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EVALUATION OF ERROR DISTRIBUTIONS IN MULTIVARIATE GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (MGARCH) MODELS: DYNAMIC CONDITIONAL CORRELATION

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ABSTRACT: *This study investigates the impact of alternative error distributions on the performance of the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH)–Dynamic Conditional Correlation (DCC) model in analysing volatility dynamics among Nigerian financial assets. Using daily data on Nigerian stock prices, Brent crude oil prices, and the All-Share Index from March 2017 to May 2025, the MGARCH-DCC(1,1) models were estimated under four distributions—Normal, Student-t, Skewed Student-t, and Generalized Error Distribution (GED). The results indicate significant volatility persistence across all assets, with beta coefficients ranging from 0.678 to 0.875, confirming long memory in conditional variances. The ARCH coefficients (α_1) ranged between 0.113 and 0.322, signifying substantial short-term volatility reactions to shocks. The DCC parameters (a_1 = 0.013–0.018 and b_1 = 0.658–0.702) reveal that correlations evolve gradually, reflecting a strong persistence in co-movement among the markets. Furthermore, the shape parameters for the t and GED distributions (e.g., v = 3.17–6.01) confirm heavy tails, while the skewness parameters close to 1.00 indicate mild asymmetry. Overall, the Skewed Student-t and Student-t error distributions outperform the Gaussian model, effectively capturing volatility clustering and fat-tailed behavior in Nigerian financial data. These findings highlight the importance of selecting appropriate error structures for accurate volatility modelling, forecasting, and risk assessment in emerging markets.*

KEYWORDS: MGARCH-DCC, Error Distributions, Volatility Persistence, Time-Varying Correlation, Nigerian Financial Assets.

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INTRODUCTION

Volatility modelling is a cornerstone of modern financial econometrics, particularly for understanding the dynamics of risk and uncertainty in financial markets. Empirical studies have shown that financial time series such as stock returns, exchange rates, and commodity prices exhibit volatility clustering, leptokurtosis, and time-varying correlations features inconsistent with the assumptions of constant variance and independence in classical linear models (Mandelbrot, 1963; Fama, 1965). To address these issues, Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, which allows conditional variance to depend on past squared errors. Bollerslev (1986) later generalized this framework into the Generalized ARCH (GARCH) model, making it more flexible for capturing persistent volatility patterns observed in financial data.

While univariate GARCH models are effective for single time series, they cannot adequately represent the interdependence among multiple financial assets. This limitation led to the development of multivariate GARCH (MGARCH) models that account for time-varying conditional correlations among series (Bollerslev, Engle, & Wooldridge, 1988; Engle & Kroner, 1995). Among the MGARCH variants, the Dynamic Conditional Correlation (DCC-MGARCH) model proposed by Engle (2002) has gained wide acceptance due to its ability to capture dynamic co-movements between assets while maintaining parsimony in estimation. The DCC-MGARCH framework is particularly valuable for applications such as portfolio optimization, risk management, and contagion analysis in global financial markets (Cappiello, Engle, & Sheppard, 2006).

However, the performance of DCC-MGARCH models might be highly sensitive to the choice of the assumed error distribution. Many empirical applications rely on the normality assumption, yet financial returns are typically characterized by fat tails, excess kurtosis, and skewness, which violate this assumption (Cont, 2001; Harvey & Siddique, 1999). Such misspecification can lead to biased parameter estimates, inaccurate volatility forecasts, and misleading inferences about market dynamics (Bauwens, Laurent, & Rombouts, 2006). To address this, alternative error distributions such as the Student-t, skewed Student-t, and Generalized Error Distribution (GED) have been proposed to better capture the heavy-tailed and asymmetric nature of financial returns (Fernandes, Medeiros, & Scharth, 2014; Haas, Mittnik, & Paolella, 2004).

Evaluating the performance of DCC-MGARCH models under different error distributions is therefore essential for improving the reliability of volatility forecasts and correlation estimates. Studies have shown that incorporating fat-tailed and skewed distributions significantly enhances the fit and predictive accuracy of MGARCH models (Theodossiou, 1998; Ardia, Hoogerheide, & van Dijk, 2019). In emerging markets, such as Nigeria, where financial returns tend to be more volatile and less predictable, proper specification of the error distribution becomes even more critical for effective policy formulation and risk assessment (Salisu & Isah, 2017).

Given the increasing integration of global financial markets and the susceptibility of asset returns to shocks, there is a growing need for robust volatility modeling frameworks that accurately reflect empirical data characteristics. Mis-specifying the error distribution within the DCC-MGARCH framework may lead to underestimation or overestimation of market risk,

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with far-reaching implications for investment strategies, asset pricing, and financial stability (Brooks, 2014; Francq & Zakoïan, 2019).

Therefore, this study aims to evaluate the impact of different error distributions on the estimation and performance of the DCC-MGARCH model, focusing on their effects on volatility dynamics, conditional correlations, and forecasting accuracy. The findings will contribute to the empirical literature on multivariate volatility modelling and offer practical insights for financial analysts, policymakers, and econometricians interested in improving risk modelling and decision-making in financial markets.

LITERATURE/THEORETICAL UNDERPINNING

The Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) framework has evolved as a powerful tool for modelling the volatility dynamics and interdependence among multiple financial time series. Building on the foundations of the ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models, the MGARCH family particularly the Dynamic Conditional Correlation (DCC) model introduced by Engle (2002) captures time-varying correlations between assets in a computationally efficient manner. The DCC-MGARCH model has been widely applied to study market co-movements, contagion effects, and portfolio diversification across global and regional markets (Cappiello, Engle, & Sheppard, 2006; Chiang, Jeon, & Li, 2007). These studies confirm that the DCC framework provides valuable insights into volatility transmission and dynamic linkages, making it a preferred approach for risk management and financial forecasting.

However, empirical evidence consistently shows that financial returns exhibit heavy tails and skewness, which violate the assumption of normally distributed errors traditionally used in MGARCH modeling (Cont, 2001; Theodossiou, 1998). To address this, several studies have incorporated alternative error distributions such as the student-t, skewed-t, and Generalized Error Distribution (GED), which better capture the leptokurtic and asymmetric nature of financial data (Bauwens, Laurent, & Rombouts, 2006; Haas, Mittnik, & Paolella, 2004). These alternative specifications have been found to improve model fit, volatility forecasts, and correlation estimates (Ardia, Hoogerheide, & van Dijk, 2019). Despite this progress, limited research has systematically compared the performance of different error distributions within the DCC-MGARCH framework, especially in emerging markets like Nigeria. This gap underscores the need for empirical evaluation of how distributional assumptions influence volatility modelling, risk assessment, and financial decision-making.

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METHODOLOGY

The Multivariate GARCH Models

The Multivariate GARCH (MGARCH) model extends the univariate GARCH framework to analyse time-varying volatilities, covariances, and correlations across multiple financial series. Unlike the Constant Conditional Correlation (CCC) model, which assumes fixed correlations, the Dynamic Conditional Correlation (DCC) model by Engle (2002) allows correlations to evolve over time, capturing dynamic interdependencies among assets. Further variants, such as VARMA-GARCH and factor MGARCH, incorporate autoregressive and latent factor structures to explain volatility transmission. Overall, MGARCH models are essential for studying co-movement, risk propagation, and volatility forecasting in finance, with broad applications in portfolio management and risk analysis. The general form of an MGARCH model can be represented as:

$$r_t = \mu_t + \varepsilon_t \quad (1)$$

$$\varepsilon_t = H_t^{\frac{1}{2}} Z_t \quad (2)$$

where:

r_t $= n \times 1$ vector of log returns of n assets at time t.

$$\mu_t = is\ the\ vector\ of\ mean\ returns\ (or\ a\ constant)$$

$\varepsilon_t = n \times 1$ is the vector of residuals (errors) at time t,

$E(\varepsilon_t) = 0 \;\; Cov(a_t) = H_t$.

$\mu_t = n \times 1$ vector of the expected value of the conditional R_t.

$H_t = n \times n$ matrix of conditional variances at time t.

