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# DYNAMIC INFLUENCE OF FOUNDATION STIFFNESS ON RECTANGULAR STRUCTURAL PLATE WITH CONSTANT BI-PARAMETRIC ELASTIC FOUNDATION WITH SIMPLE ELASTIC END CONDITIONS

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**ABSTRACT:** *This paper considers the dynamic influence of foundation stiffness such as foundation modulus, shear modulus, Young modulus as well as rotatory inertia correction factor on the vibrations of orthotropic rectangular structural plate resting on a constant bi-parametric elastic foundation with simple elastic end conditions when moving distributed loads are traversing on the structural plate. The orthotropic rectangular structural plate model is a coupled fourth order partial differential equation with variables and singular coefficients. The solutions to the model are obtained by transforming the fourth order partial differential. The fourth order partial differential equation is transformed to a set of coupled second order ordinary differential equations by using a special technique adopted by Shadnam et al [11]. This set of second order ordinary differential equations is then simplified using the modified asymptotic method of Struble. The closed form solution is analyzed, resonance conditions are obtained and the results are presented in plotted curves to show the impact of foundation parameters for both cases of moving distributed mass and moving distributed force.*

**KEYWORDS:** Bi-parametric Elastic Foundation, Moving Distributed Masses, Transvered Displacement, Resonance.

## INTRODUCTION

In structural mechanics, an orthotropic plate is a type of structure that exhibits different mechanical properties in different directions. The structural material properties such as stiffness, strength, anisotropy and material behaviour vary depending on the orientation of the orthotropic plate, which distinguishes it from isotropic plate. This orientation is very important in engineering and design, as it affects the structural behaviour and performance of the plates in various applications. Engineers and designers consider these distinctions when selecting the appropriate material for a specific structural application. The material exhibits different levels of resistance to deformation depending on the applied load direction. This variation in stiffness properties such as Young modulus, shear modulus, and Poisson ratio is due to the anisotropic nature of orthotropic materials. These stiffness properties have different values along the x-axis, y-axis and z-axis. The rectangular plate considered in this work is resting on bi-parametric elastic foundation. It is common knowledge that a bi-parametric elastic foundation is a mathematical model used to represent the response of a soil or foundation system under the application of external load. This elastic foundation considers two parameters, namely the vertical stiffness and the horizontal stiffness, to characterize the behaviour of the foundation. The vertical stiffness represents the resistance of the foundation to vertical displacements caused by the applied loads and also determines the amount of distribution of pressure beneath the foundation, while the horizontal stiffness represents the resistance of the foundation to horizontal displacements caused by lateral loads. It determines the lateral stability of the foundation and its ability to resist sliding or tilting.

In short, the bi-paametric elastic foundation model provides a more comprehensive representation of the foundation response compared to traditional models that only consider vertical stiffness. This model is commonly used in geotechnical engineering to analyze and design foundations for various structures such as buildings, bridges, and retaining walls. It is understandable that if the vertical stiffness in the foundation system is higher, the foundation is stiffer and thus resists the vertical displacements more effectively. It also allows the foundation to distribute the applied load more evenly across the underlying soil layers. The vibration catalyzed by traveling vehicles on structural members like roads, bridges, beams and rails cannot be over-emphasized and has become a significant and pressing issue in engineering and design due to the increasing high speed vehicles. The vibration problem in structures; particularly in bridges, beams and railways, when traversed by moving loads such as trains, cars, ships, cranes, and lorries is very germane in many engineering applications. The colossal deflections and vibrations of beam-like structures, especially bridges, initiated by high-speed and heavy vehicles can significantly increase the internal stresses and impact the safety and serviceability of the bridges. It is essential to have an explicit understanding of the dynamic properties of these structures for the purpose of their design and operation.

## LITERATURE/THEORETICAL UNDERPINNING

The vibration of plates is an interesting phenomenon that has been studied greatly by researchers in applied mathematics, engineering and design. Numerous authors in this field had examined the topic and are still investigating the phenomenon. Awodola and Adeoye [1] studied the vibration of orthotropic rectangular plates under the action of moving distributed masses and resting on variable elastic Pasternak foundation with clamped end conditions.


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Ugural [2] and Okafor and Oguaghamba [3] used Ritz and Galarkin methods to solve isotropic and orthotropic plate problems. Their results are approximately close to the exact solutions. A couple of researchers applied the superposition method for analysis of vibrations of plates having classical boundary conditions, for instance, Ohya et al. [4], for deformable plates. Awodola and Omolofe examined the response of concentrated moving masses of elastically supported rectangular plates resting on concentrated moving masses of elastically supported rectangular plates resting on Winkler elastic foundations by the method of separation of variables. Gbadeyan and Dada [5] improved on their proven works by reconsidering the dynamic response of a Mindlin elastic rectangular plate subjected to distributed moving load, but neglected the effect of damping. Adeoye and Awodola [6] investigated the dynamic behaviour of moving distributed masses of an orthotropic rectangular plate with clamped-clamped boundary conditions on a constant elastic foundation. Sveklo [7] suggested the contact theory for two anisotropic bodies under compression according to contact pressure distribution over an elliptical contact region. The same structural effects are real of concrete slabs in a composite girder bridge, but the steel orthotropic deck is considerably lighter, and therefore allows longer span bridges to be more expeditiously designed. Hu et al. [8] purported vibration solutions to orthotropic rectangular plates by making use of the symplectic geometry method. Oni and Awodola [9] examined the vibrations of a simple supported plate under moving masses and resting on Pasternak elastic foundations with stiffness variation. Adeoye and Adeloye [10] worked on dynamic characteristics of orthotropic rectangular plates under the influence of moving distributed masses on variable elastic foundations using the variable separable.

The aforementioned research works are based on structures with classical boundary conditions. In this research, non-classical boundaries are considered. Specifically, simple elastic end conditions are considered on rectangular plates with stiffness variation on constant elastic bi-parametric foundation.

**Governing Equation**

The transverse displacement $W(x, y, t)$ of orthotropic rectangular plates that rests on a bi-parametric elastic foundation, and traversed by distributed mass $M_r$ traversing with constant velocity $k_r$ along a straight line parallel to the x-axis issuing from point y = s on the y-axis with flexural rigidities $D_x$ and $D_y$, is governed by the fourth order partial differential equation given as

$$D_x \frac{\partial^4}{\partial x^4} W(x,y,t) + 2\beta \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) + D_y \frac{\partial^4}{\partial y^4} W(x,y,t) + \mu \frac{\partial^2}{\partial t^2} W(x,y,t) - \rho h R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] + K_0 W(x,y,t) - G_0 [\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] - \sum_{r=1}^{N} [M\_r g H(x-ct) H(y-s) - M_r (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c_r \frac{\partial^2}{\partial x \partial t} \ W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x - c_r t) H(y-s) W(x,y,t)] = 0 \tag{1}$$

where $D_x$ and $D_y$ are the flexural rigidities of the plate along x and y axes respectively.

$$D_x = \frac{E_x h^3}{12(1-\nu_x \nu_y)}, \quad D_y = \frac{E_y h^3}{12(1-\nu_x \nu_y)}, \quad B = D_x D_y + \frac{G_o h^3}{6}$$

_ $E_x$ and $E_y$ are the Young's moduli along x and y axes respectively, $G_o$ is the rigidity modulus,


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$\nu_x$ and $\nu_y$ are Poisson's ratios for the material such that $E_x\nu_y = E_y\nu_x$ , $\rho$ is the mass density per unit volume of the plate, h is the plate thickness, t is the time, x and y are the spatial coordinates in x and y directions respectively, $R_o$ is the rotatory inertia correction factor, $K_o$ is the foundation constant and g is the acceleration due to gravity, while H(.) is the Heaviside function.

