African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 4, 2025 (pp. 150-161)
www.abjournals.org

A NEW PROCEDURE FOR CONSTRUCTING N-POINT D-OPTIMAL SYMMETRIC AND ASYMMETRIC DESIGNS

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Cite this article:

A. A., Isaac, U. P., Akra, S. S., Akpan (2025), A New Procedure for Constructing N-Point D-Optimal Symmetric and Asymmetric Designs. African Journal of Mathematics and Statistics Studies 8(4), 150-161. DOI: 10.52589/AJMSS-E9WW0NZJ

Manuscript History

Received: 11 Sep 2025

Accepted: 14 Oct 2025

Published: 19 Dec 2025

ABSTRACT: *Symmetric and asymmetric N-point D-optimal response surface designs were constructed using the Balanced Confounded Design Algorithm (BCDA) proposed in the literature. The results showed that in all the constructed designs, the global optimum or best design was found to come from or located inside the designs where all the support points from* G_1 *were taken. Thus, a condition is imposed while using the BCDA to construct design. The condition is that the search for global optimum be restricted to a particular class or group of designs and that class is the class in which all the support points in* G_1 *are taken instead of sweeping through all the classes. The result of this work has shown that the new procedure, which can be named Restricted Balanced Confounded Design Algorithm (RBCDA), is superior in efficiency (fewer search time or iteration and cost) to the BCDA.*

KEYWORDS: Balanced, D-optimal, response surface, symmetric and asymmetric design.

Article DOI: 10.52589/AJMSS-E9WW0NZJ
DOI URL: https://doi.org/10.52589/AJMSS-E9WW0NZJ

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www.abjournals.org

INTRODUCTION

It was shown that for all the N-points possible symmetric and asymmetric response surface designs that could be obtained by the Balanced Confounded Designs Algorithm (BCDA) proposed by (Umoren and Isaac, 2010), the designs were not only D-optimal but were balanced designs as well; thus, the search for D-optimal designs was to be restricted only to the class of the designs that are balanced. From several research findings, the aim of all iterative search procedures or algorithms is to locate the global optimum (best design) within the shortest possible moves or iterations in order to save cost, time and computer space and not just to locate the optimal design only. Examples of such search procedures include the Variance Exchange Algorithm by (Atkinson and Donev, 1992), the Steepest Ascent Method by (Pazman, 1986), the Combinatorial Procedure proposed by (Onukogu and Iwundu, 1997), and the Balanced confounded Designs Algorithm by (Umoren and Isaac, 2010). This means that though an algorithm or procedure may locate the best design (global optimum), it is not efficient in terms of the number of iterations or moves and, by extension, the time or cost used in locating it. It is worth noting that the efficiency of a system is also a measure of the time the system used in accomplishing its goal, and one system is more efficient than the other if the time used in accomplishing its goal is shorter than the time used by the other. This has always been the reason for designing new methods of doing things or modification to the former method.

In this paper, we aim to propose a new procedure or algorithm or place a certain condition for constructing symmetric and asymmetric D-optimal N-point designs that will be superior or more efficient to the Balanced Confounded Designs Algorithm in terms of shortening the time and, by extension, space and cost used in locating the global optimum.

Symmetric Factors and Designs:

A symmetric design is the design in which the factors $\{A, B, C, \ldots N\}$ appear at the same level. For example, 3^3 design. In this type of design, Factor A appear at 3 levels $\{-, 0, +\}$, Factor B appear at 3 levels $\{-, 0, +\}$ and Factor C appear at 3 levels $\{-, 0, +\}$. (Anthony et al , 2018)

Asymmetric Factors and Designs

On the other hand, an asymmetric design is the design in which the factors $\{A, B, C, \ldots N\}$ appear at different levels. Examples of such designs include 2×3^2, $2^2 \times 3^2$, $2 \times 3 \times 5$, etc.

Response Surface

In statistics, response surface methodology explores the relationships between several explanatory variables and one or more response variables. According to Akra et al. (2024, 2025) and Usen et al (2021), and Michael et al (20170, Isaac et al (2025), RSM is an empirical model which employs the use of mathematical and statistical techniques to relate input variables, otherwise known as factors to the response (Anthony et al 2018.) for some designs. The first and second – order response surface design model was explored to obtain rotatability in a generalized case (Akpan et al 2017, 2017,) and (Akra, 2017, 2025).