$H_t^{\frac{1}{2}}$ = Any $n \times n$ matrix at time t such that $H_t$is the conditional variance matrix of a_t. $H_t^{\frac{1}{2}}$ could be gotten by Cholesky factorization of H_t

$\varepsilon_t = H_t^{\frac{1}{2}} Z_t$ where Z_t is a vector of independent and identically distributed errors (often assumed to be normal or t-distributed), and H_t is the conditional covariance matrix at time ttt.

Z_t is an $n \times 1$ vector of iid errors such that $E(Z_t) = 0$ and $E(Z_t Z_t{}') = 1$

This study employed the Dynamic Conditional Correlation (DCC) model to analyze the volatility and interdependence among multiple financial assets, such as Nigerian stock returns, the All Share Index (ASI), and crude oil prices. The DCC model's ability to capture time-varying correlations makes it ideal for understanding evolving relationships in markets affected by domestic and global shocks, including oil price fluctuations and policy changes. Unlike constant-correlation models, it provides more accurate estimates of volatility spillovers and co-movements, effectively handling large datasets. By dynamically modeling conditional

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correlations, the DCC framework enhances monitoring of changing risk structures, supports portfolio optimization, and informs hedging strategies, thereby offering a comprehensive approach to understanding the interconnectedness of Nigerian financial markets.

Dynamic Conditional Correlation (DCC) Model

The Dynamic Conditional Correlation (DCC-GARCH) model, introduced by Engle and Sheppard (2001**,** extends the traditional GARCH framework by modeling time-varying correlations among multiple financial time series. Rather than modeling the conditional covariance matrix directly, it decomposes it into conditional standard deviations and a correlation matrix, allowing for dynamic interactions between asset volatilities. This approach provides a flexible and efficient means of capturing evolving relationships and volatility co-movements across financial markets.

$$H_t = D_t R_t D_t \quad (3)$$

where:

R_t is the time-varying correlation matrix

$D_t = diag\left[h_{1t}^{\frac{1}{2}}, h_{2t}^{\frac{1}{2}}, h_{3t}^{\frac{1}{2}}, \ldots, h_{nt}^{\frac{1}{2}}\right]$ is the conditional standard deviation

Models in this class can be classified in two groups; those with a constant correlation matrix and those when the correlation matrix is time-varying. The DCC-GARCH model combines univariate GARCH models for individual series with a dynamic correlation structure. Each financial time series is modelled using a univariate GARCH model to capture its own time-varying volatility:

$$\sigma^2{}_{it} = \omega_t + \alpha_i \epsilon_{i,t-1} + \beta_i \sigma_{i,t-1}^2 \quad (4)$$

Equation (3.18) is the variance equation For each series i

where:

$\sigma^2{}_{it}$is the Conditional variance of series iii at time t

$\epsilon_{i,t-1}$ is the Innovation (shock) from the previous period

ω_t, α_i and β_i are the Model parameters

Suppose we have returns, ***at***, from *n* assets with expected value 0 and covariance matrix **H***t*. Then the mean equation for the Dynamic Conditional Correlation (DCC-) GARCH model is defined as:

$$r_t = \mu_t + \varepsilon_t \quad (5)$$

$\varepsilon_t = H_t^{\frac{1}{2}} Z_t$ and $H_t = D_t R_t D_t$

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Where:

r_t $=n \times 1$ vector of log returns of n assets at time t.

$\varepsilon_t =$ Residuals at time t. $E(\varepsilon_t) = 0$

$Cov(\varepsilon_t) = H_t$.

$\mu_t = n \times 1$ vector of the expected value of the conditional R_t.

$H_t = n \times n$ matrix of conditional variances of at time t. or Time-varying covariance matrix.

$H_t^{\frac{1}{2}}$ = Any $n \times n$ matrix at time t such that $H_t$is the conditional variance matrix of a_t. $H_t^{\frac{1}{2}}$ could be gotten by Cholesky factorization of H_t

Z_t is an $n \times 1$ vector of iid errors such that $E(Z_t) = 0$ and $E(Z_t Z_t') = 1$

Z_t could also be defined as a Vector of standardized residuals. $Z_t \sim N(0,1)$

The idea of the models in this class is that the covariance matrix, $H_t =$, can be decomposed into conditional standard deviations, D_t, and a correlation matrix, R_t. The elements in the diagonal matrix D_t are standard deviations from univariate GARCH models.

$$D_t = \left[\sqrt{\sigma^2{}_{1t}}\ 0\ 0\ \dots\ 0\ \ 0\ \sqrt{\sigma^2{}_{2t}}\ 0\ \dots\ 0\ \ 0\ \vdots\ 0\ \ 0\ \vdots\ 0\ \ \sqrt{\sigma^2{}_{3t}}\ \ddots\ \dots\ \ \dots\ \vdots\ \ddots\ \ 0\ \ 0\ \sqrt{\sigma^2{}_{nt}} \right] \quad (6)$$

$$D_t = diag(\sigma_{1t}, \sigma_{2t}, \sigma_{3t}, \dots, \sigma_{nt})$$

Where σ_{1t} is the time-varying conditional standard deviation (square root of variance) of the ith time series at time t, typically obtained from the univariate GARCH models.

The overall conditional covariance matrix H_t is constructed as $H_t = D_t R_t D_t$

Where: R_t is the time-varying correlation matrix (from the DCC part of the model). In the DCC-GARCH model, the matrix R_t is the time-varying conditional correlation matrix. It represents the dynamic correlations between the standardized residuals of the time series at time t. The residuals $Z_t = D_t{}^{-1}\varepsilon_t$ are standardized with unit variances, which are used to estimate R_t.

Since R_t is a correlation matrix it is symmetric, thus we have:

$$R_t = \left[1\ \rho_{12,t}\ \rho_{13,t}\ \cdots\ \rho_{1n,t}\ \ \rho_{12,t}\ 1\ \rho_{23,t}\ \cdots\ \rho_{2n,t}\ \ \rho_{13,t}\ \vdots\ \rho_{1n,t}\ \ \rho_{23,t}\ \vdots\ \rho_{2n,t}\ \ 1\ \ \ddots\ \dots\ \ \dots\ \rho_{3n,t}\ \ \ddots\ \rho_{n-11n,t}\ \ \vdots\ 1 \right] \quad (7)$$

After estimating the conditional variances, the model estimates the conditional correlation matrix R_t, which is dynamic over time. H_t has to be positive definite because it is a covariance matrix. To ensure H_t to be positive definite, ***Rt*** has to be positive definite (***Dt*** is positive definite since all the diagonal elements are positive). All the elements in the correlation matrix ***Rt*** have to be equal to or less than one. The standardized residuals Z_t are used to estimate the dynamic

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correlation matrix R_t. To ensure both of these requirements in the DCC-GARCH model, ***Rt*** is decomposed into:

$$R_t = {Q_t}^{*-1} Q_t \, {Q_t}^{*-1} \quad (8)$$

$$Q_t = (1 - a - b)\underline{Q} + aZ_{t-1}Z'_{t-1} + bQ_{t-1} \quad (9)$$

where:

Q_t: Dynamic covariance matrix of standardized residuals.

$\underline{Q} = Cov(Z_{t-1}Z'_{t-1}) = E(Z_{t-1}Z'_{t-1})$ is the Unconditional covariance of the standardized errors, Z_t.

a,b are the DCC parameters.

Z_{t-1} is a Vector of standardized residuals at time $t-1$.

${Q_t}^*$ is a diagonal matrix with the square root of the diagonal elements of ***Qt*** at the diagonal:

$\underline{Q}$ and ${Q_t}^*$ are estimated as follows:

$$\underline{Q} = \frac{1}{T}\sum_{t-1}^{T} Z_{t-1}Z'_{t-1} \quad (10)$$

$${Q_t}^* = \left[\sqrt{q_{11t}} \; 0 \; 0 \; \dots \; 0 \;\; 0 \sqrt{q_{22t}} \; 0 \; \dots \; 0 \;\; 0 \; \vdots \; 0 \;\; 0 \; \vdots \; 0 \;\; \sqrt{q_{33,t}} \; \ddots \; \dots \;\; \dots \; \vdots \;\; \ddots \; 0 \;\; 0 \sqrt{q_{nnt}} \right] \quad (11)$$

${Q_t}^*$ rescales the elements in *Qt* to ensure the second requirement

$$|\rho_{i,j}| = \left| \frac{q_{ijt}}{\sqrt{q_{iit}q_{jjt}}} \right| \leq 1 \quad (12)$$

Further ***Qt*** has to be positive definite to ensure ***Rt*** to be positive definite. There are some conditions imposed on the parameters a and b to guarantee H_t to be positive definite. In addition to the conditions for the univariate GARCH model to ensure positive unconditional variances the scalars a and b must satisfy: $a \geq 0, b \geq 0 \; and \; a + b < 1$.

