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MULTIVARIATE VOLATILITY MODELLING OF STOCK PRICES FOR SOME SELECTED NIGERIAN SOLID MINERALS

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ABSTRACT: *The need to provide an acceptable model and forecast for stock prices of solid minerals in Nigeria is valuable for investors and analysts. It will empower them to better understand and manage the associated risks in stock price movements. This study aimed to model and forecast the volatility of stock prices of solid minerals, like gold, tin, and zinc. The data utilized in this study was sourced from the Central Bank of Nigeria and Nigeria Stock Exchange. It is the monthly stock prices for selected solid minerals like; Gold, Tin, and Zinc. Multivariate GARCH models such as the VECH, BEKK, Diagonal VECH and Diagonal BEKK model were employed to provide the needed multivariate volatility modeling. The findings revealed that, on average, investors experienced positive returns, and a non-symmetric distribution. It was also discovered that intricate patterns exist within the volatility dynamics of these stocks. Volatility clustering, ARCH effects, and the persistence of volatility shocks over time was identified, emphasizing the non-random nature of stock returns volatility. It is recommended that investors and analysts carefully consider the implications of volatility clustering, ARCH effects, and persistence in volatility shocks when making investment decisions in the stock market, particularly regarding gold, tin, and zinc stocks.*

KEYWORDS: ARCH, Autoregression, Bekk-Garch, GARCH, Heteroskedastic, IGARCH, Skewness, Stationarity, VECH.

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INTRODUCTION

Nigeria's abundant supply of solid minerals, such as gold, tin, and zinc, has long served as the foundation of the country's economy. These minerals are essential to the country's economy, making a substantial contribution to both its GDP and foreign exchange profits. Nigeria's economic development has been greatly aided by the exploration, mining, and trading of these minerals, which have created jobs and promoted industrial expansion. Like any market-driven industry, the solid minerals sector is impacted by a number of domestic and international economic factors that affect commodity prices. For investors, legislators, and market players looking to comprehend and control the risks connected to their investments in this industry, the volatility of commodity prices presents difficulties. Since the stock prices of businesses engaged in the mining and processing of solid minerals are especially vulnerable to fluctuations in commodity prices, it is essential to precisely model and predict the volatility of these stock prices. The multivariate volatility modeling of stock prices for a few Nigerian solid minerals, like gold, tin, and zinc is the main emphasis of this study. The chosen minerals are important parts of the nation's mineral riches, and a wide range of factors, including macroeconomic indicators, geopolitical developments, global demand, and technical improvements, affect their prices. This study attempts to shed light on the dynamic nature of stock price movements within the solid minerals industry by using advanced statistical approaches for multivariate volatility modeling. There are inherent risks in Nigeria's solid minerals business, and the stock prices of companies in this sector show complicated patterns of volatility. For investors and politicians looking to make well-informed decisions, it is essential to comprehend and measure this volatility. Nevertheless, little study has been done on stock price volatility in Nigeria's solid minerals industry, and there aren't many thorough multivariate models that take into account the dynamics of several minerals at once. Therefore, the lack of a reliable multivariate volatility model for stock prices in the Nigerian solid minerals sector specifically for gold, tin, and zinc is addressed in this study. Creating such a model is essential to enhancing risk management tactics, supporting investment choices, and fostering a more robust and stable Nigerian solid minerals market.

In the univariate example, the generalized ARCH (GARCH) and autoregressive conditional heteroscedasticity (ARCH) models were effective in capturing the time-varying variances of economic data. Many academics have been inspired by this to expand these models to the multivariate level. There are numerous significant financial uses for multivariate volatilities. They can be used to calculate the value at risk of a financial position made up of several assets, and they are crucial in asset allocation and portfolio selection (Tsay, 2005). Thus, multivariate GARCH (MGARCH) models have a wide range of applications. Portfolio optimization (Kroner, 1991), asset pricing (Herwartz, 1998) and derivatives, value at Risk computation (Laurent, 2004), futures hedging Park (995), volatility transmitting (Karolyi, 1995), asset allocation, systemic risk estimation (Schröder, 2003), estimation of the leverage effect (De Goeij, 2004; Kroner, 1998), estimation of the volatility impulse response function (Hafner, 2006; Elder, 2003; nonlinear programming Leyffer, 2003), hedging the currency exposure risk (Valiani, 2004); calculating the minimum capital risk requirements for an asset portfolio (Brooks et al., 2002); determining misspecification tests for MGARCH models (Tse & Tsui, 1999); modeling of the changing variance structure in an exchange rate regime (Bollerslev, 1990); and using MGARCH models in the analysis of individual financial markets (Minović, 2007b).

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This paper aims to present the fundamental idea of multivariate volatility (GARCH) modeling, it also provides an overview of these models' theory.

LITERATURE REVIEW

Over the past 20 years, there has been a significant evolution in the modeling and explanation of the dynamics of time-series from the financial industry since Eagle's (1982) seminar work on Autoregressive Conditional Heteroscedasticity (ARCH) models. In addition to the statistical advantages of accounting for conditional heteroscedasticity and second order temporal persistence in asset return series, it is very practical to model the conditional correlation across assets and across sectors over time. A substantial body of literature on MGARCH models has developed over the past 20 years, with variations in the conditional variance-covariance matrix specifications (Silbennoinen *et al*., 2008) and conditional volatility specifications (of which a substantial body of literature has evolved (Bollerslev *et al*., 1988). Bollerslev *et al*. (1988) presented the VECH model, the first MGARCH model that explicitly measures the conditional covariance matrix between series. As the returns dimension increases, a significant number of parameters must be evaluated because the VECH approach is essentially a direct generalization of the univariate approach. The Constant Conditional Correlation (CCC) model and its later variations, as well as limited versions of Engle & Kroner's (1995) BEKK-model (Engle et al., 1995), which also explicitly guarantees positive definiteness of the covariance matrix, were the results of later attempts to make the models more parsimonious. In order to include leverage effects in the underlying correlation structure, Cappiello et al. (2006) extended the constancy of the correlation structure of the CCC model to the Asymmetric-DCC (ADCC) model, whereas Engle (2002) subsequently relaxed it with the Dynamic Conditional Correlation (DCC) version. These models have been empirically employed by numerous academics to examine return volatility, with one model outperforming the other.

Bala and Takimoto (2017) investigated stock return volatility spillovers in both developed and emerging economies using the Multivariate GARCH model and its variations. They examined how the global financial crisis of 2007–2009 affected stock market dynamics and adjusted BEKK-GARCH-type models by adding financial crisis dummies to ascertain how they affected volatility and spillovers. They discovered that, in comparison to other models, the DCC-with-skewed-t density model has a better diagnosis. This is due to the fact that financial returns frequently exhibit skewed features and fat tails. King and Botha (2014) investigated whether using Markov-switching models to account for structural changes in the conditional variance process improves estimates and forecasts of stock return volatility compared to the more traditional single-state (G)ARCH models, both within and across a subset of African markets for the years 2002–2012. They found that although some Markov-switching models have been shown to outperform models that incorporate GARCH effects in terms of forecasting and risk-adjusted returns, the simpler Markov-switching models' incapacity to adequately account for heteroscedasticity in the data continues to be an issue.

Both the study by Türkyılmaz and Balıbey (2014) and the study by Ijumba (2013) on the Multivariate GARCH models indicated that there was a persistence of volatility among the returns of the BRICS stock market. Chen and Zapata (2015) used BEKK-GARCH models to simulate volatility and spillover effects. The findings showed that the volatility in China's price hogs is explained by own-price volatility and prior unforeseen occurrences, whereas the

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volatility in America is explained by its events. Before the consequences of the global financial crisis reached Turkey, the BEKK-GARCH model was used in the work by Türkyılmaz and Balıbey (2014) to create the conditional variances of monthly stock exchange prices, exchange rates, and interest rates for Turkey. The sample period was 2002:M1 to 2009:M1. The empirical findings show that these three financial sectors are volatile, which is a sign of strong shock transmission. Sheu and Cheng (2011) examined and compared the effects of volatility for the US and Chinese stock markets on Hong Kong and Taiwan, respectively, using the VAR and the Multivariate GARCH model for two distinct time periods, 1996 to 2005 and 2006 to 2009. It is discovered that the Chinese stock market is autonomous and that its interactions with foreign markets are still negligible.

Bonga-Bonga and Nleya (2016) examined the performance of the asymmetric DCC (ADCC), dynamic conditional correlation (DCC), and constant conditional correlation (CCC) models in estimating the portfolio at risk in the BRICS nations. The study used the root mean square error, average deviations, and quadratic probability function score to measure performance error. The findings demonstrated that a portfolio can help reduce BRICS losses. Nortey et al. (2015) used a dataset of Ghana from January 1990 to December 2013 to examine the volatility and conditional link among inflation rate, currency rate, and interest rates as well as to build a model of DCC and BEKK. The study's findings demonstrate that the DCC model is strong at modeling the unconditional correlation and conditional of the exchange, interest rates, and inflation, respectively, but the BEKK model is strong at modeling and forecasting the volatility of the exchange rate, inflation rate, and interest rate. Gardebroek et al. (2013) used a Multivariate GARCH technique on a daily, weekly, and monthly basis encompassing 1998 to 2012 in order to evaluate the interdependence and dynamics of volatility in the corn, wheat, and soybean markets in the United States. The findings showed that there is weekly volatility across these commodities and that there is no cross-border reliance amongst markets. Hartman and Sedlak (2013) investigated the performance of the two Multivariate GARCH models, BEKK and DCC, using ten-year exchange rate data. The OLS regression, MAE, and RMSE are used to gauge performance. It was determined from the data that the BEKK model outperformed the DCC model. In order to evaluate the relationship between exchange rates and stock market returns, Tastan (2006) used the Multivariate GARCH model. Daily data on the Euro-Dollar exchange rate, the Dow Jones Industrial Average, and the S$P 500 index from the US economy were the series used. The study discovered that this shock determines the conditional volatility of every variable. Chen and Zapata (2015) used data from the June 1996–December 2013 sample to model volatility and spillover effects using the BEKK-GARCH model. The findings show that whereas American volatility is explained by its events, Chinese price hog volatility is documented to be explained by own-price volatility and past unforeseen events. In order to investigate and compare the impacts of two financial crises (the Asian Financial Crisis of 1997 and the Subprime Financial Crisis of 2007 to 2010), Chen and Zapata (2015) looked at the short-, short-, and long-term relationships between the equity markets in China and the US. Additionally, the volatility spill-over effects are examined by the estimation of the BEKK-GARCH model. The findings indicate that there is no cointegration between the mainland stock indexes and the US and Hong Kong stock indices. Nonetheless, there is short-term volatility and spillover effects in the various equity markets.

