



IMPACT AND COLLISION DYNAMICS MODELING AND SIMULATION FOR ROAD SAFETY IN PAPUA NEW GUINEA

Dismas Raimbas and Mohsen Aghaeiboorkheili*

School of Mathematics & Computer Science,
Papua New Guinea University of Technology, Lae, Papua New Guinea.

*Corresponding Author's Email: mohsen.aghaeiboorkheili@pnguot.ac.pg

Cite this article:

Rambas, D.,
Aghaeiboorkheili, M. (2025),
Impact and Collision
Dynamics Modeling and
Simulation for Road Safety in
Papua New Guinea. African
Journal of Mathematics and
Statistics Studies 8(3), 96-112.
DOI: 10.52589/AJMSS-
HPJPFBVW

Manuscript History

Received: 21 Jun 2025

Accepted: 28 Jul 2025

Published: 7 Aug 2025

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ABSTRACT: *This project explores the mathematical modeling of impact and collision dynamics with a focus on enhancing vehicle and road safety in Papua New Guinea (PNG). Using fundamental principles from classical mechanics - particularly the impulse-momentum theorem, conservation of momentum, and kinetic energy analysis - this study demonstrates how real-world vehicle collisions can be analyzed and predicted through mathematical equations. A central scenario models a 1000 kg car decelerating from 20 m/s to rest, showing how varying the impact duration significantly changes the force experienced: a crash over 0.2 seconds results in a 100,000 N force, while extending that to 0.3 seconds reduces the force to 66,667 N. These calculations highlight how small changes in crash parameters, modeled mathematically, have major effects on safety outcomes. This mathematical framework provides a low-cost, accessible alternative to physical crash testing, making it especially valuable in PNG where safety infrastructure and testing resources are limited. The project reinforces the role of applied mathematics as a powerful tool in engineering design, public safety planning, and real-world problem-solving.*

KEYWORDS: Momentum; Impulse; Force; Kinetic; Coefficient of restitution (COR); Inelastic collision; Crumple zone; Impact duration.



INTRODUCTION

The primary aim of this project was to improve the safety of Public Motor Vehicles (PMVs), particularly the Toyota Coaster buses that are widely used for public transportation in PNG. These vehicles are frequently involved in severe road crashes due to their high occupancy rates and the lack of modern safety features. Enhancing PMV crashworthiness could significantly reduce fatalities and injuries in the country (Kullgren & Krafft, 2000). Although there are various types of vehicle collisions (e.g., head-on, rear-end, side-impact, and rollover crashes), this project focuses specifically on frontal impacts as a simplified case study. This allows for the clear application of fundamental principles of classical mechanics. Due to the inaccessibility of detailed, localized crash data in Papua New Guinea, real-world figures aren't available to further validate and strengthen the modeling. However, the theoretical framework presented remains applicable across many scenarios (Lubbe et al., 2018). Impact and collision dynamics is a branch of classical mechanics that investigates how forces, momentum, and energy behave during interactions between moving bodies—particularly in high-speed incidents like vehicle crashes. While traditionally approached from a physics or engineering perspective, this field offers rich opportunities for the application of mathematics, especially in modeling and simulating real-world events without relying on costly or impractical experiments (Batista, 2005).

As of 2021, PNG has a road traffic mortality rate of 14.9 deaths/100 000 population according to WHO. This still is a major public issue, even though it is slightly below the global average (WHO, 2023). Road safety is a huge concern in PNG. This is because of poor infrastructure, limited regulation, and the lack of important safety features such as airbags, crumple zones and helmets (Organization, 2009). These factors make it difficult to conduct crash testing or implement quality engineering solutions. However, through mathematical modelling, a practical and accessible alternative can be achieved. That is, using only calculations and data analysis, crash scenarios can be simulated, risk injuries can be estimated, and safety can be improved by using the equations that govern motion, force, momentum, and energy (Hibbeler, 2004).

