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COST MINIMIZATION PROBLEM FOR DETERIORATING ITEM WITH STOCK LEVEL DEPENDENT DEMAND AND CONTROLLABLE DETERIORATION UNDER TRADE CREDIT AND PARTIAL BACKLOGGING

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ABSTRACT: *This paper studied cost minimization problem for deteriorating item where demand is stock level dependent, deterioration is controllable, vendor can offer trade credit and shortages are partially backlogged. We derived the objective function which is the total average cost function and solved the optimization problem by using optimization problem solver in MatLab. From the numerical examples and sensitivity analysis, we found that the optimal total average cost and the optimal ordered quantity are sensitive to the reduced deteriorating rate and initial demand parameters.*

KEYWORDS: Trade credit, partial backlogging, Deterioration, Objective function, Minimization.

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INTRODUCTION

In traditional inventory system, one would expect that the customer pays the vendor/supplier as soon as the customer gets the goods. But in real world situation, there are cases where the supplier permits trade credit. If the credit period offered by the supplier is more than the time the customer finishes selling the goods/items, the customer can accumulate revenue to earn interest. However, if the reverse is the case, then the retailer can charge interest rate on the remaining items in the stock. A contract like this have some sort of upside for the buyer and the vendor as the buyer can sell the items and earn interest before payment is settled. It may also encourage the buyer to order more quantity. And in the part of the vendor, it enables the vendor compete and as well as sell more items/goods. Therefore, it is very important to find the optimal ordering policy for the vendor and the buyer in order to minimize cost and maximize profit. Rajan and Uthayakumar (2017), studied an economic order quantity model which investigate the optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging and trade credit under inflation. Here, they discussed profit maximization problem, and determined the optimal selling price, optimal order quantity and optimal replenishment time. Banu *et al.* (2021), also considered a supply chain model with stock dependent demand rate and trade credit financing. Jie et al., (2014) presented a generalized economic order quantity model for deteriorating items where the demand rate is considered to be stock dependent and the trade credit is also dependent on the quantity ordered. So, payment for purchase of item is made immediately if the purchased item is less than the predetermined quantity but if the reverse is the case, then a fixed credit period is permissible. They maximized the average profit function to determine the optimal inventory management policies.

Studying inventory models, Inflation might be a factor to be considered. Thus, Shah and Vaghela (2018) considered an imperfect production model in which demand is effort and time dependent and studied the effects of inflation and reliability on the model. Similarly, Amalesh *et al*. (2021), in their work, an imperfect production model for breakable multi – items with dynamic demand and learning effect on rework over random planning horizon, also considered how learning effect affect rework policy, inflation and the time value of money. And concluded from their analysis, that manufacturers should adapt rework policy to avoid loss due to presence of defective items, decrease manufacturing system total cost and increase the rework rate of imperfect items. Moreover, Dipak *et al.*, (2013) presented an inventory model for deteriorating items putting into consideration random planning, inflation and time value of money. Furthermore, they assumed that the consumption rate is stock dependent and shortages are partial backlogging. Then employed genetic algorithm and fuzzy simulation – based genetic algorithm to arrive at the optimal inventory policies.

Furthermore, the work by Danjuma and Abubakar (2024a), considered inventory problem for deteriorating items where demand is stock dependent and deterioration is controllable under two warehouse storage facility. One of the warehouses is assumed to be owned by the vendor and the other warehouse is not owned by the vendor but rented. Furthermore, they assumed that goods are displayed in both warehouses but demands are fulfilled from the rented warehouse until the level of goods in the rented warehouse get to zero, then demands can be met from owned warehouse. They also obtained the optimal inventory management policies for the cases where; the demand is constant function, there is no investment in technology, there is no investment in technology and demand is also constant. Also, in the study by Khanna *et al*. (2020), they obtained the optimal preservation policies in an inventory model where

