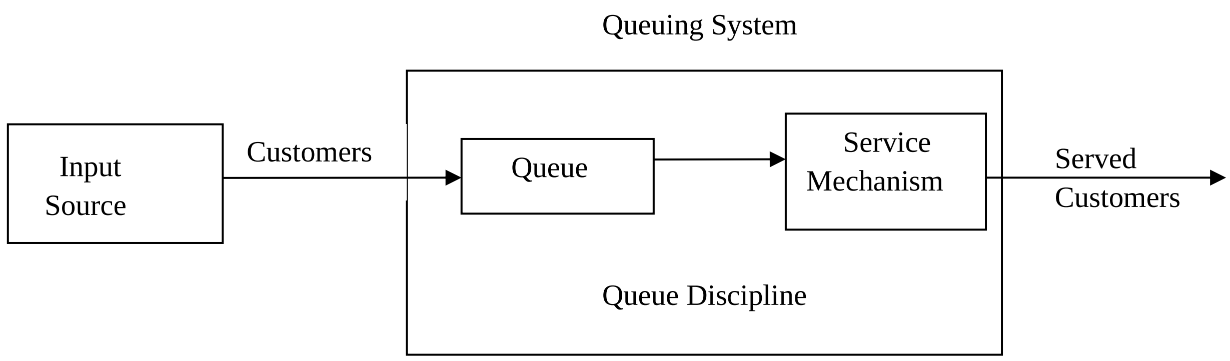

Figure 1: Structure of queuing model

Input Process

In queuing theory, the input process refers to the arrival pattern of customers or entities into the system. It describes how customers arrive at the queue, whether it follows a random or specific pattern, and at what rate customers enter the system.

Output Process

The output process in queuing theory refers to the departure pattern of customers or entities from the system. It describes how customers leave the queue after receiving service, whether they exit individually or in groups, and at what rate customers are served and exit the system. A sketch is given in Table 1.

Table 1: Input and output process

Place	Input Process	Output Process
Community Credit of Africa (CCA) bank	Arrival of customers at the ATM	Consultation and discharge

Queue Discipline

Queue discipline refers to the rules or criteria that determine the order in which customers are served from the queue. Common queue disciplines include first-come-first-served (FCFS), last-come-first-served (LCFS), shortest job next (SJN), and priority-based scheduling**.**

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Time

Service time in queuing theory is the amount of time it takes to serve a customer or complete a service transaction. It includes the time required for processing customer requests, handling transactions, and providing services to customers.

Service System

The service system in queuing theory refers to the overall structure and components of the system that provide services to customers. It includes the servers, queues, waiting areas, service processes, and customer interactions within the system.

Multiple server model with Poisson input and exponential service $M/M/S$ ($S > 1$)

In this type of queuing model, arrival time follow an exponential distribution with parameter ʎ, service time are exponentially distributed with parameter µ.The queuing discipline is first come first served and the utilization factor $p = \lambda/su$ where $S > 1$.

M/M/S/K model ($S = 1$) system with finite waiting space

In this model, it is assumed that there is a limit to which the system cannot exceed. Any customer that arrives joins the system when the queue does not exceed that limit and is rejected or loss when the system is full.

Finite source model of m/m/s

This model assumes that the input source (calling population) is limited or finite. This is against the general assumption of most queuing models that arrival rate to the system is from an infinite source.

M/M/S/C model (Erlang loss model)

Erlang loss model assumes Poisson arrival process, that is, exponentially distributed inter arrival time and exponential service time requirement. Its multiples server models with c-channels. Customer who arrives when c-channsels are busy leaves the system without waiting for service. The reason for the term is that there is no waiting room at all, the model was called Erlang loss model.

Material and Methods

In this investigation, we make use of Community Credit of Africa (CCA) bank, Commercial Avenue Bamenda in North West Region of Cameroon as case study for our analysis. Primary methods of data gathering adopted in the study were observational techniques and intensive /unstructured interviews.

The model contains one server which was heavily congested during the peak hours and but idle in the off peak hours (Manuel and Offiong, 2014). An efficient fair queuing model that is capable of reducing congestion by allocating resources on the network between contending users was developed. The proposed model gives higher priority to real time in order to allow them to have dependable performance. The primary data collection method used in this study is by direct interview and direct observation of the withdrawal section of the banking hall using a stop-watch. **One research assistant was employed and trained by the investigator on how**

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to record the service time while using a stopwatch to document both arrival time and service time. The number of customers arriving at the facility was recorded, as well as each customer's time of arrival, service time to the time of departure.