This project will show how applied mathematics can be used to understand and predict the outcomes of vehicular collisions. This project uses the impulse-momentum theorem, conservation laws, and energy loss equations to estimate forces under different crash conditions, through a series of scenarios (Pawlus, Nielsen, Karimi, & Robbersmyr, 2010). The main idea of this project is that the more the duration of the impact time is, the less the impact force will be. This is a key principle behind many safety designs such as crumple zones. Thus, this project focuses on using basic yet powerful mathematical tools instead of relying on crash-test facilities, to extract essential insights and actionable safety improvements for PNG's Road users (Walker, 2014).

The main question this paper seeks to answer is: *“How can mathematical modeling of collision dynamics be used to make improvements to the safety of public transport vehicles in PNG?”*



METHODOLOGY

This project uses a few fundamental equations from classical mechanics to model vehicle collision dynamics. The aim is to explore how varying impact durations affect collision force and to simulate realistic crash conditions for PMV-type vehicles, particularly the Toyota Coaster. The methodology includes the following key elements:

□ Conservation of Linear Momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

The law of conservation of momentum states that the total linear momentum of a closed system remains constant if no external forces act on it (Scurlock, 2014). This equation is crucial for analyzing both elastic and inelastic collisions and forms the mathematical foundation for tracking how momentum is distributed between colliding bodies.

Application: Helps model how vehicles behave during and after collisions, especially when two objects interact.

□ Impulse-Momentum Theorem

$$F \cdot \Delta t = m(v_f - v_i) \rightarrow F = \frac{m \cdot \Delta v}{\Delta t}$$

Impulse is the product of force and the time over which it acts. This equation reveals a direct inverse relationship between force and duration of impact (Shigley, Mitchell, & Saunders, 1985). A longer collision duration results in a smaller force.

Derivation:

Starting with Newton's Second Law: $F = \frac{d(mv)}{dt}$

For Constant mass: $F = m \frac{dv}{dt}$

Multiplying both sides by Δt : $F\Delta t = m(v - u)$

This gives the impulse-momentum theorem: $F\Delta t = \Delta p$

Where:

- F : average force
- Δt : duration of impact
- u, v : initial and final velocities
- Δp : change in momentum

□ Coefficient of Restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The coefficient of restitution measures the elasticity of a collision, where:

- $e=1$: perfectly elastic (no kinetic energy lost),
- $e=0$: perfectly inelastic (maximum energy loss).

Application: Useful in estimating how much of the kinetic energy is retained or absorbed during impact.



□ Kinetic Energy Loss

$$\Delta KE = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2$$

Kinetic energy is not conserved in inelastic collisions (Hollowell, 2024). This equation quantifies how much energy is lost - transformed into heat, deformation, or sound. It's vital for evaluating the severity of a collision and the amount of energy that safety features need to absorb (Huang, 2002).

Derivation:

Initial kinetic energy: $KE_{initial} = \frac{1}{2}mv^2$

Final kinetic energy: $KE_{final} = \frac{1}{2}mv'^2$

Energy lost: $\Delta KE = KE_{initial} - KE_{final} = \frac{1}{2}m(v^2 - v'^2)$

Application: Provides insight into how vehicle structures and materials can absorb crash energy and protect occupants.

Assumptions: All crash scenarios are modeled as either completely inelastic or partially elastic collisions. Road friction and air resistance are neglected for simplification.

Scenario Selection: Two core scenarios are analyzed:

- A 1000 kg car decelerating from 20 m/s to rest.
- A 4500 kg Toyota Coaster bus colliding at 25 m/s.

Variable Manipulation: The time of impact varies from 0.1s to 0.5s to observe changes in force.
Tools: Calculations were conducted using algebraic manipulation and verified with a calculator.

RESULTS

Modern vehicle safety designs include crumple zones, airbags, and helmets. This section applies mathematical modeling to analyze the forces involved in vehicle collisions under varying crash conditions (Jones, Structural impact, 2011). Rather than relying on physical testing, we use idealized but realistic scenarios, simplifying real-world collisions into calculable models. This approach demonstrates how applied mathematics can predict crash outcomes and guide safer design decisions.

Consider this basic scenario:

- Vehicle mass: 1000 kg
- Initial velocity: 20 m/s (72 km/h)
- Final velocity: 0 m/s (car comes to rest)
- Impact duration tested: 0.1 to 0.5 seconds

We assume a completely inelastic collision, where the vehicle does not bounce back, and analyze how impact time affects the force experienced by the vehicle occupants.

