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Exchange Rate Volatility Analysis: Evaluating Garch Models for Naira-Dollar Rates

Olumi Toba Timothy[1*], Lawal Fatai Kolade[2], Ehimony James Babatunde[3], Umolo David[4], Ittah Joseph Babatunde[5]

[1,4]Department of Statistics, Federal Polytechnic, Orogun, Delta State.

[2,3]Department of Statistics, Kogi State Polytechnic Lokoja, Kogi State.

[5]Department of Accountancy, Kogi State Polytechnic, Lokoja, Kogi State.

[*]Corresponding Author's Email: olumitoba@yahoo.com

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ABSTRACT: *This study examines exchange rate volatility with Generalized Autoregressive Conditional heteroscedastic (GARCH) models using daily exchange rate data obtained from the central bank of Nigeria between 1st January 2017 and 31st December 2019. The ARCH LM test of the mean equation revealed the presence of conditional heteroscedasticity. The returns were modeled using ARCH (3), GARCH (2,2), Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) (3,2), and Threshold Generalized Autoregressive Conditional Heteroscedastic (TGARCH) (1,1). The results revealed that EGARCH (3,2) was the best since it has the least AIC of -24.3197 and SIC of -24.2741. A diagnostic test of the EGARCH (3,2) model residuals with Ljung-Box and the ARCH LM tests revealed that the models were free from higher order autocorrelation and conditional heteroscedasticity respectively. The parameters of the EGARCH (3,2) model were significant and the positive value of the leverage parameter is an indication of absence of leverage effect in the returns of Naira-Dollar exchange rate. The absence of the leverage effect in the exchange rate indicates that positive shocks increase volatility than negative shocks of equal magnitude. Thus, the implication is that strengthening the Dollar (weakening the Naira) leads to higher period volatility than when the Naira is strengthened by the same amount. It is recommended that the central bank should put in place long-term measures to stabilize the Naira since weakening the Naira increases the uncertainty in the exchange market than strengthening the Naira.*

KEYWORDS: Exchange Rate, Economic Growth, Garch, Heteroscedasticity, Volatility.

INTRODUCTION

Many economies, developed and developing, have faced significant real exchange rate volatility, leading to uncertainty in achieving key macroeconomics and monetary policy goals, such as price stability and economic growth. This volatility creates real unpredictable fluctuations in relative prices, making it challenging for economies to be stabilized. Stable exchange rates play a crucial role in attracting foreign investment (both direct and portfolio), maintaining price stability, and promoting sustainable economic growth.

The exchange rate between the Nigerian Naira (NGN) and the US dollar (USD) is seen as the number of the Nigerian Naira needed to purchase one US dollar. Also, significant and unpredictable fluctuations in exchange rates pose a substantial challenge to macroeconomic stabilization policies. Having studied the effects that changes in exchange rates can have on economic conditions, policymakers seek to comprehend strategies for mitigating exchange rate volatility and understanding the potential consequences of such actions (Kuntomah, 2013).

In international trade, goods and services are exchanged across national borders, but each country's currency is typically not accepted as a legal tender in another country. This creates a payment problem, as the importer must acquire the exporter's country's currency to settle transactions. For example, Indian rupees are not acceptable as a medium of exchange in Nigeria, nor is Nigerian Naira acceptable in Japan. Therefore, to facilitate international trade, importers purchase foreign currencies in the foreign exchange market, where currencies are bought and sold.

Exchange rate volatility is said to have implications for the financial system of a country, especially the tradable sector. Changes in exchange rates have pervasive effects, with consequences for prices, incomes, interest rates, manufacturing levels, and job opportunities, and thus with direct or indirect repercussions for the welfare of virtually all economic participants.

Aim and Objectives of the Study

The aim of the study is to model exchange rate volatility between the Nigerian Naira and the United States Dollars. In addition to the aim, the following specific objectives will be explored:

1. To develop an appropriate GARCH model for the Naira/Dollar exchange rate
2. To test the adequacy of the selected model for use.

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LITERATURE REVIEW

Bollerslev (1986) acknowledged the usefulness of the ARCH process in modeling several different economic phenomena. However, he noted that in most of those applications, the introduction of a rather arbitrary linear declining lag structure in the conditional variance equation to take account of the long memory was typically found in empirical work because estimation of a totally free lag distribution would often lead to violation of the non-negativity constraints. He then came up with GARCH, a more general class of processes which allowed for a more flexible lag structure which also permitted a more parsimonious description.