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A response surface is given in matric from as:

$$Y = X\beta + e \qquad 1.1$$

Where,

Y is the $Nx1$ vector of observed values, X is the Nxp design matrix, β is the px1 vector of unknown model parameters which are estimated on the basis of N uncorrelated observations, ϵ is the random additive error associated with Y and is independently and identically distributed with zero mean and constant variance. The first order model of a Response Surface Design (RSD) is given as:

$$Y = \beta_{00}X_0 + \beta_{10}X_1 + \beta_{02}X_2 + \beta_{12}X_1X_2 + e \qquad 1.2$$

and the second order RSM is given as:

$$Y = \beta_{00}X_0 + \beta_{10}X_1 + \beta_{02}X_2 + \beta_{12}X_1X_2 + \beta_{11}X_1^2 + \beta_{22}X_2^2 + e \qquad 1.3$$

Many works have been done using second-order response surface models and designs. Lucas (1976) compared the performances of several types of second-order response surface designs in symmetric regions on the basis of D- and G-optimality criteria. Onukogu and Iwundu (2007) worked on the construction of efficient and optimal experimental designs for second-order response surface models, while (Dette and Grigoriev, 2014) investigated optimality of designs for second-order models. Optimal choices of design points have been addressed by a number of researchers, including (Chigbu and Nduka, 2006). Iwundu (2015) studied the graphical methods employed in studying the response variance property of second-order response surface designs and, recently (Akra et al., 2024, 2025) showed the use of A and D optimality criteria in solving response surface design problems. Nsikak et al (2017) and Isaac et al (2025) carried out researches using balanced designs. Akra et al., (2024) also developed a program to solve E – optimality design for quadratic response surface model. It is worth noting that second-order models serve importantly in process optimization and are very reliable low-order approximating polynomials to the true unknown response functions relating a response with several controllable variables which may be natural or coded. Michael et al (2017) and Isaac et al (2025) applied some response surface technique in fitting the weight and height of students while Isaac et al (2025) used it in the analysis of study time.

Optimality Criteria

Design optimality criterion is a variance-type criterion that involves optimizing various individual properties of the information matrix ($N'N$) of the design. An optimality criterion summarizes how good a design is and it is maximized or minimized by an optimal design (Akra et al., 2025,). A new L_P – class to establish the relationship between various optimality criteria was proposed by (Akra et al., 2017) and related optimality design (Akra and Edet, 2017). On the other hand, Optimal Designs are experimental designs that are generated based on a particular optimality criterion and these designs satisfy the specific optimality criterion. It is worth noting that design optimality is often called after the letters of the alphabet, such as A, D, E, G, etc.

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The D-optimality criterion seeks to maximize the determinant of the information matrix $N'N$ or equivalently seeks to minimize the inverse of the information matrix $(N'N)^{-1}$. Symbolically, given as $D_{opt} = \max|N'N|$ or $\min|N'N|^{-1}$ where N is the incidence matrix and N' represents the transpose.

METHODOLOGY

Balanced Confounded Design Algorithm for Constructing N-Point D-Optimal Design

The algorithm for balanced confounded procedure is the same with that of the combinatorial procedure except that the sub design measures are not split. More so, both the number of balanced and unbalanced designs are taken. To this end, the algorithm is given thus:

1. Group the support points according to the number of minus and plus $\{-+\}$ and zeros $\{0\}$ and number them as $G_1, G_2, G_3, \ldots, G_k$
2. Partition each group G_i, $i=1,2,3, \ldots, k$ into subgroups as $g_{i0}, g_{i1}, g_{i2}, \ldots, g_{ir}$ and $g_{k0}, g_{k1}, g_{k2}, \ldots, g_{kr}$ where the sub-groups are being referred to as sub- design measures.
3. Choose N number of support points that would go into the class. Let V_j be defined as a vector of points consisting of n_{11} selected from G_1, n_{22} selected from G_2 and n_{rr} selected from G_k where n_{11} is the number of support points from G_1 that will make the N number of support points and so on. For example, if N = 20, and we define a class of design as $V_{20}^{(1)} = 7:6:6:1$, this means that, take 7 , 6, 6, and 1 support points from G_1, G_2, G_3 and G_4 respectively.
4. Using combination mathematics, determine the number of designs from that class and label them as $\zeta_1^{(v_j)}, \zeta_2^{(v_j)}, \zeta_3^{(v_j)}, \ldots, \zeta_n^{(v_j)}$ without splitting the sub-design measures since they are already balanced.
5. Obtain the information matrix $X'_{\xi_1^{(v_j)}} X_{\xi_1^{(v_j)}}$ for each of the design and compute its determinant as $d^*_{\xi_1^{(v_j)}}$. Similarly, obtain $d^*_{\xi_2^{(v_j)}}$, $d^*_{\xi_3^{(v_j)}}, \ldots, d^*_{\xi_n^{(v_j)}}$ and test each design whether it is balanced or unbalanced. A design is tested for balance by calculating their loss of information L.I. (See Umoren & Isaac, 2010.)
6. Tabulate your result as balanced and unbalanced designs.
7. Define another vector of points for N and carry out Steps 4 and 5.

