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ON THE NOVEL DAMPED OSCILLATORY LOGISTIC GROWTH MODEL: A HYBRID APPROACH

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ABSTRACT: *The Damped Oscillatory Logistic Growth (DOLG) Model is introduced as a novel hybrid framework that integrates oscillatory dynamics, damping, and logistic growth into a single differential equation. This model extends classical systems such as the harmonic oscillator, logistic growth equation, and damped systems by combining their key features into a unified framework. Numerical solutions reveal rich dynamical behaviors, including damped oscillations, stabilization to carrying capacity, and phase-dependent growth patterns. The system's stability is analytically and numerically confirmed, with trajectories converging to the non-trivial equilibrium* $(x,v) = (K,0)$ *for all parameter regimes, and by extension, the trivial solution. The effects of damping, growth rate, and oscillation frequency are explored through time series and phase portraits, demonstrating the model's versatility in capturing complex phenomena. Potential applications span ecology, economics and engineering, offering new insights into oscillating populations, cyclical growth, and mechanical systems with growth constraints. This study lays the groundwork for future research on hybrid dynamical systems and their interdisciplinary applications.*

KEYWORDS: Damping coefficient; oscillatory behavior; system stability; eigenvalue analysis; phase portraits; nonlinear dynamics.

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INTRODUCTION

Dynamical systems are fundamental to understanding the behavior of complex systems across various disciplines, including biology, ecology, economics and engineering. From the oscillations of a pendulum to the growth of populations, mathematical models provide a powerful framework for predicting and analyzing system behavior [24]. Among these models, the harmonic oscillator, logistic growth equation, and damped systems are widely studied for their ability to capture oscillatory motion, bounded growth, and energy dissipation, respectively [1,3,27]. However, many real-world systems exhibit a combination of these phenomena, necessitating the development of hybrid models that integrate oscillatory dynamics, damping, and nonlinear growth into a unified framework (see [4, 5, 6, 19, 20 and 27]).

While oscillatory systems, logistic growth and damping have been extensively studied in isolation, there is a gap in the literature regarding models that combine these elements [6]. For instance, in ecological systems, populations may exhibit oscillatory behavior due to predator-prey interactions while simultaneously experiencing growth constraints due to limited resources [7, 8]. Similarly, in mechanical systems, oscillations may be damped by friction, and growth may be limited by material fatigue [9,10]. Existing models often treat these phenomena separately, leading to an incomplete understanding of systems where they coexist [11]. This gap motivates the development of a hybrid model that integrates oscillatory dynamics, damping and logistic growth.

Moreso, in oscillatory growth model, both analytical and numerical techniques can be employed to address the behavior of complex systems exhibiting damped oscillatory patterns. This of course, aligns with the growing body of literature that emphasizes the use of analytic methods and software tools such as SPSS and MATLAB for the analysis and interpretation of modeling results. For instance, [28,32,33,34,35] explored diverse modeling approaches applied to physical phenomena in respect to the sensitivity of corona virus disparities in Nigeria, while [36] adopted a fixed point methodology to analyze systems of linear Volterra integral equations of the second kind for the given oscillatory model. In [29,30,31,38], a software-assisted analysis was employed to examined the numerical stability in physical flow applications, and the principle of maximum was analytically applied to demonstrate uniqueness of solutions in metric spaces involving systems of Volterra integral equations. Furthermore, [37,39] applied modeling techniques to healthcare patient scenarios by using the MATLAB software in the analysis of HIV infection dynamics. These studies reinforce these methodologies by introducing a novel damped oscillatory framework, combining both analytical and computational implementation to yield deeper insight into system dynamics.

In this study, we seek to develop and analyze a Damped Oscillatory Logistic Growth (DOLG) Model which combines the harmonic oscillator, logistic growth, and damping into a single differential equation by formulating the DOLG Model and deriving its mathematical framework, solve the model numerically and analyze its behavior under different parameter regimes and explore potential applications of the model in fields such as ecology, economics and engineering.

The remainder of this paper is organized as follows: in section 2, definition of basic concepts is given. Section 3 presents the formulation of the DOLG Model and describes the numerical

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methods used to solve the model. Section 4 discusses the results and their implications, while Section 5 explores potential applications and limitations and Section 6 concludes the study.

Definition of Basic Concepts

Definition 2.1.

Oscillatory systems are ubiquitous in nature and engineering, describing phenomena such as mechanical vibrations, electrical circuits, and biological rhythms [7,9]. The harmonic oscillator is a cornerstone of these studies, providing a simple yet powerful framework for understanding periodic motion [10,11]. Recent work has extended this framework to include nonlinear effects and coupling with other systems [13].

Definition 2.2.