The study shall evaluate the performance of the service mechanism and to ascertain whether customers are satisfied with the banks' services based on the system's arrival and service pattern. The M/M/1queuing system shall be used to analyze the data collected.

Queuing Models

Queuing models are mathematical models used to analyze and predict the behavior of queues in various systems. These models help in studying the characteristics of queues, such as waiting times, queue lengths, service rates, and system performance. Common queuing models include M/M/1, M/M/c, M/G/1, and G/G/c, which represent different configurations of arrival and service processes in queuing systems.

Single Channel, Single phase model:

The most common and simplest case of queuing problems involves single channel, single server. This is a situation where there is only one person (or a team working as unit) providing the service at the facility. Also the service is completed in one stage. The following conditions exist in this type of system.

a. Arrivals are served on a first-come, first served (FCFS) basis and every arrival waits to be served, regardless of the length of the line or queue

b. Arrivals are independent of preceding arrivals, but the average number of arrivals (arrival rate) does not change overtime.

c. Arrivals are described by Poisson probability distribution and come from an infinite population.

d. Service times vary from one customer to the next and are independent of one another, but their average rate is known.

e. Service times occur according to the negative exponential probability distribution.

f. The service rate is faster than the arrival rate.

Queuing model notation

n : number of customers in the system (in the waiting time +service facility) at time t

λ: mean arrival rate (number of arrivals per unit of time)

μ : mean service rate per busy serve (number of customers served per unit of time)

P_n: Steady state probability of exactly n customers in the system

L_q : Expected (average) number of customers waiting in the queue

L_S : Expected number of customers in the system (waiting + being served)

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W_q :Expected waiting time per customers in the queue (expected time a customer keeps waiting in line),

W_S :Expected time a customer spends in the system (in waiting +being served).

ρ : system utilization

P_0: The probability that there are zero customers in the system;

P_w: The probability that a customer has to wait

To obtain the steady state equations, we considered that the probability that there will be n units $(n > 0)$ in the system at time $(t + \Delta t)$ may be expressed as the sum of three independent compound probabilities by using the fundamental properties of probability, Poisson arrivals and

exponential service time as in Table 2.

Table 2: Poisson arrivals and exponential service time

Time t number of units	Arrival	Service	Time $(t + \Delta t)$
N	0	0	N
n-1	1	0	N
n+1	0	1	N

By adding the above three independent compound probabilities, we obtain the probability of n units in the system at time $(t + \Delta t)$,

$$P_n(t + \Delta t) = P_n(1 - (\lambda + \mu)\Delta t) + P_{n-1}(t)\lambda\Delta t + P_{n+1}(t)\mu\Delta t + 0(\Delta t)$$

$$\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = -(\lambda + \mu)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t) + \frac{0(\Delta t)}{\Delta t}$$

$$\underset{\Delta t \to 0}{Lt} \frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = \underset{\Delta t}{Lt}\left[-(\lambda + \mu)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t) + \frac{0(\Delta t)}{\Delta t}\right]$$

$$\frac{dP_n(t)}{dt} = -(\lambda + \mu)P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t)$$

$$where\ (n > 0)\left[\therefore \underset{\Delta t}{Lt} \frac{0(\Delta t)}{\Delta t} = 0\right]$$

In a steady state

$$P_n(t) \to 0, P_n(t) = P_n$$

$$0 = -(\lambda + \mu)P_n + \lambda P_{n-1} + \mu P_{n+1} \quad (1)$$

Also the probability that there will be n units in the system at time

$(t + \Delta t)$ will be the sum of the following independent probabilities.

