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TRANSIENT DYNAMICS OF TIMOSHENKO BEAMS SUBJECTED TO MOVING LOADS IN TURBULENT ENVIRONMENTS

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ABSTRACT: *This paper investigated the transient response of Timoshenko beams under moving loads in turbulent environments. The study incorporated the effects of shear deformation, rotary inertia, and dynamic aerodynamic forces caused by turbulence. The governing equations are derived from Timoshenko beam theory and coupled with an aerodynamic force model that accounts for mean and fluctuating wind velocities. A spectral representation of turbulent forces was employed to simulate realistic wind-induced forces. Numerical simulations were conducted using both the spectral element method (SEM) and the finite element method (FEM), enabling a comparison of their accuracy and computational efficiency. Results are presented for various load velocities and turbulence intensities, highlighting the advantages and limitations of each method. This study provided valuable insights into the dynamic behavior of beams in challenging environmental conditions, offering practical applications in civil, mechanical, and aerospace engineering.*

KEYWORDS: Timoshenko beam, Turbulent environments, Shear deformation, Aerodynamic force, Finite Element Method, Spectral Element Method.

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INTRODUCTION

The dynamic behavior of beams under external loads has long been a subject of great interest in structural mechanics and engineering. Beams serve as fundamental components in various applications, including bridges, rail tracks, aerospace structures, and mechanical systems, where understanding their dynamic responses is critical for ensuring safety, reliability, and performance. While traditional studies often focus on static or quasi-static responses, modern engineering demands have shifted attention toward transient dynamics under complex loading scenarios, such as moving loads in turbulent environments.

The Bernoulli-Euler beam theory has historically served as the foundation for analyzing beam deflection and stress distribution (Inglis, 1934; Adekunle & Folakemi, 2017; Ogunbamike, 2021; Wu *et al*., 2023). This classical theory assumes that plane sections of the beam remain planar and perpendicular to the neutral axis during deformation, offering a simplified framework for modeling slender, long-span beams under static and dynamic loads. Despite its utility, this theory exhibits significant limitations when applied to thick beams, short spans, or high-frequency excitations. Under such conditions, the effects of transverse shear deformation and rotary inertia become non-negligible, resulting in inaccurate predictions of beam dynamics.

To overcome these limitations, Timoshenko (1921) introduced a more advanced beam theory that incorporates shear deformation and rotary inertia, providing a comprehensive framework for analyzing complex beam responses. Subsequent studies (Stanisic et al., 1968; Sadiku & Leipholz, 1987; Jimoh, 2021) extended Timoshenko's theory to include non-uniform cross-sections and advanced boundary conditions, enhancing its applicability to real-world problems. Further refinements by Oni (1997) and Li *et al*. (2024) investigated the theory's suitability under dynamic loading scenarios, such as moving loads. These advancements have established Timoshenko beam theory as a robust and versatile tool for modeling the transient dynamics of beams.

The behavior of beams subjected to moving loads has been extensively studied due to its relevance in applications such as vehicles crossing bridges or trains traveling on rails. Studies (Ogunbamike, 2012; Adekunle & Folakemi, 2017; Adekunle *et al.*, 2017; Ogunbamike, 2021; Wu et al., 2023; Awodola *et al.*, 2024) have shown that moving loads induce dynamic amplifications and resonance effects, significantly influencing the structural response. However, most of these studies focus on moving loads in isolation, neglecting the role of environmental forces.

In real-world scenarios, beams are frequently subjected to environmental forces, including wind-induced aerodynamic loads. These forces become particularly complex in turbulent environments, where irregular fluctuations in wind velocity introduce a stochastic component to the loading. The energy distribution of turbulent flows is commonly modeled using the Von Kármán spectrum (Von Kármán, 1948) or the Davenport spectrum (Davenport, 1961), which characterize the frequency content of turbulence. Research by Shinozuka and Deodatis (1991, 1996) has demonstrated the effectiveness of the spectral representation method in simulating stochastic wind velocity fields, enabling realistic modeling of turbulence-induced forces. However, the combined effects of moving loads and turbulent aerodynamic forces, particularly on Timoshenko beams, remain largely unexplored in existing literature.

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Modeling the transient dynamics of beams under moving loads in turbulent environments presents significant challenges. Turbulence introduces a highly dynamic and unpredictable component to the loading, necessitating advanced numerical methods for accurate simulation. The spectral representation method, developed by Shinozuka and Deodatis (1991, 1996), is widely employed to reconstruct stochastic wind velocity time-series data, enabling realistic simulations of turbulence-induced forces.

Coupling these aerodynamic forces with moving point loads further complicates the governing equations, which exhibit non-homogeneous and variable coefficients. Analytical methods, such as separation of variables or Struble's technique (Struble, 1962), often fail to yield exact solutions for such problems. Consequently, numerical approaches, including the finite element method (**FEM**) and the spectral element method (**SEM**), have become essential for solving these equations and analyzing the transient response of beams. **FEM** provides flexibility in handling complex geometries and boundary conditions, while **SEM** offers higher accuracy and computational efficiency for specific applications.