Logistic growth models are widely used in ecology, economics, and epidemiology to describe systems with bounded growth [23]. The logistic equation, first proposed by Verhulst, captures the transition from exponential growth to saturation as a system approaches its carrying capacity [14,15 and 16]. Recent studies have explored extensions of the logistic model, including time-varying parameters and stochastic effects [17,25].

Definition 2.3.

Damping plays a critical role in dissipating energy and stabilizing systems [18, 19]. In mechanical systems, damping is often modeled as a velocity-dependent force, while in ecological systems, it may represent resistance to population growth [20, 21]. Recent research has focused on the interplay between damping and nonlinearities in complex systems [22].

Definition 2.4.

Hybrid models that combine oscillatory dynamics, damping, and logistic growth are rare but have significant potential for applications in interdisciplinary research [25]. For example, in ecology, such models can describe oscillating populations with resource constraints, while in economics, they can model cyclical growth with market saturation [3,4,5]. Recent advances in numerical methods have made it feasible to analyze these complex systems [11,12].

In spite of the progress made in understanding oscillatory systems, logistic growth and damping, there is a lack of models that integrate these phenomena into a unified framework [1,23]. Existing studies often focus on isolated aspects of these systems, neglecting their interactions [15,16]. This gap underscores the need for a hybrid model like the DOLG Model, which can capture the combined effects of oscillations, damping and growth.

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MATERIALS AND METHODS

Mathematical Formulation

The Damped Oscillatory Logistic Growth (DOLG) Model is formulated as a second-order nonlinear ordinary differential equation (ODE):

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2x=rx\left(1-\frac{x}{K}\right) \tag{1}$$

where,

$x(t)$ is the state variable (e.g., population size, displacement).

ω is the natural frequency of oscillation.

γ is the damping coefficient.

r is the intrinsic growth rate.

K is the carrying capacity.

This equation combines the dynamics of a damped harmonic oscillator (left-hand side) with logistic growth (right-hand side). The damping term $\gamma\frac{dx}{dt}$ accounts for energy dissipation, while the logistic term $rx\left(1-\frac{x}{K}\right)$ introduces nonlinear growth with a saturation effect.

The DOLG Model is based on the following assumptions:

i. the parameters ω, γ, r, and K are constant over time.

ii. the system is homogeneous, with no spatial variation.

iii. there is no external forcing or stochastic noise.

In a brief qualitative analysis,

1. **Equilibrium Points**:

Set $\frac{dx}{dt}=0$ and $\frac{dv}{dt}=0$:$x=0$ (trivial equilibrium) $x=K$ (non-trivial equilibrium).

2. **Stability**:

Linearize the system around each equilibrium point and analyze the eigenvalues of the Jacobian matrix. For x=0, the system behaves like a damped harmonic oscillator. For x =K, the system may exhibit stable or unstable behavior depending on the parameters

3. No Logistic Term ($r=0$): The equation reduces to a damped harmonic oscillator:

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$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2x=0 \tag{2}$$

$$x(t)=e^{-\frac{\gamma}{2}t}\left(A\cos(\sqrt{\omega^2-\frac{\gamma^2}{4}}t)+B\sin(\sqrt{\omega^2-\frac{\gamma^2}{4}}t)\right) \tag{3}$$

4. With No Oscillation (ω=0): The equation reduces to a damped logistic growth model:

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2x=rx\left(1-\frac{x}{K}\right) \tag{4}$$

Transformation to a First-Order System

To facilitate numerical solution, the second-order ODE is transformed into a system of first-order ODEs. Let $v=\frac{dx}{dt}$, then, the system in (1) becomes:

$$v=\frac{dx}{dt} \tag{5}$$

$$\frac{dv}{dt}=rx\left(1-\frac{x}{K}\right)-\omega^2x-\gamma v \tag{6}$$

This system (2) and (3) is more amenable to numerical methods such as the Runge-Kutta method.

Numerical Solution Using Runge-Kutta Method

The system of first-order ODEs is solved numerically using the fourth-order Runge-Kutta (RK4) method. The RK4 method is chosen for its accuracy and stability in handling nonlinear systems. The method involves the following steps at each time step t_n:

1. **Compute intermediate slopes**:

$$k_1=f(t_n,y_n), \tag{7}$$

$$k_2=f(t_n+\frac{h}{2},y_n+\frac{h}{2}k_1), \tag{8}$$

$$k_3=f(t_n+\frac{h}{2},y_n+\frac{h}{2}k_2), \tag{9}$$

where $y_n=[x_n,\overline{v_n}]^T$ represents the system of ODEs.

We update the solution with:

$$y_{n+1}=y_n+\frac{h}{6}(k_1+2k_2+2k_3+k_4), \tag{10}$$

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where h is the step size.

Parameter Values and Initial Conditions

The following parameter values and initial conditions are chosen for the baseline scenario:

$\omega = 1.0$, γ = 0.1, r = 0.5, K =10.0, $x(0)$=1.0, v(0) = 0.0.