The multivariate GARCH model was added to the fractionally integrated VECM model by Yi et al. (2009) in order to concurrently disclose the return transmission and volatility spillover between market return series. The empirical findings demonstrated that the Chinese market is

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more closely linked to the Hong Kong market than the U.S. market, and that there is a fractional integration. In addition to studying the empirical application of estimating applications in the theoretical framework of GARCH models, Baybogan (2013) assessed the volatility in financial time series econometrics. The DCC-GARCH and BEKK-GARCH models are both examined. In order to estimate the returns of the expanding pension funds, Kvasnakova (2009) used both the copula and the multivariate GARCH model. The two models were once more used in the effort to estimate the VaR and compare them. The findings demonstrated that the copula model yields superior VaR estimation. Itotenaan et al. (2013) looked at the Nigerian market to see if there was a connection between stock market performance and oil prices. The enhanced Dickey-Fuller test, Johansen's cointegration model, the Vector autoregression estimation model, and the Vector error correction model were among the empirical tests used in the research work. The results showed that changes in the price of oil play a key role in explaining movements in share prices. The results also indicate that there is a strong correlation between oil prices and stock performance. Irandoost et al. (2013) conducted a study that examined dividend policy and its influence on stock volatility and investing choices for companies listed on the Tehran Stock Exchange. Multiple regressions and correlation analysis were used to examine and assess the hypotheses in this study, which included a sample of 65 firms and five years of data from 2007 to 2012. The study's findings demonstrated that dividend policies have a major short-term influence on price volatility. Dividend policy, however, has never had an impact on volatility or cash and cash accrual investment choices.

Kolawole and Olalekan (2010) looked into how the Nigerian market was affected by fluctuations in exchange rates. The study's findings demonstrated that the exchange rate volatility brought on by the GARCH process had a more detrimental effect on the Nigerian market. However, the results showed no long-term relationship between market capitalization and inflation or interest rates. This resulted from the government's substantial involvement in the market. To lessen exchange rate volatility and increase the stock market's reach, they recommended the formation of a coordinated scale and monetary strategy. Adrian (2008) examined the pricing of volatility risk by separating the equities market's inherent instability into short- and long-term components. According to the study, risk prices are adversely significant for both volatility sections, meaning that investors pay insurance premiums to lessen the likelihood of instability. The study also discovered that the short run component, which was interpreted as a tightness constraint measure, accounts for the risk of market skewness. The long-term section included the business cycle risk. According to Asteriou and Hall (2011), recent advancements in financial econometrics have prompted the use of methods, models, and processes that can assist investors in navigating risk (uncertainty) and expected return. While low volatility results in lower risk, higher volatility may yield a larger predicted return than others. The ARCH/GARCH family of models' illustrations are necessary for addressing the volatility (variance) of the series. Demers and Vega (2008) measured stock market volatility using the EGARCH model and discovered that negative shocks, or bad news, had a greater impact on volatility than positive shocks, or good news. According to Krishnamurti's (2000) paper, "Competition, Liquidity, and Volatility: A Comparative Study of the Bombay Stock Exchange and National Stock Exchange," India has two major stock exchanges: the BSE and the NSE. He noted that the two exchanges differ significantly in terms of ownership structure, internal control systems, geographic reach, and institutionalized risk management facilities. When Krishnamurti (2000) was investigating whether the notable structural differences between these stock exchanges contribute to variations in observed measures of market quality, he employed a paired comparison approach and documented significant differences in price

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volatility and liquidity between the two markets. He discovered that the NSE is higher in his department on numerous counts. Additionally, NSE's surveillance systems have a superior reputation. The BSE has a part-order-driven system, whereas the NSE has a fully order-driven system. Both exchanges have tools for price stabilization. The cement industry's profitability was compared by Bavaria (2004). He asserts that the rise in the profit percentage is proportionate to the rise in average interval profit. Thus, there is a direct correlation between Net Profit and Interval Measure in this case as well. The study concludes that the Eastern Region had the highest return on net capital employed, whereas the other regions showed a negative outcome.

MATERIALS AND METHODS

Source of Data

The data for this study is sourced from the Central Bank of Nigeria and Nigeria Stock Exchange. They provide monthly stock prices for selected solid minerals like gold, tin, and zinc. The dataset includes relevant information on stock prices, trading volumes, and other relevant financial indicators. Special attention is given to ensuring data consistency, completeness, and accuracy.

Multivariate Volatility Model Specification

To describe the volatility of multiple asset returns, numerous researches have extended the univariate generalized autoregressive conditional heteroscedastic (GARCH) model to the multivariate case in recent years. There are numerous significant uses for multivariate volatility models in statistics and finance. The GARCH model was established by Bollerslev and Taylor who extended the ARCH model by allowing the past conditional variance to be a linear function of p lagged conditional variances in addition to q past squared errors. Asteriou (2006) stated that the simplest form of the GARCH (p,q) model is the GARCH (1,1) model, which changes p = 0 and reduces the model to ARCH (q). The variance equation has the form:

$$Ln(R)_t = a + \beta' Ln(R)_{t-1} + u_t \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

Where, σ_t^2 is the conditional variance, q is the order of the ARCH terms μ^2, p is the order of the GARCH terms σ^2, u_t is the disturbance term, ω is the constant term and $\beta_0 > 0$; $\beta_i \geq 0$; i = 1,…, q; j = 1,...,p

The ARCH term is the lag of the squared residual from the mean equation. It will tell if volatility reacts to market movements i.e. if volatility from the previous period affects volatility in the current period. The GARCH parameter is the forecasted variance from the previous period. The sum of the ARCH and GARCH term will inform us if volatility shocks are persistent. If the sum is less than unity, the shocks would die out slowly, if not, it would die out quickly.

The GARCH model is easy to estimate and specifically performs very well because it has only three unknown parameters, ω, α and β. It simply illustrates that the conditional variance is allowed to depend on both q lags of the squared errors (residuals) and p lags of the conditional

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variance. The above equation is a GARCH (1,1) model, , $\sigma_i^2$is the conditional variance since is a one-period ahead estimate for the variance calculated based on past information thought relevant. The GARCH (1,1) model has the presence of (the first term in parentheses) a first-order autoregressive GARCH term and (the second term in parentheses) a first-order moving average ARCH term. The GARCH model by forming a weighted function of the long-term average (the constant dependent on ω) is often interpreted in a financial context, where an agent or trader predicts this period's variance $\alpha^2{}_i$ the fitted variance from the GARCH term($\beta\sigma_{i-1}^2$), that is, the fitted variance from the model during the previous periods and information about volatility observed during the previous periods, in the ARCH term ($\alpha\varepsilon_{i-1}^2$). If the asset return was unexpected by large in either the upward or the downward direction, then the estimate of the variance for the next period is then increased by the trader. This model is also consistent with the volatility clustering often seen in financial return data, and large returns changes are likely to be followed by further large changes. In addition, the intention of higher-order GARCH models, denoted GARCH (p, q), can be estimated by choosing either p or q larger than 1, where p is the order of the moving average GARCH terms and q is the order of the autoregressive ARCH terms. The GARCH (p, q) model can be represented as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3}$$

Where: ω, α and β are parameters, the lag of the squared residual from the mean equation of the ARCH term is ε_{t-i}^2 The ARCH parameters correspondent is α_i, the lag of the squared residual from the mean equation of the GARCH term $\alpha^2{}_{i-j}$, the GARCH parameters correspondent is β_i. For a GARCH (1,1) process, the variance can be calculated as, $\frac{\omega}{1-\alpha-\beta}$, which requires $\alpha + \beta < 1$. Otherwise, if $\alpha + \beta > 1$, the study requires an Integrated Generalized Autoregressive Conditional Heteroskedastic (IGARCH) model (Hill, *et al.* 2011). A study by (Hamilton,1994) notes that the conditional of $\omega, \alpha\ and\ \beta > 0$ are sufficient but not necessary to ensure nonnegative of σ^2. For a GARCH (1,2) process, for example, the σ^2coefficients are positive, provided that $\omega > 0,\ \beta > 0$
and $\alpha_0 + \alpha_1 > 0$. Hamilton concludes that α_1 could be negative as long as α_1 is less than α_0. The following is a description of the variance calculation.

$$\sigma_i^2 = \omega_i - \beta - \alpha \tag{4}$$

In the above equation, the mean is written as a function of exogenous variables with an error term. Since σ_i^2 is the one-period ahead forecast variance based on past information, it is called the conditional variance. This variance can be calculated to describe uncertainty by the square root of the variance and is called the standard deviation (Adams *et al.*, 2022). This model specification performs very well usually and provides a framework that is more flexible to capture various dynamic structures of conditional variance. This is because the GARCH model incorporates the time-varying conditional variance and the covariance process. According to Knight and Satchell the GARCH model allows the distributions of both the conditional variance and the observed variable (unconditionally) to be computed numerically. The conditional variance of the time series will consequently rely on the squared residuals of the process, or the lagged innovation squared (Adams *et al.*, 2023).

The ARCH model created by Engle has limits, as stated by Pagan and Schwert. Their research verified that the GARCH model outperformed other approaches in terms of stock market

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volatility. The GARCH and ARCH models are widely employed in many areas of econometrics, particularly in financial time series analysis, and they are adaptable enough to take into account changes in model specifications and requirements, according to surveys conducted by Bollerslev, Engle, and Nelson. According to an empirical study, GARCH (1,1) is the most widely used model for analyzing financial time series and is more economical than ARCH when it comes to predicting volatility in financial markets. In a similar vein, Hansen and Lunde's (2005) study investigated whether complex volatility models or parsimonious models are more effective at describing financial time series. In order to assess the one-day-ahead conditional variance, they matched 330 ARCH-type models. The main finding for the exchange rate data was that one of the best-performing models is the GARCH (1,1). Following the description above, the current study uses the GARCH (1, 1) model and multi-factor regression to investigate the volatility dynamics of the financial time series.

The core of this methodology lies in the specification and estimation of a multivariate volatility model to capture the joint dynamics of Gold, Tin, and Zinc stock prices. The model selected for this study is the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) model. The MGARCH model is particularly suited for capturing the time-varying volatility and potential spillover effects among multiple financial time series.

Multivariate time series, often the current value of a variable, depends not only on its past values but also on past and/or current values of other variables (Schmidt, 2005). Price movements in one market can spread easily and instantly to another market. Financial markets are more dependent on each other than ever before. Consequently, knowing how the markets are interrelated is of great importance in finance. For an investor or a financial institution holding multiple assets, the dynamic relationship between returns on the assets plays an important role in decision making (Tsay, 2005). Modeling of dynamic interdependent variables is done using multivariate time series. A multivariate time series $r_t = (r_{1t}, r_{2t}, ... r_{Nt})'$ is a vector of N processes that have data available for the same moments in time (Schmidt, 2005). Multivariate time series y_t is weakly stationary if its first and second moments are time-invariant. In particular, the mean vector and covariance matrix of a weakly stationary series is constant over time. For a weakly stationary time series, r_t, we define its mean vector and covariance matrix as

$$\mu = E\left(y_t\right) = \begin{bmatrix} E(y_{1t}) \\ \vdots \\ E(y_{Nt}) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix} = const \tag{5}$$

$$\Gamma_0(k) = E[(y_t - \mu)(y_{t-k} - \mu)'] = \begin{bmatrix} \gamma_0 & \gamma_{12}(k) & \cdots & \gamma_{1N}(k) \\ \gamma_{21}(k) & \gamma_1 & \cdots & \gamma_{2N}(k) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1}(k) & \gamma_{N2}(k) & \cdots & \gamma_N \end{bmatrix} \tag{6}$$

Where the expectation is taken element by element over the joint distribution of y_t. The mean μ is an N-dimensional vector consisting of unconditional expectations of the components of y_t. The covariance matrix Γ_0 is a $N \times N$ matrix. The i^{th} diagonal element of Γ_0 is the

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variance of y_{it}, whereas the element of Γ_0 is the covariance between y_{it} and y_{jt}, and it is a function of k (Tsay, 2005). A dynamic model with time-varying means, variances, and covariances for the N components of $y_t = (y_{1t}, y_{2t}, ..., y_{Nt})'$ is:

$$y_t = \mu_t(\theta) + \varepsilon_t \tag{7}$$

Here, θ is a finite vector of parameters, $\mu_t(\theta)$ is the conditional mean vector and ε_t is an $N \times 1$ vector of shock, or innovation, of the series at time t equal to:

$$\varepsilon_t = \Sigma_t^{\frac{1}{2}}(\theta) z_t, \tag{8}$$

where $\Sigma_t^{\frac{1}{2}}(\theta)$ is an $N \times N$ positive definite matrix. Furthermore, we assume the $N \times 1$ random vector z_t to have the following first two moments:

$$E(z_t) = 0,\ Var(z_t) = I_N \tag{9}$$

where I_N is the identity matrix of order N.