This study addresses the existing research gap by investigating the transient dynamics of Timoshenko beams subjected to moving loads in turbulent environments. The governing equations are derived from Timoshenko beam theory, incorporating transverse shear deformation, rotary inertia, and flexural rigidity. Aerodynamic forces are modeled using the Von Kármán turbulence spectrum and the spectral representation method, capturing mean wind velocities and stochastic fluctuations. Numerical simulations are conducted using both **FEM** and **SEM** for spatial discretization, while the Newmark-beta method is employed for time integration.

By analyzing the effects of load velocity, turbulence intensity, and beam properties on dynamic responses, this study provides valuable insights into the behavior of beams under complex loading conditions. The findings have significant implications for the design and analysis of structures in civil, mechanical, and aerospace engineering, contributing to the development of more resilient and efficient systems capable of withstanding challenging environmental conditions.

Problem Formulation

This study examines the transient response of a Timoshenko beam under a combination of a moving point load $P(x,t)$ and distributed turbulent aerodynamic forces $F_{turb}(x,t)$. The beam behavior is described by a coupled system of second-order partial differential equations accounting for transverse shear deformation, rotary inertia, and flexural rigidity.

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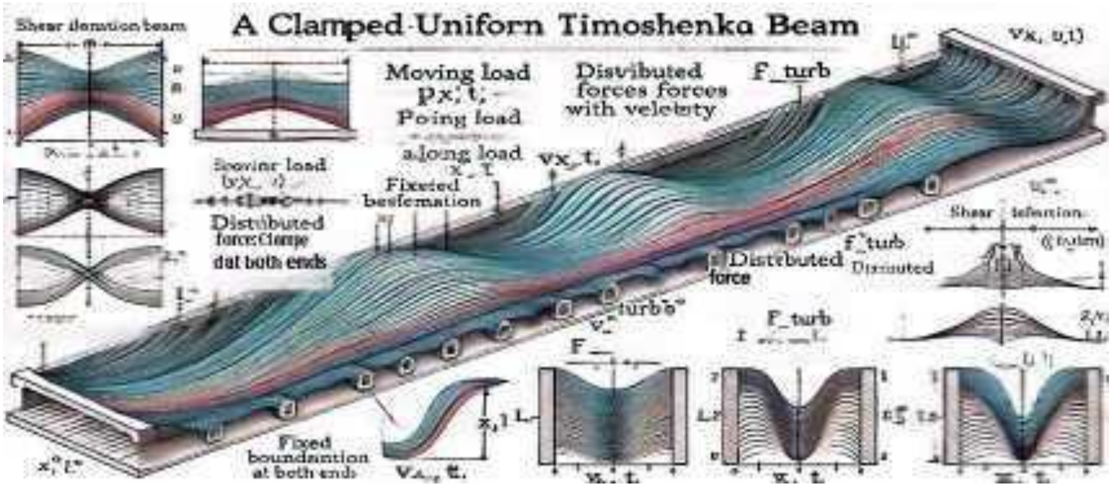

Figure 1: Geometry diagram of uniform clamped-clamped Timoshenko beam

Figure 1 above depicts the transverse displacement $V(x,t)$ of the beam as it moves at a constant speed. The equation of motion is given as:

$$\left[\rho A\frac{\partial^2}{\partial t^2}\ 0\ 0\ \rho I\frac{\partial^2}{\partial t^2}\right]\{V(x,t)\ \varphi(x,t)\ \} + \left[(k^*GA)\frac{\partial^2}{\partial x^2} - k^*GA\frac{\partial}{\partial x} - k^*GA\frac{\partial}{\partial x} - EI\frac{\partial^2}{\partial x^2} + k^*GA\right]\{V(x,t)\ \varphi(x,t)\ \} = \{Q(x,t)\ 0\ \} \qquad (1)$$

$$Q(x,t) = P(x,t) + F_{turb}(x,t) \qquad (2)$$

In this problem, the time coordinate is represented by t, while ρA denotes the mass per unit length of the beam. Additionally, EI refers to the flexural rigidity, E, is Young's modulus, I, is the moment of inertia of the cross-section of the beam, G, is the shear modulus, k^*, is the shear correction factor, ρ is the density of the beam material, ρ_{air}, is the density of air, and x represents the spatial coordinate. k^*GA, is shear rigidity, $\varphi(x,t)$ indicates beam rotation due to bending, and $Q\ (x,\ t)$ denotes total force, combining moving load $P(x,t)$ and turbulent force $F_{turb}(x,t)$. Notably, in this specific circumstance, the dispersed load traversing the beam bears a weight akin to that of the beam itself. Therefore, it must not be disregarded as its effect considerably influences the dynamic system's behavior. Thus, moving load, $P(x,t)$ and turbulent force, $F_{turb}(x,t)$ will typically modeled as follows:

$$P(x,t) = f_o\delta(x - ct) \qquad (3)$$

$$F_{turb}(x,t) = \frac{1}{2}\rho_{air}C_D A_{proj}[U(t) + u'(x,t)]^2 \qquad (4)$$