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demand is stock dependent and holding cost is time dependent. Pando *et al* (2019) derived the solution that maximizes the profitability ratio for an inventory model with nonlinear holding cost. Although, the minimization problem was not addressed. Hence, the comparison between both solutions of maximization and minimization problems was not studied. Other studies on this subject are (Danjuma and Abubakar, 2024b; Ishfaq and Bawa, 2019; Pando *et al.*, 2020). The paper by Yu – Ping and Chung – yuan, (2012) examined an inventory model for deteriorating items under stock dependent demand and controllable deteriorating rate. In their work, shortages allowed are partially backlogged. Their aim is to derive the optimal replenishment and preservation technology investment policies through the maximization of the profit function. Furthermore, the work by Jui –Jung and Kuo – Nan (2010), studied deterministic inventory model for deteriorating items that account for trade credit financing and capacity constraints. In the paper, it assumed that the excess inventory can be stored in a rented warehouse and holding cost in the rented warehouse is more than the holding cost in the owned warehouse. They maximized the average profit function to determine the optimal inventory management strategies and presented numerical examples to illustrate the derived results.

Moreover, it also has been observed that lowering the selling price of a given commodity can increase the customers demand for the commodity. Halim *et al*. (2021) explored this concept in their work. They considered demand rates that are stock dependent and nonlinear price dependent in their model. Also, Singh and Singh (2013) discussed optimal ordering strategy for deteriorating items model where demand is power – form stock dependent under two warehouse storage facility. In their assumptions, demands can be fulfilled from the rented warehouse first until the goods in the rented warehouse get to zero, after which demands can be fulfilled from the owned warehouse. Thus, in the rented warehouse goods depletion is due to deterioration and demand for the goods while depletion in the owned warehouse is due to deterioration at the initial time but later it is due deterioration and demand for the goods. The work of Pattnaik (2013) investigates instantaneous economic order quantity model. The model put into consideration the promotional effort cost, variable cost and units lost due to deterioration by allocating percentage of units lost due to deterioration in an on – hand inventory framing in promotional effort cost and variable ordering cost. The aim is to determine the optimal inventory management strategies for order quantity, promotional effort, cycle length and number of units due to deterioration by maximizing the formulated optimization problem. They also verified results by presenting some numerical examples.

Therefore, it is very important to control and maintain inventories for decaying items. Thus, our work considered cost minimization problem for deteriorating item with stock level dependent demand and controllable deterioration under trade credit and partial backlogging.

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Models Formulation

Constants and Variables

K The ordering cost per order

c the unit purchasing cost

h The holding cost per unit time (excluding interest charges)

s Unit selling price $(s > c)$

c_1 shortage cost per unit per order

c_2 Opportunity cost due to lost sales

I_e Interest earned per \$ per unit of time by the retailer

I_p Interest charges per \$ in stock per unit of time to the supplier

$I(t)$ The level of inventory at time t

I_m Maximum inventory level for each replenish cycle

I_b Maximum amount of demand backlogged per cycle

M Retailer's trade credit period offered by supplier per unit time

t_1 Time at which the inventory level falls to zero

T Inventory cycle length

Q Retailer's order quantity

ω Maximum capital constraint

ξ Preservation technology cost per unit time for reducing deterioration rate in order to preserve the products $(0 \leq \xi \leq \omega)$

Assumptions and Model

i. The replenishment rate is infinite

ii. Lead time is zero

iii. Planning horizon of the inventory system is infinite

iv. There is no repair or replacement of deteriorated items during the period under consideration

v. The inventory model deals with single item

vi. The reduced deterioration rate $N(\xi)$ is an increasing function of the preservation technology cost ξ where

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$$N(\xi) = \theta$$

vii. The demand rate function $D(t)$ is deterministic and a function of instantaneous stock level $I(t)$. When inventory is positive, $D(t)$ is given by

$$D(t) = \alpha + \beta I(t), \qquad 0 \leq t \leq t_1$$

And when inventory is negative, $D(t)$ is given by

$$D(t) = \alpha, \qquad 0 \leq t \leq T$$

$\alpha > 0\ and\ 0 < \beta < 1$ are initial demand and inventory level dependent demand factor parameters respectively

viii. Shortages are allowed. The demand that is not satisfied is partially backlogged and the fraction of shortages backordered is $b(t) = e^{-\delta t}\ \delta > 0, t$ is the waiting time before the next replenishment and $0 \leq b(t) \leq 1,\ b(0) = 1$. Note that if

$b(x) = 1\ (or\ 0)$ for all t, then the shortages are completely backlogged (or lost). We assumed that the shortages are partially backlogged.

ix. If the trade credit period M is offered, the retailer can settle the account at $t = M$ and pay for the interest charges on the items in stock with rate I_p over the interval $[M, t_1]$ if $t_1 \geq M$ and if the retailer settles the account at $t = M$, the retailer do not need to pay any interest charge on items in stock during the whole cycle if $t_1 \leq M$

x. The retailer can accumulate revenue and earn interest from the beginning of the inventory cycle until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period from

$t = 0\ to\ t = M$ with rate I_e under the trade credit agreement.