(i) Probability that there is no unit in the system at time t and no arrival in time $\Delta t = P_0(t)(1 - \lambda\Delta t)$

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(ii) Probability that there is one unit in the system at time t, one unit serviced in Δt in no arrival in $\Delta t = P_i(t)\mu\Delta t(1 - \lambda\Delta t)$

$= P_i(t)\mu\Delta t + 0\Delta t$

Adding these two probabilities we have

$$P_0(t + \Delta t) = P_0(t)(1 - \lambda\Delta t) + P_i(t)\mu\Delta t + 0\Delta t$$

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_i(t) + \frac{0(\Delta t)}{\Delta t}$$

$$\underset{\Delta t \to 0}{Lt} \frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_1(t) \text{ for n=0}$$

$$\frac{dP_n(t)}{dt} = -\lambda P_0(t) + \mu P_1(t)$$

Under steady state, we have

$$0 = -\lambda P_0 + \mu P_1 \qquad (2)$$

$Equations$ (1) and (2) are called steady state difference equations for $(M/M/1)$ $(\infty/FCFS)$ model.

$From\ equation\ (2), P_1 = \frac{\lambda}{\mu}P_0$ and from equation (1) $P_2 = \frac{\lambda}{\mu}P_1 = \frac{\lambda}{\mu} \times \frac{\lambda}{\mu}P_0 = \left(\frac{\lambda}{\mu}\right)^2 P_0$

$Therefore, in\ \ general,\ \ P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$

Since $\sum_{n=0}^{\infty} P_n = 1 \Rightarrow P_0 + \frac{\lambda}{\mu}P_0 + \left(\frac{\lambda}{\mu}\right)^2 P_0 + \cdots = 1$

$$P_0\left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \cdots\right] = 1$$

$$\Rightarrow P_0\left(\frac{1}{1-\frac{\lambda}{\mu}}\right) = 1 \qquad (3)$$

This series is a geometric series with first term 1 and common ratio $r = \left(\frac{\lambda}{\mu}\right)^2$. Since $\frac{\lambda}{\mu} < 1$, the sum of infinite GP is valid.

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho$$

Also $P_0 = \left(\frac{\lambda}{\mu}\right)^n P_0 = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$

$$P_n = \rho^n(1 - \rho)$$

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The expected number of units in the system

$$L_S = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right) = \left(1-\frac{\lambda}{\mu}\right)\frac{\lambda}{\mu}\sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^{n-1}$$

$$= \left(1-\frac{\lambda}{\mu}\right)\frac{\lambda}{\mu}\left[1+2\left(\frac{\lambda}{\mu}\right)+3\left(\frac{\lambda}{\mu}\right)^2+\cdots\right] = \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)^2$$

$$= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\rho}{1-\rho}, \rho = \frac{\lambda}{\mu} < 1$$

$$L_S = \frac{\rho}{1-\rho} \qquad (4)$$

Expected queue length $L_q = L_S - \frac{\lambda}{\mu} = \frac{\rho^2}{1-\rho} = \frac{\rho}{1-\rho}$

Expected waiting line in the queue

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{1-\rho}$$

Expected waiting line in the system

$$W_S = W_q + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$$

Expected waiting time of a customer who has to wait $(w/W > 0) = \frac{1}{\mu-\lambda} = \frac{1}{\mu(1-\rho)}$

Expected length of non-empty queue$(L/L > 0) = \frac{\mu}{\mu-\lambda} = \frac{1}{(1-\rho)}$

The probability of queue size$\geq N = \rho^N = \int_1^{\infty} \rho(\mu-\lambda)e^{(\mu-\lambda)\omega}d\omega$

Probability of waiting time in the system $\geq 0 = \int_1^{\infty} \rho(\mu-\lambda)e^{(\mu-\lambda)\omega}d\omega$

Probability of waiting time in the queue ≥ 1

Traffic intensity $\rho = \frac{\lambda}{\mu}$

Inter-relationship between L_S, L_q, W_S, W_q

We have $L_S = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\lambda}{\mu-\lambda}$

$$W_S = \frac{1}{\mu-\lambda}$$

$\therefore L_S = \lambda W_S$

Similarly, $L_q = \lambda W_q$ hold in general

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$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$W_S = \frac{1}{\mu-\lambda}$

$\therefore W_S = \frac{1}{\mu} = \frac{1}{\mu-\lambda} - \frac{1}{\mu} = \frac{\mu-(\mu-\lambda)}{\mu(\mu-\lambda)} \Rightarrow \therefore W_q - \frac{1}{\mu}$

Multiplying both sides by λ, we have

$\lambda W_q = \lambda\left(W_S - \frac{1}{\mu}\right)$

$L_q = \lambda W_S - \frac{\lambda}{\mu} = L_S - \frac{\lambda}{\mu} \Rightarrow L_q = L_S - \frac{\lambda}{\mu}$