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where $\delta(x - ct)$, is the Dirac delta function ensuring the load acts at $x = ct$, c is the velocity at any instance t, where $u'(x,t)$ is the stochastic fluctuation in wind velocity, f_o is the magnitude of the load, C_D, is the drag coefficient (depends on the shape and orientation of the beam), A_{proj}, is the projected area of the beam perpendicular to the wind direction, $U(t)$, is the mean wind velocity as a function of time.

Turbulent Wind Velocity $u'(x,t)$

The fluctuating wind velocity $u'(x,t)$ represents the variation in wind speed due to turbulence. This can be modeled using a spectral approach, where the turbulence is characterized by its power spectral density (PSD) at different frequencies. The Von Kármán turbulence spectrum is commonly used for this purpose. The Von Kármán spectrum for turbulence at a frequency f is given by:

$$S_u(f) = \frac{\sigma_u^2 L_u}{\pi U_\infty}\left[1 + 70.8\left(\frac{f L_u}{U_\infty}\right)^2\right]^{-\frac{5}{6}} \qquad (5)$$

where $S_u(f)$, is the power spectral density (PSD) of the turbulence at frequency f, which quantifies the intensity of turbulence at different scales, σ_u, is the standard deviation of the turbulent wind velocity, representing the magnitude of fluctuations in the wind, L_u, is the turbulence integral length scale, a measure of the scale over which turbulent fluctuations are correlated, and U_∞, is the mean wind velocity, representing the average steady-state wind speed. This spectrum describes how the turbulence intensity varies across different frequency ranges, providing insight into the energy distribution in the turbulent flow. The exponent $-\frac{5}{6}$ indicates the characteristic scaling of turbulence in the inertial subrange, where the energy in the turbulence follows a power-law decay with frequency.

The fluctuating wind velocity $u'(x,t)$ is then reconstructed using a stochastic simulation technique called the spectral representation method. This approach involves creating a time series of the fluctuating wind velocity that matches the given turbulence spectrum.

$$u'(x,t) = \sum_{i=1}^{N_f} \sqrt{2S_u(f_i)\Delta f}\, co(2\pi f_i t + \phi_i) \qquad (6),$$

where ϕ_i are random phases.

The boundary conditions of the problem are deemed arbitrary, thereby allowing for the adoption of any form of classical boundary conditions. In contrast, without sacrificing generality, the initial conditions are presented as follows:

$$V(x,0) = \frac{\partial V(x,0)}{\partial t} = \varphi(x,0) = \frac{\partial \varphi(x,0)}{\partial t} = 0 \qquad (7)$$

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Solution Procedure

The Partial Differential Equation (1) presents non-homogeneous variable coefficients, making the separation of variables method seem infeasible due to the difficulty of deriving separate equations with functions reliant on a single variable. It is clear that traditional methods are unlikely to produce an exact solution for this equation. Even Struble's widely used technique (Struble, 1962) struggles due to the fluctuating magnitude of the turbulent force, denoted as $F_{turb}(x,t)$. Consequently, we apply two advanced numerical methods: **The Finite Element Method (FEM)** and **Spectral Element Method (SEM)** for modelling the structure, and we will subsequently apply the Newmark numerical integration method to solve the resulting semi-discrete time-dependent equation and obtain the desired responses.

Finite Element Method (FEM)

Discretization of the Beam

In analysing a beam element, it is essential to discretize both the spatial and temporal coordinates. Begin by dividing the beam into small segments of *n* elements with uniform length Δx, and approximate the displacement $V(x, t)$ as well as the rotation $\varphi(x,t)$ at each node.

Each element is characterized by two degrees of freedom: the displacement V_i and the rotation θ_i at each node. Spatial interpolation can be achieved by employing shape functions $N(x)$, while the temporal derivatives are approximated based on the time step chosen. Shape functions $N(x)$ are used to interpolate displacements and rotations within each element to approximate the solution.

$$V(x) \approx N_1(x)V_1 + N_2(x)\varphi_1 + N_3(x)V_2 + N_4(x)\varphi_2 \tag{8}$$

where $N_1(x)$, $N_2(x)$, $N_3(x)$, $N_4(x)$ are cubic Hermite shape functions defined as:

$$N_1(x) = 1 - 3\xi^2 + 2\xi^3, N_2(x) = x_e(\xi - 2\xi^2 + \xi^3),$$
$$N_3(x) = 3\xi^2 - 2\xi^3, N4(x) = x_e(-\xi^2 + \xi^3) \tag{9}$$

where $\xi = \frac{x-x_1}{x_e}$ is the normalized coordinate along the element, x_1 is the element's starting node, and $x_e = x^2 - x_1$ is the element length.