These values are chosen to reflect a system with moderate oscillation frequency, weak damping, and logistic growth toward a carrying capacity of K=10.0.

Solutions and Analysis

Before presenting numerical results, we analyze the equilibrium points and their stability. Setting $\frac{dx}{dt} = 0$ and $\frac{dv}{dt} = 0$, the equilibrium points are:

Trivial Equilibrium: x=0, v=0. Non-Trivial Equilibrium: x=K, v=0, with respect to (1) and (2), to investigate and check stability, we linearize the system around each equilibrium point and calculate the eigenvalues of the Jacobian matrix.

Trivial Equilibrium Stability

The Jacobian matrix at (x,v)=(0,0) is:

$$J(0,0) = \begin{bmatrix} 0 & 1 \\ -\omega^2 + r & -\gamma \end{bmatrix} \tag{11}$$

The eigenvalues λ are given by the characteristic equation:

$$\lambda^2 + \gamma\lambda + (\omega^2 - r) = 0. \tag{12}$$

For $\omega^2 > r$, the eigenvalues have negative real parts, indicating stability. However, for $\omega^2 < r$, the trivial equilibrium becomes unstable.

Non-Trivial Equilibrium Stability

The Jacobian matrix at (x,v)=(K,0) is:

$$J(K,0) = \begin{bmatrix} 0 & 1 \\ -\omega^2 - r & -\gamma \end{bmatrix} \tag{13}$$

The characteristic equation is:

$$\lambda^2 + \gamma\lambda + \left(\omega^2 + r\right) = 0 \tag{14}$$

Since $\omega^2 + r > 0$ *and* $\gamma > 0$, the eigenvalues have negative real parts, indicating that the non-trivial equilibrium is stable.

The Python codes used in the implementation of the Damped Oscillatory Logistic Growth (DOLG) Model during the course of the research are made available in the appendix section.

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Numerical Results

The numerical solutions are pictured using time series plots and phase portraits. Below, we discuss the results for each scenario.

Table 1. Relationship between Time Series and Phase Portrait in Different Scenarios

Scenario	**Time Series**	**Phase Portrait**
Scenario 1: Baseline Parameters	The solution $x(t)$ exhibits damped oscillations, converging to the carrying capacity K=10.0.	The trajectory spirals toward the equilibrium point $(x,v)=(K,0)$, reflecting the system's stability.
Scenario 2: High Damping (γ=0.5)	Oscillations decay more rapidly, with the system reaching equilibrium faster.	The spiral is tighter, indicating stronger damping.
Scenario 3: High Growth Rate (r=1.0)	The system grows faster, with more pronounced oscillations before stabilization.	The oscillations are wider, reflecting the increased influence of the logistic growth term.
Scenario 4: High Frequency (ω=2.0)	Oscillations occur at a higher frequency, with smaller amplitude.	The spiral is more compact, reflecting the increased frequency.
Scenario 5: Low Carrying Capacity (K=5.0)	The system stabilizes at $x=5.0$, with oscillations converging to the new equilibrium.	The trajectory spirals toward (x,v)=(5.0,0).

From Table 1, we observe the following:

i. the system exhibits classic damped oscillations, stabilizing at the carrying capacity K = 10.0,

ii. increased damping (γ=0.5) leads to faster energy dissipation and quicker convergence to equilibrium,

iii. a higher growth rate (r=1.0) results in faster growth and more pronounced oscillations before stabilization,

iv. increased oscillation frequency (ω=2.0) leads to tighter oscillations in the phase portrait.

iv. reducing the carrying capacity (K=5.0) causes the system to stabilize at a lower equilibrium value.

Discussion of Numerical Results

The numerical results align with the analytical insights, demonstrating that the DOLG Model captures a wide variety of dynamical behaviors. The system's stability is established by the convergence of trajectories to the non-trivial equilibrium (x,v) = $(K,0)$ in all scenarios. The effects of damping, growth rate, and oscillation frequency are clearly observable in the time series and phase portraits.

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The plots illustrate the dynamics of the DOLG model under varying system parameters, particularly focusing on damping coefficient (γ), eigenvalues, and stability.