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ISSN: 2689-5323
Volume 8, Issue 4, 2025 (pp. 150-161)

Illustration of the Balanced Confounded Design Algorithm for Constructing N-Point D-Optimal Designs.

Exploration with the Asymmetric Designs

(A) Given the experimental area $\widetilde{X} = \{x_1 = -,+;\ x_2 = -,\ 0,\ +\}$. This is an asymmetric design because the number of parameters for x_1 and x_2 are not the same. The response surface model (RSM) is given by:

$$P = \beta_{00}x_0 + \beta_{10}x_1 + \beta_{20}x_2 + \beta_{12}x_1x_2 + \beta_{22}x_2^2 + \varepsilon$$

The support points are grouped as follows:

$$G_1 \quad \begin{pmatrix} - & - \\ - & + \\ + & - \\ + & + \end{pmatrix} \begin{matrix} g_{10} \\ g_{11} \\ \\ g_{12} \end{matrix} \qquad G_2 \quad \begin{pmatrix} -0 \\ \cdot \\ +0 \end{pmatrix} \begin{matrix} \\ g_{20} \\ g_{21} \end{matrix}$$

1. Define a vector of points, V_j such as $V_5^{(1)} = 4:1$.

2. For the Balanced Confounded Design Algorithm, the possible designs will be 2 since the sub-design measures are not split and they are:

$$\xi_1^{(5)} = \begin{pmatrix} g_{10} \\ g_{11} \\ g_{20} \\ g_{21} \end{pmatrix} \qquad \xi_4^{(5)} = \begin{pmatrix} g_{11} \\ g_{12} \\ g_{20} \\ g_{21} \end{pmatrix}$$

1. We obtain the information matrix $\left(X'_{\xi_1^{(s)}} X_{\xi_1^{(s)}}\right)$ for each of the designs as follows:

where $P = \beta_{00}x_0 + \beta_{10}x_1 + \beta_{20}x_2 + \beta_{12}x_1x_2 + \beta_{22}x_2^2 + \varepsilon$ as shown:

$$X_{\xi_1^{(5)}} = \begin{pmatrix} x_0 & x_1 & x_2 & x_1x_2 & x_2^2 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \text{ and } \quad X'_{\xi_1^{(5)}} X_{\xi_1^{(5)}} = \begin{pmatrix} x_0 & x_1 & x_2 & x_1x_2 & x_2^2 \\ 5 & -1 & -1 & -1 & 3 \\ & 5 & -1 & -1 & -1 \\ & & 3 & -1 & -1 \\ & & & 3 & -1 \\ & & & & 3 \end{pmatrix}$$

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2. Compute $d_1^* = \frac{\det\left(X'_{\xi_1^{(s)}} X_{\xi_1^{(s)}} \right)}{N^p}$, where N = Number of the support points and P = numbers of the parameters in the designs.

Hence we have, $d_1^* = 2.048 \times 10^{-2}$; $d_2^* = 8.182 \times 10^2$

3. Define another vector of points, V_j such as $V_5^{(1)} = 3:2$ and carry out Steps 2 and 3

and their determinants are given as follows: $d_1^* = 2.048 \times 10^{-2}$; $d_2^* = 2.048 \times 10^{-2}$; $d_3^* = 2.048 \times 10^{-2}$ and $d_4^* = 2.048 \times 10^{-2}$

4. Tabulate the result thus:

Table 1: Table for the Result of Constructing N-point D-Optimal Balanced Designs Using the BCD Algorithm

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	No of Unbalanced Designs	Max Det for Unbalanced Design
1	3:2	2	2	$2.048x10^{-2}$	-	-
2	4:1	2	2	$8.182x10^{2}$	-	-