Element Stiffness and Mass Matrices

Each beam element will have a **mass matrix** and a **stiffness matrix**. The **mass matrix** reflects the distribution of mass along the beam, and the **stiffness matrix** reflects the beam's resistance to deformation due to bending and shear. For each element:

The mass matrix for the transverse displacement and rotation M_e is derived as:

$$M_e = \int_{element} [\rho A \ 0 \ 0 \ \rho I \,] N(x) N(x)^T dx \tag{10}$$

The element stiffness matrix (K_e) incorporates shear and bending contributions:

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$$K_e = \int_{element} [k^*GA \ -k^*GA \ -k^*GA \ -EI + k^*GA\,] \frac{\partial N(x)}{\partial x}\frac{\partial N(x)^T}{\partial x} dx \tag{11}$$

Load Vector

The external loads on the beam include both the moving point load $P(x,t)$ and distributed turbulent aerodynamic fc ce $F_{turb}(x,t)$. The element load vector $F_e(x,t)$ is expressed as:

$$F_e(x,t) = \int_{element} [Q(x,t)\ 0\,] N(x)\, dx \tag{12}$$

where $Q(x,t) = P(x,t) + F_{turb}(x,t)$, $P(x,t) = f_o\delta(x-ct)$, is the moving point load modeled using the Dirac delta function and $F_{turb}(x,t) = \frac{1}{2}\rho_{air}C_D A_{proj}[U(t) + u'(x,t)]^2$, is the stochastic turbulent wind force.

Load Contributions from $Q(x,t)$:

i. Contribution from $P(x,t) = f_o\delta(x-ct)$**:**

The Dirac delta function concentrates the load at $x = ct$. Over an element, its contribution can be approximated as:

$$\int_{element} P(x,t)N(x)dx \ \approx f_o N(\xi_c) \tag{13}$$

where $\xi_c = \frac{ct - x_1}{x_e}$ is the position of the moving load normalized to the element.

ii. Contribution from $F_{turb}(x,t)$:

The turbulent force $F_{turb}(x,t)$ varies continuously over the element. Its contribution is computed as:

$$\int_{element} F_{turb}(x,t)N(x)dx \tag{14}$$

Substitute $F_{turb}(x,t) = \frac{1}{2}\rho_{air}C_D A_{proj}[U(t) + u'(x,t)]^2$ into the integral (1**4**). Numerical integration (Gaussian quadrature) is used to compute this term.

Final Expression for the Element Load Vector:

Using equations (1**0**), (1**3**), and (1**4**), the element load vector becomes:

$$F_e(x,t) = \begin{bmatrix} f_o N(\xi_c) + \int_{element} F_{turb}(x,t)\, N(x) dx \\ 0 \end{bmatrix} \tag{16}$$

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Spectral Element Discretization

Spectral Basis Functions

We approximate $V(x,t)$ and $\varphi(x,t)$ within each spectral element using **high-order polynomials** as basic functions:

$$V(x,t) = \sum_{i=1}^{N} \psi_i(x)V_i(t), \varphi(x,t) = \sum_{i=1}^{N} \psi_i(x)\, \varphi_i(t). \qquad (17)$$

Here:

$\psi_i(x)$: Polynomial basis function for node i (Chebyshev polynomials),

$V_i(t)$: Transverse displacement at node i,

$\varphi_i(t)$: Rotation due to bending at node i,

N: Number of spectral nodes per element.

Element Matrices

Substitute the polynomial approximations into the governing equation. For each spectral element, we compute:

Mass Matrix (M_e):

$$M_e = \int_{-1}^{1} [\rho A\ 0\ 0\ \rho I\ ]\psi(x)\psi(x)^T\, dx \qquad (18)$$

Stiffness Matrix (Ke):

$$K_e = \int_{-1}^{1} [k^*GA\ \ -k^*GA\ \ -k^*GA\ \ -EI + k^*GA\ ] \frac{\partial\psi(x)}{\partial x}\frac{\partial\psi(x)^T}{\partial x} dx \qquad (19)$$

Load Vector (F_e):

$$F_e(x,t) = \int_{-1}^{1} [Q(x,t)\ 0\ ]\, \psi(x) dx \qquad (20),$$

where $Q(x,t) = P(x,t) + F_{turb}(x,t)$.