Dickson and Ukavwe (2013) applied the error correction and GARCH models to investigate the impact of exchange rate fluctuations on trade variations in Nigeria, using annual time series data from 1970 to 2010. The results of the study showed that exchange rate volatility is not significant in explaining variations in import, but was found to be statistically significant and positive in accounting for variations in export. Serenis and Tsounis (2014) examined the effect of volatility on two small countries—Croatia and Cyprus—on aggregate exports during the period 1970 to 2012. ARDL methodology was adopted and results suggested that there is a positive effect of volatility on exports of Croatia and Cyprus.

Kuhe and Chiawa (2017) investigated the impact of structural breaks on the conditional variance of daily stock returns of 8 commercial banks in the Nigerian stock market for the period 17th February, 2003 to 31st September 2016. Using symmetric GARCH and asymmetric EGARCH and TGARCH models with and without dummy variables, they evaluated variance persistence, mean reversion, asymmetric and leverage effects. Results revealed high persistence in conditional volatility for the banking stocks when structural breaks were ignored. However, incorporating random level shifts into the models reduced the estimated conditional volatility, suggesting that accounting for structural breaks is crucial for accurate volatility modeling.

Oyinlola, Mutiu A. (2018) examined the volatility persistence and asymmetry of Naira/Dollar exchanges rate using monthly data between January 2004 and November 2017. The study employed Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (1, 1), TGARCH (1, 1) and EGARCH (1, 1). The findings showed that persistence is generally explosive in the Bureau de Change (BDC) market as compared to interbank market where the persistence was high but not explosive, especially under asymmetric models. Based on the model selection criteria, the symmetric GARCH model appears to be better than the asymmetric ones in dealing with exchange rate volatility in the interbank market, while asymmetric GARCH, especially TGARCH, seems to be better in the case of BDC market.

Shamiri and Hassan (2005) examined and estimated the three GARCH (1, 1) models (GARCH, EGARCH and GJR-GARCH) using the daily price data of two Asian stock indices, Strait Time Index in Singapore (STI) and Kuala and Luampur Composite Index in Malaysia (KLCI) over a 14-year period. The competing models—GARCH, EGARCH and GJR-GARCH—were developed based on three different distributions: Gaussian normal, student-t, Generalized Error Distribution. The estimated results showed that the forecasting performance of asymmetric GARCH models (GJR-GARCH and EGARCH), especially when fat-tailed asymmetric densities are taken into account in the conditional volatility, was better than symmetric GARCH. Moreover, it was found that the AR (1)-GJR model provided the best out-of-sample

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forecast for the Malaysian stock market, while AR (1)-EGARCH provided a better estimation for the Singaporean stock market.

RESEARCH METHODOLOGY

This study utilized secondary data on the exchange rate between the Nigerian Naira (NGN) and the United States Dollar (USD) spanning from January 1, 2017 to December 31, 2019, comprising 1095 data points for modeling purposes. For the in-sample forecast evaluation, a subset of 31 observations was used, covering the period from December 1, 2019 to December 31, 2019. The data were obtained from the central bank of Nigeria and were analyzed with Eviews 9 version. In this study, returns (r_t) were calculated as the continuously compounded returns which are the first difference in logarithms of the interbank exchange rate.

$$r_t = log\left(\frac{ER_t}{ER_{t-1}}\right) \quad (1)$$

where ER_tmeans Naira/Dollar exchange rate at time t and ER_{t-1} represents exchange rate at time $t-1$. The r_t in equation 3.1 will be used in investigating the volatility of the interbank exchange rate.

Unit Root Tests

A financial time series whose mean, variance and auto-covariance are constant is considered to be stationary. This means auto-covariance function as $Cov(y_t, y_{t+k})$ for any lag k is only a function of k and not time, that is, $\gamma_y(k) = Cov(y_t, y_{t+k})$. However, most of the financial time series, such as interest rate, exchange rates, or the price series of an asset tend to be non-stationary. These series do not satisfy the requirements of stationarity, so they have to be converted to stationary processes before modeling.

We shall use the Augmented Dickey-Fuller (ADF) test to establish the stationarity or otherwise of the data.

The Dickey-Fuller Test

Dickey-Fuller (DF) test is a statistic used to test whether the series contains unit root or not. (A time series that is non-stationary is said to exhibit unit root.) The test is performed by estimating regression models. The regression model can be fitted with constant and trend. The model with constant captures the non-zero mean under the alternative hypothesis.

The testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model:

$$\Delta y_t = \alpha + \beta_t + \pi y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_{p-1} y_{t-p+1} + \varepsilon_t \quad (2)$$

where α is a constant and β is the coefficient of a time trend p, the lag order of the autoregressive process, imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponding to modeling a random walk with a drift. The null hypothesis for this test is $H_0: \gamma = 0$, the existence of unit root, and the alternative hypothesis is $H_1: \gamma < 0$, the non-existence of unit root. The test statistic for the ADF test is given by

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$$ADF = \frac{\gamma}{SE(\hat{\gamma})} \tag{3}$$

where $\hat{\gamma}$ denotes the least squares estimates of γ and $SE(\hat{\gamma})$ is the standard error. The null hypothesis is rejected if the test statistic is greater than the critical value. The estimation technique is Ordinary Least Squares (OLS).

ARMA Model

ARMA models are the combination of the simple AR and MA models of order (p, q) called the Autoregressive Moving Average model (ARMA). The p represents the order of the autoregressive process and the q represents the order of the moving average process. The general form of the model is given by

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_p \varepsilon_{t-p} \tag{4}$$

Testing for Heteroscedasticity

The Lagrange Multiplier (LM) test for ARCH effects, proposed by Engle (1982), is applied.

In summary, the test procedure is performed by first obtaining the residuals from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes—(ARMA) process. For example, in ARMA (1, 1) process, the conditional mean equation is as given below:

$$y_t = \alpha_0 + \alpha y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \tag{5}$$

After obtaining the residuals, the next step is regressing the squared residuals on a constant and q lags as in the equation below:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t \tag{6}$$

where e_t is the residual.

If there exists no ARCH-effect, then it implies that the residuals of the model are homoscedastic (have constant variance).

The Autoregressive Conditional Heteroscedasticity (ARCH)

The essence of the model was that it is much more efficient to be used simultaneously for the mean and variance of a financial time series in the case that the conditional variance is not constant. The basic idea of ARCH models is that (a) the shock a_t of the financial instrument is serially uncorrelated, but depends, and (b) the dependence of a_t can be described by a simple quadratic function of its lagged values. Specifically, an ARCH (p) model assumes that

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2, \varepsilon_t = \sigma_t \epsilon_t \tag{7}$$

ARCH (1) model is given as

$$\sigma_t^2 = \alpha_0 + \alpha_1 {\varepsilon_{t-1}}^2, \quad \varepsilon_t = \sigma_t \epsilon_t \tag{8}$$

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where ε_t, defined as $\varepsilon_t = \sigma_t \epsilon_t$, is a sequence of independently and identically distributed (iid) random variables with zero mean and variance 1, $\alpha_0 > 0$ and $\alpha_i \geq 0$.

Generalized Autoregressive Conditional Heteroscedastic Models

A Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was first developed by Bollerslev in 1986. In order to capture the dynamics of volatility, high order if ARCH has to be estimated. The particular feature of this model was to introduce and use the lagged conditional variance terms as autoregressive terms. The standard GARCH (p, q) process is as specified below:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_{j\sigma_{t-j}^2}, \varepsilon_t = \sigma_t \epsilon_t \quad (9)$$

where, ε_t is $defined$ as $\varepsilon_t = \sigma_t \epsilon_t$, is a sequence of iid random variables with mean 0 and variance 1.

$$\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0 \quad and \quad \sum_{i=1}^{max(p,q)} (\alpha_i + \beta_j) < 1$$

GARCH (1, 1) model is also given below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (10)$$

$$0 \leq \alpha_1 \beta_1 \leq 1, (\alpha_1 + \beta_1) < 1$$

The persistence of the conditional variance (σ_t^2) is captured by $\alpha + \beta$ and covariance stationarity requires that $\alpha + \beta < 1$.