The conditional mean vector has the form:

$$\mu_t = E(y_t / I_{t-1}) = E_{t-1}(y_t) \tag{10}$$

Where I_{t-1} is the information available at a time $t-1$, at least containing $(y_{1t}, y_{2t}, ...)$. To make this clear we calculate the conditional variance matrix of y_t:

$$\begin{aligned} Var(y_t / I_{t-1}) &= Var_{t-1}(y_t) = Var_{t-1}(\varepsilon_t) \\ &= \Sigma_t^{\frac{1}{2}} Var_{t-1}(z_t)(\Sigma_t^{\frac{1}{2}}) \\ &= \Sigma_t \end{aligned} \tag{11}$$

Hence, $H_t^{\frac{1}{2}}$ is any $N \times N$ positive definite matrix such that Σ_t is the conditional variance matrix of y_t. Both H_t and μ_t depend on the unknown parameter vector θ, which can be split in most cases into two disjoint parts, one for μ_t and one for Σ_t (Bauwens, 2005; Bauwens et al., 2006). Multivariate volatility modeling is concerned with the time evolution of Σ_t. We refer to a model for the { H_t } process as a volatility model for the return series y_t (Tsay, 2005).

The VECH model:

The first MGARCH model was introduced by Bollerslev, Engle, and Wooldridge in 1988, which is called the VECH model. It is much more general compared to the subsequent formulations. In the VECH model, every conditional variance and covariance is a function of

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all lagged conditional variances and covariance, as well as lagged squared returns and cross-products of returns.

The model can be expressed below:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_1^* \\ c_2^* \\ c_3^* \end{bmatrix} + \begin{bmatrix} a_{11}^* & a_{12}^* & a_{13}^* \\ a_{21}^* & a_{22}^* & a_{23}^* \\ a_{31}^* & a_{32}^* & a_{33}^* \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11}^* & b_{12}^* & b_{13}^* \\ b_{21}^* & b_{22}^* & b_{23}^* \\ b_{31}^* & b_{32}^* & b_{33}^* \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \tag{12}$$

$$VEC(H_t) = C + \sum_{j=1}^{q} Avech(\varepsilon_{t-j}\varepsilon_{t-j}^{'}) + \sum_{j=1}^{p} Bvech(H_{t-j}) \tag{13}$$

where $vech(\cdot)$ is an operator that stacks the columns of the lower triangular part of its argument square matrix, H_t is the covariance matrix of the residuals, N represents the number of variables, t is the index of the t^{th} observation, C is an $N(N+1)/2\times 1$ vector, A_j and B_j are $N(N+1)/2\times N(N+1)/2A_jB_j$ parameter matrices and ε is an $N\times 1$ vector of the errors.

The condition for H_t to be positive definite for all t is not restrictive. In addition, the number of parameters equals $(p+q)\times(N(N+1)/2)^2+(N(N+1)/2$, which is large. Furthermore, It demands a large quantity of computation.

The goal is to model the conditional variance-covariance matrix H_t which is an $N\times N$ non-negative definite matrix. Different models for H_t have been proposed over the last two decades (Wang, Yao, 2005).

VECH is the operator that stacks a matrix as a column vector:

$$VEC(\textstyle\sum_t) = (\sigma_{11t}, \sigma_{21t}, \ldots, \sigma_{N1t},, \sigma_{12t}, \sigma_{22t}, \ldots, \sigma_{N2t})' \tag{14}$$

$VEC(ABC) = (C'\otimes A)VEC(B)$ (Bauwens, 2005) (15)

where A_i and B_j are parameter matrices containing $(N^*)^2$ parameters [with $N^*=[N(N+1)/2]$, whereas the vector C contains N^* coefficients. We will assume that all Eigenvalues of the matrix $\sum_{i=1}^{q} A_i + \sum_{j=1}^{p} B_j$ have modulus smaller than one, in which case the vector process e_t is covariance stationary with unconditional covariance matrix given by H_t (Hafner & Herwartz, 2006). A potentially serious issue with the unrestricted VECH model described by equation (3.8) is that it requires the estimation of a large number of parameters. This over-parameterization led to the development of the simplified diagonal VEC model, by Bollerslev, Engle, and Wooldridge (1988), where the A and B matrices are forced to be diagonal. The result is a reduction of the number of parameters in the variance and covariance equations to (16) for the trivariate case (Brooks *et al*., 2003).

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The Diagonal VECH Model

The Diagonal VECH model, the restricted version of VEC, was also proposed by Bollerslev, et al (1988). It assumes the A_j and B_j in Equation (16) are diagonal matrices, which makes it possible for H_t to be positive definite for all t. Also, the estimation process proceeds much more smoothly compared to the complete VEC model. However, the Diagonal VECH (DVEC) model with $(p+q+1)\times N\times(N+1)/2$ parameters is too restrictive since it does not take into account the interaction between different conditional variances and covariances.

Because of the simplification that it provides, the diagonal VEC model is frequently used. Each of its variance- covariance terms is postulated to follow a GARCH-type equation. The model can be written as follows (Tse, Tsui,1999):

$$\sigma_{ij,t} = c_{ij} + \sum_{h=1}^{p} a_{hij}\varepsilon_{t-h}\varepsilon_{t-h,j} + \sum_{h=1}^{q} b_{hij}\sigma_{t-j,ij} \qquad 1 \le i \le j \le k \tag{16}$$

where c_{ij}, a_{hij} and b_{hij} are parameters.

The diagonal VEC multivariate GARCH model could also be expressed as an infinite order multivariate ARCH model, where the covariance is expressed as a geometrically declining weighted average of past cross products of unexpected returns, with recent observations carrying higher weights. An alternative solution to the dimensionality problem would be to use orthogonal GARCH or factor GARCH models (Brooks, 2002). Now, the diagonal VECH model is in the form,

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_1^* \\ c_2^* \\ c_3^* \end{bmatrix} + \begin{bmatrix} a_{11}^* & 0 & 0 \\ 0 & a_{22}^* & 0 \\ 0 & 0 & a_{33}^* \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11}^* & 0 & 0 \\ 0 & b_{22}^* & 0 \\ 0 & 0 & b_{33}^* \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \tag{17}$$

$$H_t = C_0^* + \sum_{i=1}^{m} A_i^*(\varepsilon_{t-i}\varepsilon'_{t-1}) + \sum_{j=1}^{s} B_j^* H_{t-j} \tag{18}$$

Let us define the symmetric $N\times N$ matrices A_i^* and B_j^* as the matrices implied by the relations $A = diag\left[vec(A^*)\right]$, and $B = diag\left[vec(B^*)\right]$ and C_0^* is given by $C = vec(C_0^*)$ (Bauwens et al.). The model which is represented by equation (18) is DVEC (m,s)) model (Tsay, 2005). H_t must be parameter matrices, and only the lower portions of these matrices need to be parameterized and estimated. For example, Silberberg and Pafka (2001) prove that a sufficient condition to ensure the positive definiteness of the covariance matrix H_t in Eq. (3.13) is that the constant term C_0^* is positive definite and all the other coefficient matrices, A_i^* and B_j^*, are positive semidefinite (De Goeijet al., 2004). Each element of H_t depends only on its own past value and the corresponding product term in $\varepsilon_{t-i}\varepsilon'_{t-1}$. That is, each element of a DVEC model

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follows a GARCH (1,1) type model. The model is simple, but it may not produce a positive-definite covariance matrix. Furthermore, the model does not allow for dynamic dependence between volatility series (Tsay, 2005). We can construct a scalar VEC model: $A = aU$ and $B = bU$, where a and b are scalars and U is a matrix of ones (Bauwens, 2005).

The disadvantage of the Diagonal VECH Model is that it cannot capture the interaction between different variances and covariances

The BEKK model

In order for an estimated multivariate GARCH model to be plausible, H_t is required to be positive definite for all values of the disturbances. Verifying that this holds is a non-trivial issue even for VEC or diagonal VEC models of moderate size. To circumvent this problem, Engle and Kroner (1995) proposed a quadratic formulation for the parameters that ensured positive definiteness. This became known as the BEKK model (Brooks et al., 2003). Its number of parameters grows linearly with the number of assets. Therefore, this model is relatively parsimonious and suitable for a large set of assets (De Goeij et al., 2004). The BEKK model is in the form:

$$C = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad \varepsilon_t = \begin{bmatrix} \varepsilon_1^2 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_2^2 & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_3^2 \end{bmatrix}$$

$$H_t = C_0 C_0' + \sum_{k=1}^{k} \sum_{i=1}^{q} A_{ki}' \varepsilon_{t-i} \varepsilon'_{t-1} A_{ki} + \sum_{k=1}^{q} \sum_{i=1}^{p} B_{ki}' H_{t-j} B_{ki} \tag{19}$$

The elements of the variance-covariance matrix H_t, depend only on past values of itself and past values of $\varepsilon_t' \varepsilon$, indicating that the variances depend solely on past squared residuals, and the covariances depend solely on past covariances. The conditional variance for each equation, ignoring the constant terms, may be expanded for the trivariate BEKK-GARCH(1,1) as follows:

$$h_{11,t+1} = a_{11}^2 \varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{11}a_{31}\varepsilon_{i,t}\varepsilon_{3,t} + a_{21}^2\varepsilon_{2,t}^2 + 2a_{21}a_{31}\varepsilon_{2,t}\varepsilon_{3,t} + a_{31}^2\varepsilon_{3,t}^2 + b_{11}^2 h_{11,t} +$$

$$2b_{11}b_{12}h_{12,t} + 2b_{11}b_{31}h_{13,t} + b_{21}^2 h_{22,t} + 2b_{21}b_{31}h_{23,t} + b_{31}^2 h_{33,t} \tag{20}$$

$$h_{22,t+1} = a_{12}^2 \varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{12}a_{32}\varepsilon_{i,t}\varepsilon_{3,t} + a_{22}^2\varepsilon_{2,t}^2 + 2a_{22}a_{32}\varepsilon_{2,t}\varepsilon_{3,t} + a_{32}^2\varepsilon_{3,t}^2 + b_{12}^2 h_{11,t} +$$

$$2b_{12}b_{22}h_{12,t} + 2b_{12}b_{32}h_{13,t} + b_{22}^2 h_{22,t} + 2b_{22}b_{32}h_{23,t} + b_{32}^2 h_{33,t} \tag{21}$$

$$h_{33,t+1} = a_{13}^2 \varepsilon_{1,t}^2 + 2a_{13}a_{23}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{13}a_{33}\varepsilon_{i,t}\varepsilon_{3,t} + a_{23}^2\varepsilon_{2,t}^2 + 2a_{23}a_{33}\varepsilon_{2,t}\varepsilon_{3,t} + a_{33}^2\varepsilon_{3,t}^2 + b_{13}^2 h_{11,t} +$$

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$$2b_{13}b_{23}h_{12,t} + 2b_{13}b_{33}h_{13,t} + b_{23}^2 h_{22,t} + 2b_{23}b_{33}h_{23,t} + b_{33}^2 h_{33,t} \qquad (22)$$

Equations (20), (21), and (22) show how shocks and volatility are transmitted across sectors and over time.