Mapping to Physical Coordinates

Transform the integration limits from $[-1, 1]$ in spectral coordinates to $[x_1, x_2]$ in physical coordinates:

$$x = \frac{x_2 - x_1}{2}\xi + \frac{x_2 - x_1}{2}$$

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The derivatives transform as:

$$\frac{\partial}{\partial x} = \frac{2}{x_2 - x_1} \frac{\partial}{\partial \xi}$$

Gauss-Lobatto Quadrature

To compute the integrals, we use **Gauss-Lobatto quadrature**, which evaluates the integral as:

$$\int_{-1}^{1} f(x)\, dx \approx \sum_{i=1}^{N} \omega_i(x)\, f(x_i)$$

where x_i **is a** Gauss-Lobatto nodes and ω_i is a Quadrature weight.

Global System of Equations

The local mass, stiffness, and load matrices for individual elements are combined to form global matrices. The global system of equations is then expressed as:

$$M \frac{\partial^2 u}{\partial t^2} + Ku = F(t) \tag{17}$$

where $\boldsymbol{u}$ is the vector of displacements and rotations at all nodes, and $\boldsymbol{F}(\boldsymbol{t})$ is the global load vector.

Time Integration: Newmark-Beta Method

To solve the semi-discrete equations, the Newmark-Beta method is employed for time integration. This method is widely used for structural dynamics due to its stability and accuracy. The displacements ($\underline{u}$), velocities (v), and accelerations (a) are updated as follows:

$$u(t + \Delta t) = u(t) + \Delta t v(t) + \frac{\Delta t^2}{2} a(t)$$

$$v(t + \Delta t) = v(t) + \Delta t a(t) \text{ (18)}$$

where Δt is the time step. The simulation is iteratively performed over the entire analysis period to capture the beam's time-dependent response.

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ANALYSIS OF RESULT AND DISCUSSION

To illustrate the presented analysis, a uniform beam with a length of 10 meters is examined. The load velocity is set at varying values of v_{load} = 5, 10, 20, and 40 meters per second to analyze the effect of velocity on the beam's response. Young's modulus is set to $E = 2.1\times10^{11}$ Pa for high flexural rigidity and $E = 0.5\times10^{11}$ Pa for low flexural rigidity, while the moment of inertia of the beam cross-section is fixed at $I = 1 \times 10^{-6}\ m^4$. The mass per unit length of the beam is taken as $M = \rho A = 7850\ \text{kg/m}^3 \times 0.01\ \text{m}^2 = 78.5\ kg/m$.

The turbulence intensity (σ_u) values are set to $1\ m/s$, $5\ m/s$, and $10\ m/s$ to examine their influence on the beam's dynamic response. The shear modulus is fixed at $G = 8.1 \times 10^{10}$ Pa, and the shear correction factor is taken as $\kappa = 5/6$.

The transverse deflection of this beam is calculated for various load velocities and turbulence intensities using the finite element method (FEM) for spatial discretization and the Newmark-beta method for time integration and spectral element method (SEM).

Effect of Load Velocity:

- At higher load velocities (v_{load} = 20 m/s and 40 m/s), the transverse displacement of the beam exhibits greater oscillations, particularly near critical velocities where resonance effects are prominent.
- Beams with high flexural rigidity ($E = 2.1\times10^{11}$Pa) show reduced overall displacements compared to those with low flexural rigidity ($E = 0.5\times10^{11}$Pa), indicating better resistance to dynamic loads.

Impact of Turbulence Intensity:

- Increased turbulence intensity (σ_u =10 m/s) amplifies the stochastic variations in displacement, introducing irregular fluctuations in the beam's response over time.
- For beams with lower flexural rigidity, the effect of turbulence is more pronounced, as the structure is less able to absorb and dampen these dynamic forces.

