African Journal of Mathematics and Statistics Studies

ISSN: 2689-5323

Volume 8, Issue 1, 2025 (pp. 77-83)

www.abjournals.org

COMPARATIVE ANALYSIS OF FRÉCHET DISTRIBUTION VARIANTS: PARAMETER ESTIMATION AND MODEL PERFORMANCE EVALUATION

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Cite this article:

Akalagboro, I. O., Aronu, C. O., Mark, L. S. (2025), Comparative Analysis of Fréchet Distribution Variants: Parameter Estimation and Model Performance Evaluation. African Journal of Mathematics and Statistics Studies 8(1), 77-83. DOI: 10.52589/AJMSS-L3LZF5LA

Manuscript History

Received: 19 Nov 2024

Accepted: 9 Jan 2025

Published: 17 Jan 2025

ABSTRACT: *This study presents a comparative analysis of six Fréchet distribution variants: Kumaraswamy Fréchet (KF), Exponentiated Fréchet (EF), Beta Fréchet (BF), Gamma Extended Fréchet (GExF), Odd Lomax Fréchet (OLxF), and the standard Fréchet (F), focusing on their structural properties, parameter estimation, and model performance. These distributions, characterized by varying levels of complexity and flexibility, are particularly effective for modelling extreme values and heavy tails, crucial in fields like econometrics and reliability analysis. Differences in Probability Density Functions (PDFs) reveal the enhanced adaptability of BF and GExF variants, attributed to their additional beta and gamma components. The models were applied to three datasets: Jobs Made of Iron Sheets, Airborne Communication Transceiver Repairs, and Tax Revenue. The performance of the distributions under study was evaluated using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The finding showed that the standard Fréchet distribution consistently outperformed its variants, achieving the lowest AIC and BIC values across datasets, indicating a superior balance of simplicity and adaptability. EF and KF variants demonstrated competitive performance but lacked the robustness of the standard Fréchet model, while OLxF and GExF showed higher AIC and BIC values due to potential over-parameterization. This study underscores the importance of aligning model complexity with dataset characteristics and highlights the standard Fréchet distribution as a versatile choice for analyzing extreme data.*

KEYWORDS: Fréchet distribution, Parameter estimation, Model performance, Extreme values, Heavy tails, Over-parameterization.

Article DOI: 10.52589/AJMSS-L3LZF5LA

DOI URL: https://doi.org/10.52589/AJMSS-L3LZF5LA

INTRODUCTION

Probability distributions play a crucial role in statistical modelling, particularly in understanding and forecasting extreme events across diverse domains, including econometrics, reliability analysis, and environmental studies. Among these, the Fréchet distribution and its extensions have garnered significant attention due to their ability to model skewed and heavy-tailed data effectively. The Bayesian estimation of a two-component mixture of transmuted Fréchet distribution, for instance, has been highlighted for its reliability in handling dependability data, as demonstrated by Aslam et al. (2021). Their approach employed Markov Chain Monte Carlo (MCMC) techniques to enhance prediction accuracy, particularly for datasets involving right-censored sampling. The development of generalized families of distributions from the Fréchet base model has further advanced statistical methodologies, as illustrated in studies by Omekam et al. (2022) and Alyami et al. (2022). These works underscore the adaptability of Fréchet-derived distributions, such as the Fréchet binomial (FB), for diverse data types, emphasizing their utility in econometric modelling. Additionally, innovative extensions like the odd Lomax Fréchet (OLxF) distribution, proposed by Hamed et al. (2020), and the harmonic mixture Fréchet model by Ocloo et al. (2022), demonstrate enhanced performance in real-world applications by incorporating additional parameters to improve flexibility and goodness-of-fit.