Re-expressing Equation (2), one obtains

$$\mu\frac{\partial^2}{\partial t^2}W(x,y,t) + \mu\kappa_n^2 W(x,y,t) = \rho h R_0[\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t) + \frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)] - 2\beta\ \frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t) - D_x\frac{\partial^4}{\partial x^4}W(x,y,t) - D_y\frac{\partial^4}{\partial y^4}W(x,y,t) - K_0 W(x,y,t) + \mu\kappa_n^2\ W(x,y,t) + G_0[\frac{\partial^2}{\partial x^2}W(x,y,t) + \frac{\partial^2}{\partial y^2}W(x,y,t)] + \sum_{r=1}^{N}[M_r g H(x-k_r t)H(y-s) - M_r(\frac{\partial^2}{\partial t^2}W(x,y,t) + 2c\frac{\partial^2}{\partial x\partial t}W(x,y,t) + k_r^2\frac{\partial^2}{\partial x^2}W(x,y,t))H(x-k_r t)H(y-s)W(x,y,t)] \quad (2)$$

which can be represented further as

$$\frac{\partial^2}{\partial t^2}W(x,y,t) + \kappa_n^2 W(x,y,t) = R_0[\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t) + \frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)] - \frac{2B}{\mu}\ \frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}W(x,y,t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}W(x,y,t) + [\kappa_n^2 - \frac{K_0}{\mu}]W(x,y,t) + \frac{G_0}{\mu}[\frac{\partial^2}{\partial x^2}W(x,y,t) + \frac{\partial^2}{\partial y^2}W(x,y,t)] + \sum_{r=1}^{N}[\frac{M_r g}{\mu}H(x-k_r t)H(y-s) - \frac{M_r}{\mu}(\frac{\partial^2}{\partial t^2}W(x,y,t)\ + 2c\frac{\partial^2}{\partial x\partial t}W(x,y,t) + k_r^2\frac{\partial^2}{\partial x^2}W(x,y,t))\,H(x-k_r t)H(y-s)W(x,y,t)] \quad (3)$$

where $\kappa_n^2$ is the natural frequencies, $n = 1,2,3,\ldots$

The initial condition, without any loss of generality, is taken as

$$W(x,y,t) = 0 = \frac{\partial}{\partial t}W(x,y,t) \quad (4)$$

## METHODOLOGY

To obtain an expression for the solution of Equation (4), one applies the technique of Shadnam *et al.* [11] which requires that the deflection of the plates be in series form as

$$W(x,y,t) = \sum_{n=1}^{N}\delta_n(x,y)\xi_n(t) \quad (5)$$

where $\delta_n(x,y) = \delta_{ni}(x)\delta_{nj}(y)$ and

$$\delta_{ni}(x) = \sin\frac{\gamma_{ni}}{L_x}x + A_{ni}\cos\frac{\gamma_{ni}}{L_x}x + B_{ni}\sinh\frac{\gamma_{ni}}{L_x}x + C_{ni}\cosh\frac{\gamma_{ni}}{L_x}x$$

$$\delta_{ni}(y) = \sin\frac{\gamma_{nj}}{L_y}y + A_{nj}\cos\frac{\gamma_{nj}}{L_y}y + B_{nj}\sinh\frac{\gamma_{nj}}{L_y}y + C_{nj}\cosh\frac{\gamma_{nj}}{L_y}y \quad (6)$$

The right hand side of Equation (4) when written in series form takes the form


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$$\sum_{n=1}^{\infty} R_o \left[\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)\right] - \frac{2\beta}{\mu}\frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4} W(x,y,t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4} W(x,y,t + [\kappa_n^2 - \frac{K_0}{\mu}] W(x,y,t) + \frac{G_0}{\mu}[\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] + \sum_{r=1}^{N} [\frac{M_r g}{\mu} H(x - c_r t) H(y-s) - \frac{M_r}{\mu}(\frac{\partial^2}{\partial t^2} W(x,y,t) + 2k_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + k_r^2 \frac{\partial^2}{\partial x^2} \quad W(x,y,t))\, H(x - k_r t) H(y-s) W(x,y,t)] = \sum_{n=1}^{N} \delta_n(x,y) \varrho_n(t) \quad (7)$$

Multiplying both sides of equation (8) by $\delta_m(x,y)$, integrating on area A of the plate and considering the orthogonality of $\delta_m(x,y)$, one gets

$$\varrho_n(t) = \frac{1}{\eta^*}\sum_{n=1}^{\infty} \int_A [R_0(\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)) - \frac{2\beta}{\mu}\frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4} W(x,y,t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4} W(x,y,t + (\kappa_n^2 - \frac{K_0}{\mu}) W(x,y,t) + \frac{G_0}{\mu}(\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)) + \sum_{r=1}^{N} [\frac{M_r g}{\mu} H(x - k_r t) H(y-s) - \frac{M_r}{\mu}(\frac{\partial^2}{\partial t^2} W(x,y,t) + 2k_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + k_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x - c_r t) H(y-s) W(x,y,t)] \delta_m(x,y) dA$$

(8)

and zero when $n \neq m$

where

$$\eta^* = \int_A \delta_n^2(x,y) dA \quad (9)$$

Making use of equation (6), equation (8), taking into account equation (4), can be written as

$$\delta_n(x,y)[\ddot{\xi}_n(t) + \kappa_n^2 \xi_n(t)] = \frac{\delta_n(x,y)}{\eta^*}\sum_{q=1}^{\infty} \int_A [R_0(\frac{\partial^2 \delta_q(x,y)}{\partial x^2} \delta_m(x,y) \ddot{\xi}_q(t) + \frac{\partial^2 \delta_q(x,y)}{\partial y^2} \delta_m(x,y) \ddot{\xi}_q(t)) - \frac{2\beta}{\mu}\frac{\partial^2 \delta_q(x,y)}{\partial x^2 \partial y^2} \delta_m(x,y) \xi_q(t) - \frac{D_x}{\mu}\frac{\partial^4 \delta_q(x,y)}{\partial x^4} \delta_m(x,y) \xi_q(t) - \frac{D_y}{\mu} \frac{\partial^4 \delta_q(x,y)}{\partial y^4} \delta_m(x,y) \xi_q(t) + (\kappa_n^2 - \frac{K_0}{\mu}) \delta_q(x,y) \delta_m(x,y) \xi_q(t) + \frac{G_0}{\mu}(\frac{\partial^2 \delta_q(x,y)}{\partial x^2} \delta_m(x,y) \xi_q(t) + \frac{\partial^2 \delta_q(x,y)}{\partial y^2} \delta_m(x,y) \xi_q(t)) + \sum_{r=1}^{N} (\frac{M_r g}{\mu} \delta_m(x,y) H(x - k_r t) H(y-s) - \frac{M_r}{\mu}(\Psi_q(x,y) \delta_m(x,y) \ddot{\xi}_q(t) + 2k_r \frac{\partial \delta_q(x,y)}{\partial x} \delta_m(x,y) \dot{\xi}_q(t) + k_r^2 \frac{\partial^2 \delta_q(x,y)}{\partial x^2} \delta_m(x,y) \xi_q(t)) H(x - k_r t) H(y-s))] dA$$

(10)

When equation (10) is simplified further, one obtains


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$$\ddot{\xi}_n(t)+\kappa_n^2\xi_n(t)=\frac{1}{\eta^*}\sum_{q=1}^{\infty}\int_A[R_0(\frac{\partial^2\delta_q(x,y)}{\partial x^2}\delta_m(x,y)\ddot{\xi}_q(t)+\frac{\partial^2\delta_q(x,y)}{\partial y^2}\delta_m(x,y)\ddot{\xi}_q(t)$$

$$)-\frac{2\beta}{\mu}\frac{\partial^2\delta_q(x,y)}{\partial x^2\partial y^2}\delta_m(x,y)\xi_q(t)-\frac{D_x}{\mu}\frac{\partial^4\delta_q(x,y)}{\partial x^4}\delta_m(x,y)\xi_q(t)-\frac{D_y}{\mu}\frac{\partial^4\delta_q(x,y)}{\partial y^4}\delta_m(x,y)$$

$$\xi_q(t)+(\kappa_n^2-\frac{K_0}{\mu})\delta_q(x,y)\delta_m(x,y)Q_q(t)+\frac{G_0}{\mu}(\frac{\partial^2\delta_q(x,y)}{\partial x^2}\delta_m(x,y)\xi_q(t)+\frac{\partial^2\delta_q(x,y)}{\partial y^2}$$

$$\delta_m(x,y)\xi_q(t))+\sum_{r=1}^{N}(\frac{M_rg}{\mu}\delta_m(x,y)H(x-k_rt)H(y-s)-\frac{M_r}{\mu}(\delta_q(x,y)\delta_m(x,y)\ddot{\xi}_q(t)$$

$$+2k_r\frac{\partial\delta_q(x,y)}{\partial x}\delta_m(x,y)\dot{\xi}_q(t)+k_r^2\frac{\partial^2\delta_q(x,y)}{\partial x^2}\delta_m(x,y)\xi_q(t))H(x-k_rt)H(y-s))]dA$$

(11)

The system of equations in equation (11) is a set of coupled ordinary differential equations

Making use of Fourier series representation, the Heaviside functions take the form

$$H(x-k_rt)=\frac{1}{4}+\frac{1}{\pi}\sum_{r=1}^{N}\frac{\sin(2n+1)\pi(x-k_rt)}{2n+1},0<x<1 \tag{12}$$

$$H(y-s)=\frac{1}{4}+\frac{1}{\pi}\sum_{r=1}^{N}\frac{\sin(2n+1)\pi(y-s)}{2n+1},0<y<1 \tag{13}$$

