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THIN FILM FLOW OF NON-NEWTONIAN FLUID WITH TEMPERATURE-DEPENDENT VISCOSITY IN PIPE

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ABSTRACT: *The analysis of thin film flow of non-Newtonian fluid with Vogel model viscosity in cylindrical pipe is considered. Vogel model is introduced to account for the temperature-dependent viscosity. Perturbation technique is employed to solve the dimensionless nonlinear momentum and energy equations. Results indicate that an increase in the magnetic field parameter increases both the flow velocity and the temperature of the cylinder, and a little increase in the Brinkman number greatly increases the temperature of the plate. Results further show that an increase in the parameter M slightly reduced the flow velocity and a. increase in the parameter B drastically reduced the velocity of the fluid flow. It is observed that as B increases, the cylindrical wall temperature is greatly lowered which means that particles lose more thermal energy that would hitherto help in overcoming the attractive forces holding them together.*

KEYWORDS: Viscosity, Non-Newtonian, MHD, Film, Temperature-dependent.

INTRODUCTION

During the past few decades, the non-Newtonian fluids have attracted considerable attention due to their own importance in many technological applications such as the oil industries, food processing industries, chemical industries, fruit packaging and many more. Third grade fluid is a non-Newtonian fluid of the differential type. The equations involved are very complex because of the nature of the fluids. Such fluids include oil, greases, etc. Because of the complexity of these fluids, it is difficult to suggest a single model that will handle the problems involved and for this reason, it requires excessive mathematical computations and analytical procedures for a closed form solution. Many researchers in the last few decades have done some considerable research in the area of non-Newtonian fluid of the differential type. Amongst the earliest researchers are Fosdic and Rajagopal [1]. They examined the thermodynamic stability of fluid of third grade. They were concerned with the relation between thermodynamics and stability for certain non-Newtonian incompressible fluids of the differential type. They further introduced the additional thermodynamic restriction that the Helmholtz free energy density must be a minimum value when the fluid is at rest, and arrive at certain fundamental inequalities which restricts its temperature dependence. They found that these inequalities require that a body of such fluids be stable and its kinetic energy tends to zero in time. Ekmann *et al.* [2] examined the laminar flow of highly loaded suspensions in horizontal pipes. They applied the power law model for the analysis. Rajagopal and Sciubba [3] investigated the flow of a third grade non-Newtonian fluid between horizontally situated and heated parallel plates. They involved temperature-dependent viscosity in their analysis. Massoudi and Christie [4] dealt with the effects of variable viscosity and viscous dissipation on the flow of third grade fluid. The boundary layer equations of third grade fluid were treated by Pakdemirli [5].

Bejan [6] studied entropy generation in fundamental convective heat transfer. Johnson *et al.* [7] investigated a fluid flow which was infused with solid particles in a pipe, while approximate analytical solutions for flow of third grade fluid was examined by Yurusoy and Pakdemirli [8]; the velocity and temperature profiles were in close agreement with the work of Yurusoy [9]. Okedayo *et al.* [10] studied the effects of viscous dissipation, constant wall temperature and a periodic field on unsteady flow through a horrizontal channel. Okedayo *et al.* [11] analyzed the magnetohydrdynamic (MHD) flow and heat transfer in a cylindrical pipe filled with porous media. They applied the Galerkin weighted residual method for the solution of momentum equation and semi-implicit finite difference method for the energy equation. They found that an increase in Darcy number leads to an increase in the velocity profiles, while an increase in Brinkman number enhances the temperature of the system. Aiyesimi *et al.* [12] analyzed the unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. Results show that the variation of velocity and temperature profiles with the magnetic and gravitational field parameters are dependent on time.

Obi [13] on approximate analytical solution of natural convection flow of non-Newtonian fluid through parallel plates solved the coupled momentum and energy equations using the regular perturbation method. He treated cases of constant and temperature-dependent viscosities in which Reynold's and Vogel's models were considered to account for the temperature-dependent viscosity case, while third grade fluid was introduced to account for the non-Newtonian effects. Obi [14] numerically analyzed the reactive third grade fluid in a cylindrical pipe. He observed that the non-Newtonian parameters considered in the analysis: third grade

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parameter (β), magnetic field parameter (M), Eckert number (E_c) and the Brinkman number (B_r) had positive effects on the velocity and temperature profiles. Aiyesimi *et al.* [15] considered the flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall and Joule heating. Axial pressure-gradient was assumed to have induced the motion. They discovered that an increase in both Brinkman and Eckert numbers increases the temperature profiles at the boundaries.