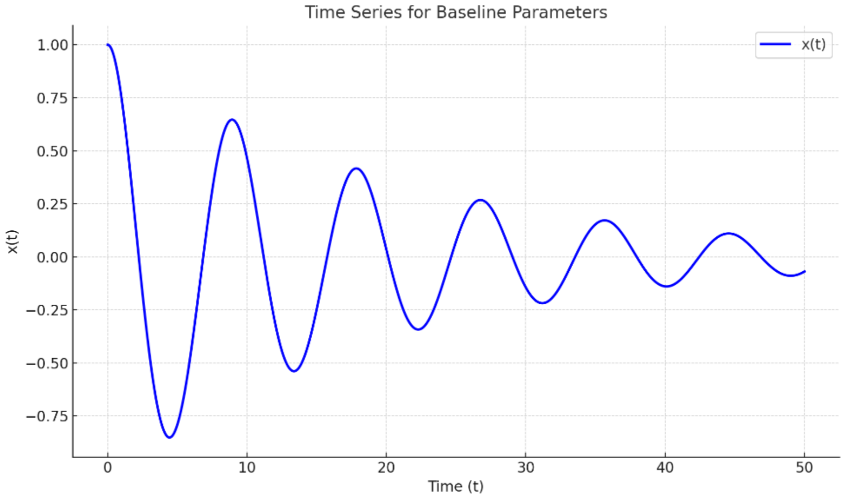

Fig 1. DOLG system, showing the time series of *x(t)*.

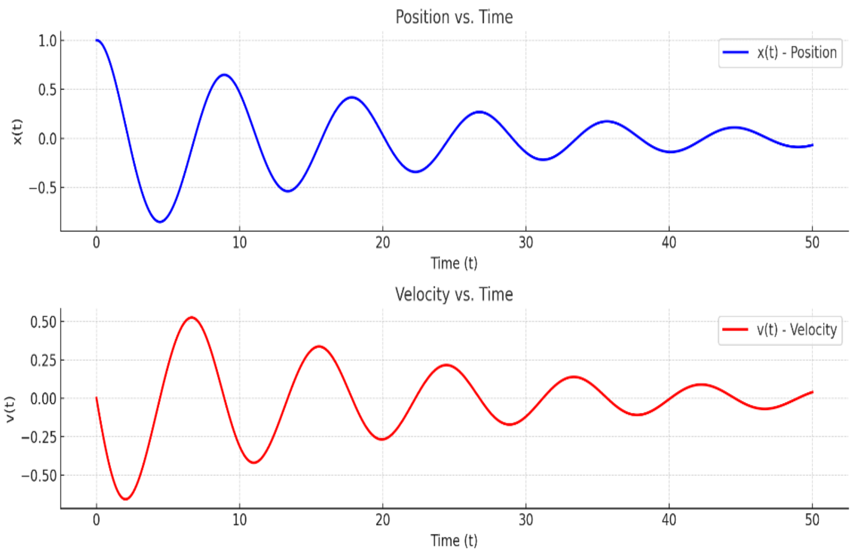

Fig 2. Time Series Plotsshowing *x(t)* (position) over time and the second plot showing *v(t)* (velocity) over time.

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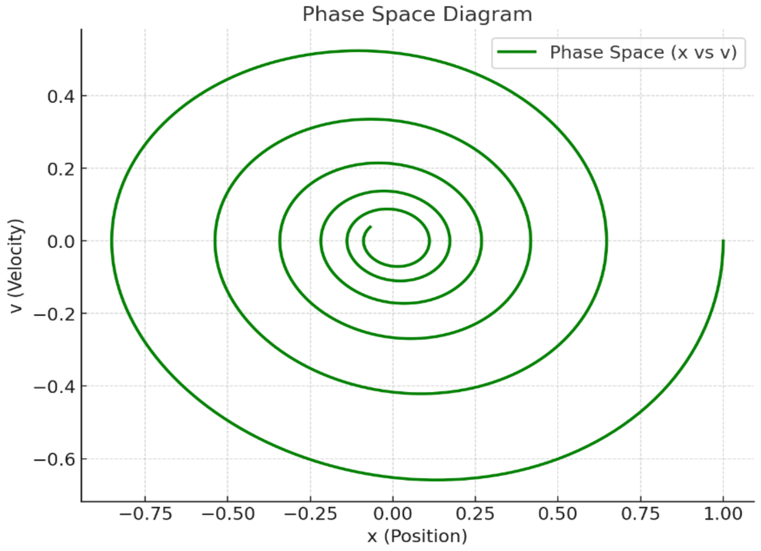
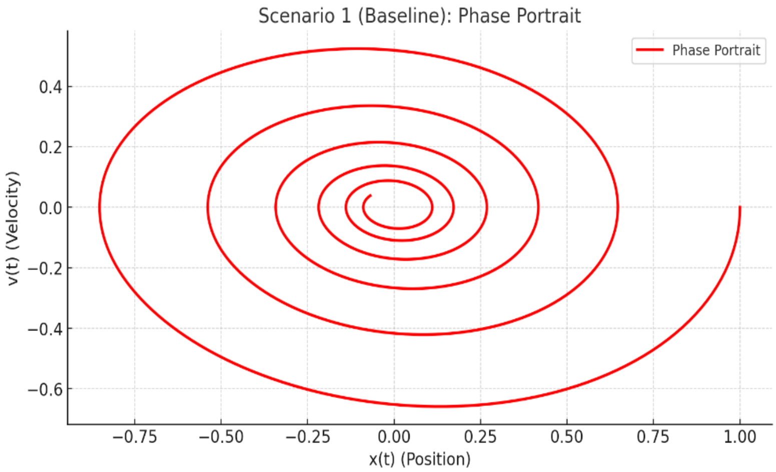


Fig 3. Phase Space Diagram plot representing the trajectory of the system in phase space, plotting velocity *v* against position *x*.