The correlation structure can be extended to the general DCC (P, M)-GARCH model:

$$Q_t = \left(1 - \sum_{i=1}^{P} a_i - \sum_{j=1}^{M} b_j\right)\underline{Q} + \sum_{i=1}^{P} a_i Z_{t-1}Z'_{t-1} + \sum_{j=1}^{M} b_j Q_{t-1} \quad (13)$$

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Distributions for the Error

In this study, we consider three different distributions for the standardized error zt; the multivariate Gaussian, the multivariate Student's *t*- and a multivariate skew Student's *t*-distribution.

Multivariate Student's t Distributed Errors

There exist many specifications of the multivariate generalization of the univariate Student's *t*-distribution. In this thesis the most commonly used distribution is considered. When the standardized errors, $z_t$are multivariate Student's *t*-distributed, the joint density of z_1, z_2 ..., z_N is:

$$f(z_i; v) = \prod_{i=1}^{N} \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\{\pi(v-2\}^{\frac{n}{2}}} \left[1 + \frac{{z_t}^T z_t}{v-2}\right]^{-\frac{v+n}{2}} \quad (14)$$

where:

$\Gamma(\cdot)$ is a Gamma function.

v is the degrees of freedom which controls the tail of the distribution. As v→∞ the distribution approaches a multivariate normal distribution.

n: Dimensionality of the random vector z_t.

N is the number of independent vectors (observations) in a dataset

The term $(v-2)$ in the denominator is related to the scaling of the covariance matrix. For $v >$ 2, this ensures a proper variance.

Assume that the residuals ϵ_t follow a multivariate Student's t-distribution with v degrees of freedom. The likelihood for each time t is given by the multivariate t-distribution density function:

$$f(\epsilon_t; H_t, v) = \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\{(v-2\}^{\frac{n}{2}} \pi^{\frac{n}{2}} {H_t}^{\frac{1}{2}}} \left[1 + \frac{{\epsilon_t}^T {H_t}^{-1} \epsilon_t}{v-2}\right]^{-\frac{v+n}{2}} \quad (15)$$

where: $H_t$is the conditional covariance matrix.

The likelihood function $(L)$is given as follows:

$$L = \prod_{t=i}^{N} \left\{ \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\{(v-2\}^{\frac{n}{2}} \pi^{\frac{n}{2}} {H_t}^{\frac{1}{2}}} \left[1 + \frac{{\epsilon_t}^T {H_t}^{-1} \epsilon_t}{v-2}\right]^{-\frac{v+n}{2}} \right\} \quad (16)$$

The log-likelihood function (LL) is the sum of the logarithms of the likelihoods for each time step t. To simplify the expression:

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$$LL = \sum_{t=1}^{N} LogLf(\epsilon_t; H_t, v) \tag{17}$$

$$= \sum_{t=1}^{N} \left\{ log \left(\frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\{(v-2\}^{\frac{n}{2}} \pi^{\frac{n}{2}} H_t^{\frac{1}{2}}} \right) - \frac{v+n}{2} log \left[1 + \frac{\epsilon_t{}^T H_t{}^{-1} \epsilon_t}{v-2} \right] \right\}$$

$$= \sum_{t=1}^{N} \left\{ \left(log\Gamma\left(\frac{v+n}{2}\right) \right) - log\Gamma\left(\frac{v}{2}\right) - \frac{n}{2} log\, log\,(v-2) \ - \frac{n}{2} log\pi - \frac{1}{2} logH_t - \frac{v+n}{2} log \left[1 + \frac{\epsilon_t{}^T H_t{}^{-1} \epsilon_t}{v-2} \right] \right\}$$

$$= N\left(log\Gamma\left(\frac{v+n}{2}\right) \right) - log\Gamma\left(\frac{v}{2}\right) - \frac{n}{2} log\, log\,(v-2) \ - \frac{n}{2} log\pi)$$

$$- \sum_{t=1}^{N} \left(\frac{1}{2} logH_t + \frac{v+n}{2} log \left[1 + \frac{\epsilon_t{}^T H_t{}^{-1} \epsilon_t}{v-2} \right] \right) \tag{18}$$

By substituting equations (3.20) and (3.33) in (3.43) we have

$$N\left(log\Gamma\left(\frac{v+n}{2}\right) \right) - log\Gamma\left(\frac{v}{2}\right) - \frac{n}{2} log\, log\,(v-2) \ - \frac{n}{2} log\pi)$$

$$- \sum_{t=1}^{N} \left(\frac{1}{2} log(D_t R_t D_t) + \frac{v+n}{2} log \left[1 + \frac{\epsilon_t{}^T \left(D_t{}^{-1} R_{\ t}{}^{-1} D_t{}^{-1} \right) \epsilon_t}{v-2} \right] \right) \tag{19}$$

The optimization of (19) is difficult. Thus, the parameters are obtained in two steps. In the first step, the parameter*s* are estimated assuming that the standardized errors are Gaussian distributed, while the parameter is estimated in the second step using the quasi log-likelihood of 19. Berg and Lunde (2001) and Bollerslev T. (1990). have shown that the change of the error distribution does not virtually affect the parameter estimate. Hence the parameter$\emptyset = \emptyset_1, \emptyset_2, \emptyset_3, \ldots, \emptyset_n$ of the univariate GARCH models are fitted using the pseudo-maximum-likelihood; assuming the errors to be Gaussian distributed. Thus, we have:

$$LL(\emptyset) = \sum_{j=1}^{n} \left(-\frac{1}{2} \sum_{i=1}^{N} \left[log\, log\,(\sigma_{i,t}^2) \ + \frac{\epsilon_{i,t}{}^2}{\sigma_{i,t}^2} \right] + constant \right) \tag{20}$$

and the parameters $\emptyset_j, j = 1,2,3, \ldots, n$ are estimated assuming univariate GARCH models with Gaussian distributed errors. The second step is the estimation of the remaining parameters a, b and v. These are estimated as follows:

Let $\vartheta = [a, b, v]$ be the parameter space, using the log-likelihood in (3.44), the second stage quasi-likelihood function is:

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$$LL(\emptyset) = \sum_{t=1}^{N}\left(log\Gamma\left(\frac{v+n}{2}\right)\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{n}{2} log\, log\, (v-2) - \frac{n}{2} log\pi)$$

$$-\sum_{t=1}^{N}\left(\frac{1}{2}log(D_tR_tD_t) + \frac{v+n}{2}log\left[1 + \frac{\epsilon_t{}^T\left(D_t{}^{-1}R_t{}^{-1}D_t{}^{-1}\right)\epsilon_t}{v-2}\right]\right)$$

Since ***Dt*** is constant when conditioning on the parameters from step one, we can exclude the constant term and maximize:

$$LL(\emptyset) = \sum_{t=1}^{N}\left(log\Gamma\left(\frac{v+n}{2}\right)\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{n}{2} log\, log\, (v-2) - \frac{n}{2} log\pi)$$

$$-\sum_{t=1}^{N}\left(\frac{1}{2}log(R_t) + \frac{v+n}{2}log\left[1 + \frac{\epsilon_t{}^T\left(R_t{}^{-1}\right)\epsilon_t}{v-2}\right]\right) \quad (21)$$

Multivariate Normal Distributed Errors

When the standardized errors, of $z_1, z_2 \ldots, z_N$, follows the multivariate normal distribution the joint distribution is given as follow:

$$L(\psi) = \prod_{t=1}^{N}\frac{1}{2\pi^{\frac{1}{2}}|H_t|^{\frac{1}{2}}}exp\left(-\frac{1}{2}\alpha_t{}^TH_t{}^{-1}\alpha_t\right) \quad (22)$$