Aksoy and Pakdemirli [16] examined the flow of a non-Newtonian fluid through a porous medium between two parallel plates. They involved Reynold's and Vogel's models' viscosity and derived the criteria for validity for the approximate solution. Shirkhani *et al*. [17] examined the unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). They transformed the Navier-Stokes equation into ordinary differential equation using similarity transformation and investigated the effects of Reynolds number and suction or injection characteristic parameter on the velocity field.

Pakdemirli and Yilbas [18] examined entropy generation in a pipe due to non-Newtonian fluid flow, a case of constant viscosity. They formulated the entropy generation number due to heat transfer and fluid friction. The influences of non-Newtonian parameters and Brinkman number on entropy generation number were examined and the results revealed that an increase in the non-Newtonian parameter reduces the fluid friction in the region close to the wall of the pipe, giving rise to low entropy generation. They further discovered that an increase in the Brinkman number enhances the fluid friction and heat transfer rate, thereby increasing the entropy number. Hayat *et al.* [19] applied homotopy perturbation and numerically obtained the solution of the third grade fluid past a porous channel with suction and injection at the walls. Obi *et al.* [20] analyzed the flow of an incompressible MHD third grade fluid in an inclined cylindrical pipe with isothermal wall and Joule heating. The governing equations of the flow field were solved using the traditional regular perturbation method. They observed that an increase in Eckert and Grashof numbers reduces the flow velocity, and increases the temperature of the system.

The distribution of this paper is in six sections. Section 1 is the introduction of the research and the literature review. Section 2 is the problem formulation. In Section 3, the analytical solutions of the constant viscosity case is presented, and velocity and temperature-dependent viscosity are considered in Reynold's model. Section 4 is the discussion of the results obtained in Section 3. Section 5 contains the concluding remarks while section 6 contains the references.

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Mathematical Formulation

Considering a steady incompressible thin film flow of third grade fluid, the non-dimensional form of equations of motion as in Aiyesimi *et al.* [12] with modifications are:

$$\frac{d^2u}{dr^2}+6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right)+Mu=-1 \quad (1)$$

$$\frac{d^2\theta}{dr^2}+\mathrm{B}_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+\mathrm{Mu}^2=0 \quad (2)$$

$$u(0)=0,u(1)=1,\theta(0)=0,\theta(1)=1 \quad (3)$$

where u is the velocity of the fluid, θ is the temperature of the cylinder; the terms are related to the non-dimensional variables

$$r=\frac{\bar{r}}{d},\theta=\frac{T}{T_0},u=\frac{\bar{u}}{u_0},\mu=\frac{\bar{\mu}}{\mu_0}, \quad (4)$$

where r is the radius of the cylinder, T_0 is the reference temperature, u_0 is the reference velocity, μ_0 is the reference viscosity.

$$\mathrm{B}_r=\frac{\mu_0u_0^2}{kT_0},\beta=\frac{\beta u_0^2}{\mu_0d^2} \quad (5)$$

Method of Solution

The semi-analytical solutions for velocity and temperature profiles can be of the form:

$$u(r)=u_0(r)+\beta u_1(r)+\mathrm{O}\left(\beta^2\right),\theta(r)=\theta_0(r)+\beta\theta_1(r)+\mathrm{O}\left(\beta^2\right),\ \mathrm{M}=\beta\mathrm{M} \quad (6)$$

Substituting Eqn (6) into Eqns (1) and (2) and separating each order of β yields

$$\frac{d^2u_0}{dr^2}=-1 \quad (7)$$

$$\frac{d^2u_1}{dr^2}+6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2}+Mu_0=0 \quad (8)$$

$$\frac{d^2\theta_0}{dr^2}+\mathrm{B}_r\left(\frac{du_0}{dr}\right)^2=0 \quad (9)$$

$$\frac{d^2\theta_1}{dr^2}+2\beta\mathrm{B}_r\frac{du_0}{dr}\frac{du_1}{dr}+Mu_0^2=0 \quad (10)$$