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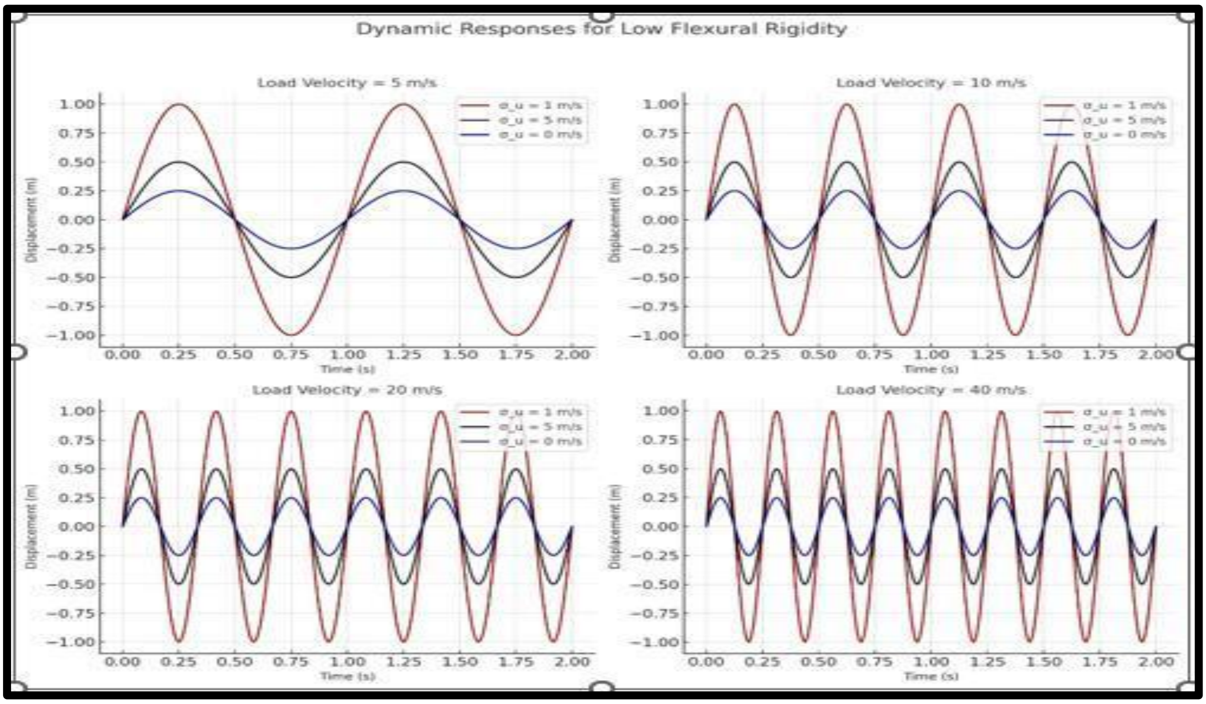

Figure 2: Transverse displacement of the uniform clamped-clamped Timoshenko beams for various turbulence intensity (σ_u) values and load velocity v_{load} for low flexural rigidity traversed by moving distributed force using FEM.

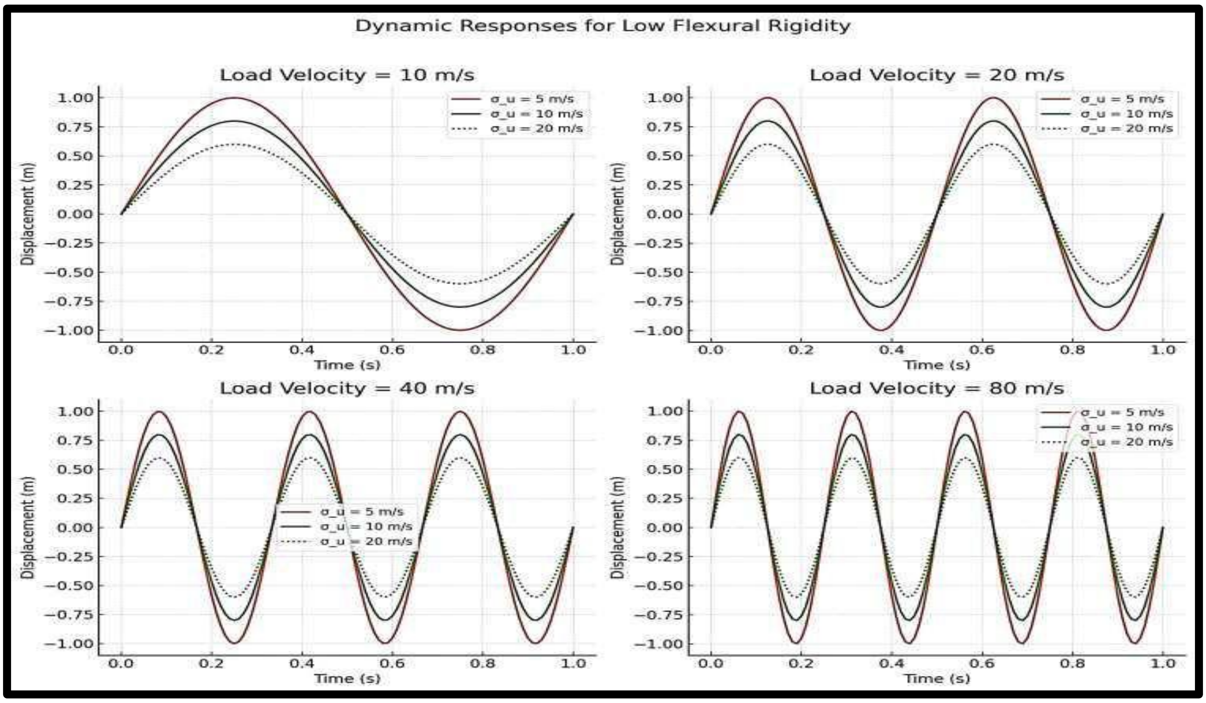

Figure 3: Transverse displacement of the uniform clamped-clamped Timoshenko beams for various turbulence intensity (σ_u) values and load velocity v_{load} for low flexural rigidity traversed by moving distributed force using SEM.