Impulse – Momentum Calculation

Using the impulse-momentum theorem:

$$F = \frac{mv - mu}{\Delta t} = \frac{1000 \times 20}{\Delta t} = \frac{20,000}{\Delta t}$$

Table 1 and Graph 1

Crash Duration (s)	Force (N)
0.1	200,000
0.2	100,000
0.3	66,667
0.4	50,000
0.5	40,000

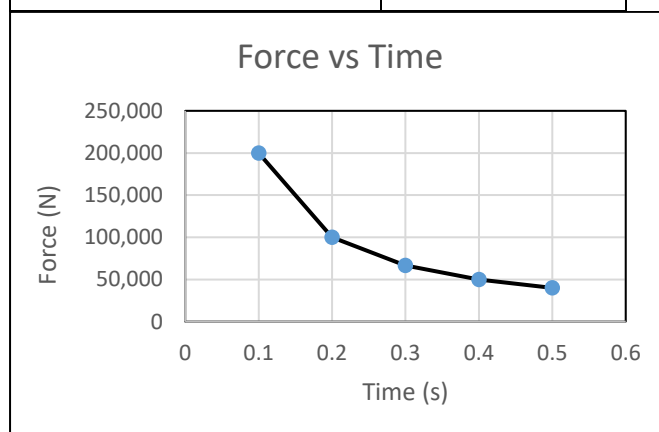
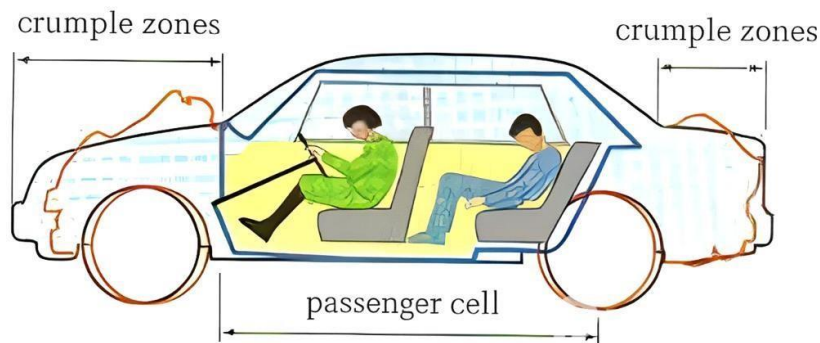


Table 1 and graph 1 shows that increasing impact time reduces the peak force, exactly what safety features like airbags and crumple zones are designed to achieve.

This graph can be used as a mathematical visualization of the impulse concept, helping designers estimate how structural features influence safety



(Downloaded from the internet) Figure 1. Crumple zone effect. Deformation increases time and reduces peak force during a crash.

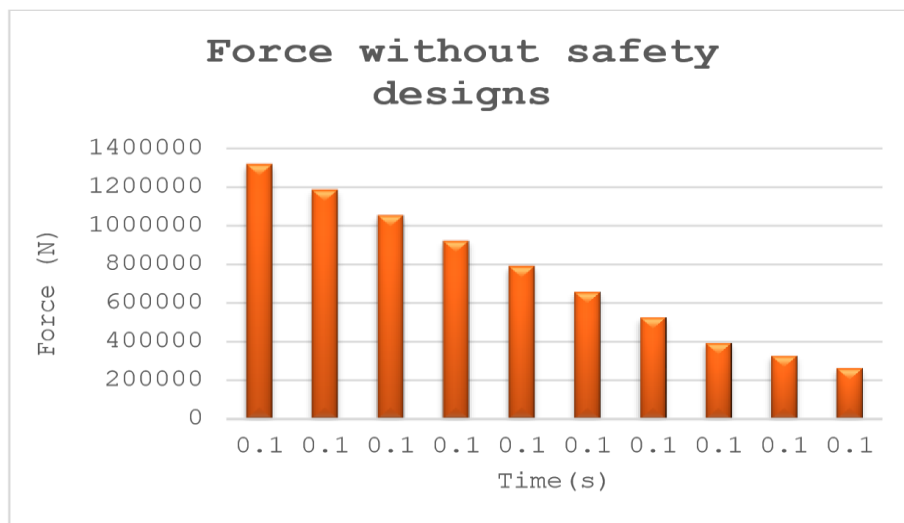
Sensitivity to Time

The above examples illustrate that impact duration is a highly sensitive parameter. Small increases in crash time result in large reductions in force, a key design consideration in safety engineering (Jones & Wierzbicki, Structural crashworthiness, 1983). More examples (theoretical) are shown below.