Substituting equations (12) and (13) into equation (11) and simplifying, one obtains

(14)

which is the transformed equation that governs the problem of an orthotropic rectangular plate resting on constant bi-parametric elastic foundation.

where

$$\tau_0=\int_A[\frac{\partial^2}{\partial x^2}\delta_q(x,y)\delta_m(x,y)+\frac{\partial^2}{\partial y^2}\delta_q(x,y)\delta_m(x,y)]dA \tag{15}$$

$$\varepsilon_1=\int_A\frac{\partial^2}{\partial x^2}[\frac{\partial^2}{\partial x^2}\delta_q(x,y)]\delta_m(x,y)dA \tag{16}$$

$$\varepsilon_2=\int_A\frac{\partial^4}{\partial x^4}[\delta_q(x,y)]\delta_m(x,y)dA \tag{17}$$

$$\varepsilon_3=\int_A\frac{\partial^4}{\partial y^4}[\delta_q(x,y)]\delta_m(x,y)dA \tag{18}$$

$$\varepsilon_4=\int_A\delta_q(x,y)\delta_m(x,y)dA \tag{19}$$

$$\varepsilon_5=\int_A[\frac{\partial^2}{\partial x^2}\delta_q(x,y)+\frac{\partial^2}{\partial y^2}\delta_q(x,y)]\delta_m(x,y)dA \tag{20}$$


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$$\varepsilon_6 = \frac{1}{16}\int_A \delta_q(x,y)\delta_m(x,y)dA \tag{21}$$

$$\tau_1^* = \int_A \delta_q(x,y)\delta_m(x,y)\sin(2j+1)\pi x dA \tag{22}$$

$$\tau_2^* = \int_A \delta_q(x,y)\delta_m(x,y)\cos(2j+1)\pi x dA \tag{23}$$

$$\tau_3^* = \int_A \delta_q(x,y)\delta_m(x,y)\sin(2k+1)\pi y dA \tag{24}$$

$$\tau_4^* = \int_A \delta_q(x,y)\delta_m(x,y)\cos(2k+1)\pi y dA \tag{25}$$

$$\tau_5^* = \tau_1^*, \quad \tau_6^* = \tau_2^*, \quad \tau_7^* = \tau_3^*, \quad \tau_8^* =_4^* \tag{26}$$

$$\varepsilon_7 = \frac{1}{16}\int_A \frac{\partial}{\partial x}\delta_q(x,y)\delta_m(x,y)dA \tag{27}$$

$$\tau_9^* = \int_A \frac{\partial}{\partial x}(\delta_q(x,y))\delta_m(x,y)\sin(2j+1)\pi x dA \tag{28}$$

$$\tau_{10}^* = \int_A \frac{\partial}{\partial x}(\delta_q(x,y))\delta_m(x,y)\cos(2j+1)\pi x dA \tag{29}$$

$$\tau_{11}^* = \int_A \frac{\partial}{\partial x}(\delta_q(x,y))\delta_m(x,y)\sin(2k+1)\pi y dA \tag{30}$$

$$\tau_{12}^* = \int_A \frac{\partial}{\partial x}\delta_q(x,y)\delta_m(x,y)\cos(2k+1)\pi y dA \tag{31}$$

$$\tau_{13}^* = \tau_9^*, \quad \tau_{14}^* = \tau_{10}^*, \quad \tau_{15}^* = \tau_{11}^*, \quad \tau_{16}^* = \tau_{12}^* \tag{32}$$

$$\varepsilon_8 = \frac{1}{16}\int_A \frac{\partial^2}{\partial x^2}(\delta_q(x,y))\delta_m(x,y)dA \tag{33}$$

$$\tau_{17}^* = \int_A \frac{\partial^2}{\partial x^2}(\delta_q(x,y))\delta_m(x,y)\sin(2j+1)\pi x dA \tag{34}$$

$$\tau_{18}^* = \int_A \frac{\partial^2}{\partial x^2}(\delta_q(x,y))\delta_m(x,y)\cos(2j+1)\pi x dA \tag{35}$$

$$\tau_{19}^* = \int_A \delta_q(x,y)\delta_m(x,y)\sin(2k+1)\pi y dA \tag{36}$$

$$\tau_{20}^* = \int_A \frac{\partial^2}{\partial x^2}(\delta_q(x,y))\delta_m(x,y)\cos(2k+1)\pi y dA \tag{37}$$

$$\tau_{21}^* = \tau_{17}^*, \quad \tau_{22}^* = \tau_{18}^*, \quad \tau_{23}^* = \tau_{19}^*, \quad \tau_{24}^* = \tau_{20}^* \tag{38}$$

$\delta_m(x,y)$ is assumed to be the products of functions $\delta_{jm}(x)\delta_{wm}(y)$ which are the beam functions in the directions of x and y axes respectively. That is

$$\delta_m(x,y) = \delta_{jm}(x)\delta_{wm}(y) \tag{39}$$

where

$$\delta_{jm}(x) = \sin\lambda_{jm}x + A_{jm}\cos\lambda_{jm}x + B_{jm}\sinh\lambda_{jm}x + C_{jm}\cosh\lambda_{jm}x$$
$$\delta_{wm}(y) = \sin\lambda_{wm} + A_{wm}\cos\lambda_{wm}y + B_{wm}\sinh\lambda_{wm}y + C_{wm}\cosh\lambda_{wm}y$$


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(40)

where $A_{jm}$, $B_{jm}$, $C_{jm}$, $A_{jm}$, $B_{jm}$ and $C_{jm}$ are constants determined by the boundary conditions while $\delta_{jm}$ and $\delta_{wm}$ are called the mode frequencies

where

$$\lambda_{jm} = \frac{\zeta_{jm}}{L_x}, \qquad \lambda_{wm} = \frac{\zeta_{wm}}{L_y} \tag{41}$$

Considering a unit mass, equation (19) can be re-expressed as

$$\ddot{\xi}_n(t) + \kappa_n^2 \xi_n(t) - \frac{1}{\eta^*}\sum_{q=1}^{\infty} [R_0\varepsilon_0\ddot{\xi}_q(t) - \frac{2\beta}{\mu}\varepsilon_1\xi_q(t) - \frac{D_x}{\mu}\varepsilon_2\xi_q(t) - \frac{D_y}{\mu}\varepsilon_3\xi_q(t) + (\kappa_n^2 - \frac{K_0}{\mu}\varepsilon_4)\xi_q(t) + \frac{G_0}{\mu}\varepsilon_5\xi_q(t) - \alpha\sigma((\varepsilon_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi kt}{2j+1} - \sum_{j=1}^{\infty}\tau_2^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_8^*$$

$$\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{\xi}_q(t) + 2k_r(\varepsilon_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}))\dot{\xi}_q(t)$$

$$+c^2(\varepsilon_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty}\tau_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}))\xi_q(t))]$$

$$= \sum_{q=1}^{\infty}\frac{Mg}{\mu\eta^*}\delta_m(ct)\delta_m(s)$$

(42)

equation (42) is the fundamental equation of the rectangular plate problem. where

$$\alpha = \frac{M}{\mu\sigma}, \quad \sigma = L_x L_y \tag{43}$$

$$\delta_m(ct) = \sin\phi_m(t) + A_m\cos\phi_m(t) + B_m\sinh\phi_m(t) + C_m\cosh\phi_m(t) \tag{44}$$

$$\delta_m(s) = \sin\iota_m + A_m\cos\iota_m + B_m\sinh\iota_m + C_m\cosh\iota_m \tag{45}$$

$$\phi_m = \frac{\gamma_m c}{L_x}, \ \iota_m = \frac{\gamma_m s}{L_y} \tag{46}$$

**Orthotropic Rectangular Plate Traversed by a Moving Force**


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In moving force, we account for only the load being transferred to the structure. In this case, the inertia effect is negligible. Setting $\phi = 0$ in the fundamental equation (43), one obtains

$$\ddot{\xi}_n(t) + (1 - \frac{\varepsilon_4}{\mu\eta^*})\kappa_n^2\xi_n(t) - \frac{1}{\mu\eta^*}[\mu R_0\varepsilon_0\ddot{\xi}_n(t) - 2\beta\varepsilon_1\xi_n(t) - D_x\varepsilon_2\xi_n(t) - D_y\varepsilon_3\xi_n(t) - K_0\varepsilon_4\xi_n(t) + G_0\varepsilon_5\xi_n(t) + \sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) + (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t))] = \frac{Mg}{\mu\eta^*}\delta_m(ct)\delta_m(s) \tag{47}$$

which is further be simplified as

$$\ddot{\xi}_n(t) + \Omega_n^2\xi_n(t) - \Psi[\mu R_0\varepsilon_0\ddot{\xi}_n(t) - 2\beta\varepsilon_1\xi_n(t) - D_x\varepsilon_2\xi_n(t) - D_y\varepsilon_3\xi_n(t) - K_0\varepsilon_4\xi_n(t) + G_0\varepsilon_5\xi_n(t) + \sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) + (\mu\Omega_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t))] = \Psi Mg\delta_m(ct)\delta_m(s) \tag{48}$$

where $\Omega_n^2 = (1 - \frac{\varepsilon_4}{\mu\eta^*})\kappa_n^2$, $\Psi = \frac{1}{\mu\eta^*}$