Considering the assumptions above, the model for the inventory level at any time is given by

$$\frac{dI(t)}{dt} = \{-\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \leq t \leq t_1\ -\alpha b(t) \qquad t_1 \leq t \leq T \qquad (1)$$

With boundary condition $I(t_1) = 0$

During the interval $[0, t_1]$, the decrease in the inventory level is due to combined effects of deterioration and demand, and the inventory drops to zero during the time interval $[0, t_1]$. But, during the interval $[t_1, T]$, shortages occurred which are partially backlogged. From equation (1), the rate of change of inventory level at any time t can be represented by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \leq t \leq t_1$$

$$\frac{dI_1}{dt} = -\alpha b(t) \qquad\qquad t_1 \leq t \leq T$$

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With the boundary conditions $I(0) = I_M, I_1(t_1) = 0$

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - [\theta - N(\xi)]I(t) \quad 0 \le t \le t_1$$

$$\frac{dI(t)}{dt} = -\alpha - [\beta + (\theta - N(\xi))]I(t)$$

$$= -(\alpha + [\beta + \theta - N(\xi)]I(t))$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = \int_t^{t_1} -dt$$

$$\int_t^{t_1} \frac{dI(t)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = -\int_t^{t_1} dt$$

$$ln\, ln \left[\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)}\right] = -(\beta + \theta - N(\xi))(t_1 - t)$$

$$\frac{\alpha + [\beta + \theta - N(\xi)]I(t_1)}{\alpha + [\beta + \theta - N(\xi)]I(t)} = e^{-(\beta+\theta-N(\xi))(t_1-t)}$$

Since $I(t_1) = 0,\ we\ Have$

$$\frac{\alpha}{\alpha + [\beta + \theta - N(\xi)]I(t)} = e^{-(\beta+\theta-N(\xi))(t_1-t)}$$

$$\alpha + [\beta + \theta - N(\xi)]I(t) = \alpha e^{(\beta+\theta-N(\xi))(t_1-t)}$$

$$[\beta + \theta - N(\xi)]I(t) = \alpha e^{(\beta+\theta-N(\xi))(t_1-t)} - \alpha$$

$$I(t) = \frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}$$

$$I_m = I(0) = \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} \tag{2}$$

The Objective Function Components

Ordering cost per cycle $= K$ (3)

Holding Cost $= h\int_0^{t_1} I(t)dt$

$$= h\int_0^{t_1} \frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)} dt$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)} \int_0^{t_1} [e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]\, dt$$

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$$= \frac{\alpha h}{\beta + \theta - N(\xi)} | \frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta + \theta - N(\xi))} - t | t_1\ 0$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)} \left(\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta + \theta - N(\xi))} - t_1 \right] - \left[\frac{e^{(\beta+\theta-N(\xi))(t_1-0)}}{-(\beta + \theta - N(\xi))} - 0 \right] \right)$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)} \left(-\frac{1}{(\beta + \theta - N(\xi))} - t_1 + \frac{e^{(\beta+\theta-N(\xi))t_1}}{(\beta + \theta - N(\xi))} \right)$$

$$= \frac{\alpha h}{\beta + \theta - N(\xi)} \left(\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - t_1 \right) \qquad (4)$$