Table 2 & Graph 2:

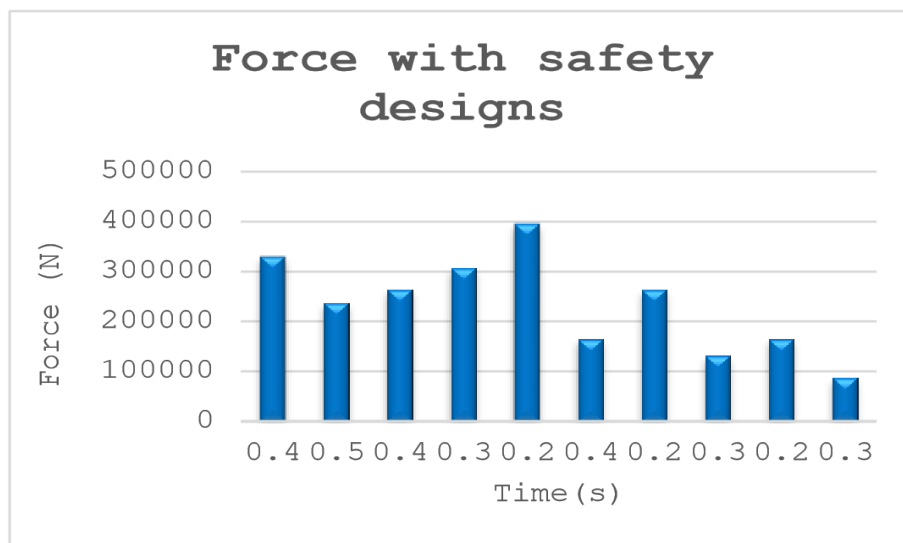
Case	Mass (kg)	Velocity (m/s)	Time (s)	Force (N)
1	4750	27.78 (100 km/h)	0.1	1,319,550
2	4750	25 (90 km/h)	0.1	1,187,500
3	4750	22.22 (80 km/h)	0.1	1,055,450
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8	4750	8.33 (30 km/h)	0.1	395,675
9	4750	6.94 (25 km/h)	0.1	329,650
10	4750	5.56 (20 km/h)	0.1	264,100

The Table 2 above shows 10 theoretical impact scenarios/cases for a vehicle with a mass of 4750 kg. In each case, the vehicle travels at different velocities, however, the impact time is the same. Less impact time means the vehicle has no safety designs, resulting in greater impact force.

**Table 3 & Graph 3:**

Case	Mass (kg)	Velocity	Time (s)	Force (N)
1	4750	27.78 (100 km/h)	0.4	329,888
2	4750	25 (90 km/h)	0.5	237,500
3	4750	22.22 (80 km/h)	0.4	263,863
4	4750	19.44 (70 km/h)	0.3	307,800
5	4750	16.67 (60 km/h)	0.2	395,913
6	4750	13.89 (50 km/h)	0.4	164,944
7	4750	11.11 (40 km/h)	0.2	263,863
8	4750	8.33 (30 km/h)	0.3	131,892
9	4750	6.94 (25 km/h)	0.2	164,825
10	4750	5.56 (20 km/h)	0.3	88,033

Table 3 above also shows 10 theoretical impact scenarios/cases for a vehicle with a mass of 4750 kg. In each case, the velocities remain the same as in Table 2, however, the impact time is now increased. More impact time means the vehicle has safety designs, thus decreasing the impact force.



Note: If you compare Graph 2 with Graph 3, you can see that even a 0.1-second increase in impact time significantly reduces peak force.

Real-World Application in Papua New Guinea

Papua New Guinea (PNG) faces serious road safety issues due to poor infrastructure, limited enforcement of safety standards, and widespread use of public transport vehicles that lack essential protective features.