If the value of λ does not remain fixed, it means some arrivals interested in joining the queue may not join due to long queue and since μ also depends on equal length, the service rate may be affected. In this situation the $P_0 = e^{-\rho}, where\ \rho = \frac{\lambda}{\mu}$

$P_n = \frac{\rho^n}{n!} e^{-\rho}$ 4

$L_S = \rho,$

$W_S = \frac{L_S}{\lambda}$

Arrival Poisson Process

Postulate of the process

1. The process $\{X(t), t > 0\}$ has independent increment $t_0 < t_1 < t_2 < t_3$ ....

Increments $X(t_1) - X(t_0), X(t_2) - X(t_1)$ ...

These increment are independent i.e. the distribution of $X(t-h) - X(t) = R(h)$ depends only on h.

2. Consider a very small interval of time $(t, t+h)$

$$P\left[X(t, t+h) - X(t) = \frac{1}{X(t)}\right] = \lambda h + 0(h), where \lim_{h-0} \frac{0(h)}{h} = 0$$

3. $P\left[X(t+h) = \frac{0}{X(t)} = 0\right] = 1 - \lambda h + 0(h)$

The value $0(h)$is so small that could be taken equal to zero

To obtain the distribution of the process

Define $P_k(t) = P[X(t) = k]$ and consider the event that $X(t+h) = k$

1. $X(t) = k\ and\ X(t+h) - X(t) = 0$

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2. $X(t) = k - 1 \; and \; X(t+h) - X(t) = 1$

3. $X(t) = k - i \; and \; X(t+h) - X(t) = i, i \geq 2$

$\Rightarrow P_k(t+h) = P_k(t)P_0(h) + P_{k+1}(t)P_0(h) + \sum_{i=2}^{k} P_{k-i}(t)P_i(h)$

$P_k(t)[1 - \lambda h + 0(h)] + P_{k+1}(t)[\lambda h + 0(h)] + \sum_{i=2}^{k} P_{k-i}(t)0(h)$

where $0(h)$ multiply by any quantity remains an $0(h)$

$P_k(t+h) = P_k(t) - \lambda h P_k(t) + 0(h) + \lambda h P_{k-1}(t) + 0(h)$

Rearrange, divide by h and take limit as h tends to zero, we obtain

$$P_k^1(t) = -\lambda P_k(t) + \lambda P_{k-1}(t), K = 1,2, \ldots \quad (5)$$

Now consider the solution of the equation (5) when $k = 0$

$P_0^1(t) = -\lambda P_0(t)$

$-\lambda = \frac{P_0^1(t)}{P_0(t)} \Rightarrow \log P_0(t) = -\lambda t + c \; Since \int \frac{dt}{P_0(t)} = -\int \lambda dt$

$P_0(t) = e^{-\lambda t + c}$

Recall $P_0(0) = P[X(0) = 0] = 1, 1 = e^c \; that \; is \; c = 0$

$$P_0(t) = e^{-\lambda t} \quad (6)$$

$when \; k = 1, we \; obtain$
$P_1^1(t) = -\lambda P_1(t) + \lambda P_0(t)$
$-\lambda P_1(t) + \lambda e^{-\lambda t}$

$$P_1(t) = \lambda t e^{-\lambda t} \quad (7)$$

In general,

$$P_k^1(t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0,1,2,3 \ldots \quad (8)$$

$which \; shows \; that \; the \; arrival \; process \; has \; poisson \; distribution$

with $\mathrm{E(t)} = \lambda t \; and \; V(t) = \lambda t$

Let X be a customer's arrival process then, X has a Poisson distribution with parameter (λ).

$f(x) = \frac{e^{-\lambda}}{x!}\lambda^{\mathrm{x}}, \; \mathrm{x} = \mathrm{o}, 1,2 \ldots$

$\mathrm{E(x)} = \lambda$ and $\mathrm{v(x)} = \lambda$

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DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

Service Process

There are no branches in the service process.