` **Fig 4. Plot for Scenario 1 Baseline for Phase Portrait**

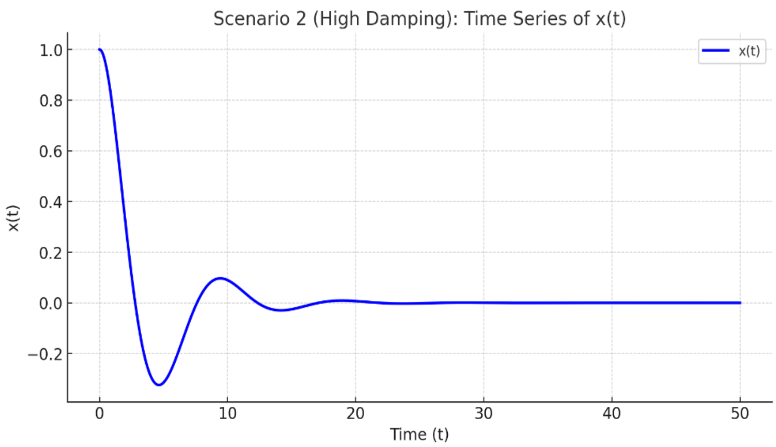


Fig 5. Plot for Scenario 2 High Damping for Phase Portrait of x(t)

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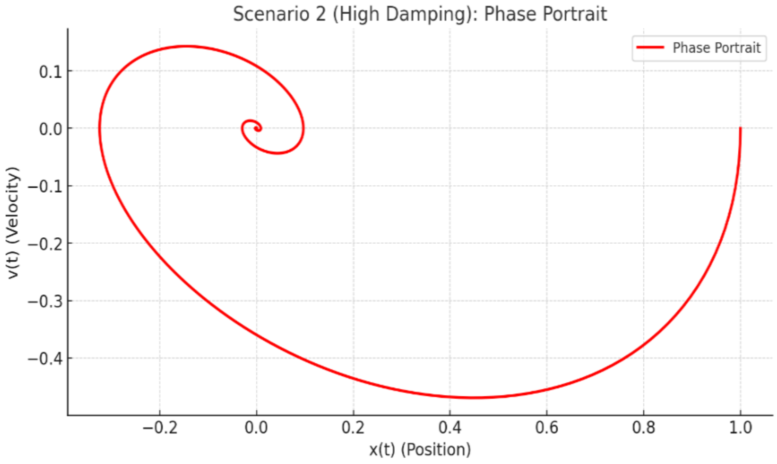

Fig 6. Plot for Scenario 2 High Damping for Phase Portrait of x(t)