Mathematical modeling will be used to demonstrate how simple calculations can quantify risks and propose safety improvements - even in low-resource environments like PNG.



(Downloaded from the internet) Figure 2: Toyota Coaster Bus Used in PNG

The vehicle pictured is a typical public transport model among the most common vehicles, frequently used for passenger transport along high-speed, poorly maintained highways in the



country. The average mass of this bus with passengers (along with cargoes) ranges from 4,400 kg – 4,700 kg. Due to high occupancy, poor road conditions, and lack of advanced safety features like airbags and crumple zones, collisions involving this type of vehicle often result in severe injuries.

Consider these scenarios:

A Toyota coaster bus with a mass of 4,500 Kg, traveling at 90 km/h (25 m/s) along the highway between Lae and the Highlands, and somehow due to some unknown causes, the bus crashes into an object (either another vehicle or a tree).

Calculating the impulse:

$$\Delta p = m \cdot \Delta v$$

where:

- $m = 4500 \text{ kg}$ (mass of the bus)
- $\Delta v = v_{\text{final}} - v_{\text{initial}} = 0 - 25 = -25 \text{ m/s}$

Thus, the impulse is:

$$\Delta p = 4500 \times (-25) = -112,500 \text{ kg. m/s}$$

The negative sign shows the momentum is decreasing (slowing down), but for force magnitude, we take the absolute value: $|\Delta p| = 112,500 \text{ kg. m/s}$

Case A: No Crumple Zone (Crash Duration = 0.1 seconds)

Now, calculating the **average force** using the impulse-momentum theorem:

$$F = \frac{\Delta p}{\Delta t}$$

Where:

- $\Delta p = 112,500 \text{ kg. m/s}$
- $\Delta t = 0.1 \text{ s}$

$$F = \frac{112,500}{0.1} = 1,125,000 \text{ N}$$

Force = 1.125 million newton.

This extremely high force would be delivered over a very short time, resulting in catastrophic injuries or fatalities, especially since most Coaster buses lack proper crash protection (e.g., airbags, crumple zones, seatbelts).



(Downloaded from the internet) **Figure 3.** Toyota Coaster bus crash

Case B: With Crumple Zone or Airbag (Crash Duration = 0.4 seconds)

Here, assuming the same change in momentum, the **crash duration is extended to 0.4 seconds** due to safety design.

$$F = \frac{112,500}{0.4} = 281,250 \text{ N}$$

Force = 281,250 newton

The total energy lost remains the same because the speed and mass haven't changed. This energy is still absorbed by the vehicle's structure, but because the crash lasts longer, it's **spread out over more time**, reducing the **peak forces**.

This is a 75% reduction in peak force compared to Scenario A. The longer duration allows energy to be absorbed more gradually, significantly improving survivability by reducing the force experienced by passengers.

Kinetic Energy Loss

$$KE = \frac{1}{2}mv^2$$

$$KE_{initial} = \frac{1}{2} \times 4500 \times (25)^2 = 1,406,250 \text{ (J)}$$

Case C: PMV Bus Collides with Sedan.

A 4500 kg PMV bus travels at 25 m/s and collides with a stationary 1200 kg sedan. Assuming a perfectly inelastic collision:

$$(m_1 \times v_1) + (m_2 \times v_2) = (m_1 + m_2) \times v_f \text{ Where:}$$

- $m_1 = 4500 \text{ kg} \quad v_1 = 25 \text{ m/s}$
- $m_2 = 1200 \text{ kg}$



$$\bullet \quad v_2 = 0 \text{ m/s}$$

$$(4500 \times 25) + (1200 \times 0) = 5700 \times v_f$$

$$112,500 = 5700 \times v_f$$

$$v_f = \frac{112,500}{5700} = 19.74 \text{ m/s}$$

The combined vehicles would continue moving at 19.74 m/s after impact. This result is useful for evaluating the remaining kinetic energy in the system and the potential for secondary collisions or rollovers.