The intercept matrix is decomposed into $C_0C_0^{'}$ which is positive definite, where C_0 is a 3×3 lower triangular matrix of constants, A_{ki} is a 3×3 square matrix that shows how conditional variances correlate with past squared errors. The elements of a matrix A_{ki} measures the effect of shocks or "news" on the conditional variance. and B_{ki} is a 3×3 square matrix which shows how past conditional variance affects the current level of conditional variance, and the degree of volatility persistence in conditional volatility among the sectors. Hence, C_0, A_{ki} and B_{ki} are parameter matrices. The BEKK representation in Equation (19) is a special case of Equation (18) (Hafner, Herwartz, 2006). Based on the symmetric parameterization of the model, H_t is almost surely positive definite provided that $C_0 \times C_0^{'}$ is positive definite (Tsay, 2005). Engle and Kroner (1995) proved that the necessary condition for the covariance stationarity of the BEKK model is that the eigenvalues, that is the characteristic roots of $H_t = C_0^* + \sum_{i=1}^{q}\sum_{k=1}^{k} A_{ik}^* \otimes A_{ik}^* + \sum_{i=1}^{p}\sum_{k=1}^{k}(B_{ik}^* \otimes B_{ik}^*)$, should be less than one in absolute value. Hence, the process can still render stationary even if there exists an element with a value greater than one in the matrix. Obviously, this condition is different from the stationarity condition required by the univariate GARCH model: that the sum of ARCH and GARCH terms has to be less than one (Pang et al., 2002). The BEKK (1,1, K) model is defined as:

$$H_t = C_0C_0^{'} + \sum_{k=1}^{k} A_{ki}^{'}\varepsilon_{t-i}\varepsilon'_{t-1} A_{ki} + \sum_{k=1}^{k} B_k^{'} \Sigma_{t-j} B_{ki} \qquad (23)$$

where C_0 , A_k and B_k are $N \times N$ matrices of parameters, but C_0 is upper triangular. One can also write $C_0 \times C_0^{'} = \Omega > 0$. Positivity of H_t is guaranteed if $H_0 \geq 0$. Here, there are 11 parameters, against 78 in the VECH model (Bauwens, 2005). This model allows for dynamic dependence between the volatility series (Tsay, 2005).

The Diagonal BEKK Model

Take A_k and B_k as diagonal matrices. For this case, the BEKK model is a restricted version of the VECH model with diagonal matrices (Bauwens, 2005; Franke et al., 2005). The scalar BEKK model, $A_k = a_k \times U$, $B_k = b_k \times U$, where a and b scalars and U is a matrix of ones (Bauwens, 2005). However, if the covariance exhibits a different degree of persistence than the volatilities, it is clear that either the volatility or the covariance process is miss-specified (Baur, 2004). The BEKK (1,1,1) model can be written as a VEC model (subject to restrictions) using formula (24)

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$$\Sigma_t = \Omega + A^{'}\varepsilon_{t-i}\varepsilon'_{t-1}A + B'\Sigma_{t-j}B \tag{24}$$

The diagonal BEKK model is given by the following equations

$$h_{11,t+1} = a_{11}^2\varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{11}a_{31}\varepsilon_{i,t}\varepsilon_{3,t} + a_{21}^2\varepsilon_{2,t}^2 + 2a_{21}a_{31}\varepsilon_{2,t}\varepsilon_{3,t} + a_{31}^2\varepsilon_{3,t}^2 + b_{11}^2h_{11,t} +$$

$$2b_{11}b_{12}h_{12,t} + 2b_{11}b_{31}h_{13,t} + b_{21}^2h_{22,t} + 2b_{21}b_{31}h_{23,t} + b_{31}^2h_{33,t} \tag{25}$$

$$h_{22,t+1} = a_{12}^2\varepsilon_{1,t}^2 + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{12}a_{32}\varepsilon_{i,t}\varepsilon_{3,t} + a_{22}^2\varepsilon_{2,t}^2 + 2a_{22}a_{32}\varepsilon_{2,t}\varepsilon_{3,t} + a_{32}^2\varepsilon_{3,t}^2 + b_{12}^2h_{11,t} +$$

$$2b_{12}b_{22}h_{12,t} + 2b_{12}b_{32}h_{13,t} + b_{22}^2h_{22,t} + 2b_{22}b_{32}h_{23,t} + b_{32}^2h_{33,t} \tag{26}$$

$$h_{33,t+1} = a_{13}^2\varepsilon_{1,t}^2 + 2a_{13}a_{23}\varepsilon_{1,t}\varepsilon_{2,t} + 2a_{13}a_{33}\varepsilon_{i,t}\varepsilon_{3,t} + a_{23}^2\varepsilon_{2,t}^2 + 2a_{23}a_{33}\varepsilon_{2,t}\varepsilon_{3,t} + a_{33}^2\varepsilon_{3,t}^2 + b_{13}^2h_{11,t} +$$

$$2b_{13}b_{23}h_{12,t} + 2b_{13}b_{33}h_{13,t} + b_{23}^2h_{22,t} + 2b_{23}b_{33}h_{23,t} + b_{33}^2h_{33,t} \tag{27}$$

$$C = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \quad \varepsilon_t = \begin{bmatrix} \varepsilon_1^2 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_2^2 & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_3^2 \end{bmatrix}$$

The BEKK model also has its diagonal form by assuming a_{kj}, b_{kj} matrices are diagonal. It is a restricted version of the DVEC model. The most restricted version of the diagonal BEKK model is the scalar BEKK one with A = aI and B = bI where a and b are scalars.

Estimation of a BEKK model still bears large computations due to several matrix transpositions. The number of parameters of the complete BEKK model is $(p+q)KN^2 + N(N+1)/2$. Even in the diagonal one, the number of parameters soon reduces to $(p+q)K \times N + N \times (N+1)/2$, but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of H_t. Under the overall consideration, it is typically assumed that $p = q = k = 1$ in the BEKK form's application. This model exhibits essentially the same problems as the Full BEKK model; there is no parameter in any equation that exclusively governs a particular covariance equation. Hence, it is not clear whether the parameters for a_{12} are just the result of the parameter estimates for a_{11} and a_{22}, or if the covariance equation alters the parameter estimates of the variance equations. In addition, the model is not very flexible and can therefore be

$$VEC(\Sigma_t) = VEC(\Omega) + (A \otimes A)^{'} VEC(\varepsilon_{t-1}\varepsilon_{t-1}^{'}) + (B \otimes B)^{'} VEC(\Sigma_{t-1}) \tag{28}$$

Hence, the BEKK model is weakly stationary if the eigenvalues of $(A \otimes A) + (B \otimes B)$ are smaller than one in modulus, and then

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$$VEC(\Sigma) = \left(I_{N^2} - (A\otimes A)^{'} - (B\otimes B)^{'}\right)^{-1} \tag{29}$$

Multivariate GARCH Model Estimation

Suppose the vector stochastic process $\{y_t\}$ (for $t = 1,...T$) has a conditional mean, conditional variance matrix, and conditional distribution $\mu_t(\theta_0), \Sigma_t(\theta_0)$ and $p(y_t / \varsigma_0, I_{t-1})$ respectively. Here, $\varsigma_0 = (\theta_0\, \eta_0)$ is the dimensional parameter vector, and η_0 is the vector that contains the parameters of the distribution of the innovations Z_t. Importantly, to justify the choice of the estimation procedure, we assume that the model to be estimated encompasses the true formulations of $\mu_t(\theta_0)$ and $H_t(\theta_0)$ (Bauwens et al., 2006). The procedure used most often in estimating θ_0 involves the maximization of a likelihood function constructed under the assumption of an $i.i.d$. distribution for the standardized innovations h_t. The likelihood function for the $i.i.d$. case can then be viewed as a quasi-likelihood function (Bauwens et al., 2006). Consequently, one has to make an additional assumption on the innovation process by choosing a density function, denoted $g(z_t(\theta);\eta)$, where η is a vector of nuisance parameters. Thus, the problem to solve is to maximize the sample log-likelihood function $L_T(\theta,\eta)$ for the L observations, with respect to the vector of parameters $\varsigma = (\theta,\eta)$

where

$$L_T(\varsigma) = \sum_{t=1}^{T} \log f(y_1 / \varsigma, I_{t-1}), \tag{30}$$

with

$$f(y_1 / \varsigma, I_{t-1}) = |\Sigma_t|^{-1/2} g(\Sigma_t^{-1/2}(y_t - \mu_t) \backslash \eta), \tag{31}$$

and the dependence with respect to θ occurs through μ_t and Σ_t. The term $\Sigma_t^{-1/2}$ is the Jacobian that arises in the transformation from the innovations to the observables. Note that as long as $g(.)$ belongs to the class of elliptical distributions, it is a function of $z_t^{'} z_t$, the maximum likelihood estimator is independent of the decomposition choice for $\Sigma_t^{-1/2}$. This is because

$$z_t^{'} z_t (y_t - \mu_t)^{'} \Sigma_t^{-1} (y_t - \mu_t) \tag{32}$$

The most commonly employed distribution in the literature is the multivariate normal, uniquely determined by its first two moments (so that ςθ = η since is empty). In this case, the sample log-likelihood, defined up to a constant, is

$$L_T(\theta) = -\frac{1}{2}\sum_{t=1}^{T} \log|\Sigma_t| - \frac{1}{2}\sum_{t=1}^{T} (y_t - \mu_t)^{'} \Sigma_t^{-1} (y_t - \mu_t) \tag{33}$$

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Under the assumption of conditional normality, the parameters of the multivariate GARCH models of any of the above specifications can be estimated by maximizing the log-likelihood function l:

$$\ell(\theta) = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log \Sigma_t + \varepsilon^{'} \Sigma_t^{-1} \varepsilon_t \tag{34}$$

where θ denotes all the unknown parameters to be estimated, N is the number of assets (the number of series in the system) and $T\varepsilon_t = y_t - u_t$ is the number of observations, and all other notation is as above. The maximum-likelihood estimate for θ is asymptotically normal. This makes the traditional procedures for statistical inference applicable (Brooks, 2002). Maximizing the log-likelihood function requires nonlinear maximization methods because it involves only first-order derivatives. The algorithm introduced by Berndt (1974) is easily implemented and particularly useful for the estimation of multivariate GARCH processes.

It is well-known that the normality of the innovations is rejected in most applications dealing with high-frequency data. In particular, the kurtosis of most financial asset returns is larger than three which means that they have too many extreme values to be normally distributed. Moreover, their unconditional distribution has often fatter tails than what is implied by a conditional normal distribution: the increase of the kurtosis coefficient brought by the dynamics of the conditional variance is not usually sufficient to match adequately the unconditional kurtosis of the data (Bauwens *et al.*, 2006).