Now, for unsatisfied demand partially backlogged, we have

$$\frac{dI_1}{dt} = -\alpha b(t) \qquad t_1 \le t \le T$$

$b(t) = e^{-\delta t}$, therefore,

$$\frac{dI_1}{dt} = -\alpha e^{-\delta t}$$

$$I_1(t) = -\alpha \int_{t_1}^{t} e^{-\delta s} ds = -\alpha \times \frac{1}{-\delta} e^{-\delta s} | t\ \ t_1$$

$$= \frac{\alpha}{\delta} e^{-\delta s} | t\ t_1 \ = \frac{\alpha}{\delta} e^{-\delta t} - \frac{\alpha}{\delta} e^{-\delta t_1}$$

$$= \frac{\alpha}{\delta} \left(e^{-\delta t} - e^{-\delta t_1} \right)$$

Maximum backordered quantity (maximum amount of demand backlogged)

$$I_1(T) = I_{1m} = \frac{\alpha}{\delta} \left(e^{-\delta T} - e^{-\delta t_1} \right) \qquad (5)$$

$$Lost\ sales\ or\ opportunity\ cost = -c_2 \alpha \int_{t_1}^{T} (1 - e^{-\delta t}) dt = -c_2 \alpha \left(t + \frac{e^{-\delta t}}{\delta} \right) | T\ \ t_1$$

$$= -c_2 \alpha \left[\left(T + \frac{e^{-\delta T}}{\delta} \right) - \left(t_1 + \frac{e^{-\delta t_1}}{\delta} \right) \right]$$

$$= c_2 \alpha \left[t_1 + \frac{e^{-\delta t_1}}{\delta} - T - \frac{e^{-\delta T}}{\delta} \right]$$

Purchase cost (case of partial backlogging)

Maximum quantity $= Q = I_m - I_{1m}$

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$$Q = I_m - I_{1m} = \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} - \frac{\alpha}{\delta}\left(e^{-\delta T} - e^{-\delta t_1}\right)$$

$$= \frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \frac{\alpha}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right) \qquad (6)$$

$$Purchase\ cost = cQ = c\left(\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \frac{\alpha}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right)\right)$$

$$= \frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta+\theta-N(\xi)} + \frac{\alpha c}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right) \qquad (7)$$

Shortage cost (case of partial backlogging)

$$Shortage\ cost = -c_1\int_{t_1}^{T} I_1(t)dt = \frac{-c_1\alpha}{\delta}\int_{t_1}^{T}\left(e^{-\delta t} - e^{-\delta t_1}\right)dt$$

$$= \frac{-c_1\alpha}{\delta}\left(-\frac{e^{-\delta t}}{\delta} - te^{-\delta t_1}\right)|T\ t_1$$

$$= \frac{-c_1\alpha}{\delta}\left[\left(-\frac{e^{-\delta T}}{\delta} - Te^{-\delta t_1}\right) - \left(-\frac{e^{-\delta t_1}}{\delta} - t_1e^{-\delta t_1}\right)\right]$$

$$= \frac{-c_1\alpha}{\delta}\left[-\frac{e^{-\delta T}}{\delta} - Te^{-\delta t_1} + \frac{e^{-\delta t_1}}{\delta} + t_1e^{-\delta t_1}\right]$$

$$= \frac{c_1\alpha}{\delta}\left[\frac{e^{-\delta T}}{\delta} + Te^{-\delta t_1} - \frac{e^{-\delta t_1}}{\delta} - t_1e^{-\delta t_1}\right] \qquad (8)$$

Preservation technology cost $= \xi T$ (9)

Sales Revenue (case of partial backlogging)

$$Sales\ revenue = s\left(\int_0^{t_1} D(t)dt - I_1(T)\right)$$

$$= s\int_0^{t_1}(\alpha + \beta I(t))dt - \frac{s\alpha}{\delta}\left(e^{-\delta T} - e^{-\delta t_1}\right)$$

$$= s\int_0^{t_1}\left(\alpha + \beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta+\theta-N(\xi)}\right)dt + \frac{s\alpha}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right)$$

$$= s\int_0^{t_1}\left(\alpha + \beta\alpha\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{\beta+\theta-N(\xi)} - \frac{\beta\alpha}{\beta+\theta-N(\xi)}\right)dt + +\frac{s\alpha}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right)$$