The Exponential GARCH Model

In the basic GARCH model, since only squared residuals ${\varepsilon_{t-1}}^2$ enter the conditional variance equation, the signs of the residuals or shocks have no effect on conditional volatility. However, a stylized fact of financial volatility is that bad news (negative shocks) tends to have a larger impact on volatility than good news (positive shocks). An EGARCH (p, q) model can be written as:

$$In(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \int \frac{2}{\pi} \right| + \sum_{j=1}^{q} \beta_j \, log(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma k \left| \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \right| \quad (11)$$

EGARCH (1, 1) is given by:

$$In(\sigma_t^2) = \alpha_0 + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int \frac{2}{\pi} \right| + \beta log(\sigma_{t-1}^2) + \gamma \left| \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \right| \quad (12)$$

where α_0, α_i, β_j, and γk are constant parameters. Note that when ε_{t-1} is positive ("good news"), the total effect of ε_{t-i} is $1 + \frac{\gamma_i}{\varepsilon_{t-i}}$, while when the ε_{t-i} is negative ("bad news"), the total effect of ε_{t-i} is $1 - \frac{\gamma_i}{\varepsilon_{t-i}}$. The EGARCH is the covariance stationary provided $\sum_{j=1}^{q} \beta_j < 1$.

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The Threshold GARCH (TGARCH) Model

Another volatility model commonly used to handle leverage effects is the threshold GARCH (TGARCH) model. In the TGARCH (1, 1) model, the specification of the conditional variance is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (13)$$

where d_{t-1} is a dummy variable, that is, $d_{t-1} = \{1 \quad if \ \varepsilon_{t-1} < 0, \ bad\ news\ 0 \quad if\ \varepsilon_{t-1} \geq 0, \ good\ news$

The coefficient γ is known as the asymmetry or leverage term. When $\gamma = 0$, the model collapses to the standard GARCH forms. Otherwise, when the shock is positive (i.e., good news), the effect on volatility is α_1, but when the news is negative (i.e., bad news), the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks. The general model for TGARCH (p, q) is as given below:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i d_{t-1}) \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_j \sigma_{t-j}^2 \quad (14)$$

ARCH-LM Test

The ARCH-LM test proposed by Engle was used to test for the presence of conditional heteroscedasticity in the model residuals. The test procedure is as follows:

$\boldsymbol{H_0}$: There is no heteroscedasticity in the model residuals

$\boldsymbol{H_1}$: There is heteroscedasticity in the model residuals

The test statistics is

$$LM = nR^2$$

where n is the number of observations and R^2 is the coefficient of determination of the auxiliary regression.

$$e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \ldots + \beta_q e_{t-q}^2 + v_t \quad (15)$$

where e_t is the residual. The null hypothesis is rejected when the p-value is less than the level of significance and the conclusion is that there is heteroscedasticity.

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ANALYSIS AND RESULTS

Table 4.1 below shows the summary of the descriptive statistics of the Naira to Dollar exchange rate returns.

Table 4.1: Descriptive Statistics for Exchange Rate Return Series

Mean	351.82
Skewness	-0.512
Kurtosis	10.59
Jarque-Bera	2674.97
Probability	0.00

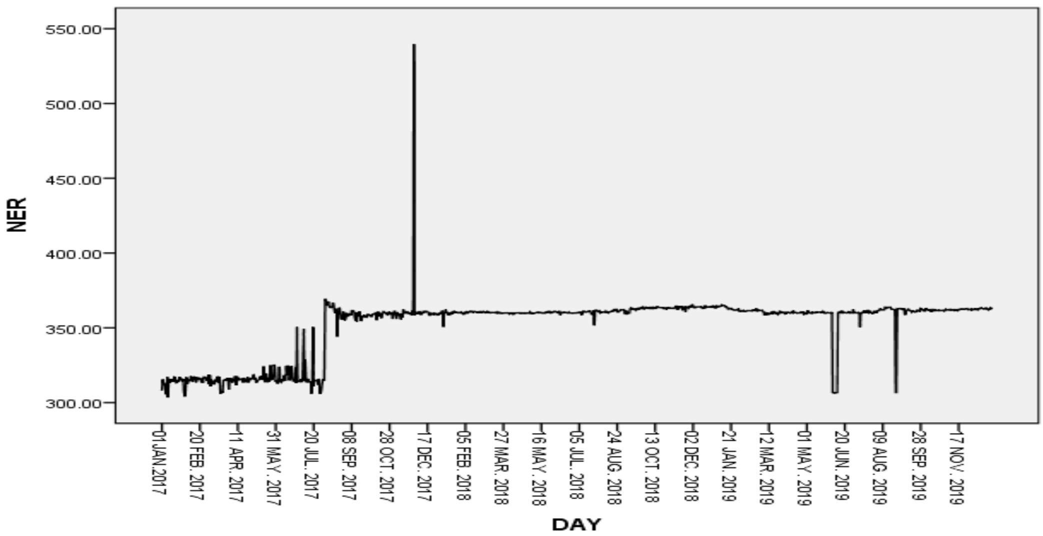

Fig 4.1: Time series plot of exchange rate in Nigeria

Figure 4.1 shows volatility clustering, i.e., periods of calm and periods of high volatility. The plot also indicates that some periods are more clustered than others.