Where ψ is the parameter vector. Let the parameters ψ be divided in two groups; (θ, Ω) where $\theta = (\alpha_{oi}, \alpha_{1i}, \alpha_{2i}, \ldots, \alpha_{qi}, \ \beta_{1i}, \beta_{2i}, \beta_{3i}, \ldots, \beta_{pi})$ are the parameters of the univariate GARCH model for the ith series, $i = 1, \ldots, n$. Ω = (a, b) are the parameters of the correlation structure. By taking the log of the likelihood function in (3.48) we obtain

$$LL(\psi) = -\frac{1}{2}\sum_{t=1}^{N}\left[log2\pi + log|H_t| + \alpha_t{}^TH_t{}^{-1}\alpha_t\right]$$

$$= -\frac{1}{2}\sum_{t=1}^{N}\left[log2\pi + log|D_tR_tD_t| + \alpha_t{}^TD_t{}^{-1}R_t{}^{-1}D_t{}^{-1}\alpha_t\right] \quad (23)$$

$$= -\frac{1}{2}\sum_{t=1}^{N}\left[log2\pi + 2log|D_t| + log|R_t| + \alpha_t{}^TD_t{}^{-1}R_t{}^{-1}D_t{}^{-1}\alpha_t\right] \quad (24)$$

Due to the complexities associated with the estimation of the estimation of the parameters using (24) which is the correctly specified log-likelihood function, A two stage estimation is adopted

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for the DCC-model. In the first stage the parameter of the univariate GARCH models $\theta = (\alpha_{oi}, \alpha_{1i}, \alpha_{2i}, \dots, \alpha_{qi}, \ \beta_{1i}, \beta_{2i}, \beta_{3i}, \dots, \beta_{pi})$ are estimated for each series. The likelihood used in the first stage results in replacing R_t with the identity matrix I_n.

$$LL(\theta) = -\frac{1}{2}\sum_{t=1}^{N}\left[log2\pi + 2log|D_t| + log|I_n| + \alpha_t{}^T D_t{}^{-1} I_n D_t{}^{-1}\ \alpha_t\right] \quad (25)$$

In the second stage, we estimate *Ω* = (*a, b)* using the correctly specified log-likelihood in (3.48), given the estimated parameters from step one. The second stage quasi-likelihood function is then:

$$LL(\Omega) = -\tfrac{1}{2}\textstyle\sum_{t=1}^{N}\left[log2\pi + 2log|D_t| + log|R_t| + \alpha_t{}^T D_t{}^{-1} R\,_t{}^{-1} D_t{}^{-1}\ \alpha_t\right] \quad (26)$$

$$LL(\Omega) = -\frac{1}{2}\sum_{t=1}^{N}\left[log2\pi + 2log|D_t| + log|R_t| + \epsilon_t{}^T D_t{}^{-1} R\,_t{}^{-1} D_t{}^{-1}\ \epsilon_t\right]$$

Since $D_t$is constant when conditioning on the parameters from step one, we can exclude the constant terms and maximize:

$$LL(\Omega) = -\frac{1}{2}\sum_{t=1}^{N}\left[log|R_t| + \epsilon_t{}^T R\,_t{}^{-1}\ \epsilon_t\right] \quad (27)$$

Multivariate Skew Student's *t*-Distributed Errors

There are several formulations for the multivariate skew Student's t-distribution; in this context, we adopt the skew Student's t-distribution introduced by Azzalini and Capitanio (2001). When the standardized errors, of z_1, z_2 ..., z_N, follows the skew student, distribution the joint distribution with parameter space $\Theta$is given as follow:

$$f(z_t;, v, \kappa) = \left\{2t_d(z_t;, v, \kappa)T_{v+n}[\ \delta^T D^{-1}](z_t - \kappa)\left[\frac{v+n}{Q_{z_t} + v}\right]^{\frac{1}{2}}, v+n\right\} \quad (28)$$

$$Q_{z_t} = (z_t - \kappa)^T D^{-1}(z_t - \kappa) \quad (29)$$

$$t_d = \frac{\Gamma\left(\frac{v+n}{2}\right)}{\left|\Omega^{\frac{1}{2}}\right|(\pi v)^{\frac{n}{2}}\Gamma(v/2)}\left[1 + \frac{Q_{z_t}}{v}\right]^{-(v+n)/2} \quad (30)$$

where ***D*** is the diagonal matrix with the square root of the diagonal elements of on the diagonal. T_{v+n} denotes the scalar Student's *t*-distribution with $v + n$ degrees of freedom and

Γ is the Gamma function. Given that:

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$$\kappa = D^{-1}\delta$$

Aas et al. (2005) have shown that Azzalini's skew Student's *t*-distribution could be standardized to have mean vector **0** and covariance matrix I_n, by letting:

$$\Omega = \{\frac{v-2}{v}\left(I_n + \kappa^T\kappa\left[-1 + \frac{\pi\Gamma\left(\frac{v}{2}\right)^2(v-(v-2)\kappa^T\kappa)}{2\kappa^T\kappa(v-2)\left[\pi\Gamma\left(\frac{v}{2}\right)^2 - (v-2)\Gamma\left(\frac{v-1}{2}\right)^2\right]}(-1+g)\kappa\kappa^T\right]\right) \quad for\ \kappa \neq$$

$$0 \quad \left(\frac{v-2}{v}\right)I_n \qquad for\ \kappa = 0 \qquad (31)$$

where:

$$g = \sqrt{1 + \frac{4v(v-2)\pi\Gamma\left(\frac{v}{2}\right)^2 - (v-2)\Gamma\left(\frac{v-1}{2}\right)^2\kappa^T\kappa}{\pi\Gamma\left(\frac{v}{2}\right)^2(v-(v-2)\kappa^T\kappa)^2}} \qquad (32)$$

Define

$$\xi = -\sqrt{\frac{v}{\pi}}\frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)}\frac{\Omega_\kappa}{\sqrt{1+\kappa^T\Omega_\kappa}} \qquad (33)$$

From (3.52) the likelihood function of $a_t = H_t^{\frac{1}{2}} Z_t$ is:

$$l(\Phi) = \prod_{i=1}^{N}\left\{2t_d\left(a_tH_t^{-\frac{1}{2}};,v,\kappa\right)T_{v+n}[\delta^TD^{-1}]\left(a_tH_t^{-\frac{1}{2}} - \kappa\right)\left[\frac{v+n}{Q_{z_t}+v}\right]^{\frac{1}{2}},v+n\right\}\frac{1}{\left|H_t^{\frac{1}{2}}\right|} \quad (34)$$

where:

$l(\Phi)$ is the parameter space and

$$Q_{a_t} = \left(a_tH_t^{-\frac{1}{2}} - \kappa\right)\Omega^{-1}\left(a_tH_t^{-\frac{1}{2}} - \kappa\right) \qquad (35)$$

We obtain the log-likelihood by taking the logarithm and substituting $H_t = D_tR_tD_t$:

$$logl(\Phi) = \sum_{i=i}^{N}\left\{Log(2) + log\left(t_da_tH_t^{-\frac{1}{2}};,v,\kappa\right) + log\left(T_{v+n}[\delta^TD^{-1}]\left(a_tH_t^{-\frac{1}{2}} - \kappa\right)\left[\frac{v+n}{Q_{z_t}+v}\right]^{\frac{1}{2}},v+n\right)\right\} - \frac{1}{2}logH_t \quad (36)$$

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The model parameters (Φ) are categorized into two groups: the parameters of the univariate GARCH model for the ith series, where $i = 1,2,3,\ldots,n$. Optimizing equation (36) poses significant challenges. Consequently, parameter estimation is carried out in two stages. Initially, parameters are estimated under the assumption that the standardized errors follow a Gaussian distribution. In the second stage, the remaining parameters are estimated using the log-likelihood function in equation (36).