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$$= s\alpha\left[\frac{t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)} + \frac{\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta + \theta - N(\xi)]^2}\right] + \frac{s\alpha}{\delta}\left(e^{-\delta t_1} - e^{-\delta T}\right) \tag{10}$$

$$Deteriorating\ cost = d_c\left[I_m - \int_0^{t_1} D(t)\,dt\right] = d_c\left[I_m - \int_0^{t_1}(\alpha + \beta I(t))\,dt\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - \int_0^{t_1}\left(\alpha + \beta\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}\right)dt\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - \left(\frac{\alpha t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)} + \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta + \theta - N(\xi)]^2}\right)\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - \frac{\alpha t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)} - \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta + \theta - N(\xi)]^2}\right]$$

$$= d_c\left[\frac{\alpha[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - \frac{\alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta + \theta - N(\xi)]^2} - \frac{\alpha t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\beta + \theta - N(\xi)][e^{(\beta+\theta-N(\xi))t_1} - 1] - \alpha\beta(e^{(\beta+\theta-N(\xi))t_1} - 1)}{[\beta + \theta - N(\xi)]^2} - \frac{\alpha t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)}\right]$$

$$= d_c\left[\frac{\alpha[\theta - N(\xi)][e^{(\beta+\theta-N(\xi))t_1} - 1]}{[\beta + \theta - N(\xi)]^2} - \frac{\alpha t_1[\theta - N(\xi)]}{\beta + \theta - N(\xi)}\right]$$

$$= \frac{d_c\alpha[\theta - N(\xi)]}{\beta + \theta - N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1} - 1]}{\beta + \theta - N(\xi)} - t_1\right] \tag{11}$$

Payable Interest and earned Interest

If the end of the credit period is shorter than or equal to the length of period in which the inventory is positive ($M \leq t_1$), payment for goods is settled and the retailer starts paying the capital opportunity cost for the items in stock with rate I_p. It is also assumed that as far as the account remains settled, the retailer can sell the goods and can accumulate sales revenue from the selling of the goods and earn interest with the rate I_e. So, the interest earned and payable per cycle for different cases are given below:

Case I: $M \leq t_1$

$$interest\ payable = cI_p\int_M^{t_1} I(t)\,dt$$

$$= cI_p\int_m^{t_1}\left(\frac{\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}\right)dt$$

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$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\int_M^{t_1}\left(e^{(\beta+\theta-N(\xi))(t_1-t)}-1\right)dt$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t)}}{-(\beta+\theta-N(\xi))}-t\right)|t_1\ M$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{-(\beta+\theta-N(\xi))}-t_1\right)-\left(\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{-(\beta+\theta-N(\xi))}-M\right)\right]$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}}{(\beta+\theta-N(\xi))}+M-\frac{1}{\beta+\theta-N(\xi)}-t_1\right]$$

$$= \frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}-1}{(\beta+\theta-N(\xi))}+M-t_1\right] \quad (12)$$

$$interest\ earned = sI_e\int_0^m\int_0^t D(s)ds\,dt$$

$$= sI_e\int_0^m\left[\int_0^t D(s)\,ds\right]dt$$

$$= sI_e\int_0^m\left[\int_0^t(\alpha+\beta I(s))ds\right]dt$$

$$= sI_e\int_0^m\left[\int_0^t\left(\alpha+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-s)}-1]}{\beta+\theta-N(\xi)}\right)ds\right]dt$$

$$= sI_e\int_0^M\left[\int_0^t\left(\alpha+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta+\theta-N(\xi)}-\frac{\beta\alpha}{\beta+\theta-N(\xi)}\right)ds\right]dt$$

$$= sI_e\int_0^M\left[\left(\alpha s-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha s}{\beta+\theta-N(\xi)}\right)|t\ 0\right]dt$$

$$= sI_e\int_0^M[\alpha t-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}]\,dt$$

$$= sI_e\left(\frac{\alpha t^2}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha t^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha t e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)|M\ 0$$

$$= sI_e[\left(\frac{\alpha M^2}{2}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3}-\frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))}+\frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right)$$

$$-\left(0+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}-0+0\right)]$$

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$$= sI_e[\frac{\alpha M^2}{2} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}$$

$$- \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}]$$

$$= sI_e[\frac{\alpha M^2}{2} - \frac{\beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-M)}}{[\beta+\theta-N(\xi)]^3} - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}$$

$$+ \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}]$$

$$= sI_e[\frac{\alpha M^2(\beta+\theta-N(\xi)) - \beta\alpha M^2}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3}$$

$$+ \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}]$$

$$= sI_e[\frac{\alpha M^2(\theta-N(\xi))}{2(\beta+\theta-N(\xi))} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}(e^{-(\beta+\theta-N(\xi))M} - 1)}{[\beta+\theta-N(\xi)]^3}$$
$$+ \frac{\beta\alpha M e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}] \quad (14)$$