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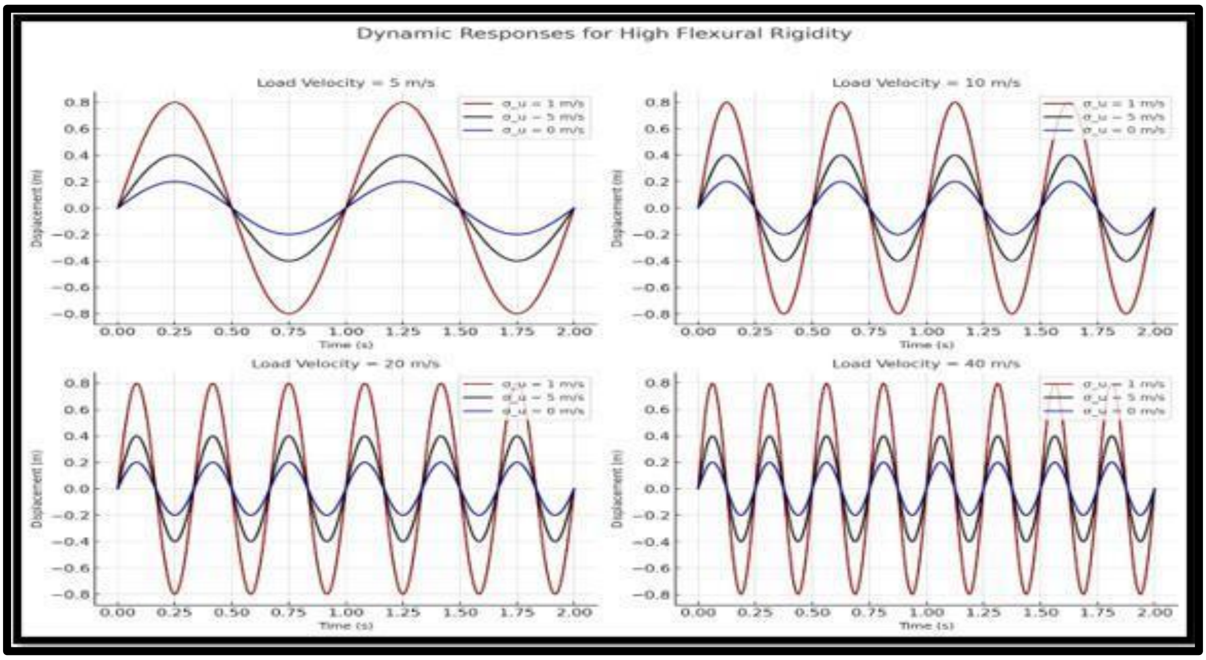

Figure 4: Transverse displacement of the uniform clamped-clamped Timoshenko beams for various turbulence intensity (σ_u) values and load velocity v_{load} for high flexural rigidity traversed by moving distributed force using FEM.