On expanding and re-arranging equation (48), one obtains

$$[1 - \Psi\mu R_0\varepsilon_0]\ddot{\xi}_n(t) + (\Omega_n^2 - \Psi J_6)\xi_n(t) - \Psi\sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) + (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t)) = \Psi Mg\delta_m(ct)\delta_m(s) \tag{49}$$

Simplifying further, one obtains

$$\ddot{\xi}_n(t) + \frac{(\Omega_n^2 - \Psi J_6)}{[1 - \Psi\mu R_0\varepsilon_0]}\xi_n(t) + \frac{\Psi}{[1 - \Psi\mu R_0\varepsilon_0]}\sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3 Q_q(t) + (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t)) = \frac{\Psi Mg}{[1 - \Psi\mu R_0 T_0]}\delta_m(ct)\delta_m(s) \tag{50}$$

where

$$J_6 = -2\beta\varepsilon_1 - D_x\varepsilon_2 - D_y\varepsilon_3 - K_0\varepsilon_4 + G_0\varepsilon_5 \tag{51}$$

For any arbitrary ratio $\Psi$, defined as

$\Psi^* = \frac{\Psi}{1+\Psi}$, one obtains

$$\Psi = \frac{\Psi^*}{1 - \Psi^*} = \Psi^* + o(\Psi^{*2}) + \ldots$$

For only $o(\Psi^*)$, one obtains

$$\Psi = \Psi^*$$


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On application of binomial expansion,

$$\frac{1}{1-\Psi^*\mu R_0\varepsilon_0} = 1 + \Psi^*\mu R_0 T_0 + o(\Psi^{*2}) + \ldots \tag{52}$$

On putting equation (52) into equation (50), one obtains

$$\ddot{\xi}_n(t) + (\Omega_n^2 - \Psi^* J_6)(1 + \Psi^*\mu R_0\varepsilon_0 + o(\Psi^{*2}) + \ldots)\xi_n(t) + \Psi^*(1 + \Psi^*\mu R_0\varepsilon_0 + o(\Psi^{*2}) + \ldots)\sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) + (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t)) = \Psi^* aMg(1 + \Psi^*\mu R_0 T_0 + o(\Psi^{*2}) + \ldots)\delta_m(ct)\delta_m(s)$$

(53)

Retaining only $o(\Psi^*)$, equation (54) becomes

$$\ddot{\xi}_n(t) + [\Omega_n^2(1 + \Psi^*\mu R_0 T_0) - \Psi^* J_6]\xi_n(t) + \Psi^*\sum_{q=1,q\neq n}^{\infty}(\mu R_0 T_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) + (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t)) = \Psi^* Mg\delta_m(ct)\delta_m(s)$$

(54)

which is simplified further as

$$\ddot{\xi}_n(t) + \Omega_n^2\xi_n(t) + \Psi^*\sum_{q=1,q\neq n}^{\infty}(\mu R_0\varepsilon_0\ddot{\xi}_q(t) - 2\beta\varepsilon_1\xi_q(t) - D_x\varepsilon_2\xi_q(t) - D_y\varepsilon_3\xi_q(t) \; (\mu\kappa_q^2 - K_0\varepsilon_4)\xi_q(t) + G_0\varepsilon_5\xi_q(t)) = \Psi^* Mg\delta_m(ct)\delta_m(s)$$

(55)

where

$$J_7 = [\Omega_n^2(1 + \Psi^*\mu R_0\varepsilon_0) - \Psi^* J_6] \tag{56}$$

Using Struble's technique, one obtains

$$\Omega_{nn} = \Omega_n - \left(\frac{\Omega_n^2 - J_7}{2\Omega_n}\right) \tag{57}$$

which is the modified frequency for moving force problem.

Using equation (57), the homogeneous part of equation (55) can be written as

$$\ddot{\xi}_n(t) + \Omega_{nn}^2\xi_n(t) = 0 \tag{58}$$

Hence, the entire equation (55) gives

$$\ddot{\xi}_n(t) + \Omega_{nn}^2\xi_n(t) = \Psi^* Mg\delta_m(ct)\delta_m(s) \tag{59}$$

On solving equation (59) one obtains

$$\xi_n(t) = \frac{Mg\Psi^*\delta_m(s)}{\Omega_{nn}(\phi_m^4 - \Omega_{nn}^4)}[(\phi_m^2 + \Omega_{nn}^2)(\phi_m\sin\Omega_{nn}t - \Omega_{nn}\sin\alpha_m t) - A_m\Omega_{nn}(\phi_m^2 + \Omega_{nn}^2)(\cos\phi_m t - \cos\Omega_{nn}t) - B_m(\phi_m^2 - \Omega_{nn}^2)(\phi_m\sin\Omega_{nn}t - \Omega_{nn}\sinh\phi_m t) + C_m\Omega_{nn}(\phi_m^2 - \Omega_{nn}^2)(\cosh\phi_m t - \cos\Omega_{nn}t)]$$

(60)


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on making use of equation (5) one obtains

$$W(x,y,t) = \sum_{jm=1}^{\infty}\sum_{wm=1}^{\infty}\frac{Mg\Psi^*\delta_m(s)}{\Omega_{nn}(\phi_m^4-\Omega_{nn}^4)}[(\phi_m^2+\Omega_{nn}^2)(\phi_m\sin\Omega_{nn}t-\Omega_{nn}\sin\alpha_m t)-A_m\Omega_{nn}(\phi_m^2+\Omega_{nn}^2)(\cos\phi_m t-\cos\Omega_{nn}t)-B_m(\phi_m^2-\Omega_{nn}^2)(\phi_m\sin\Omega_{nn}t-\Omega_{nn}\sinh\phi_m t)+C_m\Omega_{nn}(\phi_m^2-\Omega_{nn}^2)(\cosh\phi_m t-\cos\Omega_{nn}t)](\sin\frac{\zeta_{jm}}{L_x}x+A_{jm}\cos\frac{\zeta_{jm}}{L_x}x+B_{jm}\sinh\frac{\zeta_{jm}}{L_x}x+C_{jm}\cosh\frac{\zeta_{jm}}{L_x}x)(\sin\frac{\zeta_{wm}}{L_y}y+A_{wm}\cos\frac{\zeta_{wm}}{L_y}y+B_{wm}\sinh\frac{\zeta_{wm}}{L_y}y+C_{wm}\cosh\frac{\zeta_{wm}}{L_y}y) \quad (61)$$

which is the transverse displacement response to a moving force problem of orthotropic rectangular plate.

**Orthtropic Rectangular Plate Traversed by a Moving Mass**

In moving mass problem, the moving load is assumed rigid, and the weight and as well as inertia forces are transferred to the moving load. That is the inertia effect is not negligible. Thus $\alpha \neq 0$ and so it is required to solve the entire equation (42).To solve the equation,one employs analytical approximate method. This method is known as an approximate analytical method of Struble. The homogeneous part of equation (42) shall be replaced by a free system operator defined by the modified frequency $\xi_{nn}$. Thus, the entire equation becomes

$$\ddot{\xi}_n(t)+\Omega_{nn}^2\xi_n(t)+\alpha\Theta^*\sum_{q=1}^{\infty}[(\varepsilon_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{\xi}_q(t)+2k_r(\varepsilon_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}))\dot{\xi}_q(t)+c^2(\varepsilon_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{18}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}))\xi_q(t)]=\sum_{q=1}^{\infty}\frac{Mg}{\mu\eta^*}\delta_m(ct)\delta_m(s) \quad (62)$$