Case II: $t_1 \leq M$

Here, we considered the case where the cycle time t, is less than or equal to the credit period M, so no interest will be paid. Thus, the retailer pays no interest at the end of the inventory cycle. Therefore,

$$Interest\ payable = 0$$

However, from time $0\ to\ t_1$, the retailer can sell the goods and continue to accumulate sales revenue to earn interest $sI_e \int_0^{t_1}\int_0^t D(s)dsdt$. Also, from time $t_1\ to\ M$, the retailer uses the sales revenue generated in $[0, t_1]$ to earn interest $sI_e \int_0^{t_1} D(s)ds(M - t_1)$. Thus, the interest earned in this period per cycle can be described by:

$$Interest\ earned = sI_e\left[\int_0^{t_1}\int_0^t D(s)\,dsdt + \int_0^{t_1} D(t)\,dt(M - t_1)\right]$$

$$= sI_e\int_0^{t_1}\int_0^t D(s)\,dsdt + sI_e\int_0^{t_1} D(t)dt\,(M - t_1)$$

$$= sI_e\int_0^{t_1}\int_0^t D(s)\,dsdt + sI_e\int_0^{t_1} D(t)dt\,(M - t_1)$$

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$$= sI_e \int_0^{t_1} \left[\int_0^t D(s)d\,s\right] dt + sI_e \int_0^{t_1} D(t)dt\,(M - t_1)$$

$$= sI_e \int_0^{t_1} \left[\int_0^t (\alpha + \beta I(s))ds\right] dt + sI_e(M - t_1) \int_0^{t_1} (\alpha + \beta I(t))dt$$

$$= sI_e \int_0^{t_1} \left[\int_0^t \left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-s)} - 1]}{\beta + \theta - N(\xi)}\right) ds\right] dt$$

$$+sI_e(M - t_1) \int_0^{t_1} \left(\alpha + \frac{\beta\alpha[e^{(\beta+\theta-N(\xi))(t_1-t)} - 1]}{\beta + \theta - N(\xi)}\right) dt$$

$$= sI_e \int_0^{t_1} \left[\int_0^t \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)}\right) ds\right] dt$$

$$+sI_e(M - t_1) \int_0^{t_1} \left(\alpha + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{\beta + \theta - N(\xi)} - \frac{\beta\alpha}{\beta + \theta - N(\xi)}\right) dt$$

$$= sI_e \int_0^{t_1} \left[\left(\alpha s - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-s)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha s}{\beta + \theta - N(\xi)}\right) |t\ 0\right] dt$$

$$+sI_e(M - t_1) \left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)}\right) |t_1\ 0$$

$$= sI_e \int_0^{t_1} \left[\left(\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta + \theta - N(\xi)]^2} - 0\right)\right] dt$$

$$+sI_e(M - t_1) \left[\left(\alpha t_1 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t_1)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta + \theta - N(\xi)}\right) - \left(0 - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-0)}}{[\beta + \theta - N(\xi)]^2} - 0\right)\right]$$

$$= sI_e \int_0^{t_1} \left[\alpha t - \frac{\beta\alpha e^{(\beta+\theta-N(\xi))(t_1-t)}}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t}{\beta + \theta - N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta + \theta - N(\xi)]^2}\right] dt$$

$$+sI_e(M - t_1) \left[\alpha t_1 - \frac{\beta\alpha}{[\beta + \theta - N(\xi)]^2} - \frac{\beta\alpha t_1}{\beta + \theta - N(\xi)} + \frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta + \theta - N(\xi)]^2}\right]$$

$$= sI_e[\frac{\alpha t_1^2}{2} + \frac{\beta\alpha}{[\beta + \theta - N(\xi)]^3} - \frac{\beta\alpha t_1^2}{2(\beta + \theta - N(\xi))} + \frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta + \theta - N(\xi)]^2}$$