Suppose μ is the service rate

$Pr(T > t) = e^{-\mu t}$

$Pr(T < t) = 1 - e^{-\mu t}$

To obtain the pdf,

$f(t) = \frac{d}{dt}[F(t)] = \mu e^{-\mu t}, t > 0$

The service process has exponential distribution with parameter μ

$\mathrm{E(t)} = \frac{1}{\mu}$, and $\mathrm{V(t)} = \frac{1}{\mu^2}$

Proof: using MGF since y is continuous,

let y be the service process, then y follows exponential distribution with paraeter μ,

$f(y) = \mu e^{-\mu y}, 0 < y < \infty$

$E(x) = \frac{1}{\mu}, v(y) = \frac{1}{\mu^2}$

$M_y(t) = E(e^{ty}) = \int_0^\infty e^{ty}.\ f(y)dy = \int_0^\infty e^{ty}.\mu e^{-\mu y}dy = \int_0^\infty \mu e^{ty-\mu y}. = \int_0^\infty \mu e^{-y(\mu - t)}dy$

Using integration by part,

$M_y(t) = \mu[\mu - t]^{-1}$

$now\ to\ find\ the\ mean\ , E(y) = M_y^1(0) = -\mu[\mu - t]^{-2}.-1 = \frac{1}{\mu}$

$v(y) = M_y^{11}(0) - [M_y^1(0)]^2\ -2\mu[\mu - t]^{-3}.-1 = 2\mu[\mu - t]^{-3} = \frac{2\mu}{\mu^3} - [M_y^1(0)]^2 = \frac{2\mu}{\mu^3} - \frac{1}{\mu^2}$

$= \frac{2\mu - \mu}{\mu^3} = \frac{1}{\mu^2}$

This is the variance of the distribution

Conditions Determining the Effectiveness of the System

λ Is the arrival rate, $\frac{1}{\lambda}$ is the inter-arrival rate, and $\frac{1}{u}$ is the service rate.

We have the following conditions:

i. If $\frac{1}{\lambda} = \frac{1}{u}$, there is no queue and the server will not rest.

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

ii. $\frac{1}{\lambda} > \frac{1}{u}$, implies there will be no queue and the server will sometime relax.

iii. $\frac{1}{\lambda} < \frac{1}{u}$ implies there will be queue.

Data Presentation and Analysis

This section deals with data presentation, analysis and discussions of the result. The arrival and service process of the customers coming to the bank were observed and recorded in Appendix Tables 1 and 2 respectively. In Appendix Table 1, X denotes the arrival per one minute and f the number of arrivals, while in Appendix Table 2, Y denotes the service process and f the number of patients served within the intervals $0-2, 2-4, \ldots, 38-40$ minutes.

Arrival Process

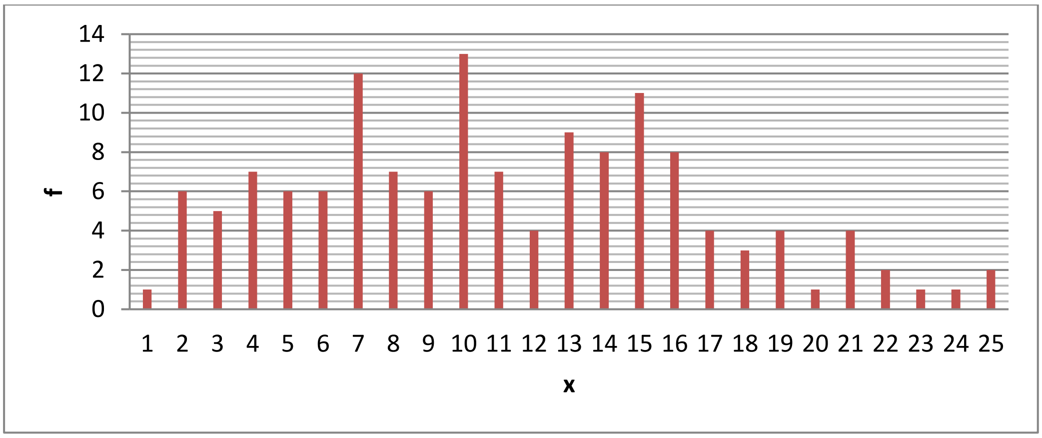

Figure 2: **Bar chart showing the arrival process (discrete data)**

Figure 2 shows that 1 customers arrive at 1 minute, 6 customers at 2 minutes and the highest number of arrival is at 10 minutes.