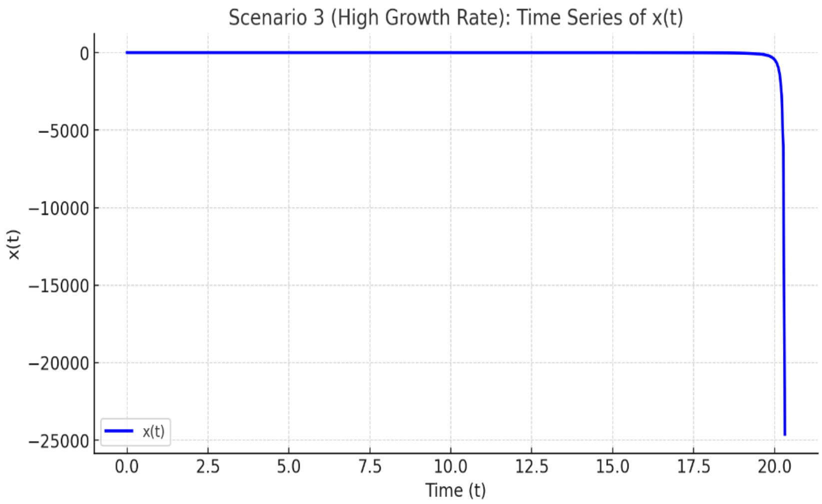

Fig 7. Plot for Scenario 3 High Growth Rate for Time Series of *x(t)*

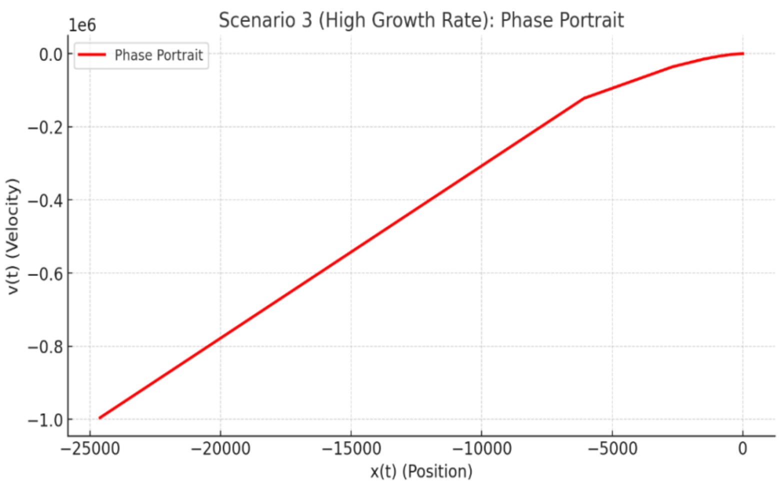

Fig 8. Plot for Scenario 3 High Growth Rate for Phase Portrait of *x(t)*

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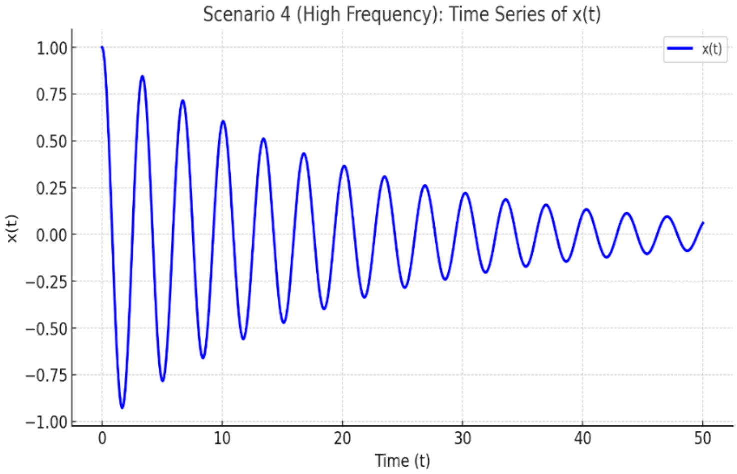