Recent advancements, such as the odd log-logistic Lindley-Weibull (OLLW) distribution introduced by Al-Sobhi (2022), have further broadened the scope of statistical modelling, offering superior adaptability for various practical domains, including survival analysis and economic decision-making. Likewise, Phaphan et al. (2023) explored innovative methodologies combining the Fréchet distribution with mixture models, presenting promising results for skewed survival data using advanced estimation techniques. Building on these developments, this study conducts a comparative analysis of Fréchet distribution variants, focusing on parameter estimation and model performance evaluation. Using Maximum Likelihood Estimation (MLE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC), the study systematically ranks the distributions based on their efficiency and consistency. Six variants of the Fréchet distribution were selected for modelling: Kumaraswamy Fréchet (KF), Exponentiated Fréchet (EF), Beta Fréchet (BF), Gamma Extended Fréchet (GExF), Odd Lomax Fréchet (OLxF), and the standard Fréchet (F) distribution. These distributions were chosen based on their theoretical flexibility and ability to model extreme-value data. This approach seeks to provide valuable insights into the suitability of these variants for real-world applications, contributing to the growing body of knowledge in econometric and statistical modelling.

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METHODS

Source of Data Collection for the Study

Secondary data from credible published works is used in this study to obtain accurate and pertinent insights into the research issue. The study expands on the findings of earlier researchers by utilising existing literature, which improves comprehension of the research subject. It contains measurements of hole diameters in iron sheet jobs from Alotaibi et al. (2022) that range from 0.02 to 0.32 millimetres. Alyami et al. (2022) provided the data on aerial communication transceiver repair timeframes. Furthermore, Tharshan and Wijekoon (2020) provided tax revenue data, which included 60 observations and provided information on fiscal trends.

Method of Data Analysis

This section describes the analytical framework, including ranking distributions for optimal data modelling, model evaluation using AIC and BIC criteria, and parameter estimation using Maximum Likelihood Estimation (MLE).

Parameter Estimation

The Maximum Likelihood Estimation (MLE) method was employed to estimate the parameters of each distribution. This method was chosen due to its efficiency and consistency in providing reliable parameter estimates.

Model Evaluation

The performance of the distributions was assessed using two model selection criteria:

i. Akaike Information Criterion (AIC): Used to evaluate model fit while penalizing for complexity to prevent overfitting.

ii. Bayesian Information Criterion (BIC): Similar to AIC but imposes a heavier penalty for the number of parameters, favoring simpler models for smaller datasets.

Ranking of Distributions

The distributions were ranked based on their AIC and BIC values for each dataset. The distribution with the lowest values was considered the best fit. Also, the results were analyzed to determine which distribution provided the best fit for each dataset. Justifications were provided based on the consistency of the performance metrics and the theoretical properties of the distributions. This methodology ensures a rigorous and comprehensive evaluation of the distribution variants, providing reliable insights into their suitability for modelling real-world data.

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RESULTS

The result presented in Table 1 shows the theoretical comparison of the variants of the Fréchet distribution.

Table 1: The Theoretical Comparison of the Variants of the Fréchet Distribution

S/No.	Distribution	PDF
1.	Kumaraswamy Fréchet (KF)	$\alpha\beta b a^b x^{-(b+1)} exp\left[-\alpha\left(\frac{a}{x}\right)^b\right]\left\{1-exp\left[-\alpha\left(\frac{a}{x}\right)^b\right]\right\}^{\beta-1}$
2	Exponentiated Fréchet (EF)	$\theta b a^b x^{-(b+1)} exp\left[-\left(\frac{a}{x}\right)^b\right]\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\theta-1}$
3	Beta Fréchet (BF)	$\frac{b a^b}{B(\alpha,\beta)} x^{-(b+1)} exp\left[-\alpha\left(\frac{a}{x}\right)^b\right]\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\beta-1}$
4	Gamma extended Fréchet (GExF)	$\frac{\alpha b a^b}{\Gamma(\beta)} x^{-(b+1)} exp\left[-\left(\frac{a}{x}\right)^b\right]\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\alpha-1}$ $\left\{-log\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\alpha}\right\}^{\beta-1}$
5	Odd Lomax Fréchet distribution (OLxF)	$\alpha\beta^{\alpha}\frac{b a^b x^{-(b+1)} exp\left[-\left(\frac{a}{x}\right)^b\right]}{\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^2}\left\{\beta+\frac{exp\left[-\left(\frac{a}{x}\right)^b\right]}{1-exp\left[-\left(\frac{a}{x}\right)^b\right]}\right\}^{-\alpha-1}$
6	Fréchet (F)	$\alpha b a^b x^{-(b+1)} exp\left[-\left(\frac{a}{x}\right)^b\right]\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\alpha-1}$ $\left(1+\beta-2\beta\left\{1-exp\left[-\left(\frac{a}{x}\right)^b\right]\right\}^{\alpha}\right)$