where $\Theta^* = \frac{\sigma}{\eta^*}$

On expanding and simplifying equation (62), one obtains


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$$
\begin{aligned}
&\ddot{\xi}_n(t) + \Omega_{nn}^2\xi_n(t) + \alpha\Theta^*[(\varepsilon_6 + \tfrac{1}{\pi^2}(\textstyle\sum_{j=1}^{\infty} \tau_1^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_2^* \\
&\tfrac{\sin(2j+1)\pi k_r t}{2j+1})(\textstyle\sum_{k=1}^{\infty} \tau_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_5^* \\
&\tfrac{\cos(2j+1)\pi k_r t}{2j+1} - \textstyle\sum_{j=1}^{\infty} \tau_6^* \frac{\sin(2j+1)\pi k_r}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_8^* \\
&\tfrac{\sin(2k+1)\pi s}{2k+1}))\ddot{\xi}_n(t) + 2k_r(\varepsilon_7 + \tfrac{1}{\pi^2}(\textstyle\sum_{j=1}^{\infty} \tau_9^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_{10}^* \frac{\sin(2j+1)\pi k_r t}{2j+1} \\
&)(\textstyle\sum_{k=1}^{\infty} \tau_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} \tau_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_{13}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} \\
&-\textstyle\sum_{j=1}^{\infty} \tau_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{16}^* \frac{\sin(2k+1)\pi s}{2k+1} \\
&))\dot{\xi}_n(t) + k_r^2(\varepsilon_8 + \tfrac{1}{\pi^2}(\textstyle\sum_{j=1}^{\infty} \tau_{17}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_{18}^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) \\
&(\textstyle\sum_{k=1}^{\infty} \tau_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_{21}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} \\
&-\textstyle\sum_{j=1}^{\infty} \tau_{22}^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \\
&))\xi_n(t)] + \alpha\Theta^* \textstyle\sum_{q=1,q\neq n}^{\infty} [(\varepsilon_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} \tau_1^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_2^* \\
&\tfrac{\sin(2j+1)\pi k_r t}{2j+1})(\textstyle\sum_{k=1}^{\infty} \tau_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_5^* \\
&\tfrac{\cos(2j+1)\pi k_r t}{2j+1} - \textstyle\sum_{j=1}^{\infty} \tau_6^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_8^* \\
&\tfrac{\sin(2k+1)\pi s}{2k+1}))\ddot{\xi}_q(t) + 2k_r(\varepsilon_7 + \tfrac{1}{\pi^2}(\textstyle\sum_{j=1}^{\infty} \tau_9^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_{10}^* \frac{\sin(2j+1)\pi k_r t}{2j+1} \\
&)(\textstyle\sum_{k=1}^{\infty} \tau_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} \tau_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_{13}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} \\
&-\textstyle\sum_{j=1}^{\infty} \tau_{14}^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{16}^* \frac{\sin(2k+1)\pi s}{2k+1} \\
&))\dot{\xi}_q(t) + k_r^2(\varepsilon_8 + \tfrac{1}{\pi^2}(\textstyle\sum_{j=1}^{\infty} \tau_{17}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty} \tau_{18}^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) \\
&(\textstyle\sum_{k=1}^{\infty} \tau_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} \tau_{21}^* \frac{\cos(2j+1)\pi k_r t}{2j+1} \\
&-\textstyle\sum_{j=1}^{\infty} \tau_{22}^* \frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \\
&))\xi_q(t)] = \Psi^* M g \delta_m(ct)\delta_m(s)
\end{aligned} \tag{63}
$$

On further rearrangements and simplifications,one obtains



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$$(1+\alpha\Theta^*(\varepsilon_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_2^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*$$
$$\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-$$
$$\sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi s}{2k+1})$$
$$))\ddot{\xi}_n(t)+2k_r\alpha\Theta^*(\varepsilon_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}$$
$$)(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}$$
$$-\sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}$$

$$))\dot{\xi}_n(t)+(\Omega_{nn}^2+\alpha\Theta^*k_r^2(\varepsilon_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{18}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})$$
$$(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}$$
$$-\sum_{j=1}^{\infty}\tau_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}$$
$$)))\xi_n(t)+\alpha\Theta^*\sum_{q=1,q\neq n}^{\infty}[(\varepsilon_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_2^*$$
$$\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*$$
$$\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_8^*$$

$$\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{\xi}_q(t)+2k_r(\varepsilon_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}$$
$$)(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}$$
$$-\sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}$$
$$))\dot{\xi}_q(t)+k_r^2(\varepsilon_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}-\sum_{j=1}^{\infty}\tau_{18}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})$$

$$(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1}$$
$$-\sum_{j=1}^{\infty}\tau_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}$$
$$))\xi_q(t)]=\Psi^*Mg\delta_m(ct)\delta_m(s)$$

(64)


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On further expression of equation (64), one obtains

$$\ddot{\xi}_n(t) + 2k_r\alpha\Theta^*(\varepsilon_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + (\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}))\dot{\xi}_n(t) + (\Omega_{nn}^2(1 - \alpha\Theta^*(\varepsilon_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_2^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + (\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi k_r t}{2k+1})))) + k_r^2\alpha\Theta^*(\varepsilon_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{18}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}E_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + (\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}))\xi_n(t)$$
$$+\alpha\Theta^*\sum_{q=1,q\neq n}^{\infty}[(\varepsilon_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_1^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_2^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_3^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_4^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_5^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_6^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + (\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi k_r t}{2k+1}))\ddot{\xi}_q(t) +$$

$$2c(\varepsilon_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_9^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{10}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{11}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{12}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{13}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{14}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + (\sum_{k=1}^{\infty}\tau_{15}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{16}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}))\dot{\xi}_q(t) + c^2(\varepsilon_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty}\tau_{17}^*$$

$$\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi k_r t}{2j+1})(\sum_{k=1}^{\infty}\tau_{19}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{20}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty}\tau_{21}^*\frac{\cos(2j+1)\pi k_r t}{2j+1} - \sum_{j=1}^{\infty}\tau_{22}^*\frac{\sin(2j+1)\pi k_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi k_r t}{2k+1}))\xi_q(t)] = \Psi^* Mg\delta_m(k_r t)\delta_m(s) \quad (65)$$

Making use of modified asymptotic method of Struble, equation (65) can be rewritten as for homogeneous case

$$\ddot{\xi}_n(t) + \omega_n^2\xi_n(t) = 0 \quad (66)$$

Hence, the entire equation becomes

$$\ddot{\xi}_n(t) + \omega_n^2\xi_n(t) = \Psi^* Mg\delta_m(k_r t)\delta_m(s) \quad (67)$$

where


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$$\omega_n = \Omega_{nn} - \frac{1}{2\Omega_{nn}}(\Omega_{nn}^2 \alpha \Theta^*(\varepsilon_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} \tau_8^* \frac{\sin(2k+1)\pi s}{2k+1}))$$
$$-k_r^2 \alpha \Theta^*(\varepsilon_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} \tau_{23}^* \frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty} \tau_{24}^* \frac{\sin(2k+1)\pi k_r t}{2k+1})))$$
(68)

which gives the modified frequency representing the frequency of the free system.

Rewriting equation (67), one obtains

$$\ddot{\xi}_n(t) + \omega_n^2 \xi_n(t) = \Psi^* Mg\delta_m(s)[\sin\phi_m(t) + A_m \cos\phi_m t + B_m \sinh\phi_m t + C_m \cosh\phi_m t]$$
(69)

Following the procedures applied to solve equation (59), one obtains

$$\xi_n(t) = \frac{\Psi^* Mg\delta_m(s)}{\_n(\phi_m^4 - \omega_n^4)}[(\phi_m^2 + \omega_n^2)(\phi_m \sin\omega_n t - \omega_n \sin\phi_m t) - A_m \omega_n(\phi_m^2 + \omega_n^2)(\cos\phi_m t$$
$$-\cos\omega_n t) - B_m(\phi_m^2 - \omega_n^2)(\phi_m \sin\omega_n t - \omega_n \sinh\phi_m t) + C_m \omega_n(\phi_m^2 - \omega_n^2)(\cosh\phi_m t$$
$$-\cos\omega_n t)]$$
(70)

On making use of equation (5)

$$W(x,y,t) = \sum_{jm=1}^{\infty} \sum_{wm=1}^{\infty} \frac{\Psi^* Mg\delta_m(s)}{\_n(\phi_m^4 - \omega_n^4)}[(\phi_m^2 + \omega_n^2)(\phi_m \sin\omega_n t - \omega_n \sin\phi_m t) - A_m \omega_n(\phi_m^2$$
$$+\omega_n^2)(\cos\phi_m t - \cos\omega_n t) - B_m(\phi_m^2 - \omega_n^2)(\phi_m \sin\omega_n t - \omega_n \sinh\phi_m t) + C_m \omega_n(\phi_m^2$$
$$-\omega_n^2)(\cosh\phi_m t - \cos\omega_n t)](\sin\frac{\zeta_{jm}}{L_x}x + A_{jm}\cos\frac{\zeta_{jm}}{L_x}x + B_{jm}\sinh\frac{\zeta_{jm}}{L_x}x + C_{jm}$$
$$\cosh\frac{\zeta_{jm}}{L_x}x)(\sin\frac{\zeta_{wm}}{L_y}y + A_{wm}\cos\frac{\zeta_{wm}}{L_y}y + B_{wm}\sinh\frac{\zeta_{wm}}{L_y}y + C_{wm}\cosh\frac{\zeta_{wm}}{L_y}y)$$
(71)

which is the transverse displacement response to a moving mass of an orthotropic rectangular plate.