Fitting the GARCH models

From Table 4.2, the ADF test was employed to affirm the stationarity of the returns. The tests performed with constant, and constant with trend, both affirm that the values were stationary.

Table 4.2: Augmented Dickey-Fuller Test

	t-Statistic	Prob.
Augmented Dickey-Fuller test statistic	-14.68612	0.0000
Test Critical Values: 1% level	-3.436171	
5% level	-2.863998	
10% level	-2.568130	

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Table 4.3: Selecting an Appropriate Mean Equation

ARMA (p, q)	ARMA (1,1)	ARMA (1,2)	ARMA (1,3)	ARMA (2,1)	ARMA (2,2)	ARMA (2,3)	ARMA (3,1)	ARMA (3,2)	ARMA (3,3)
AIC	6.9798	7.2310	7.2295	7.2006	7.1461	7.3775	7.4005	7.4449	7.2386
SIC	6.9980	7.2493	7.2478	7.2188	7.1644	7.3934	7.4188	7.4632	7.2569

Several mean equations were fitted to the returns and the ARMA (1, 1) was selected as the best mean equation based on the AIC and SIC, as shown in Table 4.3.

Table 4.4: Heteroscedasticity Test: ARCH LM Test

F-Statistic	35.6429	Prob.F(1,1092)	0.0000
Obs R-Squared	34.5795	Prob. Chi-Square(1)	0.0000

The ARCH-LM test shown in Table 4.4 was used to test for the presence of ARCH effects.

Table 4.5: Test of Heteroscedasticity- Ljung Box Test

Lags	**6**	**12**	**18**	**24**	**36**
Test statistic	139.41	154.59	170.09	182.99	209.96
p-value	0.000	0.000	0.000	0.000	0.000

The Ljung Box test results shown in Table 4.5 were used to test for the presence of ARCH effects. The result revealed that there was an ARCH effect in the residual of the ARMA model.

Table 4.6: Selecting the Best ARCH Model

MODEL	**AIC**	**SIC**
ARCH(1)	3.1937	3.2211
ARCH(2)	3.6619	3.3981
ARCH(3)	3.1551	3.1916

Several ARCH models were estimated and the best model was selected based on the AIC and the SIC, as shown in Table 4.6. ARCH (3) has the lowest AIC and SIC values of (3.1551) and (3.1916) respectively.

Equation 3.8 is the ARMA (1, 1) – ARCH (3) model.

$$y_t = 359 + 0.9999_{t-1} + \varepsilon_t - 0.5118\varepsilon_{t-1}$$

$$\sigma_t^2 = 0.5058 + 0.8886\varepsilon_{t-1}^2 + 0.5607\varepsilon_{t-2}^2 + 0.3857\varepsilon_{t-3}^2,\ \varepsilon_t = \sigma_t\epsilon_t$$

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Table 4.7: Estimates of the Best GARCH Model

GARCH(p,q)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
AIC	6.760	6.753	6.774	6.724	5.904	5.952	5.906	6.150	7.364
SIC	6.788	6.785	6.810	6.756	5.940	5.993	5.943	6.191	7.410

Several GARCH models were estimated and the best model selected as shown in Table 4.7 was the **GARCH (2,2)**, which has the smallest AIC and SIC values of 5.9039 and 5.9404 respectively and hence is the most appropriate among the GARCH(p, q) models.