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$$-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e[\frac{\alpha t_1^2}{2}-\frac{\beta\alpha t_1^2}{2(\beta+\theta-N(\xi))}-\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^3}$$

$$+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}]$$

$$+sI_e(M-t_1)\left[\alpha t_1-\frac{\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e[\frac{\alpha t_1^2\big(\beta+\theta-N(\xi)\big)-\beta\alpha t_1^2}{2\big(\beta+\theta-N(\xi)\big)}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\beta+\theta-N(\xi)]-\beta\alpha t_1}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right]$$

$$=sI_e[\frac{\alpha t_1^2\big(\theta-N(\xi)\big)}{2\big(\beta+\theta-N(\xi)\big)}+\frac{\beta\alpha t_1 e^{(\beta+\theta-N(\xi))t_1}}{[\beta+\theta-N(\xi)]^2}-\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^3}]$$

$$+sI_e(M-t_1)\left[\frac{\alpha t_1[\theta-N(\xi)]}{\beta+\theta-N(\xi)}+\frac{\beta\alpha[e^{(\beta+\theta-N(\xi))t_1}-1]}{[\beta+\theta-N(\xi)]^2}\right] \qquad (15)$$

Cost Minimization Problem for the Case of Partial Backlogging

Our problem here is to determine the optimal value of t which minimizes $X(t)$. The necessary condition for minimization of the Total cost function $Y(t)$ are:

$$\frac{d}{dt}Y(t)=0 \qquad (16)$$

Equations (16) can be solved for t to obtain the optimal value of t (say t^*). The sufficient condition for $X(t)$ to be a minimum is that the

$$\frac{d^2}{dt^2}Y(t)>0 \qquad (17)$$

If we put the above components into consideration, then the cost function can be described by:

$$Y(t)=\frac{1}{T}\{odering\ cost+holding\ cost+purchasing\ cost+deteriorating\ cost$$
$$+shortage\ cost+opportunity\ cost+preservation\ cost$$
$$+interest\ payable\}$$

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Case I: $M \leq t_1$

$$Here, opportunity\ cost \neq 0, therefore,$$

$$Y(t_1) = \frac{1}{T}\{K + \frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} - t_1\right)$$
$$+\left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} + \frac{\alpha c}{\delta}\left(e^{-\delta t_1}-e^{-\delta T}\right)\right)$$
$$+\frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} - t_1\right]$$
$$+\frac{c_1\alpha}{\delta}\left[\frac{e^{-\delta T}}{\delta} + Te^{-\delta t_1} - \frac{e^{-\delta t_1}}{\delta} - t_1 e^{-\delta t_1}\right] + c_2\alpha\left[t_1 + \frac{e^{-\delta t_1}}{\delta} - T - \frac{e^{-\delta T}}{\delta}\right] + \xi T$$
$$+\frac{cI_p\alpha}{\beta+\theta-N(\xi)}\left[\frac{e^{(\beta+\theta-N(\xi))(t_1-M)}-1}{(\beta+\theta-N(\xi))} + M - t_1\right]\} \tag{18}$$

Case II: $t_1 \leq M$

$$Y_1(t_1) = \frac{1}{T}\{odering\ cost + holding\ cost + purchasing\ cost + deteriorating\ cost$$
$$+ shortage\ cost + opportunity\ cost + preservation\ cost$$
$$+ interest\ payable\}$$

$$interest\ payable = 0$$

$$Y_1(t_1) = \frac{1}{T}\{K + \frac{\alpha h}{\beta+\theta-N(\xi)}\left(\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} - t_1\right)$$
$$+\left(\frac{\alpha c[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} + \frac{\alpha c}{\delta}\left(e^{-\delta t_1}-e^{-\delta T}\right)\right)$$
$$+\frac{d_c\alpha[\theta-N(\xi)]}{\beta+\theta-N(\xi)}\left[\frac{[e^{(\beta+\theta-N(\xi))t_1}-1]}{\beta+\theta-N(\xi)} - t_1\right]$$
$$+\frac{c_1\alpha}{\delta}\left[\frac{e^{-\delta T}}{\delta} + Te^{-\delta t_1} - \frac{e^{-\delta t_1}}{\delta} - t_1 e^{-\delta t_1}\right]$$
$$+c_2\alpha\left[t_1 + \frac{e^{-\delta t_1}}{\delta} - T - \frac{e^{-\delta T}}{\delta}\right] + \xi T\} \tag{19}$$

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NUMERICAL ANALYSIS AND SENSITIVITY ANALYSIS

Case I: $M \leq t_1$

For numerical examples, we solved Equ. (18)., an inventory system with the following parameter set. α=40units, β=0.04, T=40days, M=20days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order, θ=0.7, K=30, N=0.02, ξ=0.3, d_c=0.3, I_p=0.14, $\sigma = 0.03$.