## RESULTS/FINDINGS

### Orthotropic Rectangular Plate with Simple Elastic Boundary Conditions

For an orthotropic rectangular plate with simple elastic boundary conditions, the plate is simply supported at $x = 0$, $x = L_x$ and elastically supported at $y = 0$, $y = L_y$ the boundary conditions are expressed below

$$W(0,y,t) = 0 = \frac{\partial^2}{\partial x^2} W(0,y,t) \quad (72)$$

at the end $x = 0$ and


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$$\frac{\partial^2}{\partial^2 x}W(L_x,y,t)-\rho_1\frac{\partial}{\partial x}W(L_x,0,t)=0=\frac{\partial^3}{\partial^3 x}W(L_x,0,t)+\rho_2 W(L_x,0,t) \quad (73)$$

at the other end $x=L_x$

$$W(x,0,t)=0=\frac{\partial W(x,L_y,t)}{\partial y} \qquad \frac{\partial^2 W(x,0,t)}{\partial^2 y}=0=\frac{\partial^3 W(x,L_y,t)}{\partial^3 y} \quad (74)$$

Thus,for the normal modes

$$\zeta_{jm}(0)=0=\frac{\partial^2\zeta_{jm}(0)}{\partial x^2} \quad (75)$$

at $x=0$ and

$$\frac{\partial^2}{\partial^2 x}\zeta_{jm}(L_x)-\rho_1\frac{\partial}{\partial x}\zeta_{jm}(L_x)=0=\frac{\partial^3}{\partial^3 x}\zeta_{jm}(L_x)+\rho_2\zeta_{jm}(L_x) \quad (76)$$

at the other end $x=L_x$ Making use of equations (75) and (76) in equation (40), it can be shown that

$$B_{jm}=\frac{\varphi_1\cos\zeta_{jm}-\frac{\zeta_{jm}}{L_x}\sin\zeta_{jm}}{\frac{\zeta_{jm}}{L_x}\sinh\zeta_m-\varphi_1\cos\zeta_{jm}}=\frac{\frac{\zeta_{jm}^3}{L_x^3}\cos\zeta_{jm}-\sin\zeta_{jm}}{\frac{\zeta_{jm}^3}{L_{jm}^3}\cosh\zeta_{jm}+\varphi_1\sinh\zeta_{jm}} \quad (77)$$

$$B_{wm}=\frac{\varphi_1\cos\zeta_{wm}-\frac{\zeta_{wm}}{L_x}\sin\zeta_{wm}}{\frac{\zeta_{wm}}{L_x}\sinh\zeta_{wm}-\varphi_1\cos\zeta_{wm}}=\frac{\frac{\zeta_{wm}^3}{L_x^3}\cos\zeta_{wm}-\sin\zeta_{wm}}{\frac{\zeta_{wm}^3}{L_{wm}^3}\cosh\zeta_{wm}+\varphi_1\sinh\zeta_{wm}} \quad (78)$$

In the same vein, we have

$$B_m=\frac{\varphi_1\cos\zeta_m-\frac{\zeta_m}{L_x}\sin\zeta_m}{\frac{\zeta_m}{L_x}\sinh\zeta_m-\varphi_1\cos\zeta_m}=\frac{\frac{\zeta_m^3}{L_x^3}\cos\zeta_m-\sin\zeta_m}{\frac{\zeta_m^3}{L_m^3}\cosh\zeta_m+\varphi_1\sinh\zeta_m} \quad (79)$$

and

$$A_{jm}=0=C_{jm}, \quad A_{wm}=0=C_{wm}, \quad \Rightarrow A_m=0=C_m \quad (80)$$

From equation (79) , one obtains

$$\tan\zeta_m=\tanh\zeta_m \quad (81)$$

which is termed the frequency equation for the dynamical problem, such that

$$\zeta_1=3.927, \quad \zeta_2=7.069, \quad \zeta_3=10.210,\ldots \quad (82)$$

On using equations (79), (80) and (82) in equations (61) and (71), one obtains the displacement response to a moving force and a moving mass of orthotropic rectangular plate with simple elastic end conditions and resting on bi-parametric condition respectively.


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## DISCUSSION

### DISCUSSION OF THE ANALYTICAL SOLUTIONS

For this undamped system, it is expedient to examine the phenomenon of resonance. From equation (61), it is explicitly shown that the orthotropic rectangular plate with simple elastic end conditions and resting on constant elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance whenever

$$\phi_m = \Omega_{nn} \tag{83}$$

while equation (71) shows that the same orthotropic rectangular plate with simple elastic end conditions and resting on constant elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance when

$$\phi_m = \omega_n \tag{84}$$

where

$$\omega_n = \Omega_{nn} - \frac{1}{2\Omega_{nn}}(\Omega_{nn}^2\alpha\Theta^*(\varepsilon_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi s}{2k+1})) \\ -k_r^2\alpha\Theta^*(\varepsilon_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi k_r t}{2k+1})))$$
(85)

Comparing equations (84) and (85), one obtains

$$\vartheta_n = \Omega_{nn}[1 - \frac{1}{2\Omega_{nn}^2}(\Omega_{nn}^2\alpha\Theta^*(\varepsilon_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_7^*\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty}\tau_8^*\frac{\sin(2k+1)\pi s}{2k+1})) \\ -k_r^2\alpha\Theta^*(\varepsilon_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty}\tau_{23}^*\frac{\cos(2k+1)\pi k_r t}{2k+1} - \sum_{k=1}^{\infty}\tau_{24}^*\frac{\sin(2k+1)\pi k_r t}{2k+1})))] = \Omega_{nn}$$
(86)

### GRAPHICAL REPRESENTATION OF VARYING PARAMETERS

To illustrate the analysis presented in this work, orthotropic rectangular plate is taken to be of length $L_y = 0.923m$, breadth $L_x = 0.432m$ the load velocity c = 0.8123 m/s and $s = 0.4m$.The results are presented on the various graphs below for the simply supported boundary conditions.

Figures 3.1 and 3.2 display the effect of foundation modulus, $\mathbf{K_o}$ on the deflection profile of orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of foundation modulus $\mathbf{K_o}$ increases.


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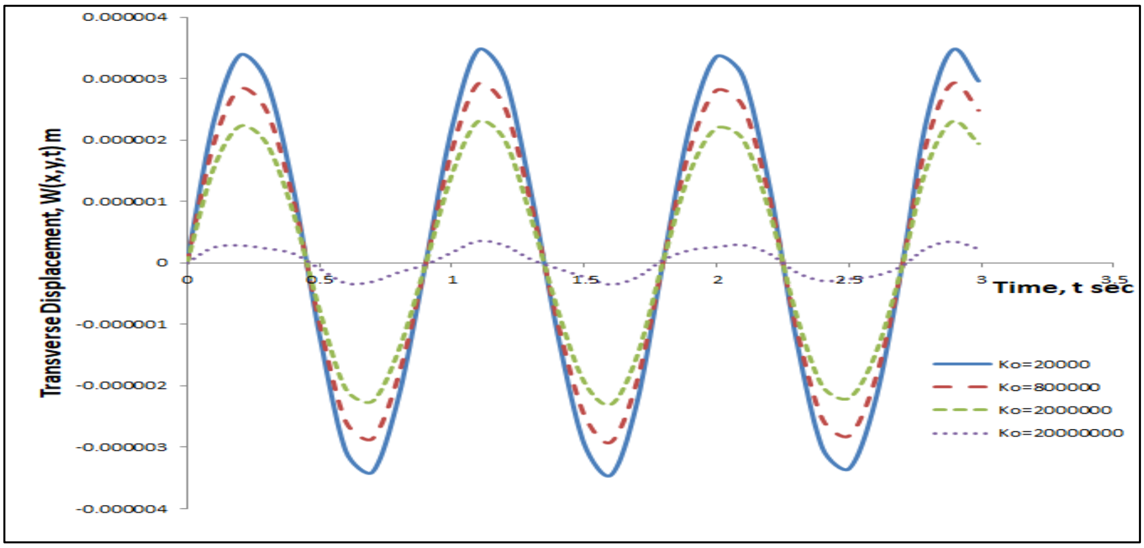