Table 1 compares six variants of the Fréchet distribution, emphasizing their Probability Density Functions (PDFs). The Kumaraswamy Fréchet, Exponentiated Fréchet, Beta Fréchet, Gamma Extended Fréchet, Odd Lomax Fréchet, and standard Fréchet distributions exhibit structural similarities but differ in parameterization and complexity, enhancing flexibility for modelling diverse data.

Each variant incorporates shape, scale, and exponentiation factors that fit specific data sets. The Beta Fréchet and Gamma Extended Fréchet, with additional beta and gamma functions, provide enhanced adaptability for heavy-tailed phenomena. Such flexibility justifies their relevance in applications requiring precise modelling of extreme events.

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Table 2: Summary Result of the Performance of the Variants of the Fréchet Distribution

Data	Distributions	Parameter estimates	AIC	BIC	Rank
Jobs made of Iron Sheet	KF	$\alpha = 0.7295$, $\beta = 149.1105$, a = 43.7049, b = 0.3493	112.1345	119.7826	3
	EF	a = 28.3148, b = 0.3588, $\theta = 502.3551$	107.6745	113.4105	2
	BF	a = 2.3126e-11, b = 0.0732, $\alpha = 0.159$, $\beta = 8.1366$	971.2671	978.9152	5
	GExF	a = 1.0, b = 1.0, $\alpha = 1.1$, $\beta = 1.0$	554.8456	547.1975	4
	OLxF	a = 1.0943e-18, b = 0.5756, $\alpha = 1.2752$, $\beta = 2.7303$	7726.076	7733.724	6
	F	a = 4.0242, b = 1.1488, $\alpha = 12.5059$, $\beta = -0.1402$	0.3346	7.9827	1
Airborne communication transceiver's active repairs	KF	$\alpha = 1.1372$, $\beta = 0.9561$, a = 1.2664, b = 1.2409	170.8966	164.1411	2
	EF	a = 1.4070, b = 0.0732, $\theta = 0.9582$	172.8966	167.83	3
	BF	a = 0.024, b = 0.0003, $\alpha = 391.941$, $\beta = 705.69$	25267.95	25274.7	6
	GExF	a = 0.0049, b = 4.3900, $\alpha = 0.0311$, $\beta = 3.2524$	228.7159	235.4714	4
	OLxF	a = 7.0406e-10, b = 1.5420, $\alpha = 1.6109$, $\beta = 1.0657$	9781.074	9787.83	5
	F	a = 14.8058, b = 0.3554, $\alpha = 21.7172$, $\beta = 123.0675$	8.6940	55.4495	1
Taxes revenue	KF	$\alpha = 8.8667$, $\beta = 2.0892$, a = 3.1427, b = 1.5453	368.7503	360.4401	3
	EF	a = 12.9084, b = 1.5444, $\theta = 2.0919$	370.7503	364.5177	4
	BF	a = 3.2255, b = 2.1952, $\alpha = 5.7689e-12$, $\beta = 7.5761e-7$	189.4616	181.1514	2
	GExF	a = 0.0426, b = 5.4837, $\alpha = 0.0389$, $\beta = 10.9661$	1643.81	1652.12	6
	OLxF	a = 5.0564e-09, b = 1.6437, $\alpha = 1.4649$, $\beta = 0.9638$	14298.12	14306.43	5
	F	a = 2.5867, b = 13.6471, $\alpha = 0.1079$, $\beta = 27.8796$	147.9172	139.6071	1