Equation 3.9 is the **ARMA (1, 1) – GARCH (2, 2)** model.

$$y_t = 345.0156 + 0.9498y_{t-1} + \varepsilon_t + 0.0202$$

$$\sigma_t^2 = 0.2524 + 0.2115\varepsilon_{t-1}^2 + 16.9448\varepsilon_{t-2}^2 - 0.0000207\sigma_{t-1}^2 - 0.0000132\sigma_{t-2}^2,$$

Table 4.8: Estimating the Parameters of EGARCH (p, q) Model

EGARCH (p,q)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)
AIC	6.658	5.728	5.509	6.222	6.019	5.931	5.994	-24.320	6.676
SIC	6.690	5.770	5.550	6.259	6.060	5.977	6.035	-24.274	6.806

Table 4.8 displays several EGARCH models fitted to the returns. EGARCH (3, 2) was the best among the EGARCH models as it has the least values of AIC and SIC. From Equation 3.11, the ARMA (1,1) – EGARCH(3,2) model can be written as:

$$y_t = 360.0061 - 0.9485y_{t-1} + 5310083\varepsilon_{t-1}$$

$$In(\sigma_t^2) = -9.0273 + 2.5111\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int\frac{2}{\pi}\right| + 1.8373\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}} - \int\frac{2}{\pi}\right| + 0.4419\left|\frac{\varepsilon_{t-3}}{\sigma_{t-3}} - \int\frac{2}{\pi}\right| + 0.0052\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + 0.1703\,log\,(\sigma_{t-1}^2) + 0.6248\,log\,(\sigma_{t-2}^2), \qquad \varepsilon_t = \sigma_t\epsilon_t$$

Table 4.9: Selecting the Best TGARCH Model

TGARCH(p,q)	(1,1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)	(3, 1)	(3, 2)	(3, 3)
AIC	6.113	6.114	6.769	6.722	6.697	6.758	6.772	6.236	6.756
SIC	6.145	6.150	6.810	6.75	6.738	6.804	6.813	6.281	6.806

Table 4.10 displays the various TGARCH models fitted to the returns. From Table 4.9, TGARCH (1, 1) was the best among the TGARCH models since it has the least values of AIC and SIC. Equation 3.13, the ARMA (1, 1) – TGARCH (1, 1) model can be written as:

$$y_t = 339.6241 + 0.9904y_{t-1} + \varepsilon_t - 0.5670\varepsilon_{t-1}$$

$$\sigma_t^2 = 0.7520 + 66.0457\varepsilon_{t-1}^2 - 60.6059d_{t-1}\varepsilon_{t-1}^2 + 0.0023\sigma_{t-1}^2$$

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Table 4.10: Heteroscedasticity Test: ARCH LM for ARCH (3)

F- Statistic	0.020242	Prob. F	1.0000
$Obs^{*}R - Squared$	0.204452	Prob. Chi-Square	1.0000

Table 4.10 displays the results of Lagrange Multiplier test for ARCH effects and the null hypothesis of no ARCH effects cannot be rejected since the p-value was greater than 5%. There are no additional ARCH effects in the residual of ARCH (3).

Table 4.11: Heteroscedasticity Test: ARCH LM for GARCH (2, 2)

F-Statistic	0.000516	Probability	0.9819
Obs^{*} R-Square	0.000517	Probability	0.9819

Table 4.11 displays the results of Lagrange Multiplier test for ARCH effects in the GARCH (2, 2) model. The null hypothesis of no ARCH effects cannot be rejected since the P-values are greater than 5%. There are no additional ARCH effects in the residual of GARCH (2, 2).

Table 4.12: Heteroscedasticity Test: ARCH LM for EGARCH (3, 2)

F-Statistic	0.652760	Probability	0.4193
Obs^{*} R-Square	0.653565	Probability	0.4188

Table 4.12 displays the results of the Lagrange Multiplier test for ARCH effects in the EGARCH (3, 2) model. The null hypothesis of no ARCH effects cannot be rejected since the p-values are greater than 5%. There are no additional ARCH effects in the residual of EGARCH (3, 2).

Table 4.13: Heteroscedasticity Test: ARCH LM for TGARCH (1, 1)

F-Statistic	0.035371	Probability	0.8509
Obs^{*} R-Square	0.035434	Probability	0.8507

Table 4.13 displays the results of the Lagrange Multiplier test for ARCH effects in the EGARCH (3, 2) model. The null hypothesis of no ARCH effects cannot be rejected since the p-values are greater than 5%. There are therefore no additional ARCH effects in the residual of TGARCH (1, 1).

Table 4.14: Selecting the Most Appropriate Model

Model	ARCH(3)	GARCH(2, 2)	EGARCH(3, 2)	TGARCH(1, 1)
AIC	3.1551	5.9039	-24.3197**	6.1128
SIC	3.3981	5.9404	-24.2741**	6.1448

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The diagnosing test for all fitted models revealed that all the models fitted were adequate. A complete analysis of all the models revealed that **ARMA (1,1) – EGARCH(3,2)** was the best model, as shown in Table 4.14, since it has the least values of the AIC and SIC.