**Figure 1: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $K_o$ and Traversed by Moving Force**

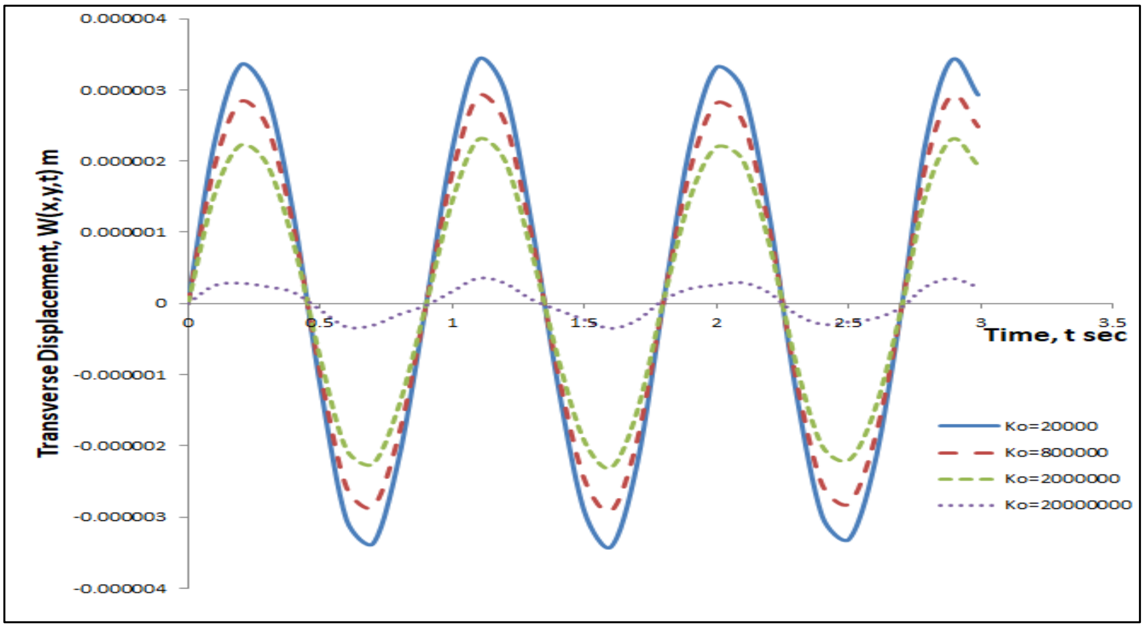


**Figure 2: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $K_o$ and Traversed by Moving Mass**

Figures 3.3 and 3.4 display the effect of shear modulus, $G_o$ on the deflection profile of the orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of shear modulus $G_o$ increases.


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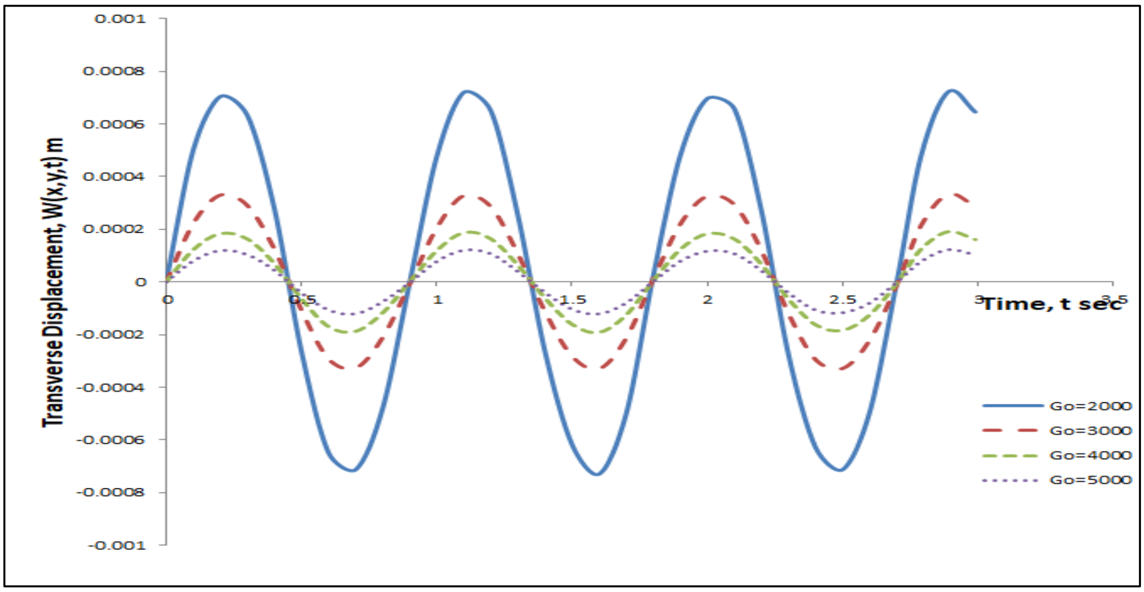


**Figure 3: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $G_o$ and Traversed by Moving Force**

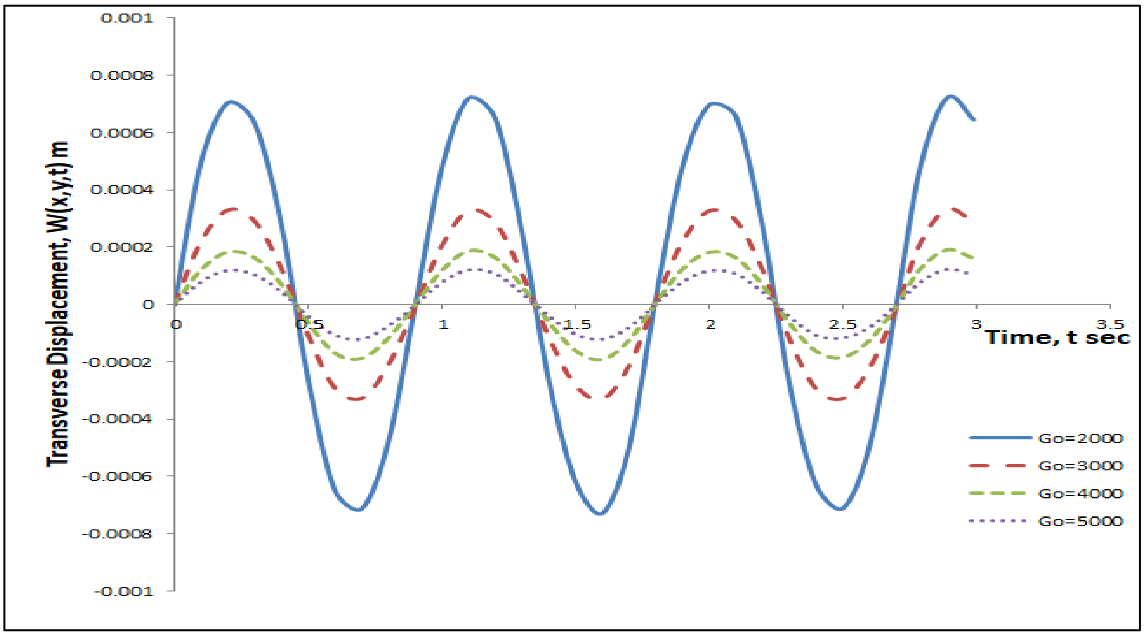


**Figure 4: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $G_o$ and Traversed by Moving Mass**

Figures 3.5 and 3.6 display the effect of rotatory inertia, $R_o$ on the deflection profile of the orthotropic rectangular plate under the action of load moving at constant velocity in both cases


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of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the $R_o$ increases.