If the conditional distribution of e_t is not normal, then maximizing equation (34) is interpreted as quasi maximum-likelihood (QML) (Hafner, 2006). The (QML) - estimator is consistent under the main assumption that the considered multivariate process is strictly stationary and ergodic (Cízek *et al.*, 2005). This quasi-maximum likelihood (QML) estimator is suitable for models which specify conditional covariances and variances because it correctly specifies the conditional mean and the conditional variance (Bauwens, Laurent, 2002). Estimation of multivariate GARCH models is troublesome, because the number of parameters may be large even for a moderate vector dimension N. Suppose there is enough data available for estimation, the likelihood might still be relatively “flat” as a function of many parameters. Thus, it might be hard for optimization routines to find the global maximum. Therefore, constraints on the parameter space are in many cases indispensable (Deistler, 2006).

3.8 Model Evaluation

The effectiveness of the multivariate volatility model is evaluated through statistical measures, including:

i. **Model Fit:** The goodness-of-fit is assessed using information criteria such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), aiming to strike a balance between model complexity and explanatory power.

ii. **Forecasting Accuracy:** The model's ability to accurately forecast volatility is evaluated through out-of-sample forecasting exercises. Mean Squared Error (MSE) and Root Mean Squared Error (RMSE) are employed to assess the accuracy of volatility forecasts.

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RESULTS AND DISCUSSION

Data Description

The dataset used in this study comprises daily stock prices for companies involved in the extraction and processing of Gold, Tin, and Zinc within the Nigerian solid minerals sector. The data spans over six decades, from 1960 to 2024, capturing various economic regimes, geopolitical events, and technological advancements. Stock prices were sourced from the Central Bank of Nigeria Statistical Bulletin and authenticated by National Bureau of Statistics, to ensure the accuracy and consistency of the data.

Descriptive Statistics

Table 1 provides descriptive statistics of the study stocks. The descriptive statistics for the stock return prices of Gold, Tin, and Zinc from January 1960 to May 2024 provide key insights into the behavior of these commodities in the Nigerian market. Gold has the highest average monthly return, indicating a stronger long-term performance compared to Tin and Zinc. However, Gold's returns are also characterized by significant positive skewness and high kurtosis, suggesting that while the returns tend to be positive, they also experience extreme fluctuations more frequently than a normal distribution would predict. Tin and Zinc show different return dynamics. Tin has a slightly lower average return and exhibits negative skewness, meaning that extreme negative returns are more common. Zinc, with the lowest average return, shows the highest volatility, as indicated by its standard deviation, and a distribution that is nearly symmetrical but still prone to extreme returns, as suggested by its leptokurtic nature. All three commodities demonstrate non-normal distribution patterns, as confirmed by the significant Jarque-Bera test results, which reject the hypothesis of normality. This suggests that models capturing volatility in these commodities need to account for the observed heavy tails and skewness in their return distributions. Zinc's particularly high volatility highlights the need for careful risk management when dealing with this commodity in the Nigerian market.

Table 1: Descriptive Statistics for Stock Prices of Gold, Tin, and Zinc

Statistic	RGOLD	RTIN	RZINC
Mean	0.005440	0.003519	0.003146
Median	0.000000	0.001867	0.001606
Maximum	0.394699	0.182757	0.334505
Minimum	-0.183862	-0.251711	-0.287297
Std. Dev.	0.043697	0.053552	0.061338
Skewness	1.288918	-0.468583	-0.144474
Kurtosis	13.30868	6.252485	6.049859
Jarque-Bera	3632.068	368.5316	301.8883
Probability	0.000000	0.000000	0.000000
Sum	4.199619	2.716317	2.428897
Sum Sq. Dev.	1.472162	2.211126	2.900730
Observations	772	772	772

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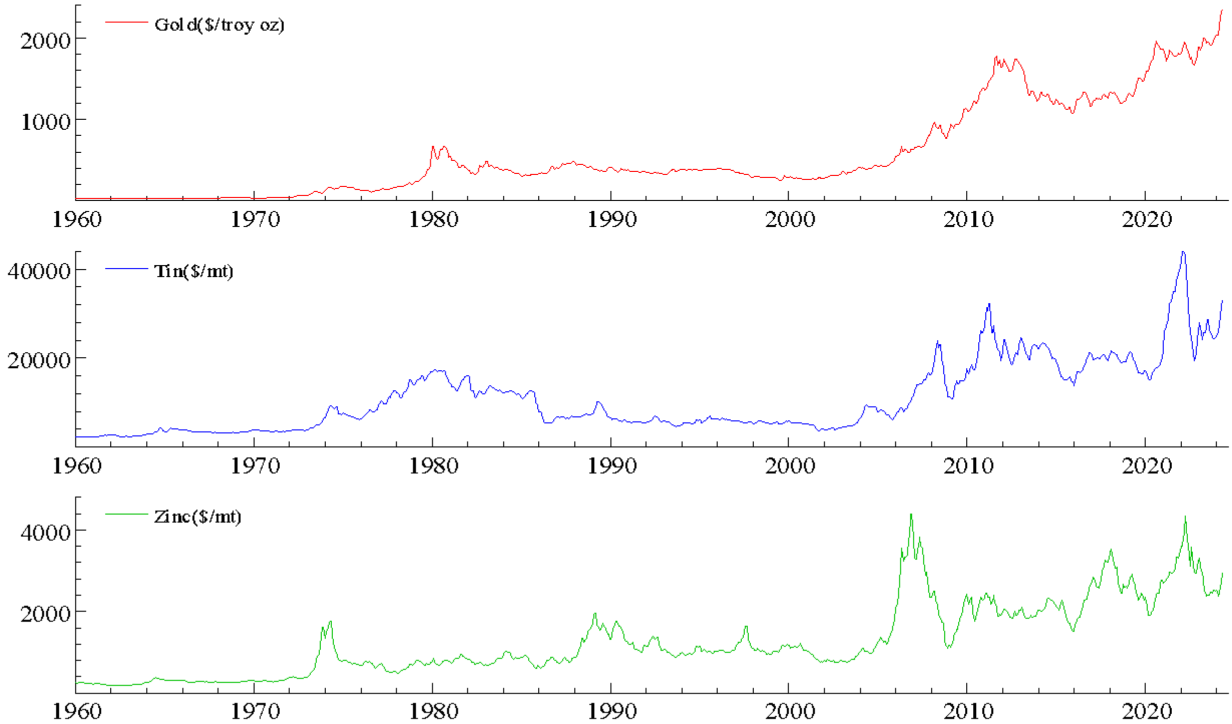
Time Series Analysis

Time series analysis is crucial for understanding the historical behavior and patterns of stock prices. Figures 1 present time series plots for Gold, Tin, and Zinc, respectively, illustrating the trends, fluctuations, and potential structural breaks. The time series plots facilitate the visual inspection of trends, seasonality, and volatility patterns, laying the groundwork for the subsequent multivariate volatility modeling. Figure 2 shows the time plot of stock returns over the study periods. All the stocks experienced volatility clustering, taking positive and negative values with different magnitude. These movements in returns throughout the study are an indication of volatility in the stock market. but, merely looking at the trends, a strong conclusion may not be drawn until a full statistical analysis is done



Figures 1: Time Series Plots for Gold, Tin, and Zinc

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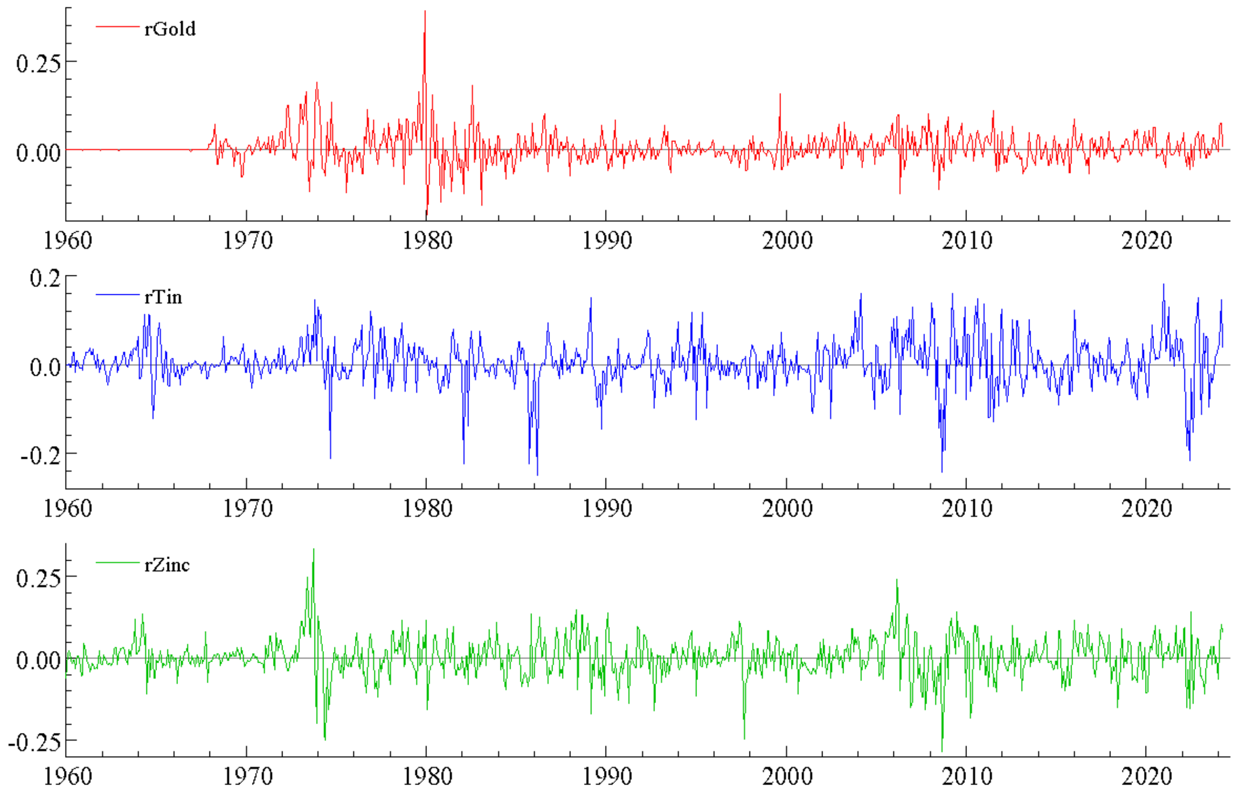

Figure 2: Time-Plots for Stock Returns

Test for Stationarity

Table 2 presents the results of the Augmented Dickey-Fuller (ADF) test statistic for Gold, Tin and Zinc stock prices respectively. There is evidence against the presence of a unit root at level for all the study stock. The results suggest that stock prices may require further investigation or transformation to achieve stationarity. Table 3 displays the results of the Augmented Dickey-Fuller (ADF) test statistic for the first differences of Gold, Tin and Zinc stock prices respectively. The extremely negative ADF test statistics along with low p-values and the rejection of the null hypothesis at different significance levels indicate that the first differences of all study socks prices are stationary.