The first-stage quasi-likelihood is typically defined as the log-likelihood function assuming that the standardized errors are Gaussian distributed. It is given by:

$$logl_1 = \sum_{j=1}^{n}\left(-\frac{1}{2}\sum_{i=1}^{N}\left[log\, log\,\left(\sigma_{i,t}^2\right) + \frac{{\epsilon_{i,t}}^2}{\sigma_{i,t}^2}\right] + constant\right)$$

The constant in the given equation typically accounts for terms that do not depend on the parameters being estimated. These terms are often omitted in likelihood calculations because they do not affect the optimization process, as they are invariant with respect to the parameters. The remaining parameters $a, b, v, and\ \kappa$ are estimated in the second step. This step typically involves maximizing the full log-likelihood function, which incorporates the standardized errors from the first step. The goal is to refine the model by estimating these parameters under a potentially more accurate assumption about the error distribution (skewed Student's t-distribution) Mathematically, the second-step log-likelihood might look like:

$$logl_2 = \sum_{j=1}^{N}\sum_{j=1}^{T} logf\left(\epsilon_{i,t}; a, b, v, \kappa\right) \tag{37}$$

$$logl(\Phi) = \sum_{i=i}^{N}\left\{Log(2) + log\left(t_d a_t H_t^{-\frac{1}{2}};, v, \kappa\right) + log\left(T_{v+n}[\,\delta^T D^{-1}]\left(a_t H_t^{-\frac{1}{2}} - \kappa\right)\left[\frac{v+n}{Q_{z_t}+v}\right]^{\frac{1}{2}}, v+n\right)\right\} - \frac{1}{2}log|D_t R_t D_t| \tag{38}$$

$$= \sum_{i=i}^{N}\left\{Log(2) + log\Gamma\left(\frac{v-1}{2}\right) - \frac{1}{2}log|\Omega| - \frac{n}{2}log\, log\,(\pi v)\, log\Gamma\left(\frac{v}{2}\right) - \left(\frac{v+n}{2}\right) log\,\left(1 + \frac{Q_{\alpha_t}}{n}\right) + log\left(T_{v+n}[\,\delta^T D^{-1}]\left(a_t H_t^{-\frac{1}{2}} - \kappa\right)\left[\frac{v+n}{Q_{z_t}+v}\right]^{\frac{1}{2}}, v+n\right)\right\} - \frac{1}{2}log|R_t| - log\,(D_t) \tag{39}$$

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When $D_t$is constant, given the parameters estimated in the first step, the focus shifts to maximizing the terms dependent on a, b, v, κ. Excluding constant terms, the objective function becomes:

$$\sum_{i=i}^{N} \left\{ Log(2) + log\Gamma\left(\frac{v-1}{2}\right) - \frac{1}{2}log|\Omega| - \frac{n}{2}log\, log\,(\pi v)\, log\Gamma\left(\frac{v}{2}\right) - \left(\frac{v+n}{2}\right) log\left(1+\frac{Q_{\alpha_t}}{n}\right) + log\left(T_{v+n}[\delta^T D^{-1}]\left(a_t H_t^{-\frac{1}{2}} - \kappa\right)\left[\frac{v+n}{Q_{z_t}+v}\right]^{\frac{1}{2}}, v+n\right)\right\} - \frac{1}{2}log|R_t| \quad (40)$$

Multivariate Generalized Error Distribution (MGED),

Multivariate Generalized Error Distribution (MGED), sometimes also called the multivariate exponential power distribution. This distribution is often used as an error distribution in MGARCH models when we want to allow for heavy-tailed (leptokurtic) or light-tailed (platykurtic) innovations.

Suppose $\varepsilon_t \epsilon$ k-dimensional error vector with mean zero and covariance matrix Σ.

The probability density function (pdf) of the MGED is:

$$f(\varepsilon_t) = \frac{\lambda^{\frac{k}{2}}}{2^{\frac{k}{2}}\pi^{\frac{k}{2}}|\Sigma|^{\frac{k}{2}}\Gamma\left(\frac{k}{2\lambda}\right)} exp\left(-\frac{1}{2}({\varepsilon_t}^T\Sigma^{-1}\varepsilon_t)^{\lambda}\right) \quad (41)$$

where:

λ is the shape parameter

k is the dimension,

Σ is the positive-definite scale covariance matrix,

Γ (·) is the Gamma function.

The models discussed so far provide a foundation estimating the Dynamic Conditional Correlation (DCC)-GARCH model, a prominent approach in multivariate volatility modelling. Initially, univariate GARCH models were introduced to capture time-varying volatility in individual series, estimating conditional variances based on past returns and variances. These models were extended to multivariate settings, where the challenge lies in modelling not just individual volatilities but also the dynamic correlations among multiple series. Simple extensions like the VECH or BEKK models became computationally demanding as the number of series increased, motivating the need for more scalable and parsimonious approaches.

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The DCC-GARCH model will addresses these challenges by separating the estimation of conditional variances and dynamic correlations into two steps. First, univariate GARCH models are applied to estimate the conditional variances for each series. Then, the standardized residuals from these models are used to estimate the time-varying conditional correlation matrix, allowing for dynamic and flexible modelling of relationships between series. This structure significantly reduces computational complexity while maintaining the ability to capture intricate dependence structures, making the DCC-GARCH model a practical and powerful tool for multivariate volatility analysis.

SOURCE OF DATA

This study employed secondary data comprising daily price observations spanning the period from March 3, 2017, to March 12, 2025. The data was sourced from the financial market database available on www.investing.com, a globally recognized platform for comprehensive financial market information. The dataset includes the following variables,

Nigerian All-Share Index (ASI), Crude Oil Price (Brent**)** reflecting global crude oil market trends, and Stock price. The choice of secondary data is justified by its reliability, accessibility, and the historical coverage required for time-series modelling. Using data from a reputable source like investing.com ensures accuracy and consistency, which are critical for volatility modelling and forecasting. Before analysis, the raw price series were cleaned and transformed into continuously compounded returns using the logarithmic difference formula:

$$Return = Log\left(\frac{P_t}{P_{t-1}}\right) * 100 \qquad (42)$$

where:

P_t is the price at time t

P_{t-1} is the price at time, $t-1$

RESULTS/FINDINGS

Table 4.1: Descriptive Statistics of the Daily Return Series of Nigeria Stock Prices Brent Crude Oil Price and All Share.

Descriptive Measure	Stock Prices	Bret Crude Oil Price	All share
Mean	0.03236	69.534	0.0429
Median	-0.00186	68.710	0.0065
Maximum	9.36002	127.98	7.9848
Minimum	-6.25452	19.330	-5.0329
Std. Dev.	1.02789	20.181	0.9723
Skewness	0.5327	0.3115	0.37154
Kurtosis	10.822	2.6651	8.6392
Jarque-Bera	7611.1	61.120	3951.1
Probability	0.000000	0.0000	0.0000

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Table 4.1 presents the descriptive statistics of the daily return series for Nigerian stock prices, Brent crude oil prices, and the All Share Index. The results show that all series have positive mean values, indicating overall average gains. Brent crude oil exhibits the highest level of volatility as reflected by its large standard deviation, while stock prices and the All Share Index display relatively moderate fluctuations. The positive skewness for all variables suggests the presence of occasional large positive returns. Moreover, the high kurtosis values for stock prices (10.822) and the All Share Index (8.6392) reveal heavy-tailed distributions, indicating the occurrence of extreme events. The Jarque–Bera test results with significant p-values (0.000) confirm that none of the return series follow a normal distribution, thereby justifying the application of volatility models such as MGARCH for further analysis.