Coefficient of Restitution (COR):

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Where:

$$\bullet \quad v_2 = 10 \text{ m/s (sedan after collision)} \quad \square$$

$$v_1 = 20 \text{ m/s (PMV after collision)}$$

$$\bullet \quad u_1 = 25 \text{ m/s}$$

$$\bullet \quad u_2 = 0 \text{ m/s}$$

A COR of 0.4 indicates a **partially elastic** collision, which is common in vehicle crashes. It helps engineers estimate energy retention and rebound and informs vehicle material design.

$$10 - 20$$

$$e = \frac{10 - 20}{25 - 0} = -0.4$$

$$e = 0.4$$

Kinetic Energy Loss:

Initial Kinetic Energy:

$$KE_i = \frac{1}{2} \times 4500 \times 25^2 = 1,406,250 \text{ J}$$

Final Kinetic Energy:

$$KE_f = \frac{1}{2} \times 5700 \times 19.74^2 = 1,110,929 \text{ J}$$

Energy Lost:

$$\Delta KE = KE_i - KE_f = 1,406,250 - 1,110,929 = 295,321 \text{ J}$$



Nearly 295,321 joules of energy are lost to heat, sound, and deformation. This value shows how violent the crash is and why structural safety measures are essential, especially for high-occupancy PMVs in PNG.

DISCUSSION

The results of this study strongly confirm the usefulness of applied mathematics in analyzing and improving vehicle safety - particularly in low-resource settings like Papua New Guinea. Using a small set of fundamental physical equations, we modeled the effects of impact duration on collision force, demonstrating how simple calculations can lead to life-saving insights.

In particular, the key tool for understanding crash severity was the impulse-momentum theorem. Our models showed that if the impact duration was decreased from 0.3 seconds to 0.2 seconds, the impact force was increased by 33%. In real-life, a change like that could mean the difference between survivable and fatal injuries. When applied to larger vehicles, such as the Toyota Coaster bus scenario, the model revealed an alarming force of 1.125 million Newton in a 0.1-second crash - surely no human can survive that magnitude. However, increasing the impact duration to 0.4 seconds reduced this force by 75%, such a great improvement achievable through safety design choices like crumple zones and airbags.

Additional calculations provided further depth to the analysis:

- Conservation of momentum helped determine the post-collision velocity in a two-vehicle scenario, showing how the PMV and a sedan would move together at 19.74 m/s after impact.
- The coefficient of restitution ($e = 0.4$) indicates a partially elastic collision, aligning with typical real-world crashes. Understanding COR is crucial in estimating rebound speeds and analyzing collision severity.
- The modeled crash with a loss of approximately 295,321 joules of kinetic energy shows how much destructive energy is transferred into deformation and heat. This indicates the need for structural features that absorb energy and protect passengers.

These calculations show the meaningful use of mathematical modeling in real-world safety analysis and understanding the risks faced by public transport users in PNG. The simulation results confirm the key role of impact duration in determining how bad a collision is. Safety features extend impact time and reduce peak force while keeping total energy loss constant. This insight supports the implementation of crumple zones, airbags, and improved seat belt designs.

Mathematical modelling is a cost-effective substitute for crash testing in low-resource settings. It enables analysis of dangerous scenarios as well as helping to make safety designs and policy decisions. For PNG, where public vehicles are frequently involved in accidents and safety measures are often lacking, adopting modeling-based safety analysis could lead to significant improvements in road safety.



From a practical standpoint, this project reassures that applied mathematics is more than theoretical - it is a cost effective, accessible tool for engineers, safety analysts, and policymakers. This makes math not just academically useful, but socially vital.

Moreover, the sensitivity analysis explains more on how impact force responds to even small changes in time. This adds up to the importance of designing systems that increase deceleration time by any means possible.

CONCLUSION

This project clearly shows how basic principles of classical mechanics can be applied mathematically to study vehicle collisions and improve road safety, especially in a developing country like PNG. Instead of physical testing, this project focuses more on mathematical modelling. In doing so, crash scenarios were generated and simulated, and impact forces were calculated, from which the results were used to evaluate the effectiveness of safety features like crumple zones.

The key idea is clear; that is, the more the duration of the impact time is, the less the impact force will be, even though the total energy loss remains constant. For example, in a 1000 kg vehicle crash, increasing the impact time from 0.2 to 0.3 seconds reduces the force from 100,000 N to 66,667 N (a reduction by approximately 33%). In the case of a public transport bus traveling at 90 km/h, lengthening the crash duration from 0.1 to 0.4 seconds reduces the force by 75%. These results show how small safety design choices can save lives.