Table 2: Augmented Dickey-Fuller Test Statistic at Level

		Gold Stock Price t-Statistic	Tin Stock Price t-Statistic	Zinc Stock Price
Augmented Dickey-Fuller test statistic		1.894060	-1.857091	-1.928758
Test critical values:	1% level	-3.438616	-3.438616	-3.438605
	5% level	-2.865078	-2.865078	-2.865073
	10% level	-2.568709	-2.568709	-2.568707
	Prob.*	0.9998	0.3528	0.3191

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Table 3: Augmented Dickey-Fuller Test Statistic at First Difference

		Gold Stock Price t-Statistic	Tin Stock Price t-Statistic	Zinc Stock Price
Augmented Dickey-Fuller test statistic		-18.95926	-13.51625	-21.75348
Test critical values:	1% level	-3.438616	-3.438616	-3.438605
	5% level	-2.865078	-2.865078	-2.865073
	10% level	-2.568709	-2.568709	-2.568707
	Prob.*	0.0000	0.0000	0.0000

ARCH Effect Test

The test statistics for all stock returns are extremely significant, according to Table 4. We agree that there is the presence of the ARCH effect in the residuals of the time series since p-values < 0.05 allow us to reject the null hypothesis of "no arch effect" at the 5% level. As a result, we can now proceed with the estimate of the GARCH family Model.

Table 4: Heteroskedasticity Test: ARCH

Gold			
F-statistic	54155.85	Prob. F(1,770)	0.0000
Obs*R-squared	761.1774	Prob. Chi-Square(1)	0.0000
Tin			
F-statistic	11913.41	Prob. F(1,770)	0.0000
Obs*R-squared	725.1325	Prob. Chi-Square(1)	0.0000
Zinc			
F-statistic	7692.096	Prob. F(1,770)	0.0000
Obs*R-squared	701.7526	Prob. Chi-Square(1)	0.0000

Multivariate Volatility Modeling

Building on the univariate analysis, a multivariate volatility model was developed to capture the interdependencies and spillover effects among the stock prices of Gold, Tin, and Zinc. The proposed model considered joint volatility dynamics and potential cross-market linkages. Table 5 presents the estimation results for a VECH (Vectorized Heteroscedasticity) model applied to the stock returns. The estimation results from the Diagonal VECH model for the stock returns of Gold, Tin, and Zinc reveal significant volatility dynamics in the Nigerian solid minerals market. The coefficients related to the mean equations indicate that each commodity's returns exhibit autocorrelation, with Gold (M(1,1) = 1.17E-06, p = 0.0000) and Zinc (M(3,3) = 0.000116, p = 0.0001) showing significant effects. Some interaction effects are also observed, particularly between Gold and Tin (M(1,2) = 1.62E-12, p = 0.0000), but other cross-commodity interactions, such as between Tin and Zinc (M(2,3) = -4.54E-13, p = 0.9325), are not significant, indicating that not all linkages between these commodities are equally influential. The ARCH coefficients demonstrate that past shocks (squared returns) significantly impact current volatility across all three commodities. For instance, the ARCH coefficient for Gold (A1(1,1) = 0.226015, p = 0.0000) indicates that past shocks have a strong effect on its volatility. Similarly, Tin (A1(2,2) = 0.150000, p = 0.0000) and Zinc (A1(3,3) = 0.181447, p = 0.0000)

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also exhibit significant responses to past shocks. The cross-commodity effects are noteworthy, with coefficients like A1(1,2) = 0.184126 (p = 0.0000) indicating that shocks in Tin can influence the volatility of Gold, underscoring the interconnectedness of these markets. The GARCH coefficients show that volatility is highly persistent in each commodity, with values such as B1(1,1) = 0.755472 (p = 0.0000) for Gold and B1(3,3) = 0.802817 (p = 0.0000) for Zinc, indicating that once volatility is elevated, it tends to remain high for some time. This persistence is especially pronounced in Zinc, suggesting that its market is particularly prone to sustained periods of high volatility. Overall, the model captures the complex nature of volatility in these markets, driven by both individual commodity factors and cross-commodity interactions, which are crucial for risk management and forecasting.

Table 5: Estimation of Diagonal VECH Model for Stock Returns

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.17E-06	2.00E-07	5.857865	0.0000
M(1,2)	1.62E-12	3.61E-13	4.495900	0.0000
M(1,3)	1.09E-06	1.15E-06	0.948305	0.3430
M(2,2)	4.51E-17	4.78E-18	9.420104	0.0000
M(2,3)	-4.54E-13	5.36E-12	-0.084729	0.9325
M(3,3)	0.000116	2.97E-05	3.905439	0.0001
A1(1,1)	0.226015	0.025506	8.861235	0.0000
A1(1,2)	0.184126	0.019925	9.240933	0.0000
A1(1,3)	0.202508	0.019115	10.59425	0.0000
A1(2,2)	0.150000	0.024595	6.098865	0.0000
A1(2,3)	0.164976	0.018067	9.131523	0.0000
A1(3,3)	0.181447	0.021796	8.324796	0.0000
B1(1,1)	0.755472	0.019682	38.38325	0.0000
B1(1,2)	0.673263	0.026887	25.04071	0.0000
B1(1,3)	0.778785	0.014608	53.31352	0.0000
B1(2,2)	0.600000	0.043836	13.68736	0.0000
B1(2,3)	0.694039	0.027401	25.32904	0.0000
B1(3,3)	0.802817	0.018927	42.41599	0.0000

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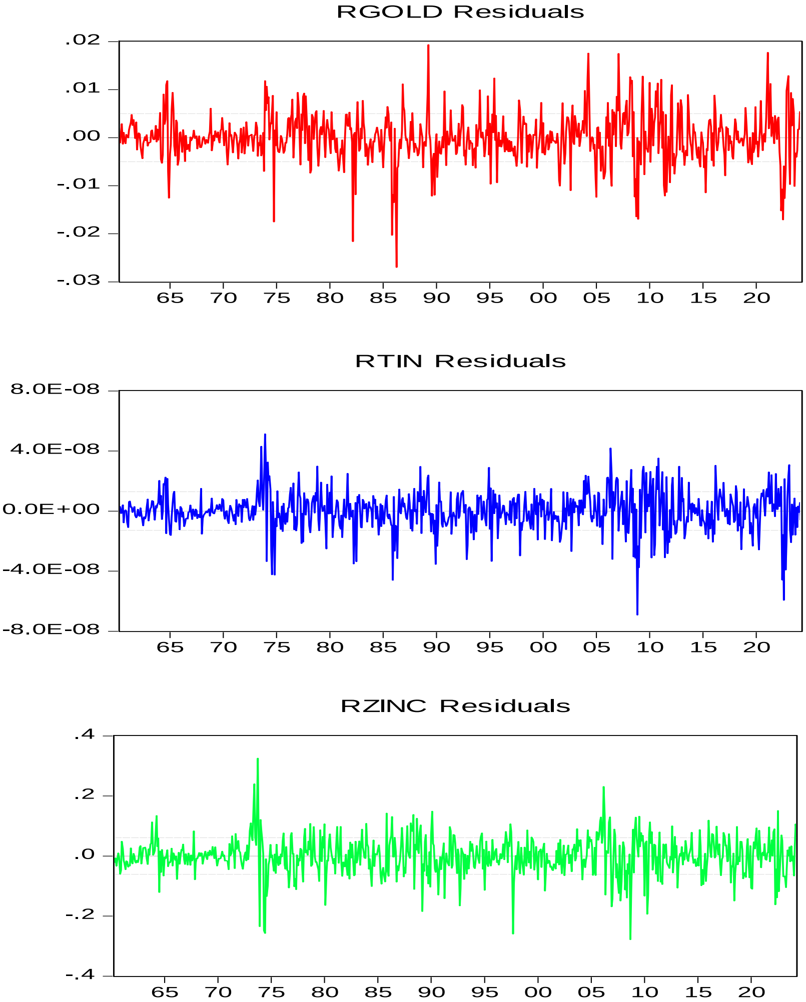

Figure 3: VECH Residual Plots of the Gold, Tin and Zinc Stock Returns respectively.

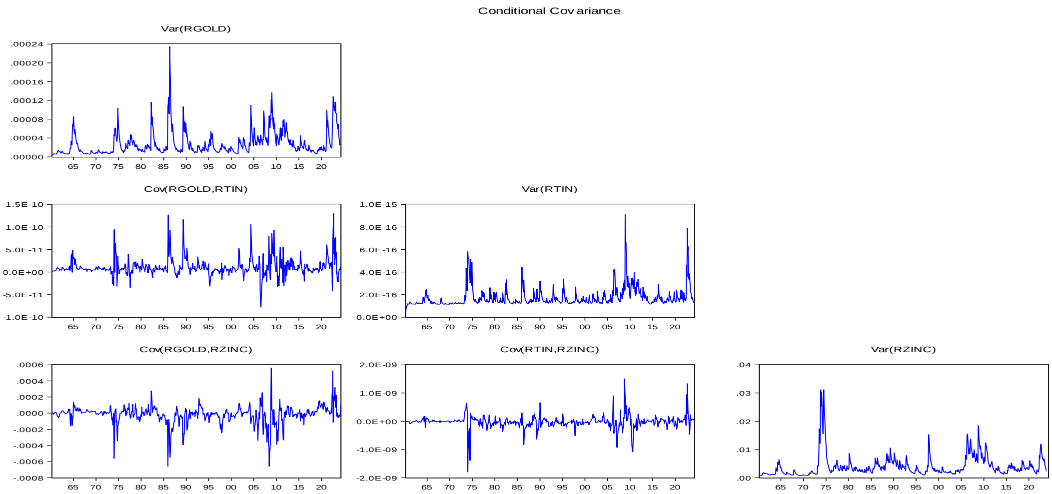

Figure 4: VECH Conditional Variance Plots.