SUMMARIES OF FINDINGS

The exchange rate is negatively skewed, i.e., -0.5180, which clearly indicates lack of symmetry in the returns. In a normally distributed series, skewness must be zero (0) and kurtosis must be around 3. From our results, the skewness is -0.5180 (i.e., negatively skewed), which implies that the distribution has a long left tail and a deviation from normality. In addition, the exchange rate returns are leptokurtic caused by large kurtosis statistics of 10.5866 that exceeds the normal value of 3, indicating that the return is fat tailed. Regarding Jarque and Bera test for normality, it is consistent with the outcome provided by both statistics of kurtosis and skewness, since the JB test is significant at 1%, 5% and 10%, i.e., JB test statistic is 2674.965 with an associated p-value of zero, as shown in Table 4.1. This means we reject the null hypothesis and accept the alternative hypothesis which states that returns are not normally distributed. A check for stationarity was done using the Augmented Dickey Fuller test in Table 4.2, and the test confirmed that the data were stationary. Consequently, all the pre-mentioned statistical analyses give more support to the suitability of applying ARCH/GARCH model for our data, since the selected observations can be described as leptokurtic, fat tailed, stationary and not normally distributed.

The results from the GARCH (2,2) revealed that the volatility in the current day exchange rate is explained by approximately 78% of the volatility in the previous day's exchange rate. The sum of the ARCH term and the GARCH term is greater than unity. This shows that the conditional variance is unstable and the entire process is non-stationary. This indicates over persistence of shocks in the exchange rate which can eventually explode to infinity. Exchange rate with explosive shock is not conducive for long-term investments as investors can lose or gain indefinitely.

The results from the EGARCH (3, 2) revealed that the volatility in the current day exchange rate is explained by 100% of the volatility in the previous day's exchange rate. EGARCH (3,2) was not covariance stationary since the sum of the GARCH parameters was more than one. The parameters of the EGARCH (3,2) were all significant at the 1%, 5% and 10% significance level, except the leverage parameter. The significance of the ARCH and GARCH parameters in the EGARCH model indicates that previous period squared residual and previous period variance of the residual have an influence on current variance of the residuals. There was the existence of asymmetric effects on the volatility of the daily exchange rate returns. However, there was no evidence of leverage effects in the asymmetric model as the leverage parameter was positive. The positive and insignificant leverage effect indicates that positive shocks (good news) increase volatility more than negative shocks (bad news).

The results from the TGARCH (1, 1) revealed that the volatility in the current day exchange rate is explained by approximately 84% of the volatility in the previous day's exchange rate. TGARCH (1,1) was not covariance stationary since the sum of the GARCH parameters was more than one. This implies that the shocks to volatility are very high and will remain so forever as the variances are not stationary. In summary, the Nigerian exchange rate market is

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characterized by high persistent volatility. The parameters of the mean equation were significant at the 1%, 5% and 10% significant levels. In the variance equation, all the parameters were significant at the 1% significance level except the GARCH term. This means that it adds little explanatory power to the model. The asymmetric (leverage) parameter was significant but negative, indicating the absence of leverage effects. Leverage effects are said to exist in the TGARCH model if the leverage parameters are significant and positive.

The selected model EGARCH (3,2) was diagnosed using the Univariate LM test and it was found to be adequate.

CONCLUSION

The conclusion of this study is as stated below:

The exchange rate volatility between the Nigeria Naira and the US Dollar is highly persistent as the sum of the ARCH and the GARCH models were greater than one.

Based on these studies, the best model for modeling exchange rate volatility between Naira and Dollar for the period covered is the EGARCH (3, 2).

Positive shocks increased volatility than the negative effects of equal magnitude.

RECOMMENDATIONS

This study focused on a few GARCH models in modeling exchange rate returns. However, after almost three decades, different extensions of the ARCH models have been proposed. These include multivariate ARCH, GARCH-in-mean (GARCH-M) models, and Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH) models.

It is therefore recommended that further expository studies on modeling these extensions should be carried out.

It is also recommended that the central bank should put in place long-term measures to stabilize the Naira since weakening the Naira increases the uncertainty in the exchange market than strengthening the Naira, as depicted by the EGARCH and the TGARCH models.

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