The optimal time $t_1^* = 26.4681$

Optimal Total average cost $Y^* = 184.1948$

Optimal order quantity $Q^* = 3127.2$

Table 1. The effect of the parameter N on Y^* and Q^*

N	Y^*	Q^*
0.02	184.1948	3127.2
0.04	187.1180	3127.9
0.06	190.2955	3129
0.08	193.7489	3130.5
0.10	197.5025	3132.3
0.12	201.5832	3134.4
0.14	206.0207	3136.6

α=40units, β=0.04, T=40days, M=20days, c=1Naira/unit,

h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order,

K=30, , ξ=0.3, d_c=0.3, I_p=0.14, $\sigma = 0.03$.

Table 2. The effect of the parameter α on Y^* and Q^*

α	Y^*	Q^*
20	92.6060	1563.6
30	138.4004	2345.4
40	184.1948	3127.2
50	229.9893	3909.0
60	275.7837	4690.8
70	321.5781	5472.6

β=0.04, T=40days, M=20days, c=1Naira/unit, h=1Naira/unit time,

c_1=1Naira/unit/order, c_2=1Naira/unit/order, θ=0.7, K=30,

N=0.02, ξ=0.3, d_c=0.3, $\sigma = 0.03$ I_p=0.14.

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Case II: $t_1 \leq M$

For numerical examples, we solved Equ. (19)., an inventory system with the following parameter set. α=40units, β=0.04, T=40days, c=1Naira/unit, h=1Naira/unit time, c_1=1Naira/unit/order, c_2=1Naira/unit/order, θ=0.7, K=30, N=0.02, ξ=0.3, d_c=0.3, I_p=0.14, $\sigma = 0.03$.

The optimal time $t_1^* = 26.9788$

Optimal Total average cost $Y_1^* = 182.1915$

Optimal order quantity $Q^* = 3215.6$

Table 3. The effect of the parameter N on Y_1^* *and* Q^*

$N(\xi)$	Y_1^*	Q^*
0.02	182.1915	3175.5
0.04	185.1058	3177.9
0.06	188.2784	3180.9
0.08	191.7322	3184.4
0.10	195.4926	3188.5
0.12	199.5882	3193.0
0.14	204.0511	3197.9

α=40units, β=0.04, T=40days, c=1Naira/unit, θ=0.7, h=1Naira/unit time,

c_1=1Naira/unit/order, c_2=1Naira/unit/order, K=30, ξ=0.3, d_c=0.3, I_p=0.14,

$\sigma = 0.03$.

Table 4. The effect of the parameter α on Y_1^* *and* Q^*

α	Y_1^*	Q^*
20	91.6043	1587.8
30	136.8979	2381.6
40	182.1915	3175.5
50	227.4850	3969.4
60	272.7786	4763.3
70	318.0722	5557.2

β=0.04, T=40days, c=1Naira/unit, h=1Naira/unit time, c_2=1Naira/unit/order,

c_1=1Naira/unit/order, θ=0.7, K=30, $N(\xi)$=0.02, ξ=0.3, d_c=0.3, $\sigma = 0.03$.

From Tables (1) and (3), we can see that the parameter N (controllable deterioration rate parameter) is sensitive to the optimal total average cost and the optimal order quantity. That is, as the parameter N increases, the total average cost also increases. The implication is that if the cost of preservation technology rises, the total average cost also rises. Similarly, from Tables (2) and (4), we observed that the parameter α (initial demand parameter) is also sensitive to the optimal total average cost and the optimal ordered quantity. That is, as the parameter α increases, the optimal quantity and the optimal total average cost also increases. By implication, if the demand for the goods increases the vendor increases the quantity purchased.

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CONCLUSION

We have considered an inventory cost minimization problem for deteriorating item with stock level dependent demand and controllable deterioration under trade credit and partial backlogging. Furthermore, we obtained the objective function which is the total average cost and minimized the objective function to obtained the optimal inventory management policies for the inventory management problem. We also presented some numerical illustrations and sensitivity analysis.

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