Service Process

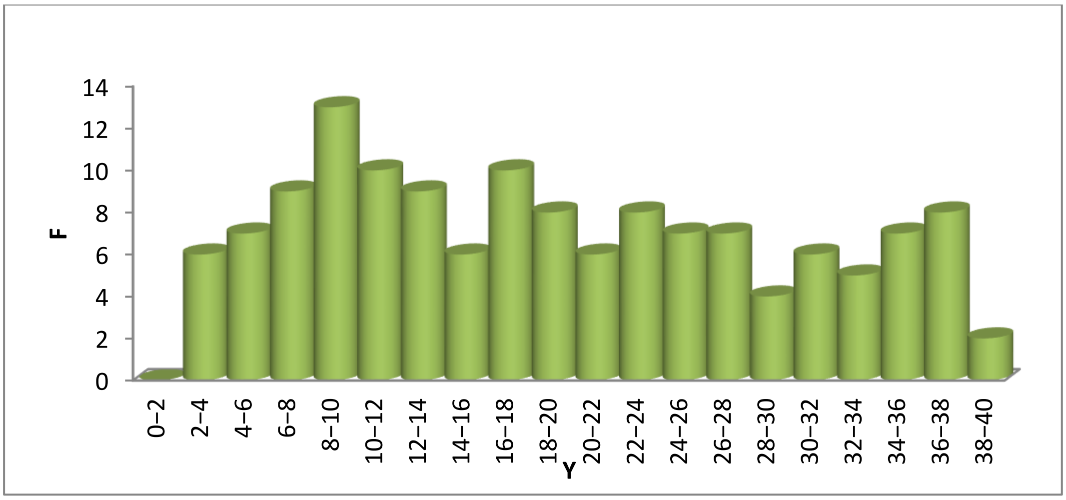

Figure 3: A histogram showing the service process (continuous data)

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

Based on Figure 3, at 0-2 minutes no customer is served, and the highest number of customers are served at 8 - 10 minutes.

DATA ANALYSIS

Let X denote the arrival process, then X is said to have a Poisson distribution with parameter (λ),

$$f(x) = \begin{cases} \frac{e^{-\lambda}}{x!}\lambda^{x}, & x = 0,1,2 \ldots, \lambda > 0 \\ 0, & Otherwise \end{cases}$$

With $E(x) = \lambda$, where λ is the mean arrival rate and $v(x) = \lambda$

$$E(x) = \lambda = \frac{\sum fx}{\sum f} = \frac{1398}{138} = 10.13$$

dividing λ by 6 to get the daily average we have $\frac{10.13}{6} = 1.69$

On average we have two arrivals per minute.

Let $\frac{1}{\lambda}$ be the inter arrival rate, then $\frac{1}{\lambda} = \frac{1}{1.69} = 0.59\ (30\ sec)$

This is the time between successive arrivals of the patient.

Also, let y be the service process, then y follows exponential distribution with parameter μ,

$$f(y) = \begin{cases} \mu e^{-\mu y}, & 0 < y < \infty, \mu > 0 \\ 0, & Otherwise \end{cases}$$

$$E(y) = \frac{1}{\mu}, v(y) = \frac{1}{\mu^2}$$

$$E(y) = \frac{1}{\mu} = \frac{\sum fx}{\sum f} = \frac{2632}{138} = 19.07$$

Dividing it by 6 to get daily average, we have $\frac{19.07}{6} = 3.18$, which is the expected service time.

Since, $\frac{1}{\lambda} = 0.59 < \frac{1}{u} = 3.18$, it implies that queue exist, which should be eliminated.

Optimum Number of ATM Needed to Eliminate the Queue

Let C be the number of service point (ATM) needed to reduce the queue; then an additional

$C - 1$ ATMs, will be needed,

where $C = \frac{\lambda}{\mu}, \lambda = 1.69\ and\ \mu = \frac{1}{3.18} = 0.31,$

$$C = \frac{1.69}{0.31} = 5.45 \approx 5$$

Therefore, the number of ATMS to be added is $c - 1 = 4$ for an effective service delivery.