Figure 1: Time Plot of the Nigeria Daily Stock Prices from 2017-2025

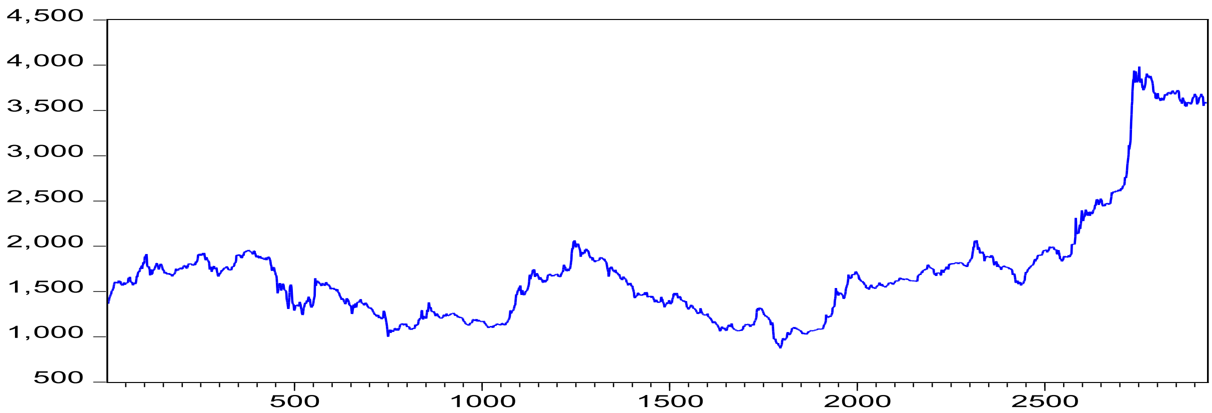

Figure 1 depicts Nigeria's daily stock prices from March 2017 to May 2025, showing notable fluctuations over time. From 2017 to 2025, the series experienced moderate volatility and a noticeable decline, likely linked to economic disruptions such as COVID-19. Beginning in early 2021, stock prices exhibited a sustained upward trend, reflecting market recovery and improved performance.
The plot highlights persistent volatility with stronger growth momentum in the post-2020 period.

Figure 2: Time Plot of Bent Crude Oil Price from 2017-2025

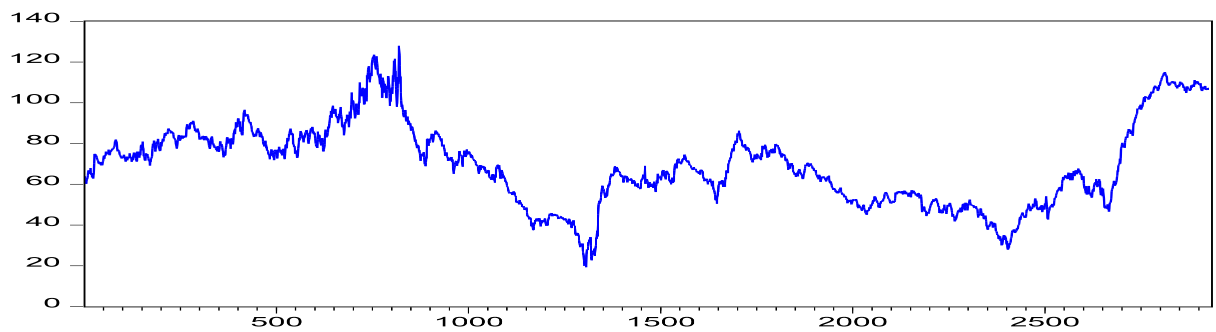

Figure 2 illustrates the Brent crude oil price trends from 2017 to 2025, revealing substantial volatility influenced by global market forces. Prices fluctuated between $60 and $80 per barrel until mid-2019, followed by a sharp surge above $120 and a dramatic crash in early 2020 due to the COVID-19 pandemic. From late 2020 onward, prices gradually recovered, oscillating between $60 and $90 amid global economic recovery, OPEC+ decisions, and inflation

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pressures. These fluctuations highlight the oil market's sensitivity to geopolitical tensions, especially the Russia–Ukraine conflict, and post-pandemic adjustments.

Figure 3: Time Plot of Time All Share Index (ASI) from 2017-2025

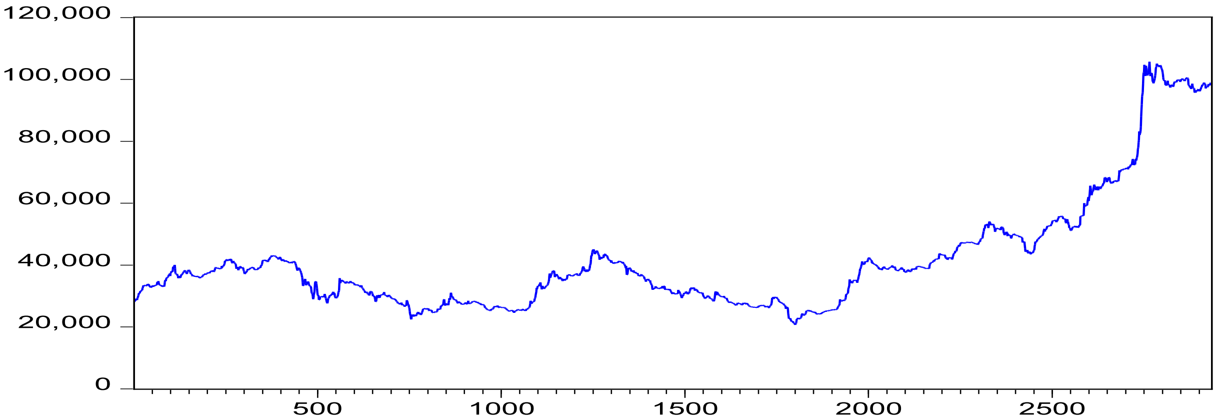

Figure 3 depicts the Nigerian All Share Index from 2017 to 2025, showing alternating periods of calm and heightened volatility. Short bursts of increased variability occur in late 2018 and early 2020, with a sharp COVID-19-induced decline followed by a rapid rebound an example of volatility clustering.
From 2021 to early 2024, the index shows a strong but uneven upward trend, marked by rapid gains and abrupt corrections. These patterns highlight persistent short-term volatility clustering and dynamic market adjustments over time.

Table 4.2: Unit Root Test for the Nigeria Daily Stock Price, Stock Prices, Brent Crude Oil Price and All Share

Null Hypothesis: Price has a Unit Root				
		Stock Prices	**Brent Crude Oil Price**	**All share**
Augmented Dickey-Fuller test statistic		0.964348	-1.574216	2.091968
Test critical values:	1% level	-3.432391	-3.432391	-3.432391
	5% level	-2.862328	-2.862327	-2.862328
	10% level	-2.567234	-2.567233	-2.567234
	p-value	**0.9963**	**0.4957**	**0.9999**

Table 4.2 presents the unit root test results for Nigeria's daily stock prices, Brent crude oil prices, and the All Share Index, showing that all three series are **non-stationary in their levels**. The Augmented Dickey-Fuller (ADF) test statistics for each variable are less negative than the critical values, and their corresponding p-values exceed 0.05, indicating failure to reject the null hypothesis of a unit root. Specifically, the stock price (ADF = 0.9643, p = 0.9963), Brent

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crude oil price (ADF = -1.5742, p = 0.4957), and All Share Index (ADF = 2.0919, p = 0.9999) all exhibit non-stationarity. Consequently, these series must be **differenced or transformed into returns** to achieve stationarity, a necessary condition for reliable MGARCH volatility and correlation modeling.

Table 3: Unit Root Test for the Return on Daily Stock Price

Null Hypothesis: PRICE has a Unit Root				
		Stock Prices	**BrentCrude Oil Price**	**All share**
ADF Test Statistic		-39.18259	-53.50654	-39.74618
Test critical values:	**1% level**	-3.432391	-3.432391	-3.432391
	5% level	-2.862328	-2.862328	-2.862328
	10% level	-2.567234	-2.567234	-2.567234
	p-value	**0.0000**	**0.0001**	**0.0000**

Footnote: * = significant at 5%

Table 4.3 presents the unit root test results for the return series of the All Share Index, Brent crude oil price, and stock prices, showing that all three return series are **stationary**. The Augmented Dickey-Fuller (ADF) test statistics for the All Share Index (-39.1826), Brent crude oil (-53.5065), and stock prices (-39.7462) are all far below their respective critical values at the 1%, 5%, and 10% significance levels, with corresponding **p-values of 0.0000**, confirming strong rejection of the null hypothesis of a unit root. This implies that each return series maintains a constant mean and variance over time, satisfying the stationarity requirement for time series modeling. Hence, these results validate the use of the stationary return series in subsequent MGARCH modeling and volatility analysis.