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Solving Eqns (7–10) with Condition (3) yields

$$u(r)=\frac{3}{2}r-\frac{1}{2}r^2+\beta\left(\frac{27}{4}r^2-3r^3+\frac{1}{2}r^4-M\left(\frac{1}{4}r^3-\frac{1}{24}r^4\right)-\frac{13}{4}r+\frac{5}{24}rM\right) \quad (11)$$

$$\theta(r)=-\mathrm{B}_r\left(-\frac{17}{24}r+\frac{9}{8}r^2-\frac{1}{2}r^3+\frac{1}{12}r^4\right)-r+\beta\left(-\beta\mathrm{B}_r\left(\frac{47}{6}r^3-\frac{9}{2}r^4+\frac{6}{5}r^5-M\left(\frac{5}{16}r^5-\frac{1}{40}r^5\right.\right.\right.$$
$$\left.-\frac{39}{8}r^2+\frac{5}{16}r^2M-\frac{2}{15}r^6+M\left(\frac{3}{40}r^5-\frac{1}{60}r^6\right)-\frac{5}{24}r^2M\right)-M\left(\frac{3}{16}r^4-\frac{3}{40}r^5+\frac{1}{120}r^6\right)+r$$
$$\left.+\beta\mathrm{B}_r\left(-\frac{57}{120}r+\frac{78}{240}rM\right)+\frac{29}{240}rM\right) \quad (12)$$

TEMPERATURE-DEPENDENT VISCOSITY

In temperature-dependent viscosity, two models are involved, which are Reynold's and Vogel's models, but for this present paper, we consider only the Vogel's model.

The Vogel's Model

The dimensionless momentum and energy equations are:

$$\frac{d\mu}{dr}\frac{du}{dr}+\frac{d}{dr}\left(\mu\frac{du}{dr}\right)+6\beta\left(\frac{du}{dr}\right)^2+Mu=-1 \quad (13)$$

$$\frac{d^2\theta}{dr^2}+\mu B_r\left(\frac{du}{dr}\right)^2+2\beta B_r\left(\frac{du}{dr}\right)^4+Mu^2=0 \quad (14)$$

Assuming the expansion for the velocity and energy equations is of the form Eqn (6),

where β is a small parameter. In Vogel's model, viscosity can be considered in the direction of Massoudi and Christie (1995) as

$$\mu=\mu\exp\left(\frac{A}{B+\theta}-\theta_w\right) \quad (15)$$

Applying Taylor's series expansion on Eqn (15) yields

$$\mu=\alpha\left(1-\frac{A\theta}{B^2}\right) \quad (16)$$

where

$$\alpha=\mu\exp\left(\frac{A}{B}-\theta_w\right) \quad (17)$$

A and B being parameters relating to Vogel's model.

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$$\frac{d\mu}{dr} \equiv -\alpha \frac{A}{B^2}\frac{d\theta}{dr} \tag{18}$$

Substituting Eqns (6), (16) and (18) into Eqns (13) and (14), and expanding and choosing the zeroth order and the order 1 of the small parameter β for the Vogel's momentum and energy equations yields:

$$\beta^0 : \frac{d^2u_0}{dr^2} = -1 \tag{19}$$

$$\beta : \frac{d^2u_1}{dr^2} - \alpha\frac{n}{B^2}\frac{d\theta_0}{dr}\frac{du_0}{dr} + 6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2} + Mu_0 \tag{20}$$

$$\beta^0 : \frac{d^2\theta_0}{dr^2} + \alpha\left(\frac{du_0}{dr}\right)^2 = 0 \tag{21}$$

$$\beta : \frac{d^2\theta_1}{dr^2} - 2\alpha\frac{du_0}{dr}\frac{du_1}{dr} - \alpha\frac{n\theta_0}{B^2}\left(\frac{du_0}{dr}\right)^2 + 2\left(\frac{du_0}{dr}\right)^4 + Mu_0^2 \tag{22}$$