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 1, 2025 (pp. 167-186)

Distribution of the Developed Queue

Since queue exist, we seek to find the distribution of the developed queue. Let Z denote the waiting time of customers in the queue, then Z is said to have Poisson distribution with parameter (λ) and

$$f(z) = \begin{cases} \frac{e^{-\lambda}}{z!}\lambda^{z}, & z = o, 1,2 \dots \\ 0, & Otherwise \end{cases}$$

with $E(z) = \lambda$, where λ is the mean waiting time and $v(z) = \lambda$

$$p(Z = z) = \frac{e^{-1.69}}{z!}(1.69)^{z}, \quad z = 0,1,2 \dots$$

With this, we compute the probability of waiting at $z = 0,1,2 \dots$

Table 3: Probability of waiting time

Z	$p(Z = z)$
1	0.311838
2	0.263503
3	0.14844
4	0.062716
5	0.021198
6	0.005971
7	0.001442
8	0.000305
9	5.72E-05
10	9.66E-06
11	1.48E-06

Table 3 shows that the computed probability values decreases exponentially with an increasing waiting time. This implies that the inter-arrival time between customers has an exponential distribution. A graph of Table 3 is shown in Figure 4.

Figure 4: Probability of waiting time against waiting time

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DISCUSSION

This study has revealed that the expected service time $\frac{1}{u} = 3.18$ is greater than the inter-arrival rate $\frac{1}{\lambda} = 0.59$ which means that a queue exists. The traffic intensity C which is the number of ATMs needed for optimum service is found to be 5 based on the arrival pattern of the customers. That is, 4 additional ATMs are required for efficient service delivery. The inter-arrival time between customers was found to have an exponential distribution.

CONCLUSION

Queuing theory is a useful statistical technique for solving peculiar problems; its application in organization is indispensable. The queuing characteristic at CCA Bank, Bamenda, Cameroon was analyzed to determine the optimal number of ATM needed to reduce the waiting time of the customers. The result of the analysis showed that the waiting time of customers and over utilization of ATMs could be reduced when the service capability level of the ATMs at the Bank is increased to 5. As service increases the time spent waiting on line decreases. This study can be applied to queuing situations in similar organisations. This study has shown that the management of the CCA bank require additional 4 ATM machines to increase the service rate, as one ATM machine cannot cope with the number of customers that visit the ATM service point.

REFERENCES

Ahmad, M., & Naseer, H. (2018). Delays in Services and Customer Service Evaluation: A Study of Family Dining Restaurants of Pakistan. Journal of Investment Management, 7, 108.

Ajiboye, M. R. (2020). A simulation Model for Customer Flow Analysis in A commercial Bank in Nigeria. European Journal of Technique (EJT), 10(1), 64-74.

Augustine, A. N. (2013). Queuing model as a technique of queue solution in Nigeria banking industry. Developing Country Studies, 3(8), 188-195.

Bello, Ahmadu & Adeniran, Abdulazizmaruf & Sani Burodo, Muhammad & Suleiman, Shamsuddeen. (2022). Application of Queuing Theory and Management of Waiting Time Using Multiple Server Model: Empirical Evidence From. International Journal of Scientific and Management Research. 05. 2022. 10.37502/IJSMR.2022.5412.

Bielen, F., & Demoulin, N. (2007). Waiting time influence on the satisfaction-loyalty relationship in services. Journal of Service Research.

Bunday, B. D. (1996). An introduction to queuing theory. Arnold.

Davis, M. M., & Heineke, J. (1993). Understanding the roles of the customer and the operation for better queue management. International Journal of Service Industry Management, 4(3), 18-34.

De Vries, Jelle & Roy, Debjit & De Koster, René. (2018). Worth the wait? How restaurant waiting time influences customer behavior and revenue. Journal of Operations Management. 63. 10.1016/j.jom.2018.05.001.

Duggirala, M., Rajendran, C., & Anantharaman, R. N. (2008). Patient-perceived dimensions of total quality service in healthcare. Benchmarking: An International Journal.

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

Gorney, M. (1981). The role of queuing theory in the design of telecommunication systems. Bell System Technical Journal.

Gross, D., & Harris, C. M. (1998). Fundamentals of queueing theory (3rd ed.). John Wiley & Sons.

Gumus, S., Bubou, G. M., & Oladeinde, M. H. (2017). Application of queuing theory to a fast food outfit: a study of blue meadows restaurant. Independent Journal of Management & Production, 8(2), 441-458.

Hillier, S.F and Lieberman, J.G (2005) *introduction to operation research,* Boston: McGraw Hill, Eight Edition.

Ji, J. (2023, November). Application of Queuing Theory in Public Service Counters. In 2023 International Conference on Image, Algorithms and Artificial Intelligence (ICIAAI 2023) (pp. 509-517). Atlantis Press.