Figure 4: Time Plot of the Return Series for the Nigeria Daily Stock Prices

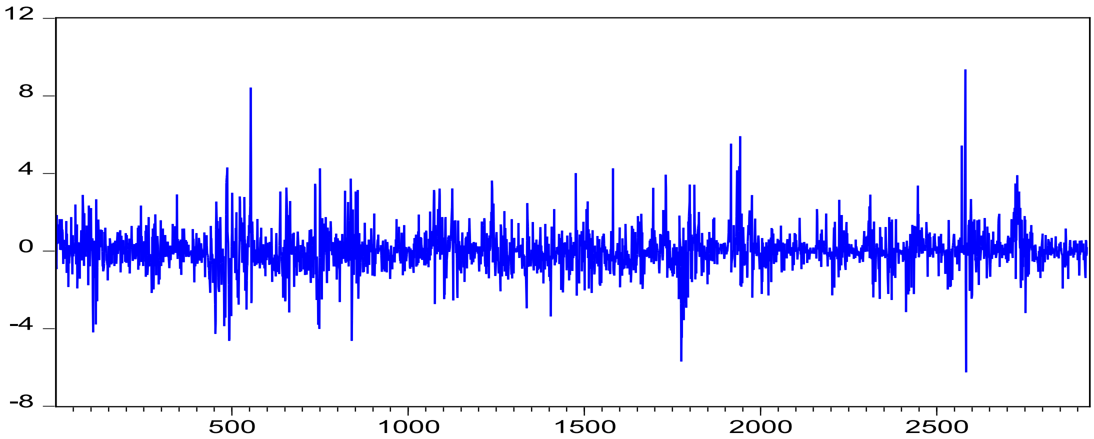

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Figure 5: Time Plot of the Return Series for Nigeria Daily Brent Crude Oil Price

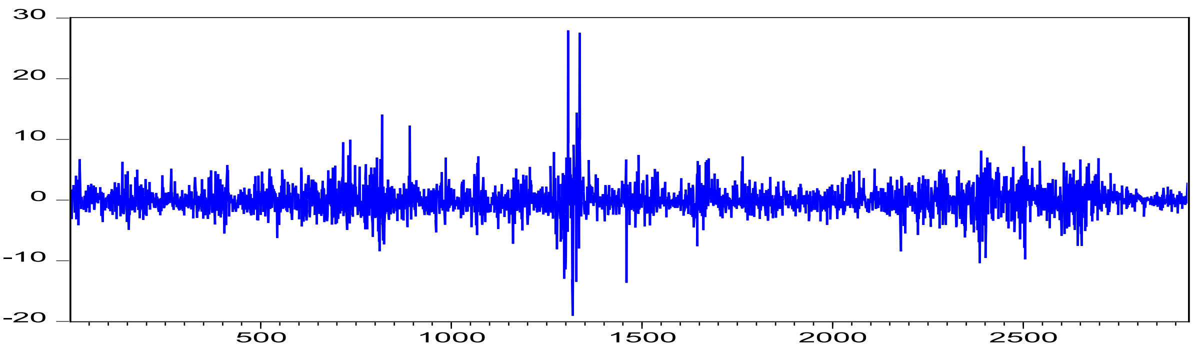
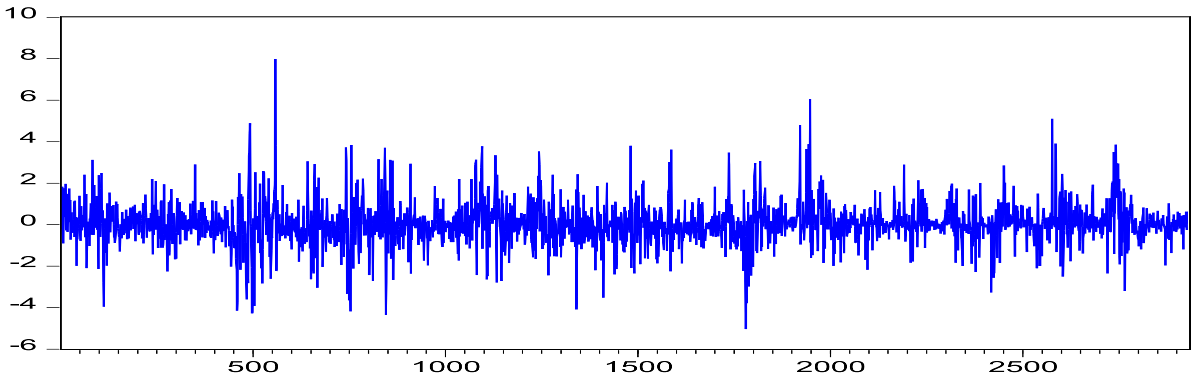


Figure 6: Time Plot of the Return Series for Nigeria Daily All Share Price



Figures 4–6 present the daily return series for Nigerian stock prices, crude oil prices, and the All Share Index from March 2017 to mid-2025. The returns fluctuate around a mean of zero, confirming mean adjustment of the data. The plots reveal alternating periods of high and low volatility, demonstrating **volatility clustering**, where large price changes positive or negative occur in close succession. This pattern reflects shifts in market uncertainty and external shocks, indicating that the mean is stable but volatility varies over time, thereby justifying the application of GARCH-type models to effectively capture the dynamics of time-varying volatility in the financial return series.

Table 4.4: ARCH Effect Tests for the Return on Daily Stock Price, All Share Index (ASI) and Crude Oil Prices (OIL)

Series	Chi-squared Statistic	Degrees of Freedom (df)	p-value	Conclusion
Return on Stock Prices (RSP)	349.62	12	0.0000**	Significant ARCH effects (reject H_0)
Return on All Share Index (RASI)	375.02	12	0.0000**	Significant ARCH effects (reject H_0)
Return on Crude Oil Prices (ROIL)	365.97	12	0.0000**	Significant ARCH effects (reject H_0)

Footnote: ** = significant at 5%

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The results of the ARCH LM test for stock returns, the All-Share Index (ASI), and crude oil prices all reveal highly significant ARCH effects, with Chi-squared statistics of 349.62, 375.02, and 365.97 respectively, each having p-values below 0.0000. These findings strongly reject the null hypothesis of no ARCH effects, indicating that the conditional variances of the return series are time-varying and exhibit volatility clustering a common feature of financial time series. This means that periods of high volatility tend to follow high volatility, while calm periods follow calm periods. Therefore, the presence of significant heteroskedasticity across all three-return series justifies the use of ARCH or GARCH-type models to appropriately capture and model their dynamic volatility behaviour.

Table 4.5: Estimated Parameters for MGARCH-DCC(1,1) Models Using Different Error Distributions

Parameter	mvst			mvt			mvnorm			mged		
	Est.	SE	P-Val	Est.	SE	P-Val	Est.	SE	P-Val	Est.	SE	P-Val
[RSP].mu	-.004	0.015	0.985	-.001	0.011	0.905	-.004	0.016	0.779	-.008	0.001	0.000*
[RSP].omega	0.061	0.025	0.014*	0.061	0.025	0.014*	0.083	0.03	0.006*	0.058	0.019	0.003*
[RSP].alpha1	0.297	0.05	0.000*	0.298	0.05	0.000*	0.22	0.047	0.000*	0.27	0.05	0.000*
[RSP].beta1	0.702	0.061	0.000*	0.701	0.061	0.000*	0.712	0.063	0.000*	0.711	0.053	0.000*
[RSP].shape	3.542	0.178	0.000*	3.541	0.179	0.000*	–	–	–	0.974	0.037	0.000*
[RSP].skew	1.003	0.023	0.000*	–	–	–	–	–	–	–	–	–
[RASI].mu	0.002	0.015	0.9	0	0.01	0.98	-.001	0.015	0.931	-.007	0.001	0.000*
[RASI].omega	0.076	0.04	0.054	0.076	0.04	0.053	0.091	0.04	0.024*	0.064	0.026	0.013*
[RASI].alpha1	0.321	0.064	0.000*	0.322	0.064	0.000*	0.211	0.051	0.000*	0.267	0.059	0.000*
[RASI].beta1	0.678	0.091	0.000*	0.677	0.09	0.000*	0.7	0.085	0.000*	0.702	0.071	0.000*
[RASI].shape	3.169	0.134	0.000*	3.17	0.134	0.000*	–	–	–	0.909	0.032	0.000*
[RASI].skew	1.005	0.022	0.000*	–	–	–	–	–	–	–	–	–
[ROIL].mu	-.036	0.032	0.265	-0.061	0.031	0.050*	-0.02	0.033	0.546	-.064	0.034	0.048*
[ROIL].omega	0.09	0.028	0.001*	0.086	0.027	0.002*	0.094	0.03	0.002*	0.091	0.027	0.001*
[ROIL].alpha1	0.113	0.015	0.000*	0.115	0.015	0.000*	0.122	0.018	0.000*	0.117	0.015	0.000*
[ROIL].beta1	0.875	0.015	0.000*	0.875	0.015	0.000*	0.866	0.018	0.000*	0.871	0.016	0.000*
[ROIL].shape	6.009	0.587	0.000*	5.932	0.578	0.000*	–	–	–	–	–	–
[ROIL].skew	1.06	0.026	0.000*	–	–	–	–	–	–	–	–	–