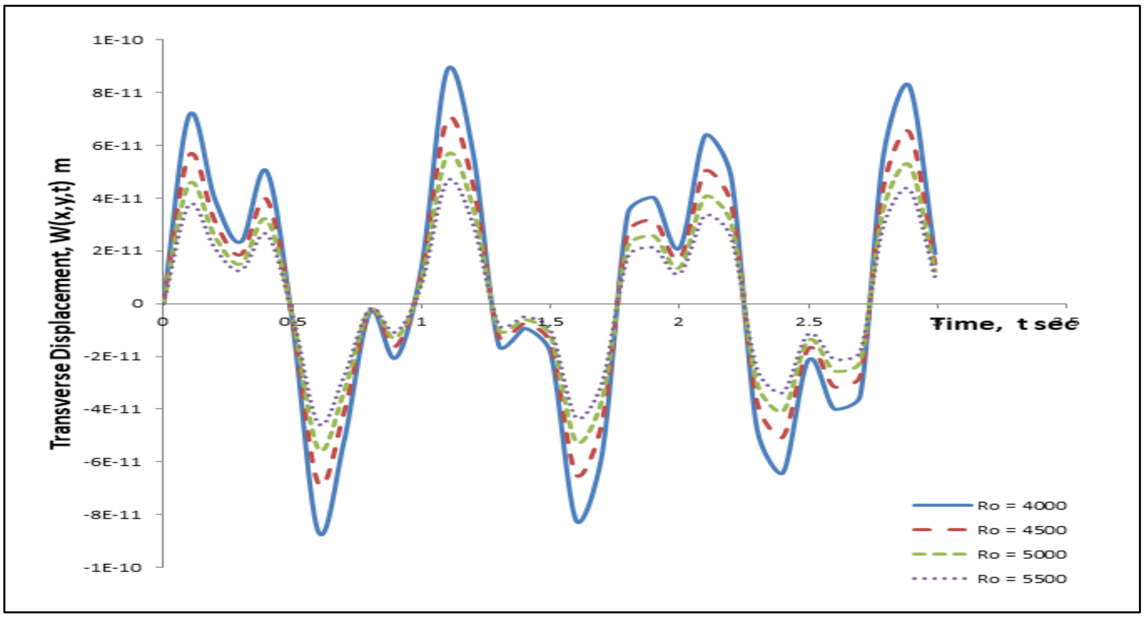


**Figure 5: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $R_o$ and Traversed by Moving Force**

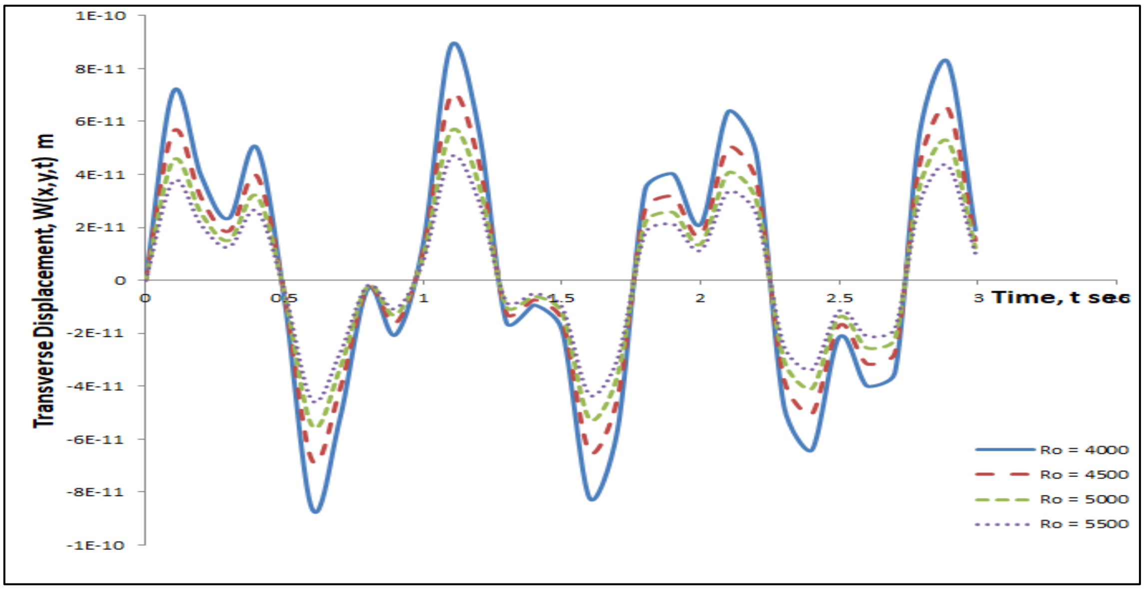


**Figure 6: Displacement Profile of Simple Elastic Orthotropic Rectangular Plate with Varying $R_o$ and Traversed by Moving Mass**


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## IMPLICATION TO RESEARCH AND PRACTICE

### IMPLICATION TO RESEARCH

#### 1. Advanced Foundation-Structure Interaction Modeling

The research can be extended to an advanced foundation-structure interaction modeling. This will investigate the interaction between structural plates and two-parameter (bi-parametric) foundations which may be typically represented by Winkler and Pasternak or Vlasov models.

This will also enable researchers to analyze how shear layer stiffness (second parameter) influences vibration characteristics, which is crucial for dynamic analysis under transient loads.

#### 2. Exploration of Boundary Condition Effects

Consideration of simple elastic end conditions bridges the gap between idealized (clamped, simply supported) and real-life support scenarios. This can promote research into semi-rigid supports and their realistic impact on dynamic response.

#### 3. Parametric Studies and Optimization

The research can also be extended parametric studies and optimization. This facilitates parametric investigations of the effects of varying foundation stiffness, shear modulus, and elastic edge stiffness on natural frequencies, mode shapes, and deflection profiles. This can contribute to optimization-based research thus minimizing vibration or deflection by tuning foundation or edge stiffness.

#### 4. Extension to Multi-Physics and Smart Materials

This framework can be extended to smart plates (e.g., piezoelectric, magneto-elastic) and multi-field coupling problems, offering new frontiers for multi-physics FEM modeling. This opens up research into adaptive foundation systems using variable stiffness materials.

#### 5. Extension to Numerical Methods

Future research can consider extension to numerical methods (e.g., FEM, spectral methods, Chebyshev collocation). This serves as a benchmark problem for developing and validating numerical methods. It also enables researchers to test accuracy, convergence, and stability of computational solvers in dynamic plate analysis.

### IMPLICATIONS TO ENGINEERING PRACTICE

#### 1. Structural Health and Vibration Control

The research can be extended to structural health and vibration control. This could inform engineers about how foundation stiffness affects vibration behavior, critical for the design of floors, pavements, bridge decks, and machine foundations. The will be useful in diagnosing problems like excessive deflection or resonance and proposing retrofitting strategies (e.g., foundation reinforcement or support stiffening).


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**2. Realistic Support Modeling**

Engineers often deal with supports that are not ideally pinned or clamped; incorporating simple elastic conditions yields more accurate predictions of structural behavior under dynamic loads.

**3. Foundation Design and Material Selection**

This will offer guidance on choosing foundation parameters (e.g., modulus of subgrade reaction, shear layer stiffness) for structures resting on soils, foams, or layered substrates. It will also help tailor the type and depth of foundation system to control deflection and frequency response in critical installations.

**4. Seismic and Impact Load Applications**

Understanding dynamic behavior under different stiffness conditions is vital in seismic engineering, impact mitigation, and blast-resistant design. It will help in designing resilient infrastructure on varying soil types or layered media.

**5. Smart Foundation Implementation**

Future research would encourage the integration of variable stiffness supports (e.g., magneto rheological or piezoelectric foundations) that adjust dynamically to reduce vibration or extend structural life.

## CONCLUSION

The dynamic influence of foundation stiffness on the rectangular structural plate with constant bi-parametric elastic foundation with simple elastic end conditions has been examined in this research article. The closed form solutions to the fourth order partial differential equations with variable and singular coefficients governing the orthotropic rectangular plates has been obtained for both cases of moving force and moving mass using the solution technique adopted by Shadnam et al. [11], which was used to remove the singularity in the governing fourth order partial differential equation, thereby reducing it to a sequence of second order ordinary differential equations. After making use of the modified asymptotic technique of Struble and Laplace transformation, the analytical solution is obtained. The solutions are then analyzed. From the analyses, it was evident that for the same natural frequency, the critical speed for moving mass problem is smaller than that of moving force problem. Hence, resonance is attained earlier in the moving mass system than in the moving force system. The results in the plotted curves show that an increase in rotatory inertia correction factor $R_o$, foundation modulus $K_o$ and shear modulus $G_o$ resulted in a decrease in the amplitudes of the plates for both cases of moving force and moving mass problems. It is also depicted in the curves that response amplitude of moving mass problem is greater than that of moving force problem, which implies that resonance is reached earlier in moving mass problem than in moving force problem of dynamic influence of foundation stiffness on rectangular structural plate, with constant bi-parametric elastic foundation with simple elastic end conditions.

## FUTURE RESEARCH


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The above research could be extended to the tabulated future research directions as depicted below.

| S/N | Future Research Area | Focus |
|---|---|---|
| 1 | Nonlinear Foundation | Soil plasticity, large deflection, bifurcation |
| 2 | Smart Foundation | Adaptive stiffness, control strategies |
| 3 | Complex Loadings | Seismic, moving, or impact loads |
| 4 | Variable Geometry | Functionally graded materials (FGMs), tapered, or perforated plates |
| 5 | Numerical Methods | Finite element method (FEM), Chebyshev collocation method (CCM), High-order solvers, reduced order models (ROMs), meshfree methods |
| 6 | Experimental Validation | Lab setups, field data, vibration tests |
| 7 | Soil-structure Interaction | Multi-layer soil, (Soil-structure interaction) SSI effects |
| 8 | Damage/health monitoring | Crack detection, SHM techniques |
| 9 | Micro/nano electromechanical systems(MEMS/NEMS) and bio-plates | Cross-disciplinary applications |
| 10 | Multi-physics | Thermoelastic, hygroelastic effects |

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