Kamau, G. W. (2012). Waiting lines management and customer satisfaction in Commercial banks in Kenya (Doctoral dissertation, University of Nairobi).

Katz, K. L., Larson, B. M., & Larson, R. C. (1991). Prescription for the waiting-in-line blues: Entertain, enlighten, and engage. Sloan Management Review.

Mittal, A. (2016). The Influence of Waiting Time Satisfaction on Customer Loyalty towards multi-stage Services in a full-service restaurant: Evidence from India. Вісник Київського національного університету ім. Тараса Шевченка. Серія: Економіка, (6 (183)), 22-26.

Mohammad Salameh, A. A., Ahmad, H., Zulhumadi, F., & Abubakar, F. M. (2018). Relationships between system quality, service quality, and customer satisfaction: M-commerce in the Jordanian context. Journal of Systems and Information Technology, 20(1), 73-102.

Nosek Jr, R. A., & Wilson, J. P. (2001). Queuing theory and customer satisfaction: a Review of terminology, trends, and applications to pharmacy practice. Hospital pharmacy, 36(3), 275-279.

Obamiro, J. K. (2010). The role of queuing in customer satisfaction in Nigerian banks. Journal of Retail and Consumer Services.

Pierskalla, W. P., & Wilson, J. R. (1989). The effectiveness of different scheduling policies in reducing waiting times in public services. Management Science.

Prabhu, N. U. (2012). Foundations of queueing theory (Vol. 7). Springer Science & Business Media.

Prabhu, N. U. (1997). Foundations of queuing theory. Springer-Verlag.

Sagayarag, M. R., Raguvaran, R., & Daisy, M. (2015). A study on queuing model to reduce the waiting time in fuel stations. International Journal of Technical Research and Applications, 3(2), 104-106.

Shortle, J. F., Thompson, J. M., Gross, D., & Harris, C. M. (2018). Fundamentals of queueing theory (Vol. 399). John Wiley & Sons.

Stallings, W. (2000). Queuing analysis. WilliamStallings. com/StudentSupport. html.

Taha, H. A. (1997). Operations research: An introduction (6th ed.). Prentice Hall.

Taha, H. A. (2001). Operations research: An introduction (7th ed.). Prentice Hall.

Taylor, S. (1994). Waiting for service: the relationship between delays and evaluations of service. Journal of marketing, 58(2), 56-69.

Ting, C. T., Huang, Y. S., Lin, C. T., & Pan, S. C. (2019). Evaluation of Consumers' WTP for Service Recovery in Restaurants: Waiting Time Perspective. Administrative Sciences, 9(3), 63.

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

Willcox, S., Seddon, M., Dunn, S., Edwards, R. T., Pearse, J., & Tu, J. V. (2007). Measuring and reducing waiting times: a cross-national comparison of strategies. Health Affairs, 26(4), 1078-1087.

APPENDIX TABLE 1: ARRIVAL PROCESS

S/N	Arrival Per Minute (X)	Number of Arrival (F)	(XF)
1	0	1	0
2	1	6	6
3	2	5	10
4	3	7	21
5	4	6	24
6	5	6	30
7	6	12	72
8	7	7	49
9	8	6	48
10	9	13	117
11	10	7	70
12	11	4	44
13	12	9	108
14	13	8	104
15	14	11	154
16	15	8	120
17	16	4	64
18	17	3	51
19	18	4	72
20	19	1	19
21	20	4	80
22	21	2	42
23	22	1	22
24	23	1	23
25	24	2	48
	TOTAL	138	1398

Article DOI: 10.52589/AJMSS-IU8RVUTA
DOI URL: https://doi.org/10.52589/AJMSS-IU8RVUTA

African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 1, 2025 (pp. 167-186)
www.abjournals.org

APPENDIX TABLE 2: SERVICE PROCESS

X	*F*	*YF*
0−2	0	0
2−4	6	18
4−6	7	35
6−8	9	63
8−10	13	117
10−12	10	110
12−14	9	117
14−16	6	90
16−18	10	170
18−20	8	152
20−22	6	126
22−24	8	184
24−26	7	175
26−28	7	189
28−30	4	116
30−32	6	186
32−34	5	165
34−36	7	245
36−38	8	296
38−40	2	78

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