Solving the dimensionless momentum and energy Eqns (19–22), with the Condition (3) yields

$$u(r) = \frac{3}{2}r - \frac{1}{2}r^2 + \beta\left(\frac{\alpha n}{B^2}\left(-\alpha\left(\frac{9}{16}r^3 - \frac{3}{16}r^4 + \frac{11}{80}r^5 - \frac{1}{90}r^6\right) + \frac{3}{4}r^2 - \frac{1}{6}r^3 + \frac{51}{96}r^2\alpha - \frac{17}{144}r^2\alpha\right)\right.$$
$$\left. + \left(\frac{27}{4}r^2 - 3r^3 + \frac{1}{2}r^4\right) - M\left(\frac{1}{4}r^3 - \frac{1}{24}r^4\right) - r\left(-1 + \frac{\alpha n}{B^2}\left(\frac{7}{12} - \frac{307}{720}\right) + \frac{17}{4} - \frac{5}{24}M\right)\right) \tag{23}$$

$$\theta(r) = -\alpha\left(\frac{9}{8}r^2 - \frac{1}{2}r^3 + \frac{1}{12}r^4\right) + r\left(1 + \frac{17}{24}\alpha\right) + \beta\left(-\alpha\left(\frac{\alpha n}{B^2}\left(-\alpha\left(\frac{27}{64}r^4 - \frac{9}{40}r^5 - \frac{51}{960}r^6 - \frac{1}{168}r^7\right)\right.\right.\right.$$
$$\frac{3}{4}r^3 - \frac{1}{8}r^4 + \frac{51}{91}r^3\alpha - \frac{17}{192}r^4\alpha\Big) + \left(\frac{27}{4}r^3 - \frac{9}{4}r^4 + \frac{3}{10}r^5\right) - M\left(\frac{3}{16}r^4 - \frac{1}{40}r^5\right) + \frac{3}{2}r^2 - \frac{\alpha n}{B^2}\left(\frac{7}{8}r^2 - \frac{1}{5}r^2\alpha\right.$$
$$\left. - \frac{51}{8}r^2 + \frac{5}{16}r^2M\right) - \frac{\alpha n}{B^2}\left(-\alpha\left(\frac{27}{160}r^5 - \frac{1}{10}r^6 - \frac{17}{672}r^7 - \frac{1}{336}r^8\right) - \frac{1}{4}r^4 - \frac{1}{20}r^5 + \frac{17}{96}r^4\alpha - \frac{17}{480}r^5\alpha\right)$$
$$\left.\left. - \frac{9}{4}r^4 - \frac{9}{10}r^5 + \frac{2}{15}r^6 - M\left(\frac{3}{40}r^5 - \frac{1}{90}r^6\right) - \frac{1}{6}r^4 + \frac{\alpha n}{B^2}\left(-\alpha\left(\frac{7}{36}r^3 - \frac{2}{45}r^3\alpha\right) - \frac{17}{12}r^3 - \frac{5}{72}r^3M\right)\right)\right)$$
$$+ \frac{\alpha n}{B^2}\left(-\alpha\left(\frac{27}{128}r^4 - \frac{9}{4}r^5 + \frac{3}{32}r^6 - \frac{1}{56}r^7 + \frac{1}{672}r^8\right) + \frac{3}{8}r^3 - \frac{1}{4}r^4 + \frac{1}{20}r^5 + \frac{17}{64}r^3\alpha - \frac{17}{96}r^4\alpha + \frac{17}{480}r^5\alpha\right)$$
$$\frac{81}{16}r^2 + \frac{9}{2}r^3 - \frac{7}{3}r^4 + \frac{3}{5}r^5 - \frac{1}{15}r^6 - M\left(\frac{3}{16}r^4 - \frac{3}{40}r^5 + \frac{1}{30}r^6\right) + r\left(1 + \frac{\alpha n}{B^2}\left(-\frac{2729}{6720}\alpha - \frac{24}{5} + \frac{41}{120}M\right)\right.$$
$$\left. - \frac{\alpha n}{B^2}\left(\frac{163}{2688}\alpha + \frac{7}{40}\right) + \frac{567}{240} - \frac{7}{48}M\right) \tag{24}$$