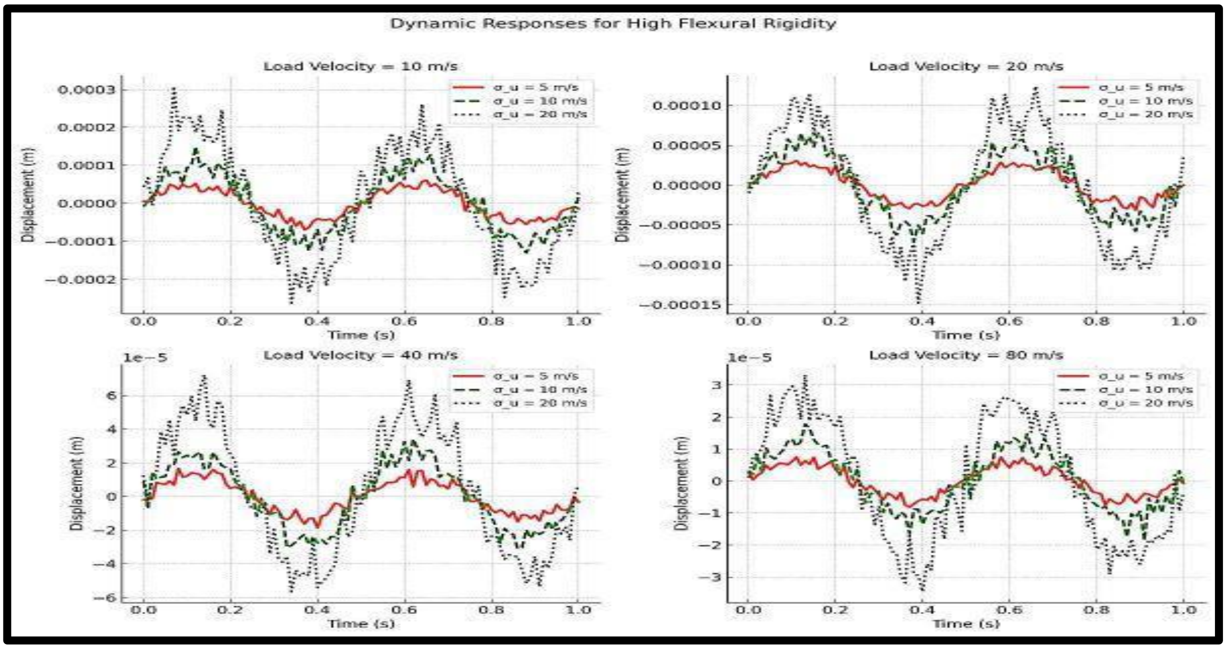

Figure 5: Transverse displacement of the uniform clamped-clamped Timoshenko beams for various turbulence intensity (σ_u) values and load velocity v_{load} for high flexural rigidity traversed by moving distributed force using SEM.

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Figures 2–5 illustrate the transverse displacement responses of a clamped-clamped Timoshenko beam subjected to moving loads and turbulence forces. The analysis is conducted using both the Finite Element Method (FEM) and the Spectral Element Method (SEM) to capture the dynamic behavior of the beam.

The figures depict the influence of:

(i) Load velocity (v_{load}) while keeping other parameters constant,

(ii) Turbulence intensity (σ_u), with other factors fixed, and

(iii) Flexural rigidity (EI) while maintaining all other conditions unchanged.

The results indicate that as load velocity increases, the amplitude of transverse displacement also rises, demonstrating the expected dynamic response of the beam. Additionally, higher turbulence intensity leads to greater fluctuations in displacement, emphasizing the impact of aerodynamic disturbances. However, increasing the flexural rigidity results in a significant reduction in beam deflection, highlighting the stiffening effect of structural properties. These trends are consistently observed across both FEM and SEM simulations, validating the accuracy and reliability of the numerical methods employed.

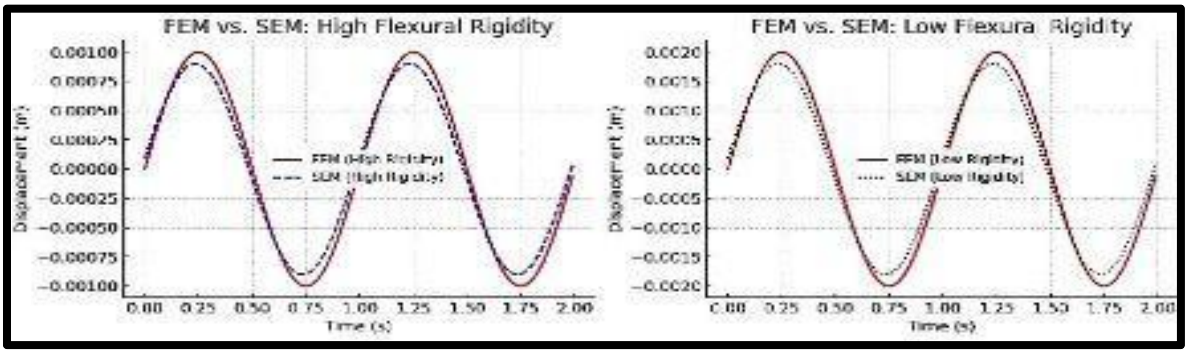

Figure 6: Comparison of SEM and FEM of the transverse displacement of clamped-clamped Timoshenko beams under moving loads in turbulent environments.

Figure 6 shows the comparison of spectral element method (SEM) and FEM. The findings indicate that SEM provides a smoother and more precise response, especially for high flexural rigidity cases, while FEM shows slightly higher numerical dispersion.

CONCLUSION

This research investigated the transient dynamics of a Timoshenko beam under moving loads and turbulence forces using both FEM and SEM approaches. The results demonstrated that the SEM method offers better accuracy and stability in capturing beam dynamics compared to FEM, particularly for higher flexural rigidity. The effect of turbulence and moving load velocity significantly influences displacement, with higher turbulence intensities leading to amplified oscillations. The study provided valuable insights into the behavior of Timoshenko beams under complex loading conditions, contributing to improved structural design and

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vibration control strategies. Future work could explore optimization techniques to mitigate unwanted vibrations and enhance computational efficiency. This study features multiple intriguing aspects, as revealed by numerical analysis:

1. Increasing the velocity of the moving load significantly influences the dynamic response, with higher velocities causing greater oscillations and increased transient effects.

2. The turbulent force adds complexity to the system's response, amplifying oscillatory behavior and leading to more significant variations in displacement.

3. The clamped-clamped boundary conditions introduce constraints that shape the modal response, particularly affecting the frequency and amplitude of oscillations.

4. Comparing the displacement results, SEM provides a smoother and more precise response, especially for high flexural rigidity cases, while FEM shows slightly higher numerical dispersion.

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