(B) Given $f(x_1, x_2, x_3) = \beta_{00}x_0 + \beta_{10}x_1 + \beta_{20}x_2 + \beta_{30}x_2 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 +$

$\beta_{23}x_2x_3 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \beta_{33}x_3^3 + \varepsilon$

Given $\widetilde{X} = \{x_1 = -,+;\ x_2 = -,\ 0,\ +; x_3 = -, 0,\ +\}$ to be the experimental region, the support points are grouped as:

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$$G_1 \begin{pmatrix} ---\\ --+\\ -+-\\ +--\\ ++-\\ +-+\\ -++\\ +++ \end{pmatrix} \begin{matrix} g_{10}\\ \\ g_{11}\\ \\ g_{12}\\ \\ \\ g_{13} \end{matrix} \qquad G_2 \begin{pmatrix} --0\\ -0-\\ -+0\\ -0+\\ +0-\\ +-0\\ +0+\\ ++0 \end{pmatrix} \begin{matrix} g_{20}\\ \\ g_{21}\\ \\ g_{22}\\ \\ g_{33}\\ \\ \end{matrix} \qquad G_3 \begin{pmatrix} -00\\ +00 \end{pmatrix} \begin{matrix} g_{30}\\ g_{31} \end{matrix}$$

A tabulation of the result of the analysis is given in the table below:

Table 2: Table for the Result of Constructing 13-point D-Optimal Designs for 2 x 3^2 Response Surface Design Using BCDA

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	No of Unbalanced Designs	Max Det for Unbalanced Design
1	7:6:0	6	4	$2.88x10^{-3}$	2	$2.66x10^{-3}$
	7:4:2	12	4	$3.32x10^{-3}$	8	$2.66x10^{-3}$
2	8:4:1	12	4	$5.78x10^{-3}$	8	$4.26x10^{-3}$
Total		30	12		18	

Exploration with the Symmetric Designs

Given $f(x_1x_2) = \beta_{00}x_0 + \beta_{10}x_1 + \beta_{20}x_2 + \beta_{12}x_1x_2 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \varepsilon$

to be the experimental model for the series, for 3^2 designs, the support points are grouped as:

$$G_1 \begin{pmatrix} --\\ -+\\ +-\\ ++ \end{pmatrix} \begin{matrix} g_{10}\\ g_{11}\\ \\ g_{12} \end{matrix} \qquad G_2 \begin{pmatrix} -0\\ 0-\\ +0\\ 0+ \end{pmatrix} \begin{matrix} \\ g_{20}\\ \\ g_{21} \end{matrix} \qquad G_3 \begin{pmatrix} 00\\ \\ \\ \\ \end{pmatrix} \begin{matrix} g_{30}\\ \\ \\ \\ \end{matrix}$$

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ISSN: 2689-5323
Volume 8, Issue 4, 2025 (pp. 150-161)

Table 3: Table of the Result for Constructing 7-point D-Optimal Designs for 3^2 Response Surface Designs Using BCD Algorithm

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	No of Unbalanced Designs	Max Det for Unbalanced Design
1	3:4:0	2	2	$3.26x10^{-3}$	-	-
2	4:2:1	2	2	$8.155x10^{-3}$	-	-
Total		4	4			

Exploration with 2^k Series

Let $f(x_1, x_2, x_3) = \beta_{00}x_0 + \beta_{10}x_1 + \beta_{20}x_2 + \beta_{30}x_2 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 +$

$\beta_{23}x_2x_3 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \beta_{33}x_3^3 + \varepsilon$

Give $\widetilde{X} = \{x_1, x_2, x_3, x_4 = -, +\}$ to be the experimental region, the support points are grouped as:

$$G_1 \quad \begin{pmatrix} - - - - \\ - - - + \\ - - + - \\ - + - - \\ + - - - \\ - - + + \\ - + - + \\ - + + - \\ + - - + \\ + - + - \\ + + - - \\ + + + - \\ + + - + \\ + - + + \\ - + + + \\ + + + + \end{pmatrix} \quad \begin{matrix} \\ g_{10} \\ \\ \\ g_{11} \\ \\ \\ \\ g_{12} \\ \\ \\ \\ g_{13} \\ \\ \\ g_{14} \end{matrix}$$

A tabulation of the result for balanced and unbalanced designs for a 12-point designs for a 2^4 response surface design is given in the table below:

Article DOI: 10.52589/AJMSS-E9WW0NZJ
DOI URL: https://doi.org/10.52589/AJMSS-E9WW0NZJ