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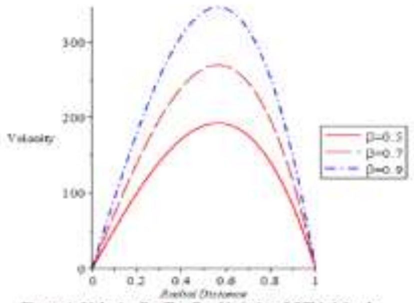

Figure 1: Velocity Profiles For Variation Of Third Grade Parameter (β)

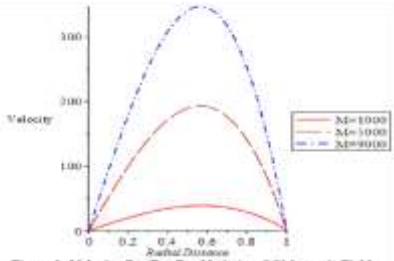

Figure 2: Velocity Profiles For Variation Of Magnetic Field Parameter (M)

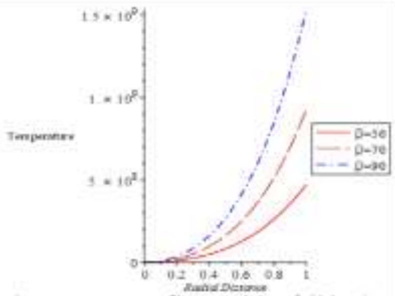

Figure 3: Temperature Profiles For Variation Of Third Grade Parameter (β)

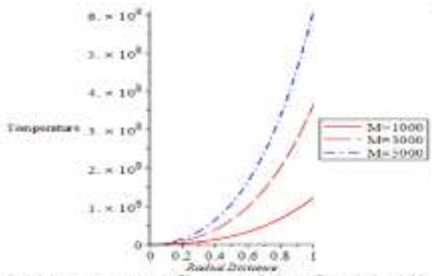

Figure 4: Temperature Profiles For Variation Of Magnetic Field Parameter (M)

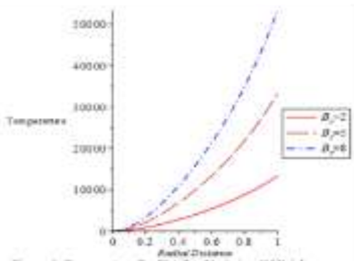

Figure 5: Temperature Profiles For Variation Of Brinkman Parameter (B_r)

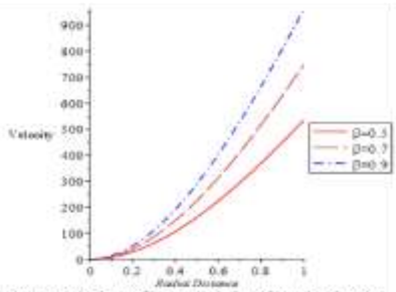

Figure 6: Velocity Profiles For Variation Of Vogel's Viscosity Index (β)

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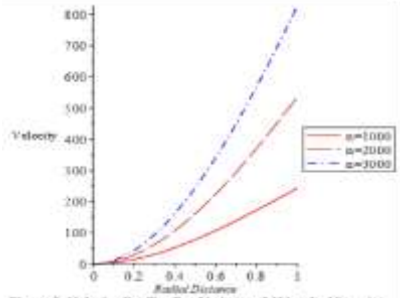

Figure 7: Velocity Profiles For Variation Of Vogel's Viscosity Index (n)

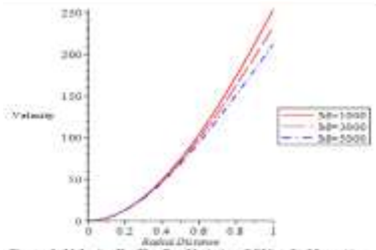
Figure 8: Velocity Profiles For Variation Of Vogel's Viscosity Index (M)

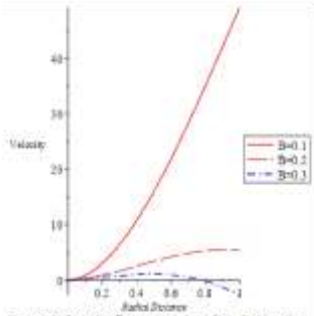

Figure 9: Velocity Profiles For Variation Of Vogel's Viscosity Index (B)