Table 2 presents the performance of six Fréchet distribution variants applied to three datasets: Jobs Made of Iron Sheet, Airborne Communication Transceiver's Active Repairs, and Tax Revenue. The models were evaluated using AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion), with rankings assigned based on the best fit.

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The standard Fréchet (F) model consistently ranks highest, showing the lowest AIC and BIC values, indicating superior performance across datasets. The Exponentiated Fréchet (EF) and Kumaraswamy Fréchet (KF) variants also perform well in certain cases but lack the consistency of the F model. Variants like OLxF and GExF often rank lowest due to their complexity and higher parameter sensitivity, making them less suited for practical applications. The superior performance of the standard Fréchet model is attributed to its balance between simplicity and flexibility, making it efficient for fitting diverse datasets while avoiding over-parameterization.

CONCLUSION

This study conducted a comprehensive comparative analysis of six Fréchet distribution variants, emphasizing their structural properties, parameter estimation techniques, and model performance. The distributions examined Kumaraswamy Fréchet (KF), Exponentiated Fréchet (EF), Beta Fréchet (BF), Gamma Extended Fréchet (GExF), Odd Lomax Fréchet (OLxF), and the standard Fréchet (F) demonstrate notable structural similarities while offering varying levels of complexity and flexibility. These characteristics make them well-suited for modelling data with extreme values and heavy tails, a critical need in fields such as econometrics, reliability analysis, and extreme value theory.

The study highlighted the differences in Probability Density Functions (PDFs), showcasing how variants like BF and GExF, through the inclusion of additional beta and gamma functions, provide enhanced adaptability for complex datasets. This flexibility is vital for applications involving extreme events, where precise modelling of heavy-tailed phenomena is essential. Also, the study presented the model performance across three datasets: Jobs Made of Iron Sheet, Airborne Communication Transceiver’s Active Repairs, and Tax Revenue. Performance was assessed using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), with the standard Fréchet (F) distribution emerging as the most consistent performer. It achieved the lowest AIC and BIC values across all datasets, reflecting its superior balance of simplicity and flexibility. The EF and KF variants also showed competitive performance but lacked the standard Fréchet model's robustness across datasets. Conversely, variants like OLxF and GExF often exhibited higher AIC and BIC values. While their additional parameters provide greater flexibility for specific scenarios, this complexity can lead to over-parameterization and reduced practical applicability, especially for smaller datasets or less complex phenomena.

Hence, the standard Fréchet distribution ranks highest in terms of overall performance, offering an optimal trade-off between simplicity and adaptability. This makes it a versatile choice for various applications requiring efficient modelling of extreme values. The Beta Fréchet (BF) and Gamma Extended Fréchet (GExF) variants provide enhanced flexibility for modelling heavy-tailed phenomena but may require careful parameter tuning to avoid overfitting. The Exponentiated Fréchet (EF) and Kumaraswamy Fréchet (KF) variants provide intermediate flexibility and performance, making them viable alternatives when additional shape control is needed. This analysis underscores the importance of selecting a distribution model that aligns with the specific characteristics of the dataset. While advanced variants may offer improved adaptability, their increased complexity can be a drawback in practical applications. Future studies could explore hybrid approaches, combining the simplicity of the standard Fréchet distribution with the adaptability of its extensions to optimize performance further. By

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providing a detailed evaluation of these distributions, this study contributes valuable insights for practitioners and researchers, enhancing their ability to choose appropriate models for analyzing extreme data in econometric and statistical applications.

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