Table 4: Table for the Result of Constructing 12-point, D-Optimal Balanced and Unbalanced Designs for 2^4 Response Surface Design Using BCDA

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det For Balanced Designs	No of Unbalanced Designs	Max Det For Unbalanced Design
1	12	2	2	$1.8x10^{-1}$	-	-

Proposed New Procedure for Constructing N-Point D-Optimal Designs

Under the proposed new procedure or algorithm, Column 6 and Column 7 respectively in Tables 1, 2, 3 and 4 will be removed, that is, the columns for Unbalanced Designs and Max determinant for Unbalanced Designs will be removed while the column for Global Optimum will be added. The new tables will thus become:

Table 5: Table of the New Procedure for constructing 5-point D-Optimal Balanced Designs

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	Global Optimum
1	3:2	2	2	$2.048x10^{-2}$	
2	4:1	2	2	$8.182x10^{2}$	$8.182x10^{2}$

Table 6: Table for the New Procedure for Constructing 13-point D-Optimal Designs for 2×3^2 Response Surface Design

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	Global Optimum
1	7:6:0	6	4	$2.88x10^{-3}$	
	7:4:2	12	4	$3.32x10^{-3}$	
2	8:4:1	12	4	$5.78x10^{-3}$	$5.78x10^{-3}$
Total		30	12		

Article DOI: 10.52589/AJMSS-E9WW0NZJ
DOI URL: https://doi.org/10.52589/AJMSS-E9WW0NZJ

Table 7: Table of the New Procedure for constructing 7-point D-Optimal Designs for 3^2 Response Surface Designs

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det for Balanced Designs	Global Optimum
1	3:4:0	2	2	$3.26x10^{-3}$	
2	4:2:1	2	2	$8.155x10^{-3}$	$8.155x10^{-3}$
Total		4	4		

Table 8: Table of the New Procedure for Constructing 12-point, D-Optimal Designs for 2^4 Response Surface Designs

Class (s)	Group	Class Size (s_j)	No of Balanced Designs	Max Det For Balanced Designs	Global Optimum
1	12	2	2	$1.8x10^{-1}$	$1.8x10^{-1}$

DISCUSSION OF FINDINGS

A careful examination of Tables 5, 6, 7 and 8 respectively reveals that the Global optimum or best design for all the Symmetric and Asymmetric N-Point D-optimal designs are found or located in the class of balanced designs in which all the support points from G_1 were taken. With this, a new procedure for constructing N-Point D-optimal designs using the BCDA can be proposed. The new proposed procedure for constructing asymmetric and symmetric N-Point D-optimal design is that the search for the global optimum (best design) should be restricted to the class of design in which all the support points in G_1 are taken. With this, the search time is reduced to the barest minimum, needless searching through all the combinations. For instance, in Table 5, by restricting the search for the global optimum to the class of designs in which all the support points from G_1 are taken, it will take only two iterations or moves as against four iterations to locate the global optimum. Again, from Table 6, if we restrict the search for the global optimum to the class of designs in which all the support points from G_1 are taken, it will take only four moves or iterations as against twelve moves which the Balanced Confounded Design Algorithm used before locating the global optimum. Similarly, from Table 7, with only two moves as against four, the global optimum would have been located. Finally, from Table 8, the global optimum is found in the set of balanced designs in which all the support points came from G_1. In this case, 2 moves are required to reach the global optimum—the same number of moves the Balanced Confounded Design Algorithm used.

Article DOI: 10.52589/AJMSS-E9WW0NZJ
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CONCLUSION

We have shown that in constructing both symmetric and asymmetric designs, such as 3^2, 2×3^2 and a 2^4 N-points D-optimal response surface designs using the BCDA, the global optimum were all found to come from the class of designs in which all the support points in G_1 were taken. This is a new discovery. Hence, we propose that in using the BCDA for constructing symmetric and asymmetric designs, the condition is that the search for global optimum or best design should be restricted to a particular class and not sweep through all the classes as before. With this condition, fewer steps or iterations or moves will be used in locating the global optimum. This will make the new procedure, which may be called Restricted Balanced Confounded Algorithm (RBCDA), to be better or more efficient than the former procedure (BCDA).

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Article DOI: 10.52589/AJMSS-E9WW0NZJ
DOI URL: https://doi.org/10.52589/AJMSS-E9WW0NZJ