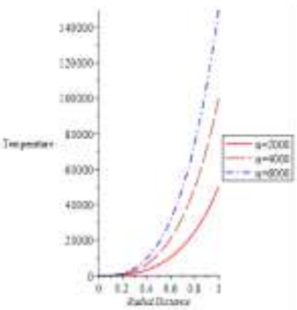

Figure 10: Temperature Profiles For Variation Of Vogel's Viscosity Index (n)

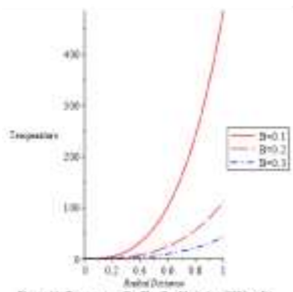

Figure 11: Temperature Profiles For Variation Of Vogel's Viscosity Index (B)

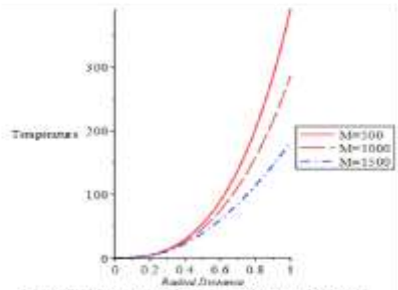

Figure 12: Temperature Profiles For Variation Of Vogel's Viscosity Index (M)

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RESULT AND DISCUSSIONS

In this section, the solutions for velocity and temperature profiles are obtained by specifying the values of the thermo-solutal parameters to ascertain the effects of various dimensionless numbers on the velocity field and energy balance. Two cases were considered: Constant viscosity and Vogel model viscosity.

Constant Viscosity Case

The solutions of velocity and energy equations of motion are in Eqns (11) and (12). Figures (1) and (3) show the velocity and temperature profiles for various values of the parameter β. Results show that increase β at the same rate increases the velocity and temperature profiles. Figures (2) and (4) show the velocity and temperature profiles for various values of the magnetic field parameter, M, when the other parameters are held constant. Results indicate that an increase in the magnetic field parameter increases both the velocity of the fluid flow and the temperature of the cylinder. In Figure (5), the Brinkman parameter was varied while the other parameters were held constant. Results show that a little increase in the Brinkman number greatly increased the temperature of the plate.

The Vogel Model Case

The effects of the viscosity indices β and n are shown in Figures (6) and (7). Results show that an increase in these viscosity parameters increases the velocity. This indicates a low rate of fluid strain which results in low mean and maximum flow velocity. Figures (8) and (9) show the velocity profiles for different Vogel model indices M and B. Results indicate that an increase in the parameter M slightly reduced the flow velocity. Results further show that an increase in the parameter B drastically reduced the velocity of the fluid flow. This is as a result of the shear strain which reduces the flow velocity. Figure 10 shows the temperature profiles for various values of the Vogel index n, and the results reveal that as the index n increases at a regular rate, the temperature is enhanced. Figure 11 contains the temperature profiles for various values of the Vogel model index B. It is observed that as B increases, the temperature of the cylindrical pipe is greatly lowered. It is also seen that low temperature means that particles lose more thermal energy that would hitherto help in overcoming the attractive forces holding them together. Figure 12 contains the temperature profiles for Vogel model viscosity index M. It is observed an increase in the parameter M slightly lowers the temperature of the cylinder.

CONCLUSIONS

The analysis of thin film flow of non-Newtonian fluid with Vogel model viscosity in cylindrical pipe is considered with. Vogel model viscosity is introduced to account for the temperature-dependent viscosity. Results indicate that an increase in the magnetic field parameter increases both the velocity of the fluid flow and the temperature of the cylinder and that a little increase in the Brinkman number greatly increases the temperature of the plate. Results indicate that an increase in the parameter M slightly reduces the flow velocity. Results further show that an increase in the parameter B drastically reduces the velocity of the fluid flow. It is observed that as B increases, the temperature of the cylindrical pipe is greatly lowered which means that

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particles lose more thermal energy that would hitherto help in overcoming the attractive forces holding them together.

Declarations

1. Funding: Not applicable

2. Informed Consent Statement: Not applicable

3. Data Availability: Not applicable

4. Conflict of Interest Statement: No conflict of interest.

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