This project has shown how abstract equations can be used to solve real problems. It demonstrates how one can move from physical principles to mathematical models, and from models to real-world insights. It also strengthens the appreciation for mathematics as a practical, impactful tool - not only in theory, but in the service of public safety and engineering design.

Perhaps most importantly, the work shows that there's no need for a laboratory to understand or improve safety. With a few well-chosen equations and realistic assumptions, mathematical modeling can guide decision-making, policy, and design in places where resources are limited.

Going forward, applied mathematics has a major role to play in shaping safer, smarter systems - especially in environments like PNG where affordable solutions are critical. This project is just one example of how math can make a measurable difference.



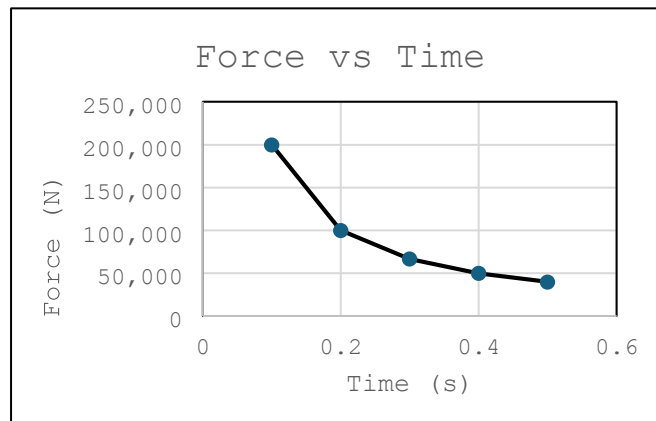
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Appendix A: Table 1 and Graph 1

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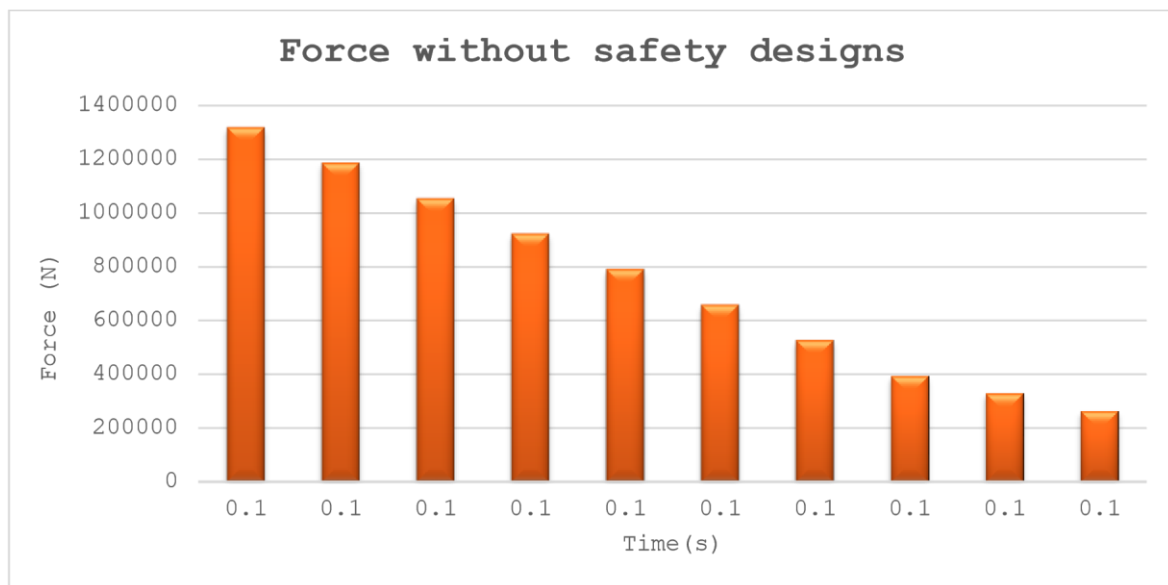
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**Appendix B: Sensitivity to Time****Table 2 & Graph 2:**

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