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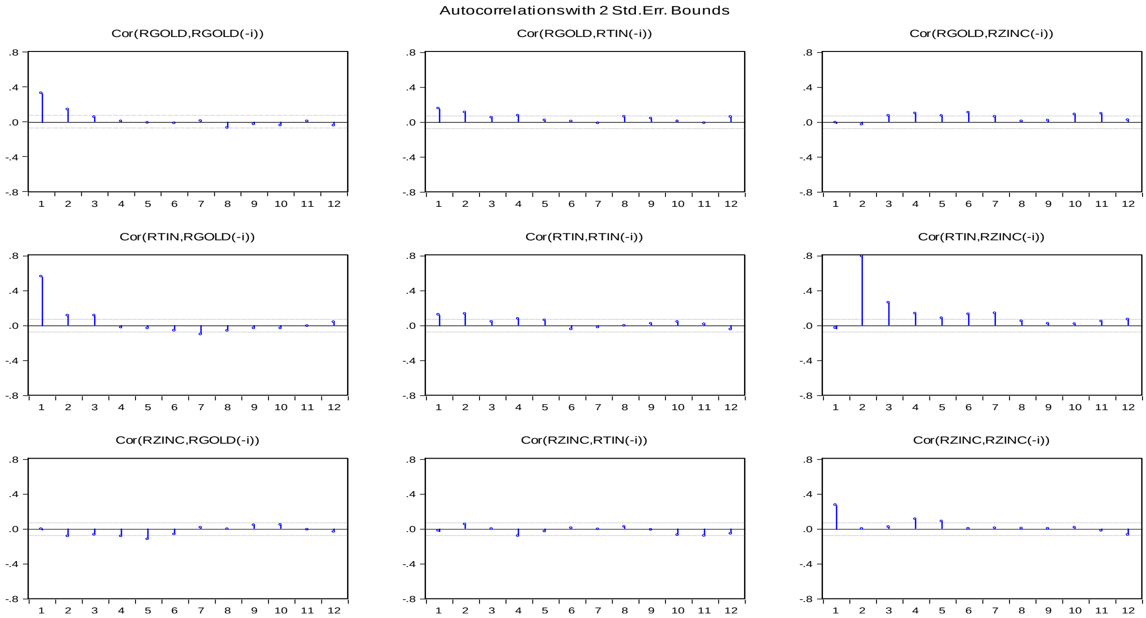

Figure 5: VECH Plot of Residual Correlogram

Table 6 BEKK Model for Stock Returns

Models	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
M(1,1)	1.17E-06	2.00E-07	5.857865	0.0000
M(1,2)	1.62E-12	3.61E-13	4.495900	0.0000
M(1,3)	1.09E-06	1.15E-06	0.948305	0.3430
M(2,2)	4.51E-17	4.78E-18	9.420104	0.0000
M(2,3)	-4.54E-13	5.36E-12	-0.084729	0.9325
M(3,3)	0.000116	2.97E-05	3.905439	0.0001
A1(1,1)	0.475410	0.026825	17.72247	0.0000
A1(2,2)	0.387298	0.031752	12.19773	0.0000
A1(3,3)	0.425965	0.025584	16.64959	0.0000
B1(1,1)	0.869179	0.011322	76.76651	0.0000
B1(2,2)	0.774597	0.028296	27.37473	0.0000
B1(3,3)	0.896000	0.010562	84.83197	0.0000

Table 6 shows the estimation results for a BEKK (Baba, Engle, Kraft, and Kroner) model applied to the stock returns. he BEKK model results reveal significant interactions and volatility dynamics among the stock returns of Gold, Tin, and Zinc. The mean equation coefficients indicate that the conditional mean of Gold is significantly influenced by its own past values (M(1,1) = 1.17E-06, p = 0.0000) and by the returns of Tin (M(1,2) = 1.62E-12, p = 0.0000). However, Zinc does not significantly affect Gold’s returns (M(1,3) = 1.09E-06, p = 0.3430). Tin’s returns also exhibit strong autocorrelation (M(2,2) = 4.51E-17, p = 0.0000), while the influence of Zinc on Tin is not significant (M(2,3) = -4.54E-13, p = 0.9325). Zinc's own returns show significant autocorrelation as well (M(3,3) = 0.000116, p = 0.0001). The ARCH coefficients show that past shocks have a substantial impact on the current volatility of all three commodities. Gold has a highly significant response to past shocks (A1(1,1) = 0.475410, p = 0.0000), followed by Zinc (A1(3,3) = 0.425965, p = 0.0000), and Tin (A1(2,2)

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= 0.387298, p = 0.0000). This indicates that unexpected price movements in these commodities strongly influence their future volatility, with Gold and Zinc being particularly sensitive to past shocks. The GARCH coefficients highlight the persistence of volatility across the three commodities, with Zinc showing the highest persistence (B1(3,3) = 0.896000, p = 0.0000), followed by Gold (B1(1,1) = 0.869179, p = 0.0000), and then Tin (B1(2,2) = 0.774597, p = 0.0000). This suggests that periods of high volatility in these commodities are likely to be prolonged, especially in Zinc, indicating the need for careful monitoring and risk management in these markets.

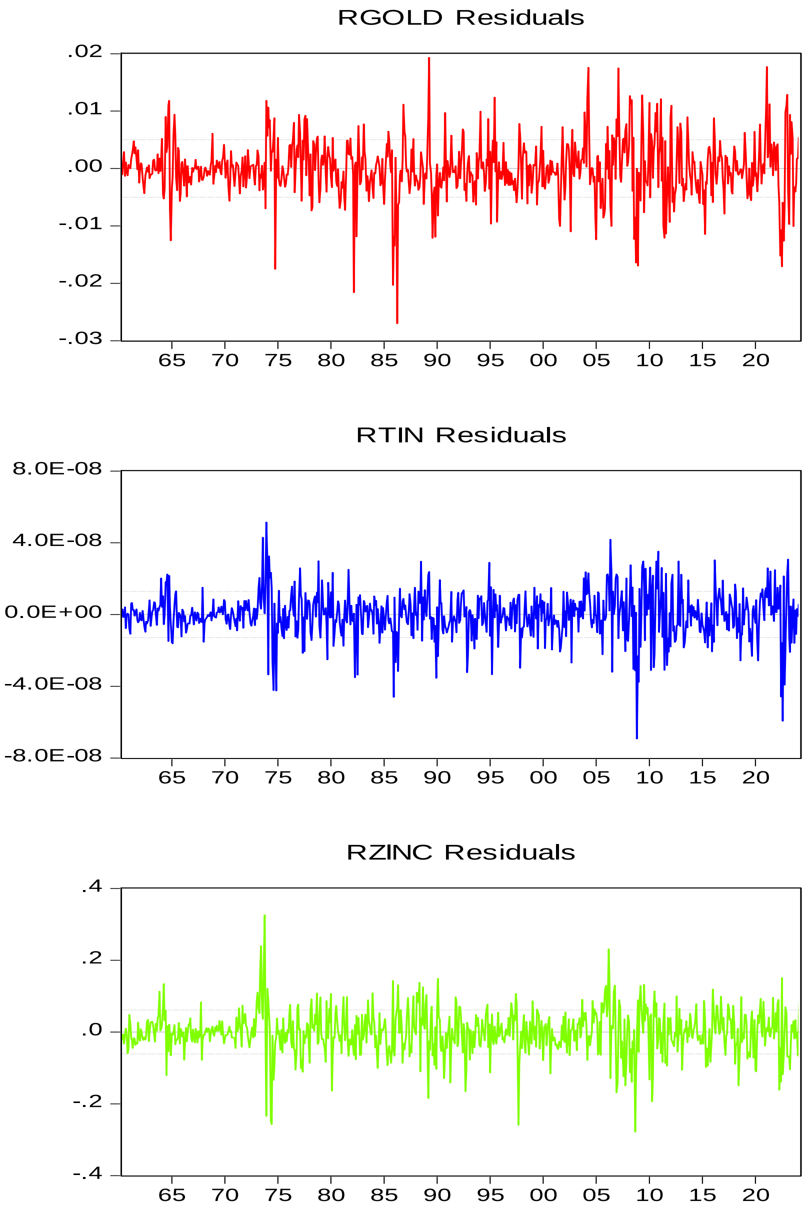

Figure 6: BEKK Residual Plots of the Gold, Tin and Zinc Stock Returns respectively.

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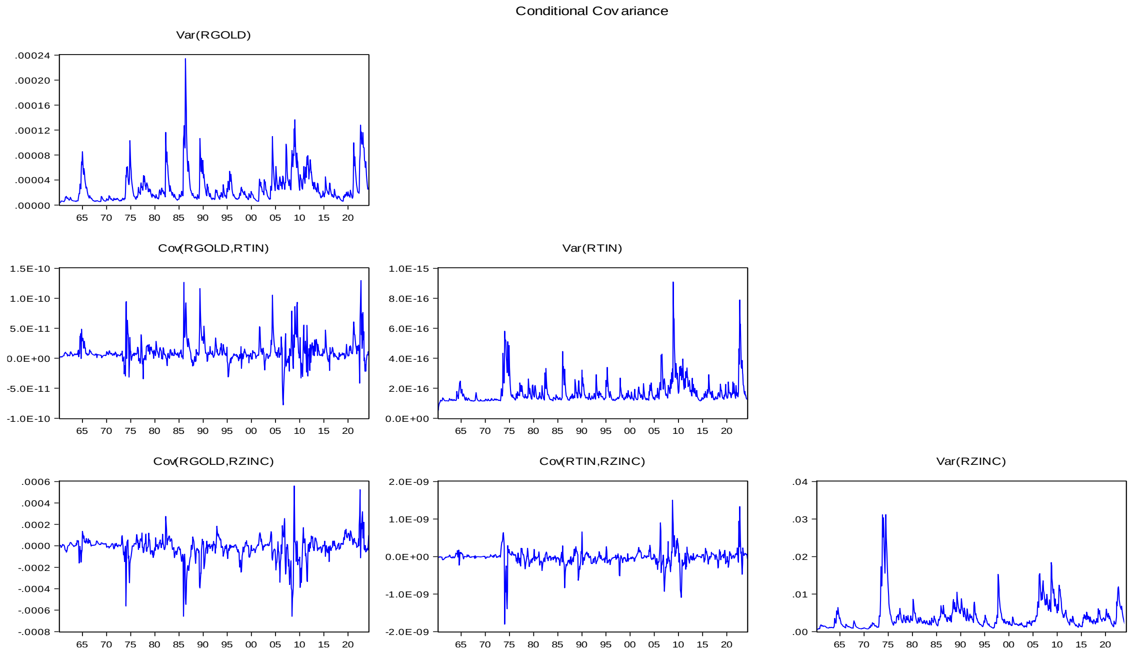

Figure 7: BEKK Conditional Variance Plots.

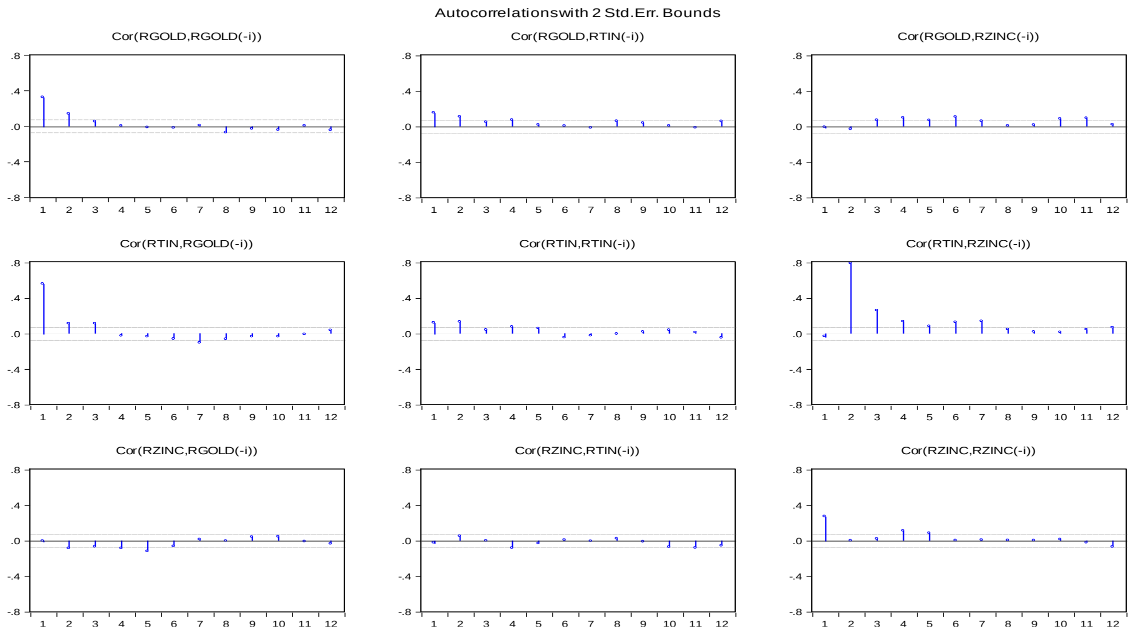

Figure 8: BEKK Plot of Residual Correlogram

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Diagnostic Check

The diagnostic check tests as shown in Table 7 suggest that there is no significant ARCH effect in the residuals of the models for Gold, Tin, and Zinc stock returns. This means that the volatility clustering or conditional heteroskedasticity patterns are not prominent in the residuals, and the models adequately capture the variance dynamics. Investors and analysts can be more confident in the reliability of the models for predicting these stock returns, at least in terms of capturing ARCH effects. The multivariate modeling results offer insights into the cross-market interactions and simultaneous volatility dynamics among the selected solid minerals.