Footnote: * = significant at 5%

Table 4.5 presents the estimated parameters of the MGARCH-DCC(1,1) models under four different error distributions multivariate skewed Student-t (mvst), multivariate Student-t (mvt), multivariate normal (mvnorm), and multivariate generalized error distribution (mged) for Nigerian stock prices (RSP), All Share Index (RASI), and Brent crude oil prices (ROIL). Across all models, the ARCH (α_1) and GARCH (β_1) coefficients are positive and statistically significant at the 5% level, indicating strong evidence of volatility persistence in all return series. This means that periods of high volatility tend to be followed by further volatility,

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confirming the presence of volatility clustering consistent with financial market behaviour. The ω (omega) parameters are also significant in most cases, confirming that the models appropriately capture the conditional variance dynamics.

The shape and skew parameters for the mvst, mvt, and mged distributions are highly significant, suggesting that non-normal error distributions provide a better fit to the data than the normal distribution. The significant skewness and heavy-tail parameters indicate that the return distributions of the Nigerian financial assets exhibit asymmetry and leptokurtosis, implying that extreme events (such as sharp price increases or crashes) occur more frequently than under normality assumptions. This finding validates the use of fat-tailed distributions like Student-t and GED in modelling financial returns, as they better capture the empirical characteristics of Nigerian market volatility and risk.

Comparatively, the mvst and mged models show more statistically significant parameters and better alignment with observed data characteristics, suggesting their superiority over the mvnorm model in describing conditional volatility and correlation dynamics. The results for Brent crude oil returns (ROIL) show particularly high β_1 values (around 0.87), indicating strong volatility persistence and slow decay of shocks in oil price volatility. Overall, the findings demonstrate that the MGARCH-DCC framework with fat-tailed and skewed distributions effectively captures the dynamic correlations and volatility transmission among Nigerian stock prices, the All Share Index, and global crude oil prices, thereby providing deeper insights into the interdependence and risk behaviour in the Nigerian financial market.

Table 4.6: DCC Parameter Estimates

Parameter	mvnorm			mvt			mvsst			mged		
	Est.	SE	P-Val	Est.	SE	P-Val	Est.	SE	P-Val	Est.	SE	P-Val
[Joint]dcca1	0.013	0.009	0.168	0.018	0.01	0.098	0.018	0.01	0.098	0.013	0.01	~0.100
[Joint]dccb1	0.679	0.107	0.000	0.658	0.131	0.000	0.658	0.131	0.000	0.702	0.106	0.000
[Joint]mshape	–	–	–	5.365	0.219	0	5.365	0.219	0.000	–	–	–

Footnote: * = significant at 5%

Table 4.6 presents the estimated parameters of the Dynamic Conditional Correlation (DCC) component under four error distributions multivariate normal (mvnorm), multivariate Student-t (mvt), multivariate skewed Student-t (mvsst), and multivariate generalized error distribution (mged). The α_1 (dcca1) parameter measures the effect of short-term shocks on dynamic correlations, while β_1 (dccb1) captures the persistence of correlations over time. The α_1 estimates are small (0.013–0.018) and mostly insignificant at the 5% level, suggesting that short-run innovations have a limited immediate impact on correlation dynamics. In contrast, β_1 values (0.658–0.702) are high and statistically significant, indicating strong persistence in conditional correlations—meaning that past correlations strongly influence current correlations among the financial assets.

For the Student-t and skewed Student-t distributions, the shape parameter (mshape ≈ 5.37) is significant, confirming the presence of fat tails in the joint distribution. This implies that

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extreme co-movements (e.g., simultaneous large changes in oil and stock prices) are more likely than predicted by the normal distribution.

Overall, the results demonstrate that the dynamic correlations among Nigerian stock prices, the All Share Index, and crude oil prices are persistent and better captured by fat-tailed distributions (mvt and mvsst) than by the normal distribution, reinforcing the importance of using non-Gaussian MGARCH-DCC models for financial volatility analysis.

DISCUSSION

This study evaluated the performance of different error distributions within the framework of the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH)–Dynamic Conditional Correlation (DCC) model using Nigerian daily stock prices, Brent crude oil prices, and the All-Share Index. The analysis began with preliminary tests confirming that all price series were non-stationary in levels but became stationary after first differencing, indicating the suitability of using returns for volatility modelling. The return series displayed volatility clustering—periods of persistent high and low volatility stressing the presence of time-varying variance, which justifies the application of MGARCH-type models.

The estimation results from the DCC models under different error distributions (Normal, Student-t, Skewed Student-t, and Generalized Error Distribution) revealed that the dynamic correlations among the financial assets were time-varying and highly persistent. The DCC parameters, particularly β_1, were positive, significant, and close to unity across all models, indicating that correlations between the markets tend to evolve slowly over time and respond more to long-term dynamics than to short-term shocks. However, the α_1 coefficients were small and mostly insignificant, implying that immediate shocks had limited effects on correlation adjustments.

Furthermore, the shape parameter in the student-t and Skewed Student-t distributions was significant, confirming the presence of heavy tails and asymmetric behaviour in the data. This finding suggests that the assumption of normality underestimates the likelihood of extreme market events, such as simultaneous large swings in crude oil and stock prices. Hence, models with fat-tailed distributions (Student-t and Skewed Student-t) provided a more accurate characterization of the joint return dynamics. The results underscore that financial data in emerging markets like Nigeria often exhibit non-Gaussian properties leptokurtosis and skewness—which must be captured for realistic modelling and forecasting.

Overall, the findings demonstrate that while all MGARCH-DCC models can describe time-varying correlations, those assuming fat-tailed error distributions outperform the Gaussian specification in capturing volatility persistence and co-movement during turbulent periods. This provides critical insights for portfolio diversification, risk management, and policy formulation, especially in resource-dependent economies sensitive to oil price fluctuations.

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CONCLUSION

The study concludes that the choice of error distribution plays a crucial role in modelling and forecasting financial market volatility using MGARCH-DCC models. Empirical evidence shows that Nigerian stock returns, crude oil prices, and the All-Share Index are characterized by volatility clustering, persistence, and fat-tailed distributions. Among the evaluated models, the Student-t and Skewed Student-t distributions provided superior fits, effectively capturing excess kurtosis and asymmetry inherent in financial time series.

The persistence of dynamic correlations implies that shocks to one market can have long-lasting effects on others, underscoring the interconnectedness of financial and commodity markets. Therefore, assuming normality may lead to underestimation of risk and correlation spillovers, while using heavy-tailed distributions yields more reliable inferences for investment and policy decisions.

In conclusion, the MGARCH-DCC framework with non-Gaussian error distributions, particularly the student-t and Skewed Student-t, is best suited for modelling volatility and correlation dynamics in Nigeria's financial markets. Future research could extend this analysis to higher-dimensional portfolios or incorporate regime-switching features to capture structural breaks and nonlinear dependencies in financial data.

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