Fig 9. Plot for Scenario 4 High Frequency for Time Series of *x(t)*

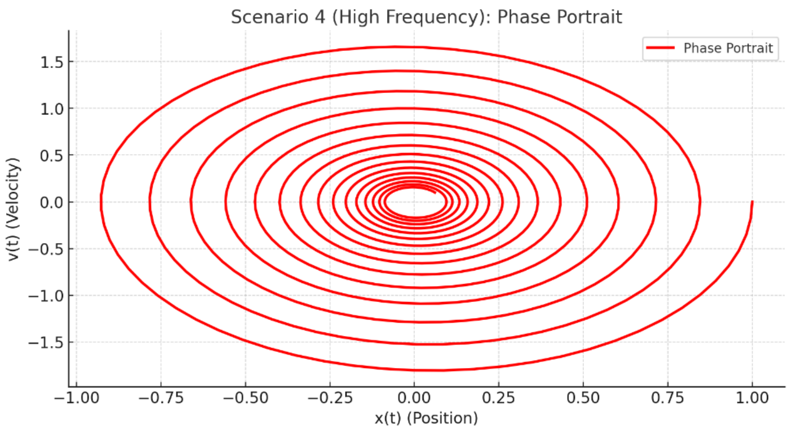

Fig 10. Plot for Scenario 4 High Frequency for Phase Portrait of *x(t)*

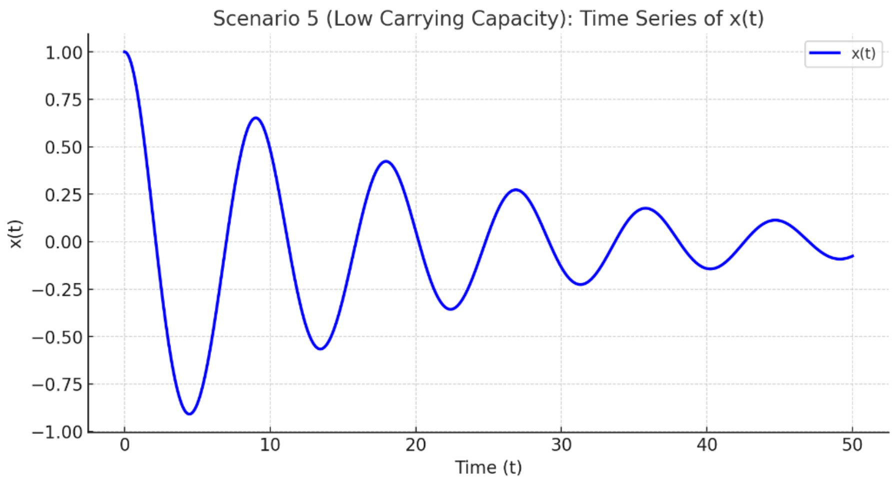

Fig 11. Plot for Scenario 5 Low Carrying Capacity for Time Series of *x(t)*

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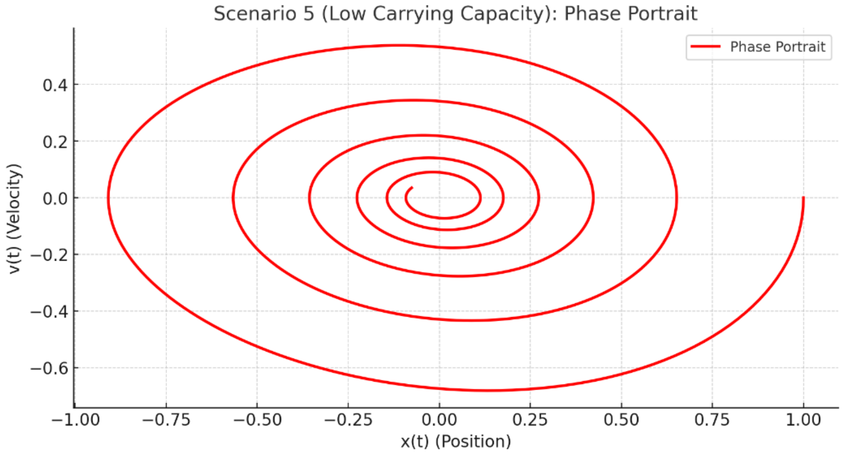

Fig 12. Plot for Scenario 5 Low Carrying Capacity for Phase Portrait of *x(t)*

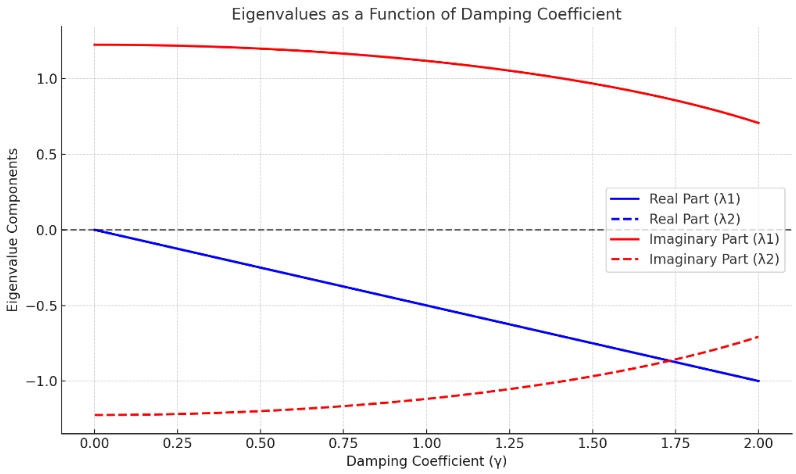

Fig 13. Eigenvalues as a function of damping coefficient of *x(t)*

In **Fig** 13, the plot shows how the real and imaginary parts of the eigenvalues vary with the damping coefficient (γ).Blue lines indicate Real parts of the eigenvalues (λ_1, λ_2), indicating stability. Red lines indicate Imaginary parts of the eigenvalues, representing oscillatory behavior, dashed lines represent Second eigenvalue components while Black dashed line show zero threshold for stability.

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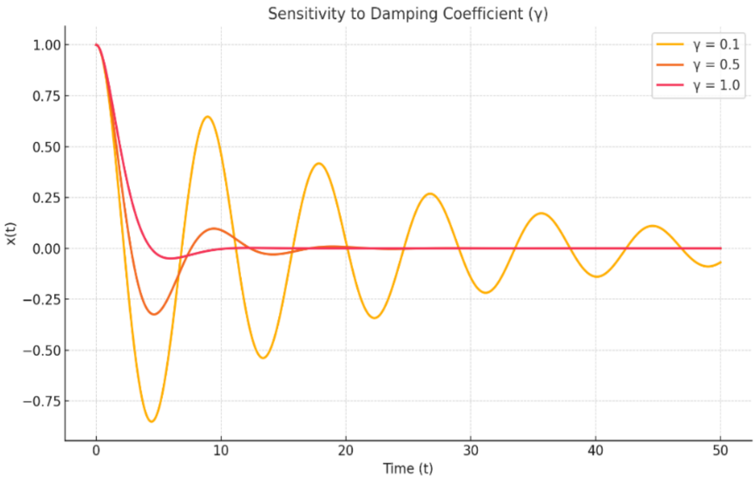

Fig 14. Plot Showing How *x(t)* Behaves For Different Damping Coefficient (*γ*) Values

In **Fig** 14, Lower damping ($\gamma=0.1$) results in sustained oscillations, higher damping (γ=1.0) leads to earlier decay and stabilization and the system's response smooths out as damping increases.This shows how damping influences oscillatory behavior and system stability

Fig 3-4 results highlight the comparison between oscillatory behavior and stability. Low damping allows oscillations to persist, while high damping suppresses fluctuations and promotes stability. Understanding these dynamics is crucial in designing systems where stability and response time are key factors, such as in mechanical dampers, population dynamics, and control systems.