Table 7: Diagnostic Check Tests for the presence of ARCH effect in the residuals of the models.

Gold Stock Returns			
F-statistic	0.050448	Prob. F(1,770)	0.8223
Obs*R-squared	0.050579	Prob. Chi-Square(1)	0.8221
Tin Stock Return			
F-statistic	0.177141	Prob. F(1,749)	0.6740
Obs*R-squared	0.177572	Prob. Chi-Square(1)	0.6735
Zinc Stock Return			
F-statistic	0.044670	Prob. F(1,770)	0.8327
Obs*R-squared	0.044786	Prob. Chi-Square(1)	0.8324

Source: Eviews 12 Output

Forecasting

The forecast evaluations indicate that the predictions for Gold and Tin are highly accurate, with very low error metrics and Theil coefficients close to zero, reflecting minimal deviations from the actual returns. However, the forecast for Zinc shows considerable errors, with high RMSE, MAE, and an especially high MAPE, indicating significant inaccuracies in predicting Zinc's returns.

Table 8: Forecast Evaluation

Forecast	RMSE	MAE	MAPE	Theil
RGOLD	0.009405	0.006800	0.000541	3.74E-06
RTIN	0.006376	0.004654	0.003710	2.54E-05
RZINC	0.061299	0.044363	110.8937	0.000050

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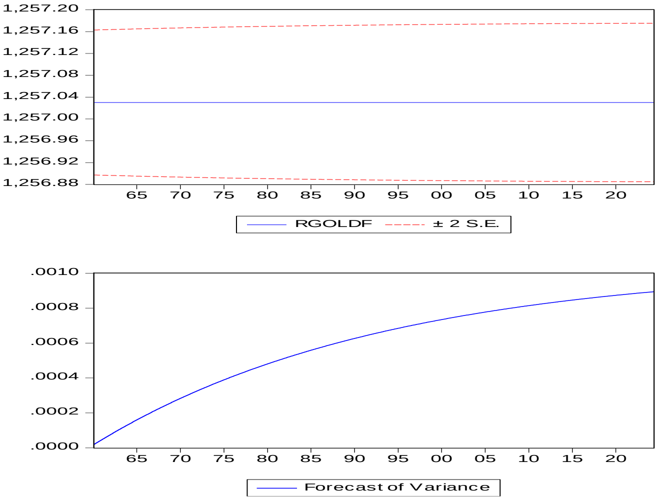
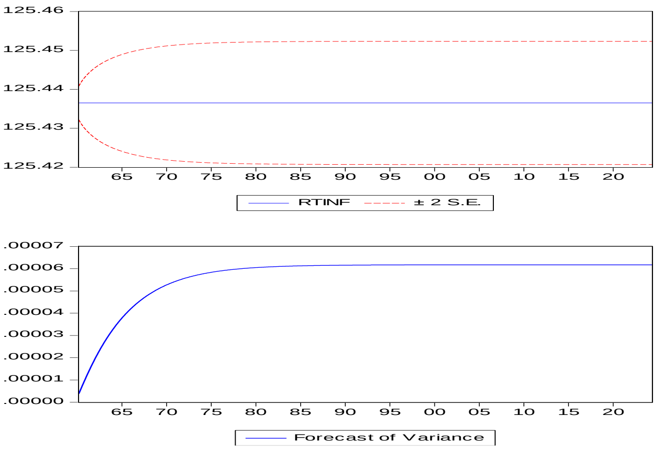


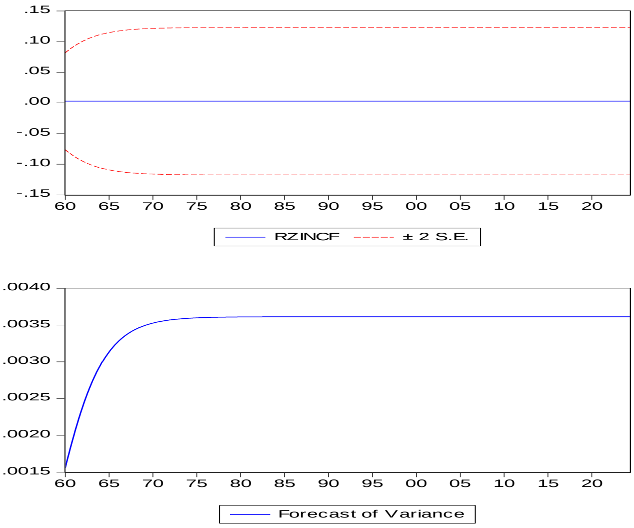
Forecast: RGOLDF	
Actual: RGOLD	
Forecast sample: 1960M01 2024M05	
Adjusted sample: 1960M02 2024M05	
Included observations: 772	
Root Mean Squared Error	0.009405
Mean Absolute Error	0.006800
Mean Abs. Percent Error	0.000541
Theil Inequality Coefficient	3.74E-06
Bias Proportion	0.001558
Variance Proportion	NA
Covariance Proportion	NA

Figure 9a



Forecast: RTINF	
Actual: RTIN	
Forecast sample: 1960M01 2024M05	
Adjusted sample: 1960M03 2024M05	
Included observations: 771	
Root Mean Squared Error	0.006376
Mean Absolute Error	0.004654
Mean Abs. Percent Error	0.003710
Theil Inequality Coefficient	2.54E-05
Bias Proportion	0.000351
Variance Proportion	NA
Covariance Proportion	NA

Figure 9b



Forecast: RZINCF	
Actual: RZINC	
Forecast sample: 1960M01 2024M05	
Included observations: 772	
Root Mean Squared Error	0.061299
Mean Absolute Error	0.044363
Mean Abs. Percent Error	110.8937
Theil Inequality Coefficient	0.956408
Bias Proportion	0.000050
Variance Proportion	NA
Covariance Proportion	NA

Figure 9c

Figure 9(a, b and c): Forecast plot for gold, Tin and Zinc respectively

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DISCUSSION OF FINDINGS

The findings presented in the analysis offer valuable insights into the behavior of stock returns for Gold, Tin, and Zinc. The descriptive statistics in Table 1 reveal significant implications regarding the historical performance of these stocks. Despite variations in their average returns, the positive mean returns across all three stocks suggest that investors generally experienced positive returns over the study period. However, the notable negative skewness and high kurtosis values indicate that the return distributions for all three stocks are asymmetric and possess heavier tails than a normal distribution. This implies that extreme returns, both positive and negative, are more likely than what would be expected under a normal distribution. Visualizing the time plots of stock returns in Figure 2 further reinforces these observations, demonstrating volatility clustering across all three stocks. The fluctuations in returns throughout the study period suggest a dynamic stock market environment characterized by periods of both positive and negative volatility. However, a thorough statistical analysis draws definitive conclusions from these trends.

The application of the VECH model reveals intricate dynamics within the stock returns. The M Matrix Coefficients indicate that while lagged squared returns may not significantly impact current volatility for certain variables, they exhibit significance for others. Specifically, positive coefficients suggest volatility clustering, indicating that periods of high volatility tend to persist. This finding is consistent with the result from Ajayi *et al*. (2019) and Mohammed *et al.* (2022). The presence of significant ARCH and GARCH coefficients underscores the non-random patterns in stock returns volatility and the persistence of volatility shocks over time, respectively. Similarly, the BEKK model reaffirms these observations, providing further evidence of volatility clustering, ARCH effects, and persistence in volatility shocks in stock returns. The diagnostic check tests in Table 6 offer additional validation, indicating that there is no significant ARCH effect in the residuals of the models for Gold, Tin, and Zinc stock returns. This result is corroborated with the findings from Yahaya *et al.* (2022). This suggests that the models adequately capture the variance dynamics, enhancing confidence in their reliability for predicting stock returns. Overall, the multivariate modeling results offer comprehensive insights into the cross-market interactions and simultaneous volatility dynamics among the selected solid minerals. These findings are invaluable for investors and analysts, empowering them to better understand and manage the associated risks in stock price movements. Armed with this knowledge, traders can refine their risk management strategies, while investors can make more informed portfolio allocation decisions.

CONCLUSION

The comprehensive analysis of stock returns for gold, tin, and zinc stocks has provided valuable insights into their historical behavior and volatility dynamics. The findings reveal that, on average, investors experienced positive returns, but the presence of significant negative skewness and high kurtosis values suggests a non-symmetric distribution with heavier tails than a normal distribution. This implies a higher likelihood of extreme returns, both positive and negative. The application of advanced statistical models, including the VECH and BEKK models, has uncovered intricate patterns within the volatility dynamics of these stocks. Volatility clustering, ARCH effects, and the persistence of volatility shocks over time have been identified, emphasizing the non-random nature of stock returns volatility.

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Investors and analysts can leverage these insights to enhance their risk management strategies and portfolio allocation decisions. By acknowledging the presence of volatility clustering and non-random patterns, market participants can make more informed choices in mitigating risks and optimizing their investment portfolios. The diagnostic check tests further validate the reliability of the models in capturing variance dynamics, providing a level of confidence in their predictive capabilities. The absence of significant ARCH effects in the residuals underscores the adequacy of the models in capturing the complexity of stock returns, boosting their reliability for future predictions. This analysis would contribute valuable knowledge for market participants, enabling them to navigate the dynamic landscape of stock market volatility. As the financial landscape continues to evolve, these insights will be crucial for investors and analysts in making sound decisions, managing risks effectively, and achieving their financial objectives.

Based on the findings of the analysis, it is recommended that investors and analysts carefully consider the implications of volatility clustering, ARCH effects, and persistence in volatility shocks when making investment decisions in the stock market, particularly regarding gold, tin, and zinc stocks.

i. Investors should develop robust risk management strategies that account for the non-random patterns observed in stock returns volatility. This may include diversification across asset classes, hedging strategies, and setting appropriate stop-loss levels to mitigate potential losses during periods of heightened volatility.

ii. Consideration should be given to incorporating the insights gained from the analysis into portfolio allocation decisions. Investors may opt to allocate their portfolios based on the observed volatility dynamics, ensuring a balanced exposure to assets that exhibit varying degrees of volatility clustering and persistence in volatility shocks.

iii. Given the dynamic nature of the stock market, it is imperative for investors to continuously monitor the volatility dynamics of their portfolios and adjust their strategies accordingly. Regular review of the models and diagnostic tests can help identify changes in volatility patterns and guide timely portfolio adjustments.

iv. For individuals less experienced in financial markets or those managing larger portfolios, seeking professional advice from financial advisors or portfolio managers with expertise in risk management and investment strategies may be beneficial. These professionals can provide personalized recommendations tailored to individual investment objectives and risk tolerances.

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