This study sightsees the dynamic behavior of the DOLG model under varying damping coefficients (γ), with a focus on system stability, oscillatory behavior, and eigenvalue transitions. Through time-series analysis, phase portraits, and eigenvalue evolution, we establish how damping influences the system's trajectory. Results specify that lower damping coefficients allow for sustained oscillations, while increasing γ progressively suppresses fluctuations, leading to a faster return to equilibrium. The eigenvalue analysis further reveals the transition from underdamped to overdamped regimes, marking a critical shift in system stability. These findings underscore the fundamental trade-off between oscillatory persistence and stabilization, offering valuable comprehensions for applications in mechanical systems, biological populations, and control theory. Understanding these dynamics provides a framework for optimizing stability in real-world systems where damping plays a central role in performance and flexibility.

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DISCUSSION OF RESULTS

Interpretation of Results

The results of the DOLG Model establish its ability to capture complex dynamical behaviors, including damped oscillations, stabilization to carrying capacity, and phase-dependent growth patterns. The system's behavior is governed by the interplay of three key components:

i. the term $\omega^2 x$ introduces oscillations with a natural frequency ω, which are observed in the time series and phase portraits.

ii. the term $\gamma \frac{dx}{dt}$ dissipates energy, leading to the decay of oscillations over time. This is evident in the convergence of trajectories to the equilibrium point $(x, v) = (K, 0)$.

iii. the term $rx\left(1-\frac{x}{K}\right)$ introduces nonlinear growth with a carrying capacity K, which determines the long-term behavior of the system.

The stability analysis confirms that the non-trivial equilibrium $(x, v) = (K, 0)$ is stable for all parameter regimes, while the trivial equilibrium $(x,v) = (0,0)$ is unstable when $\omega^2 < r$. This aligns with the numerical results, which show convergence to $x = K$ in all scenarios.

Comparison with Existing Models

The DOLG Model extends classical models such as the harmonic oscillator, logistic growth equation, and damped systems by integrating their key features into a unified framework [3, 25, and 10]. Unlike existing models, which often treat oscillations, damping, and growth separately, the DOLG Model provides a more comprehensive representation of systems where these phenomena coexist [12,24]. For example:

In ecology, the model can describe oscillating populations with resource constraints, where oscillations arise from predator-prey interactions and growth is limited by carrying capacity.In economics, the model can capture cyclical growth with market saturation, where oscillations result from economic cycles and growth is bounded by market size [24, 26].

Applications

The DOLG Model has potential applications in various fields:

Modeling oscillating populations with carrying capacity constraints. Analyzing cyclical growth with market saturation.Studying oscillating structures with material fatigue.

Limitations and Future Work

While the DOLG Model provides valuable insights, it has several limitations:

The parameters ω, γ, r, and K are assumed to be constants. Future work could explore time-varying parameters to model more realistic systems.The model does not account for external forces or stochastic effects. Incorporating these factors could enhance its applicability to real-world systems.

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The model assumes a homogeneous system with no spatial variation. Extending the model to include spatial dynamics could open new avenues for research.

CONCLUSION

This study introduced the Damped Oscillatory Logistic Growth (DOLG) Model, a novel hybrid framework that combines oscillatory dynamics, damping, and logistic growth into a single differential equation. The model was solved numerically using the Runge-Kutta method, and its behavior was analyzed under various parameter regimes. Key findings include:

i. the system exhibits damped oscillations that converge to the carrying capacity K.

ii. the non-trivial equilibrium $(x,v)=(K,0)$ is stable for all parameter regimes.

iii. the effects of damping, growth rate, and oscillation frequency are clearly visible in the time series and phase portraits.

CONTRIBUTIONS

The DOLG Model makes several key contributions:

By combining logistic growth, damping, and oscillatory behavior into a cohesive framework, this study integrates important dynamical concepts and advances our knowledge of complex systems. It highlights the value of computational tools in contemporary research by highlighting the function of numerical techniques in the analysis of nonlinear dynamics. The paper also demonstrates the model's versatility and wide applicability by highlighting interdisciplinary applications in disciplines including ecology, economics and engineering.

Conflicts of Interest

The authors declare that there is no potential conflicts of interest.

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